Data-Driven Discovery of Emergent Behaviors

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Abstract

We extend the learning framework in [1, 2] to a set of **more complicated** agent-based dynamics in [3] and apply it to NASA JPL's Development Ephemeris in [4]. Our non-parametric approach is used to derive, validate and improve modeling in **collective dynamics**. The first order dynamics is modeled as follows,

$$\begin{cases} \dot{\boldsymbol{x}}_i(t) &= \boldsymbol{F}^{\boldsymbol{x}}(\boldsymbol{x}_i(t), \xi_i(t)) + \sum_{i'=1}^N \frac{1}{N_{\mathcal{k}_{i'}}} \phi^E_{\mathcal{k}_i,\mathcal{k}_{i'}}(\mathcal{F}^E_{i,i'}(t)) \boldsymbol{r}_{i,i'}(t), \\ \dot{\xi}_i(t) &= \boldsymbol{F}^{\xi}(\boldsymbol{x}_i(t), \xi_i(t)) + \sum_{i'=1}^N \frac{1}{N_{\mathcal{k}_{i'}}} \phi^\xi_{\mathcal{k}_i,\mathcal{k}_{i'}}(\mathcal{F}^\xi_{i,i'}(t)) \boldsymbol{r}_{i,i'}(t). \end{cases} \tag{1}$$

and second order dynamics as,

$$\begin{cases} m_i\ddot{\boldsymbol{x}}_i(t) &= \boldsymbol{F}^{\boldsymbol{x}}(\boldsymbol{x}_i(t), \dot{\boldsymbol{x}}_i(t), \xi_i(t)) + \sum_{i'=1}^N \frac{1}{N_{\mathcal{k}_{i'}}} \left(\phi^E_{\mathcal{k}_i,\mathcal{k}_{i'}}(\mathcal{F}^E_{i,i'}(t)) \boldsymbol{r}_{i,i'}(t) + \phi^A_{\mathcal{k}_i,\mathcal{k}_{i'}}(\mathcal{F}^A_{i,i'}(t)) \dot{\boldsymbol{r}}_{i,i'}(t)\right), \\ \dot{\xi}_i(t) &= \boldsymbol{F}^{\xi}(\boldsymbol{x}_i(t), \dot{\boldsymbol{x}}_i(t), \xi_i(t)) + \sum_{i'=1}^N \frac{1}{N_{\mathcal{k}_{i'}}} \phi^\xi_{\mathcal{k}_i,\mathcal{k}_{i'}}(\mathcal{F}^\xi_{i,i'}(t)) \xi_{i,i'}(t). \end{cases} \tag{2}$$

$x_i(t) \in \mathbb{R}^d$	state of agent i	$\xi_i(t) \in \mathbb{R}$	phase, excitation, emotion
N_k	Num. of agents in the k^{th} type	$\boldsymbol{r}_{i,i'}(t)$	$\boldsymbol{x}_{i'}(t) - \boldsymbol{x}_i(t)$
$\mathcal{k}_i$	type index of agent i	$\dot{\boldsymbol{r}}_{i,i'}(t)$	$\dot{\boldsymbol{x}}_{i'}(t) - \dot{\boldsymbol{x}}_i(t)$
$r_{i,i'}(t)$	$\lVert \boldsymbol{r}_{i,i'}(t) \rVert$	$\xi_{i,i'}(t)$	$\xi_{i'}(t) - \xi_i(t)$
m_i	mass of agent i	$\boldsymbol{F}^{\boldsymbol{x}}, \boldsymbol{F}^{\xi}$	non-collective forces
$\mathcal{F}^E_{i,i'}, \mathcal{F}^A_{i,i'}, \mathcal{F}^\xi_{i,i'}$	Feature maps	$\phi^E_{\mathcal{k}_i,\mathcal{k}_{i'}}, \phi^A_{\mathcal{k}_i,\mathcal{k}_{i'}}, \phi^\xi_{\mathcal{k}_i,\mathcal{k}_{i'}}$	unknown interaction laws

The Empirical Error Functional

Given a set of continuous trajectories, $\{\boldsymbol{x}_i(t)^m, \dot{\boldsymbol{x}}_i(t)^m, \xi_i(t)^m\}_{i,m=1}^{N,M}$ for $t \in [0, T]$, we learn the interaction laws by minimizing the following **Empirical Error Functional** for $\phi^E_{k,k'}$ and $\phi^A_{k,k'}$,

$$\frac{1}{MT} \sum_{m,i=1}^{M,N} \frac{1}{N_{\mathcal{k}_i}} \int_{t=0}^{T} \left| m_i \ddot{\boldsymbol{x}}_i^m - \boldsymbol{F}^{\boldsymbol{x}}(\boldsymbol{x}_i^m, \dot{\boldsymbol{x}}_i^m, \xi_i^m) - \sum_{i'=1}^N \frac{1}{N_{\mathcal{k}_{i'}}} \left(\varphi^E_{\mathcal{k}_i,\mathcal{k}_{i'}}(\mathcal{F}^E_{i,i'}) \boldsymbol{r}^m_{i,i'} + \varphi^A_{\mathcal{k}_i,\mathcal{k}_{i'}}(\mathcal{F}^A_{i,i'}) \dot{\boldsymbol{r}}^m_{i,i'}\right) \right|^2 dt. \tag{3}$$

For $\phi^\xi_{k,k'}$, we use a de-coupled **Empirical Error Functional**,

$$\frac{1}{MT} \sum_{m,i=1}^{M,N} \frac{1}{N_{\mathcal{k}_i}} \int_{t=0}^{T} \left| \dot{\xi}_i^m - \boldsymbol{F}^{\xi}(\boldsymbol{x}_i^m, \dot{\boldsymbol{x}}_i^m, \xi_i^m) - \sum_{i'=1}^N \frac{1}{N_{\mathcal{k}_{i'}}} \varphi^\xi_{\mathcal{k}_i,\mathcal{k}_{i'}}(\mathcal{F}^\xi_{i,i'}) \xi^m_{i,i'} \right|^2 dt. \tag{4}$$

$\varphi^E_{k,k'}, \varphi^A_{k,k'}, \varphi^\xi_{\mathcal{k}_i,\mathcal{k}_{i'}} \in \mathcal{H}_{k,k'} = \{\psi \in L^2([0, R_{k,k'}]) : \lVert\psi\rVert_{L^\infty([0,R_{k,k'}])} + \lVert\psi'\rVert_{L^\infty([0,R_{k,k'}])} < \infty\}$, with $R_{k,k'}$ being the maximum interaction radius between agents in types k and k'.

The Algorithm

Given $\{\boldsymbol{x}_i(t_l)^m, \dot{\boldsymbol{x}}_i(t_l)^m, \xi_i(t_l)^m\}_{i,m,l=1}^{N,M,L}$,

1. Choose appropriate finite dimensional subspaces for $\mathcal{H}_{k,k'}$'s;
2. Discretize (3) and (4) in t_l;
3. Form the learning matrix and the learning vector;
4. Solve the linear system.

Details for the actual algorithms, see [3, 4]

Numerical Results

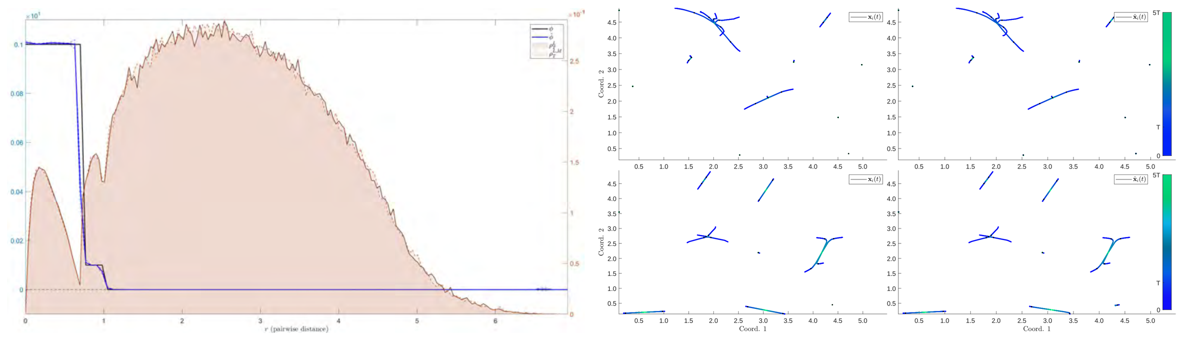

Figure 1: Opinion Dynamics

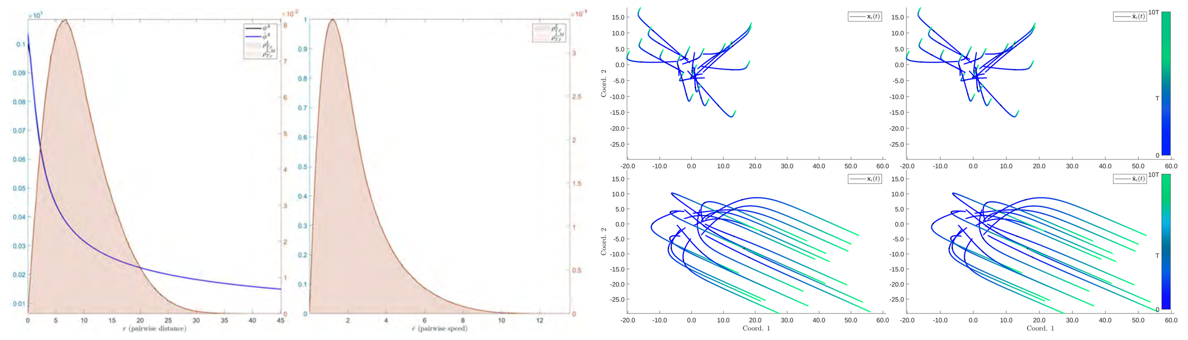

Figure 2: Cucker-Smale Dynamics

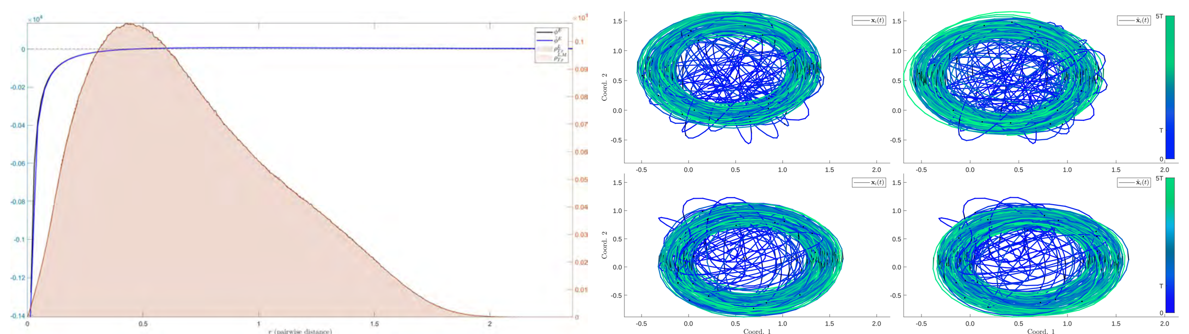

Figure 3: Fish Mill $2D$

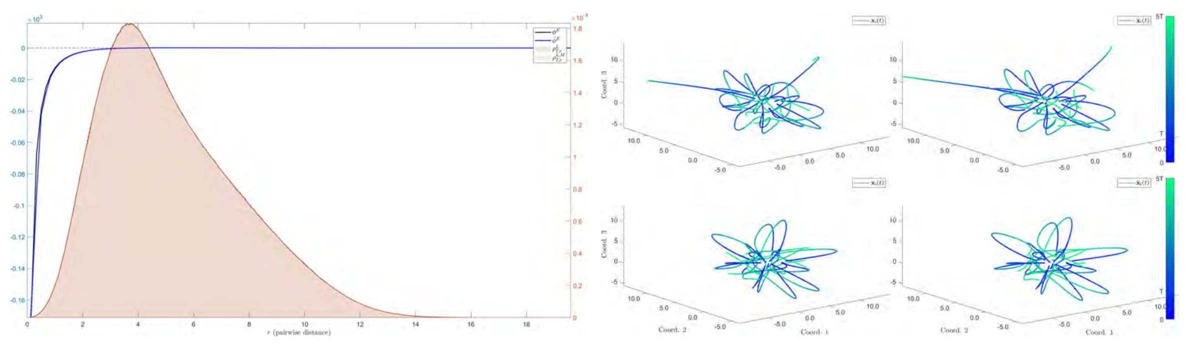

Figure 4: Fish Mill $3D$

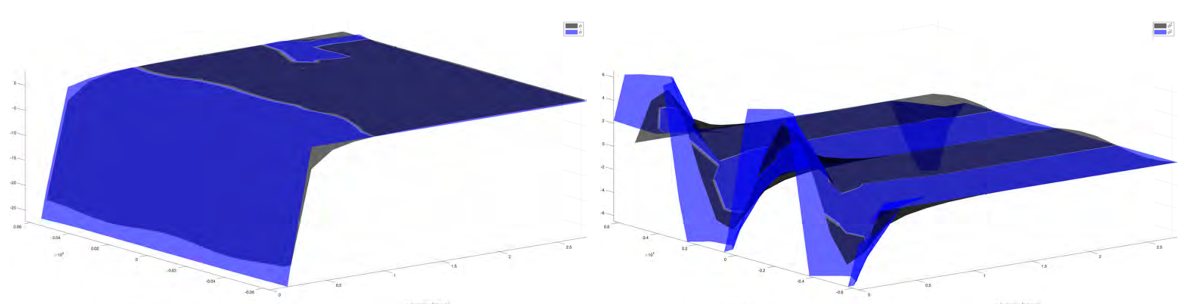

Figure 5: Synchronized Oscillator Dynamics

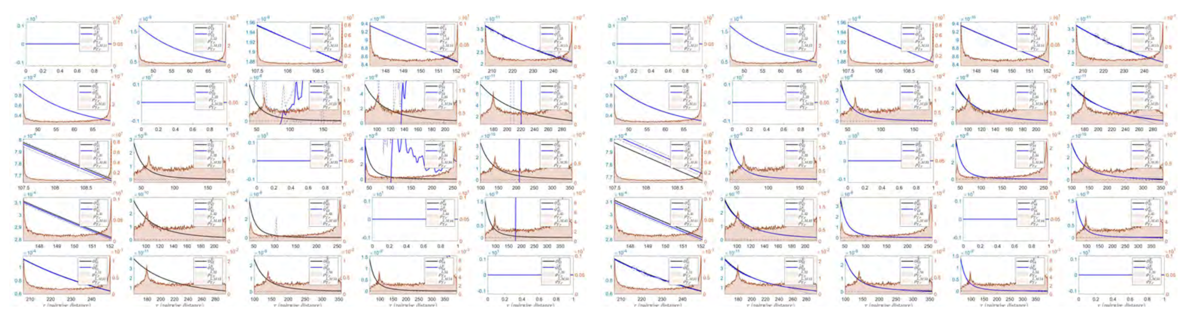

Figure 6: Gravitational Solar System

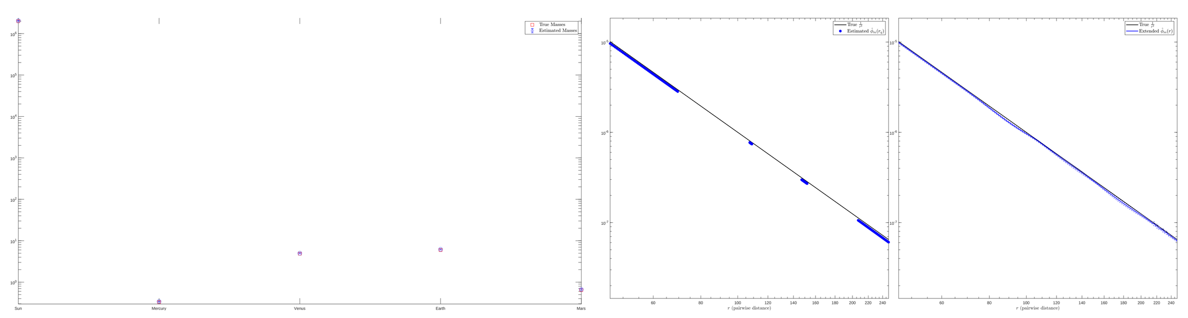

Figure 7: Learning of Parametric Form

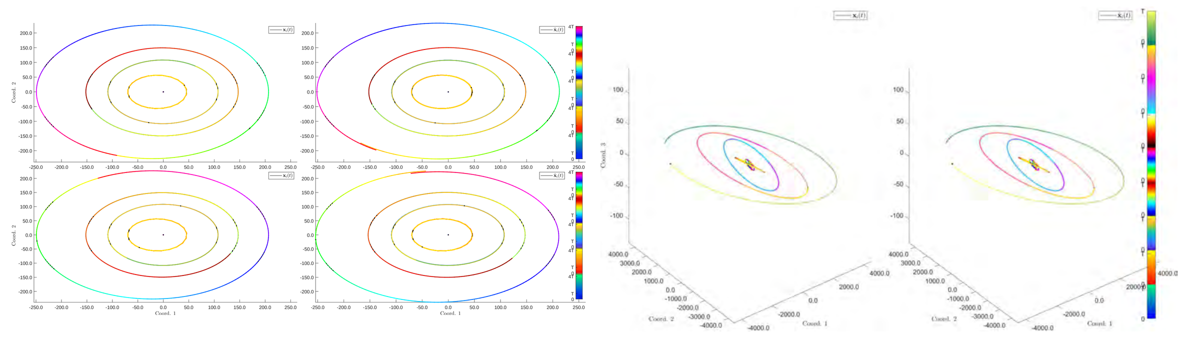

Figure 8: Solar System Trajectories

References

[1] M. Bongini, M. Fornasier, M. Hansen, and M. Maggioni. Inferring interaction rules from observations of evolutive systems I: The variational approach. *Math Mod Methods Appl Sci*, 27(05):909–951, 2017.

[2] Fei Lu, Ming Zhong, Sui Tang, and Mauro Maggioni. Nonparametric inference of interaction laws in systems of agents from trajectory data. *PNAS*, 116(3):14424 – 14433, 2019.

[3] Maruo Maggioni, Jason Miller, and Ming Zhong. Data-driven discovery of emergent behaviors in collective dynamics. Submitted to Physica D, 2019.

[4] Maruo Maggioni, Jason Miller, and Ming Zhong. Data-driven discovery of important astronomical events. In Preparation, 2020.