

Learning Switching Diffusions for Extreme and Rare Material Responses

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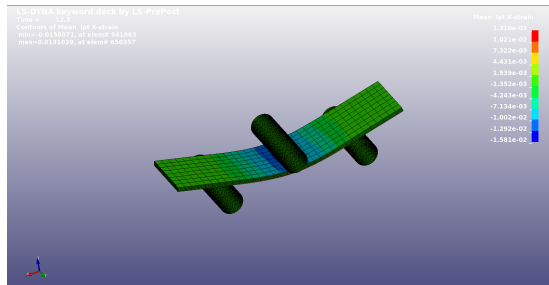
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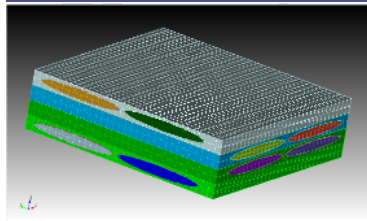
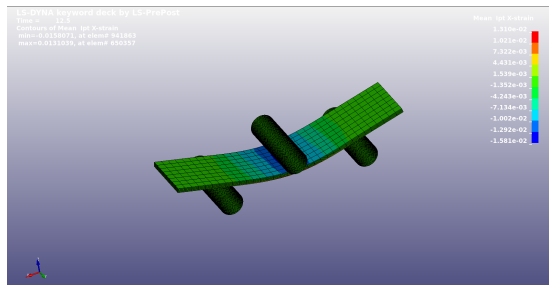
Acknowledgment

- this is an extension of a portion of Zheming Gou @ amazon 's PhD dissertation.
- also an extension of a collaboration with Sergey Lototsky @ math.usc

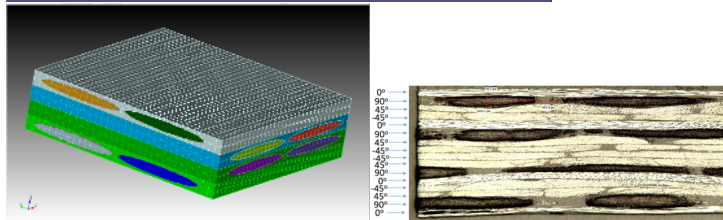
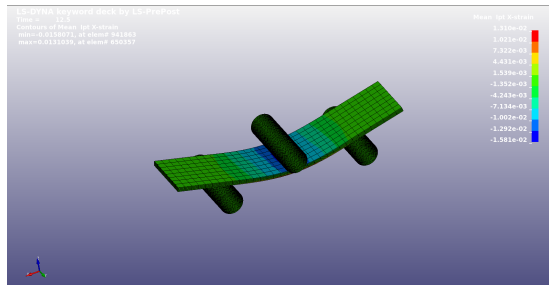
Composite Beam under Bending



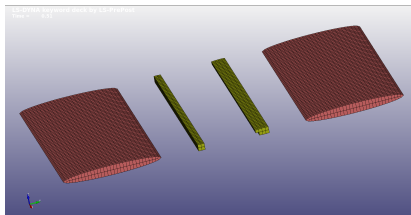
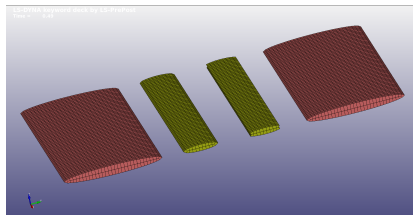
Composite Beam under Bending



Composite Beam under Bending

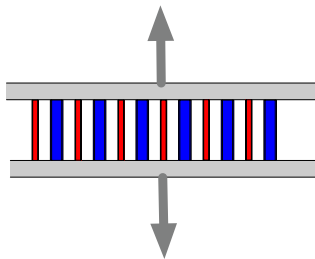


Microscale Damage

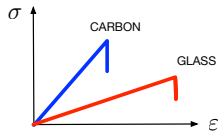


Physics-Based Model

Jenkins-Iwan Model



MICRO-STRUCTURE

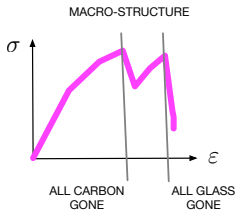
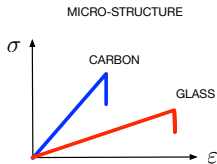
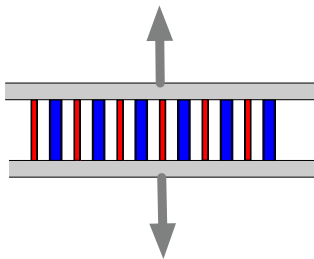


MACRO-STRUCTURE



Physics-Based Model

Jenkins-Iwan Model

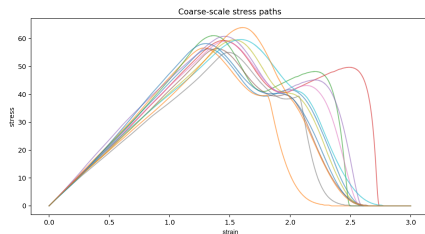


Constructing the experimental dataset

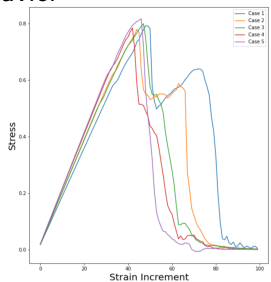
Effect of random micro-elements:

200 Carbon and 200 Glass elements sampled from beta distributions

Model-Based Behavior



Experimentally Observed Behavior



Jenkins / Iwan model: from microstructure to a physical generator

The deterministic simulator becomes a *physical random generator* by sampling the latent element properties at initialization.

For realization ω ,

$$\{k_i^C(\omega), f_{y,i}^C(\omega), k_i^G(\omega), f_{y,i}^G(\omega)\}_{i=1}^{200}$$

are fixed, and the complete response is generated by the loading history.

The resulting ensemble provides, for every strain ε ,

$$C_\omega(\varepsilon), \quad G_\omega(\varepsilon), \quad S_\omega(\varepsilon) = C_\omega(\varepsilon) + G_\omega(\varepsilon),$$

plus latent failure quantities such as constituent failure fractions.

Key modeling point

The HSD is intended to reproduce the *distribution of trajectories generated by the construction mechanism*, not to identify or reconstruct the 400 Jenkins elements.

HSD representation

We replace the high-dimensional physical process by a hybrid switching diffusion

$$(S_\varepsilon, Z_\varepsilon), \quad Z_\varepsilon \in \{0, 1, \dots, K-1\}.$$

Within state j ,

$$dS_\varepsilon = \mu_j(S_\varepsilon) d\varepsilon + \sigma_j dW_\varepsilon$$

and the latent state switches monotonically,

$$j \longrightarrow j+1,$$

with intensity

$$\lambda_j(S_\varepsilon, \varepsilon)$$

(or, in the present simplified pipeline, $\lambda_j(S_\varepsilon)$).

The HSD therefore separates

continuous degradation from discrete latent transitions.

The latent switch is *not* defined to coincide with a physical stress peak.

Generator of the switching diffusion

For a sufficiently smooth test function $f(s, j)$, the infinitesimal generator is

$$\mathcal{L}f(s, j) = \mu_j(s) \partial_s f(s, j) + \frac{\sigma_j^2}{2} \partial_{ss} f(s, j) + \lambda_j(s) [f(s, j+1) - f(s, j)]$$

for $j < K - 1$.

For the terminal state,

$$\mathcal{L}f(s, j) = \mu_j(s) \partial_s f + \frac{\sigma_j^2}{2} \partial_{ss} f, \quad j = K - 1$$

This is the continuous-time mathematical object that all later calculations seek to represent numerically.

Fitted diffusion component

For each state j we fitted

$$\mu_j(s) = \sum_{m=0}^p \beta_{jm} P_m(\xi(s)), \quad \xi(s) = 2 \frac{s - S_{\min}}{S_{\max} - S_{\min}} - 1,$$

with Legendre basis P_m .

The fit was obtained from the physical trajectories using a monotone state-path representation and EM-type iteration:

E-step: latent state/path probabilities $\longleftrightarrow$ M-step: diffusion parameters.

The convergence diagnostics

- changes in latent paths
- changes in the objective.

Distribution of a transition time

Let

$$T_j = \inf\{\varepsilon : Z_\varepsilon = j + 1 \mid Z_0 = j\}$$

denote a transition time.

The transition-time survival function is

$$S_j(\varepsilon) = \mathbb{P}(T_j > \varepsilon).$$

The corresponding density is

$$f_j(\varepsilon) = -\frac{d}{d\varepsilon} S_j(\varepsilon).$$

Equivalently, if $p_j(s, \varepsilon)$ denotes the joint density of being at stress s while still in state j ,

$$f_j(\varepsilon) = \int \lambda_j(s) p_j(s, \varepsilon) ds.$$

Thus the transition PDF is the *switching flux* out of state j .

Switching law as a counting-process hazard

Conditional on S_ε , the state jump is a counting-process event with intensity

$$\lambda_j(S_\varepsilon)$$

and the integrated hazard along a path is

$$H_j(\varepsilon) = \int_0^\varepsilon \lambda_j(S_u) \, du.$$

The conditional survival probability is

$$\mathbb{P}(T_j > \varepsilon \mid S_{[0,\varepsilon]}) = \exp[-H_j(\varepsilon)].$$

A convenient simulation representation is:

$$E \sim \text{Exp}(1), \quad T_j = \inf\{\varepsilon : H_j(\varepsilon) \geq E\}.$$

This is the continuous-hazard formulation used for the HSD sampling.

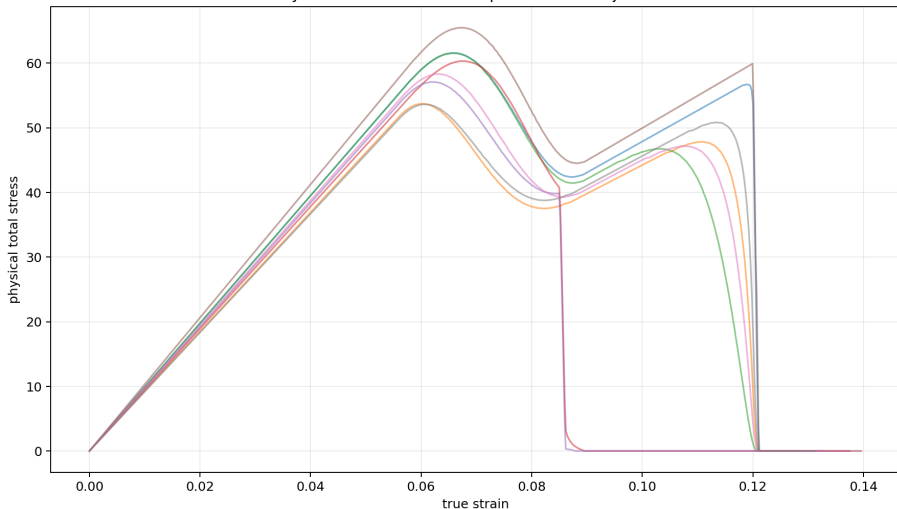
We also deduce joint probability densities for the all the transition times

- Joint distribution of transition times
- conditional distribution of transition times
- distribution of switching conditional on diffusion

Also deduce probability of extremes of the transition times

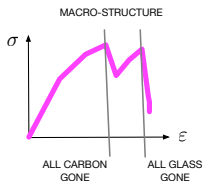
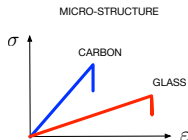
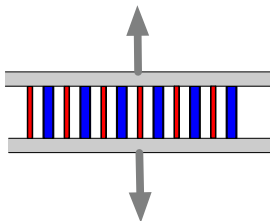
RESULTS

Physical test ensemble: 8 representative trajectories

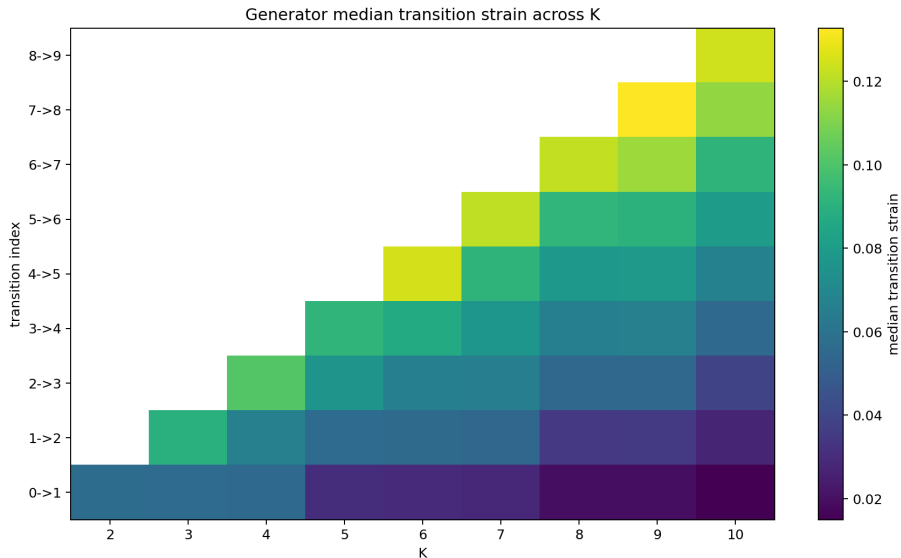


Physics-Based Model

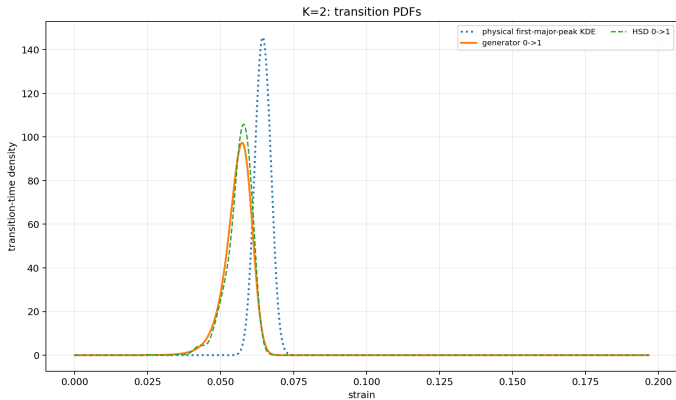
RECALL Jenkins-Iwan Model



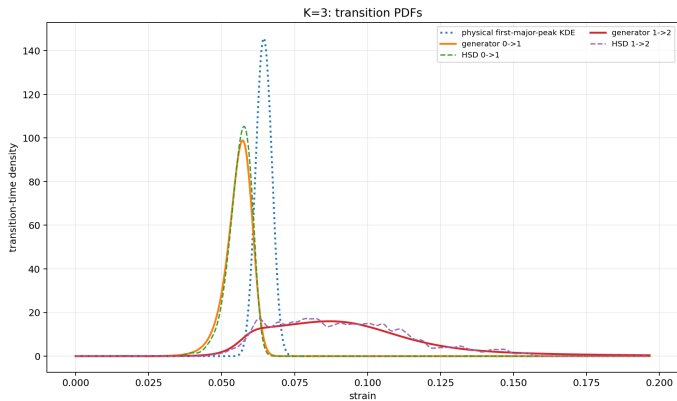
Comparison of transitions as function of number of states



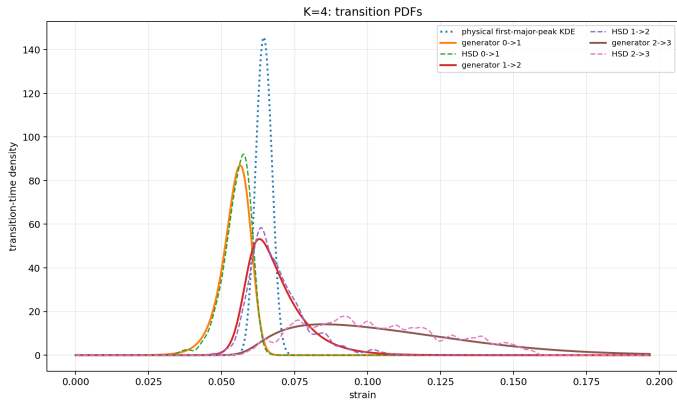
Probability of transition times



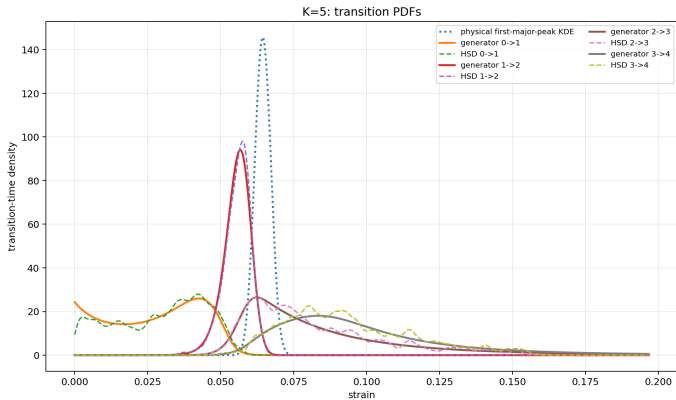
Probability of transition times



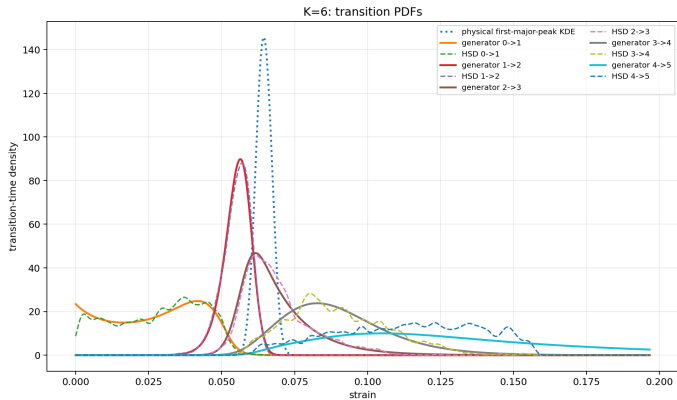
Probability of transition times



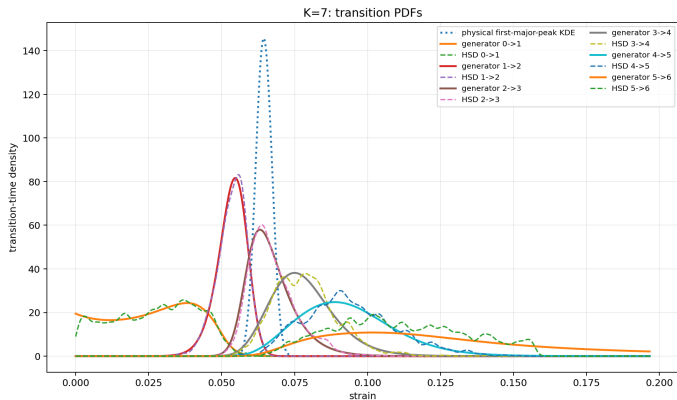
Probability of transition times



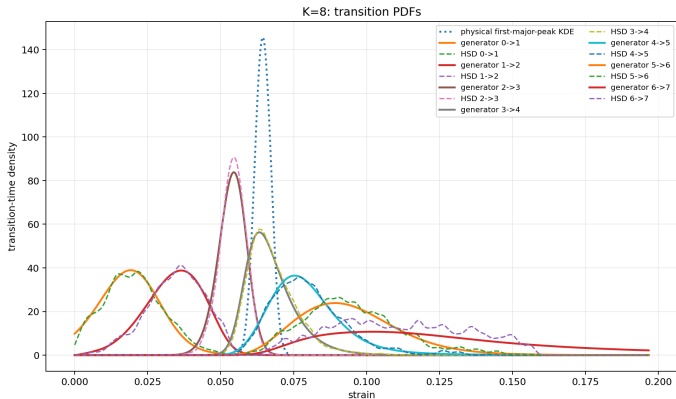
Probability of transition times



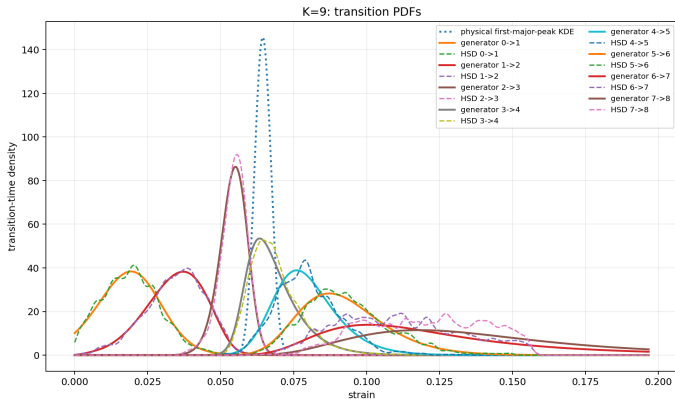
Probability of transition times



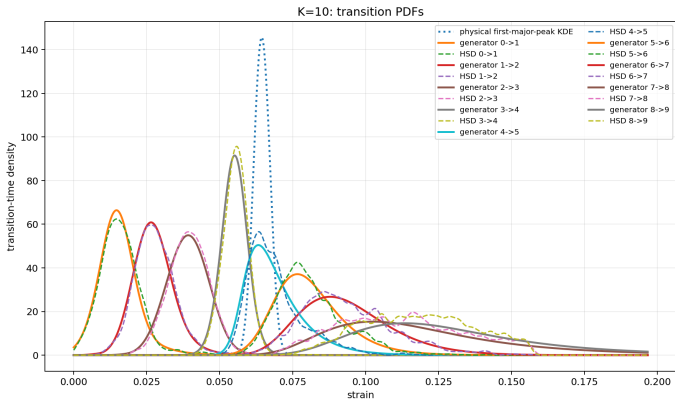
Probability of transition times



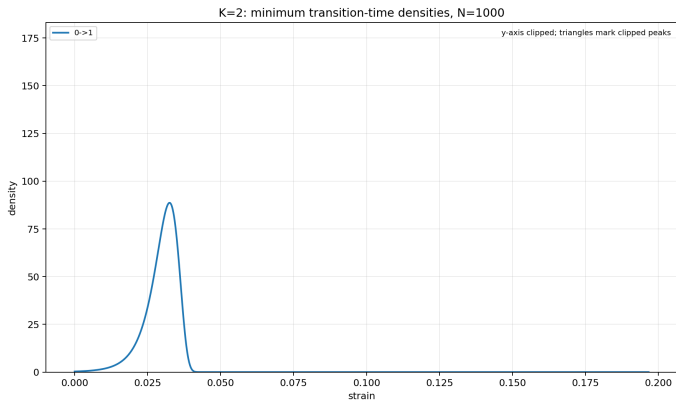
Probability of transition times



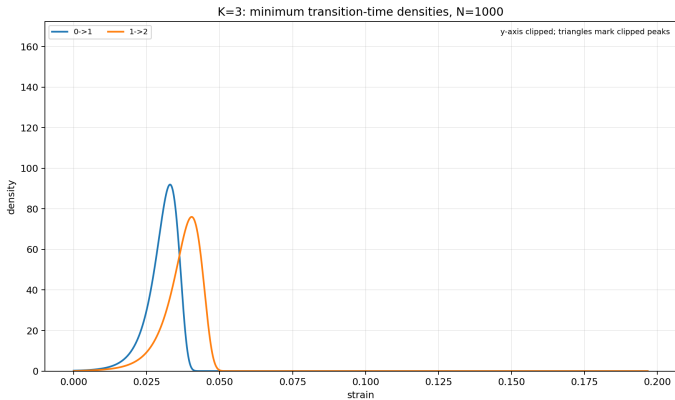
Probability of transition times



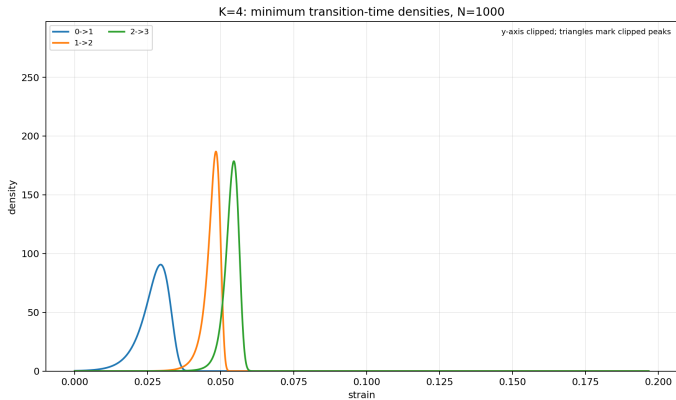
Probability of transition times extremes (minima)



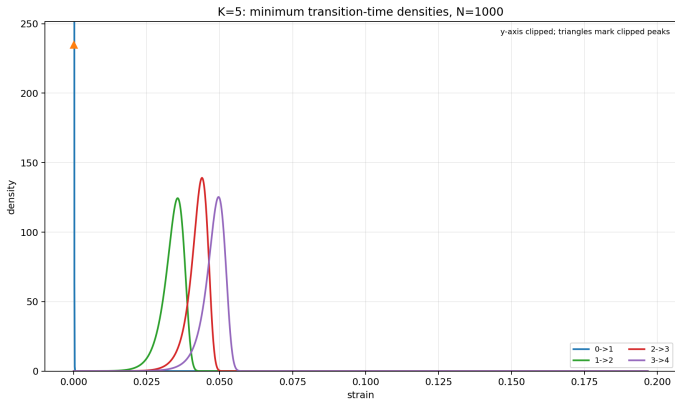
Probability of transition times extremes (minima)



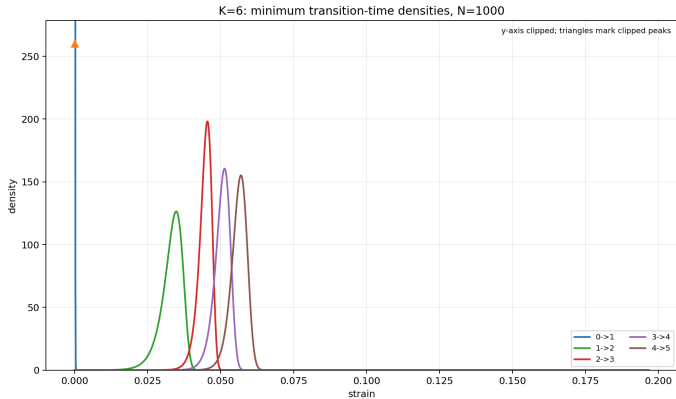
Probability of transition times extremes (minima)



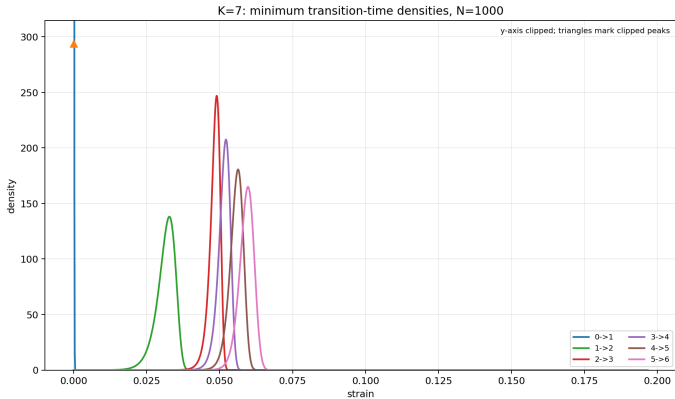
Probability of transition times extremes (minima)



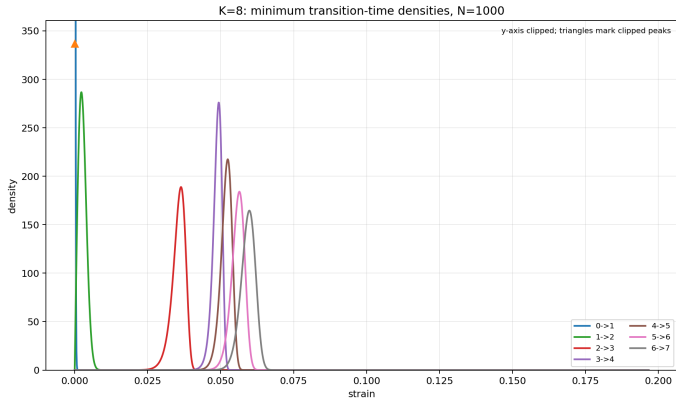
Probability of transition times extremes (minima)



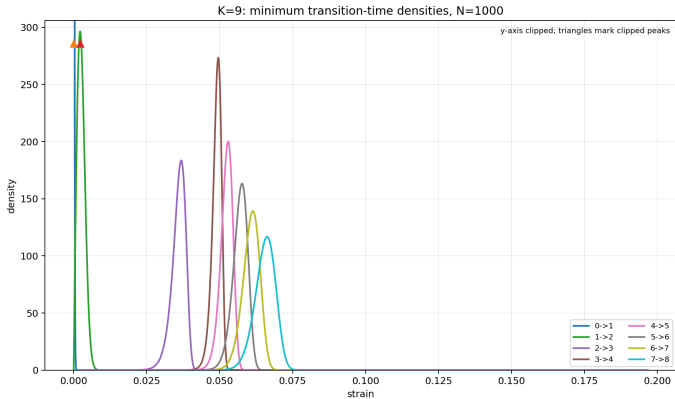
Probability of transition times extremes (minima)



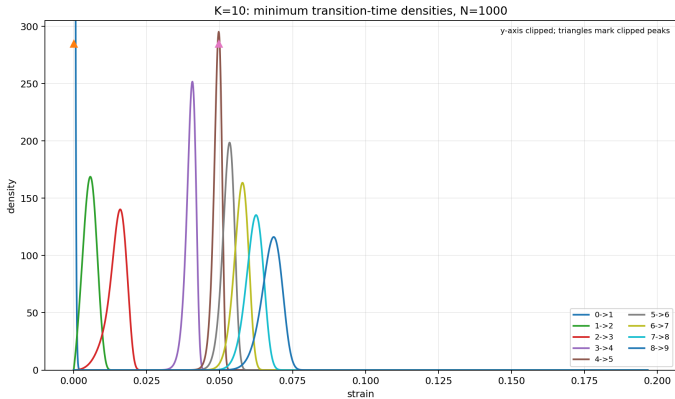
Probability of transition times extremes (minima)



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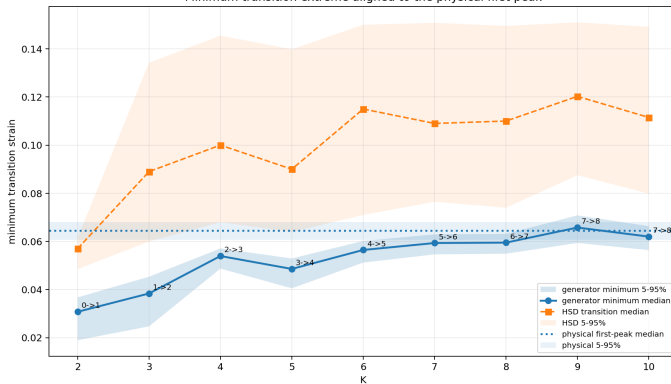


Probability of transition times extremes (minima)



Significance of using the generator to evaluate extremes

Minimum transition extreme aligned to the physical first peak



Next step

- The transitions are intrinsic to the data.
- Our data is synthetic, with pre-engineered transitions.
- Next step is to explore experimental data