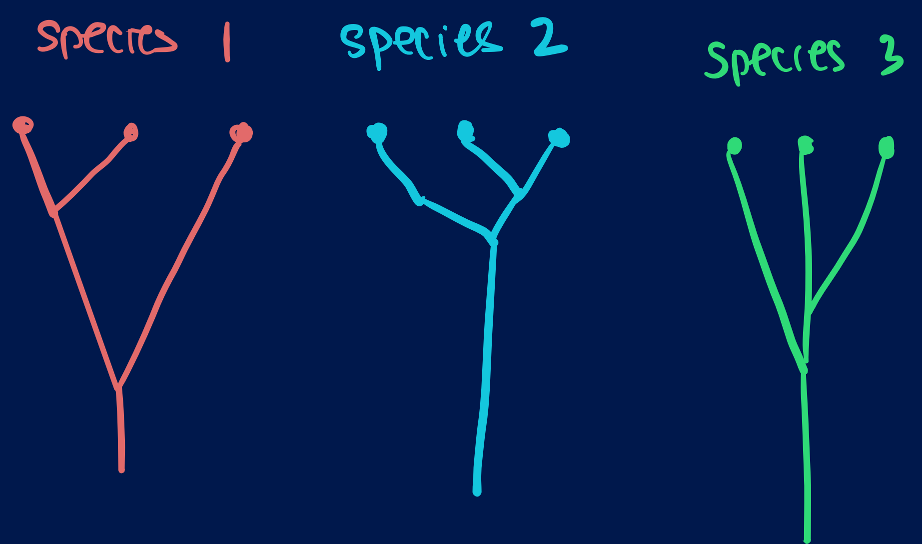
Patterns in Joint Genealogies

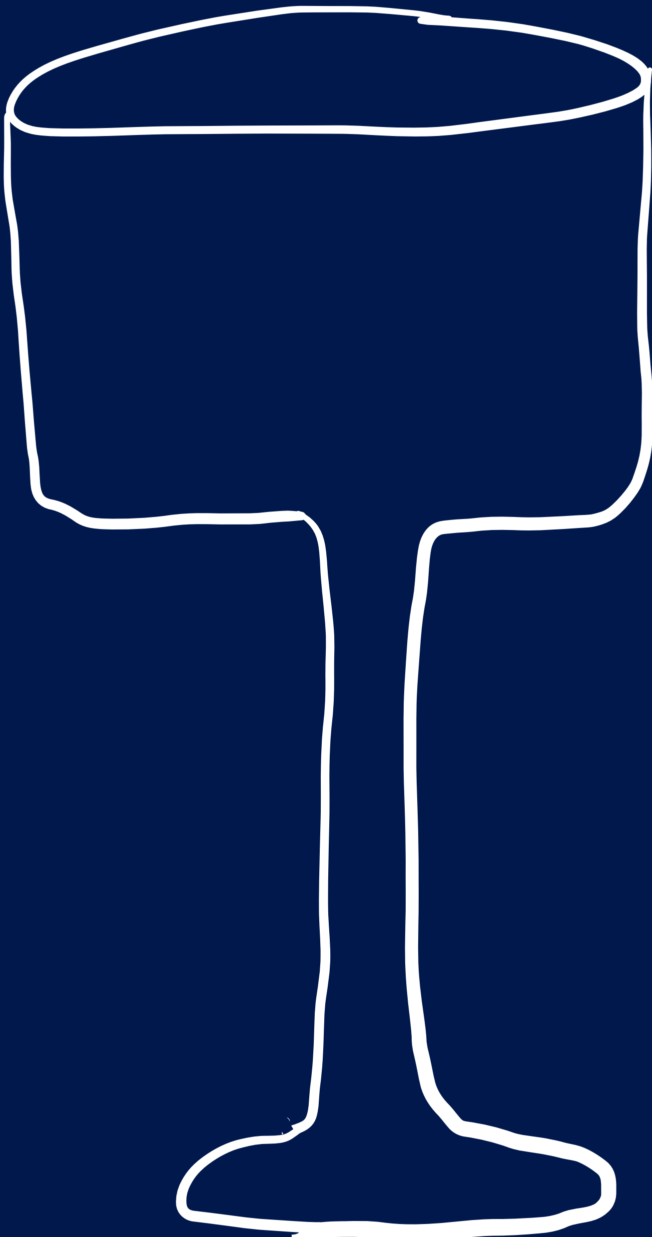
Linking the coalescent into community ecology



Swati Patel
Oregon State Univ.

SMTPB conference
2026

Outline



Broad: Coexistence Theory
Community Ecology

Broad: Joint Coalescent

Narrow: Lattice Percolation

$\boxed{\text{Main Result: Law of Large Numbers type}}$

Some Final Remarks

Joint Work w/

Axel Saenz Rodriguez

Lucas Allen
(grad student)

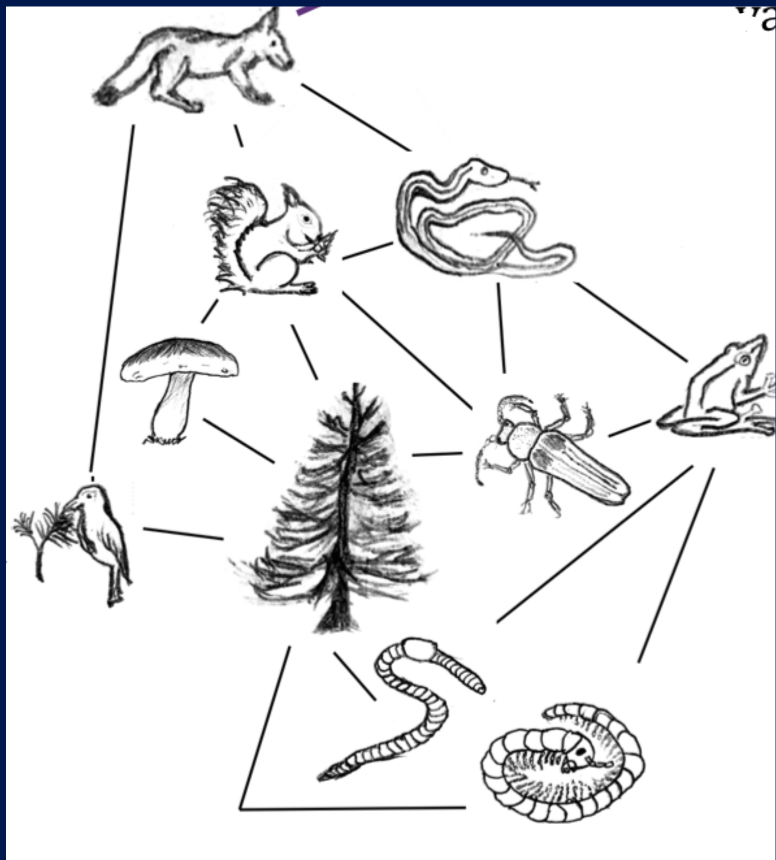
Also benefitted from discussions w/

- Peter Ralph (U. of Oregon)
- Sebastian Schreiber (UC Davis)
- Mark Novak (OSU)
- Lauren Shoemaker (U of Wyoming)
- Nick Kortessis (Wake Forest)

(original)

Motivation: Coexistence Theory

Question: when & how do species coexist?



(original)

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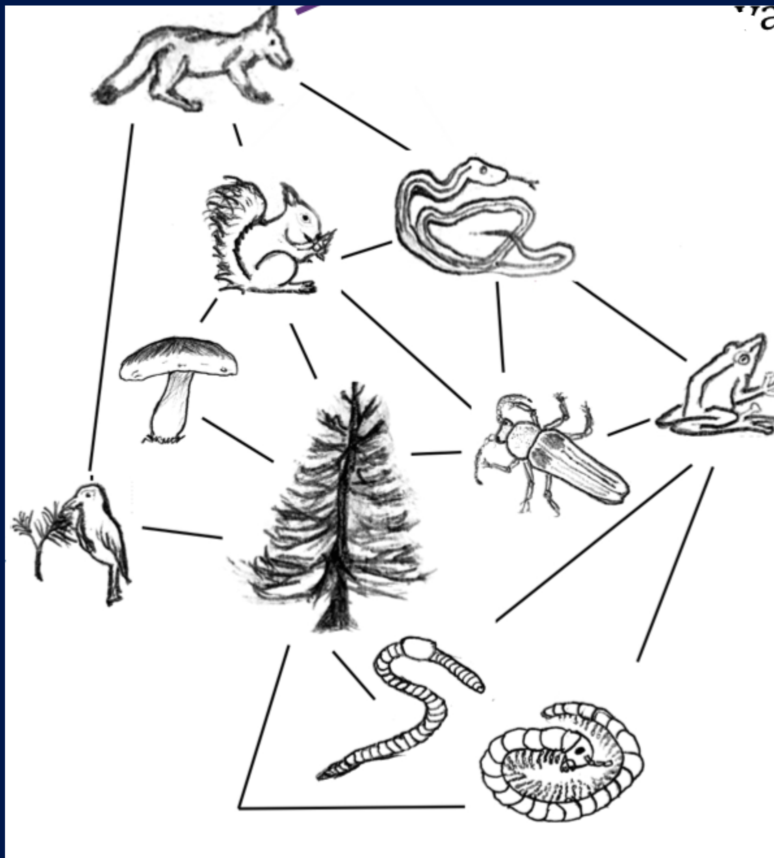

Forward Model

population dynamic equations

e.g. Lotka-Volterra ODEs

(original)

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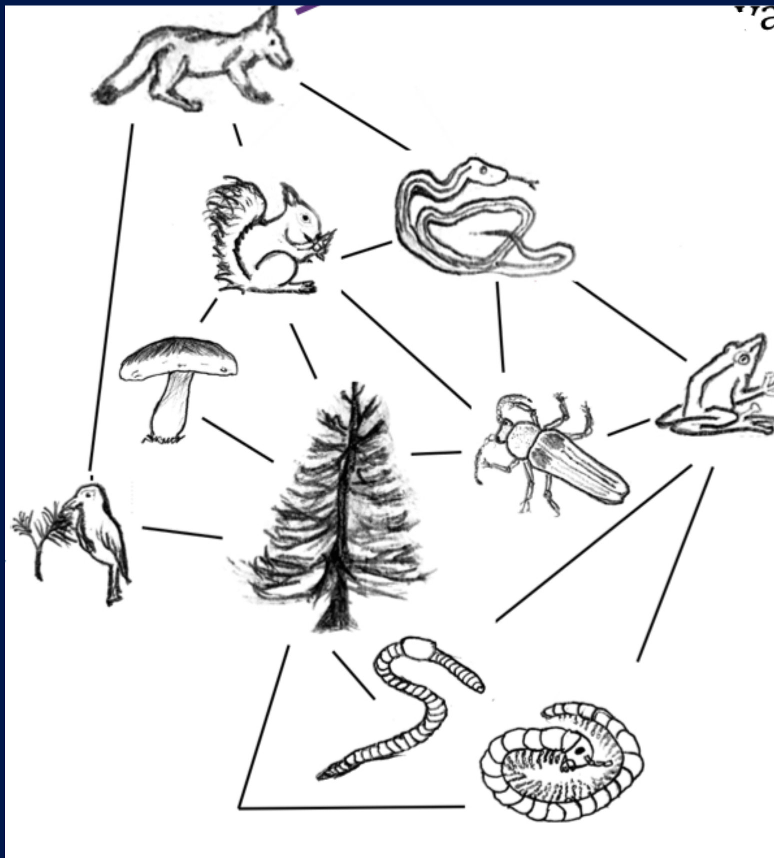
Question: when & how do species coexist?



Forward Model

population dynamic equations → predict behaviors

e.g. coexistence

(original)

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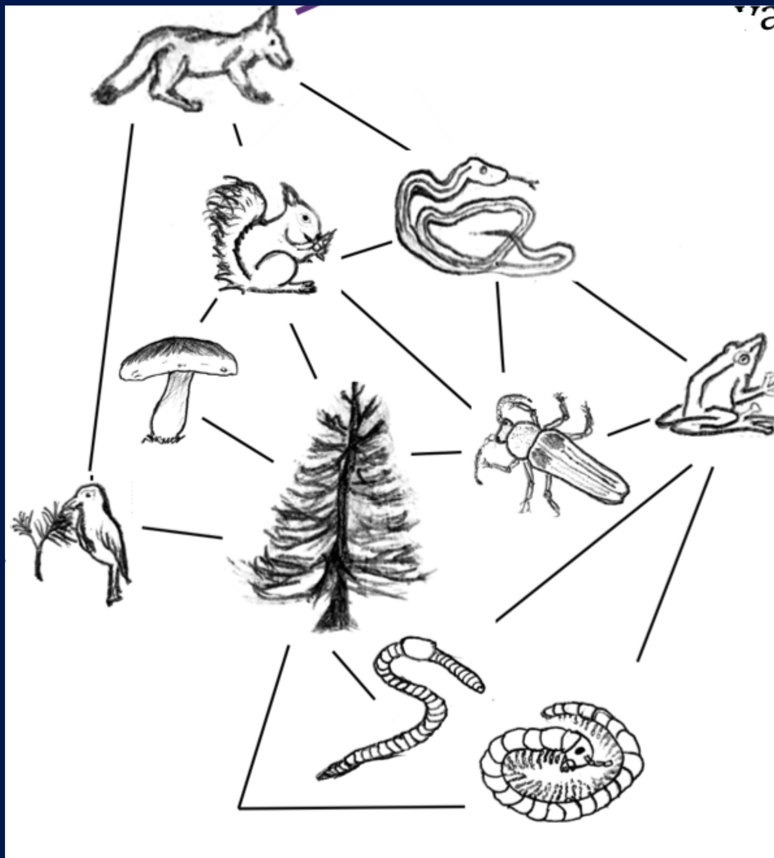

Forward Model → Theory

e.g. Ostling's talk

tradeoffs → coexist

(original)

Motivation: Coexistence Theory

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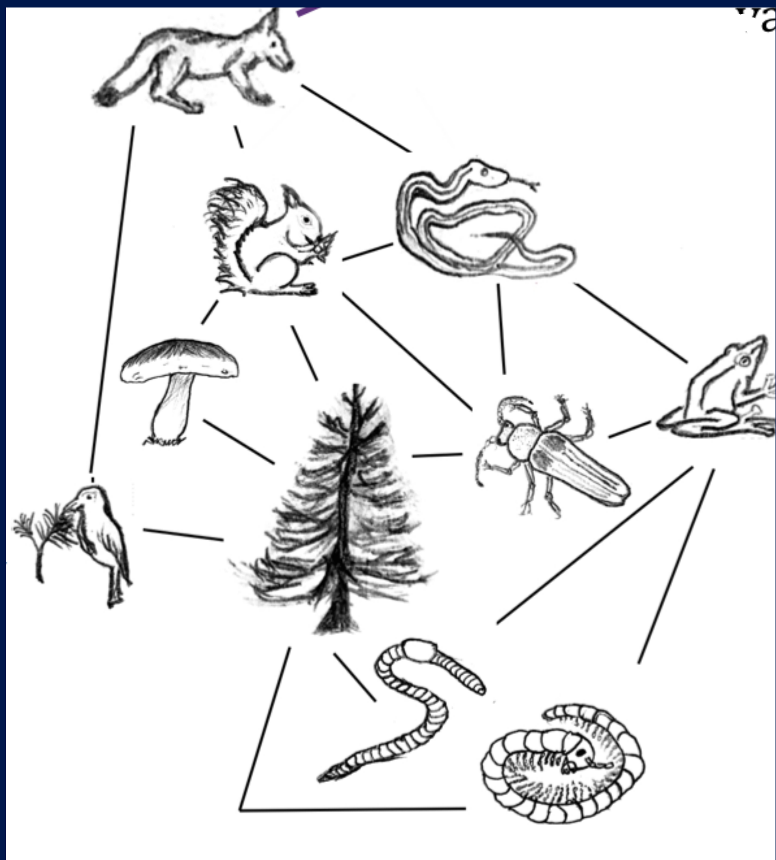
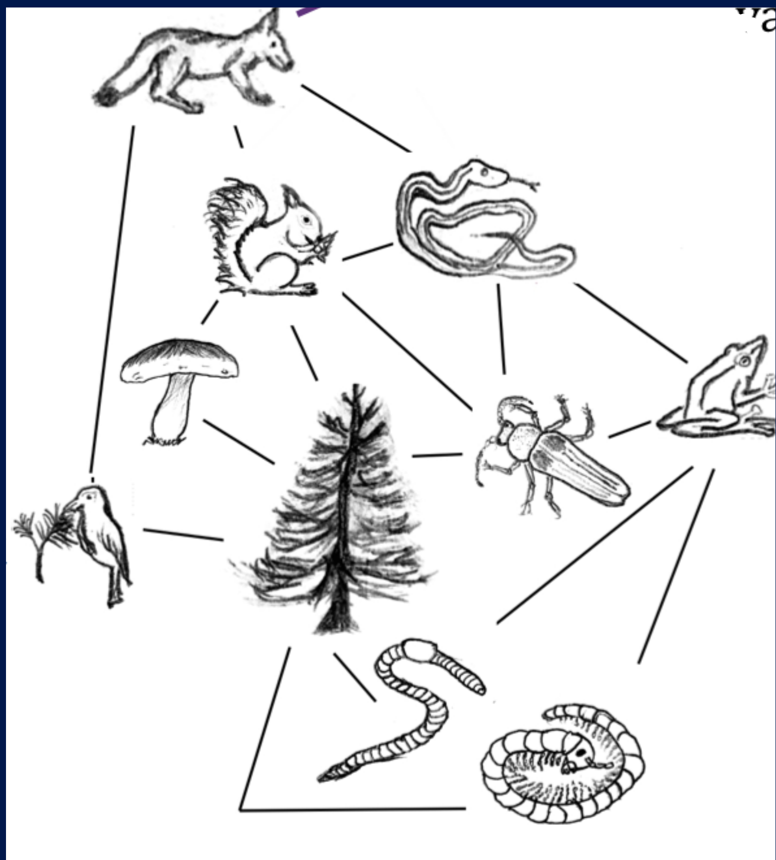


Forward Model → Theory → empirical systems

(original)

Motivation: Coexistence Theory

Question: when & how do species coexist?

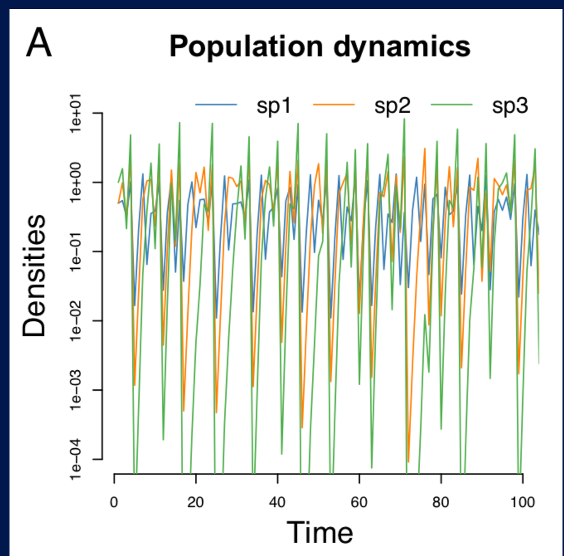


Forward Model → Theory → empirical systems

parameterize from

• time series

e.g. Nguyen et al 2025



(original)

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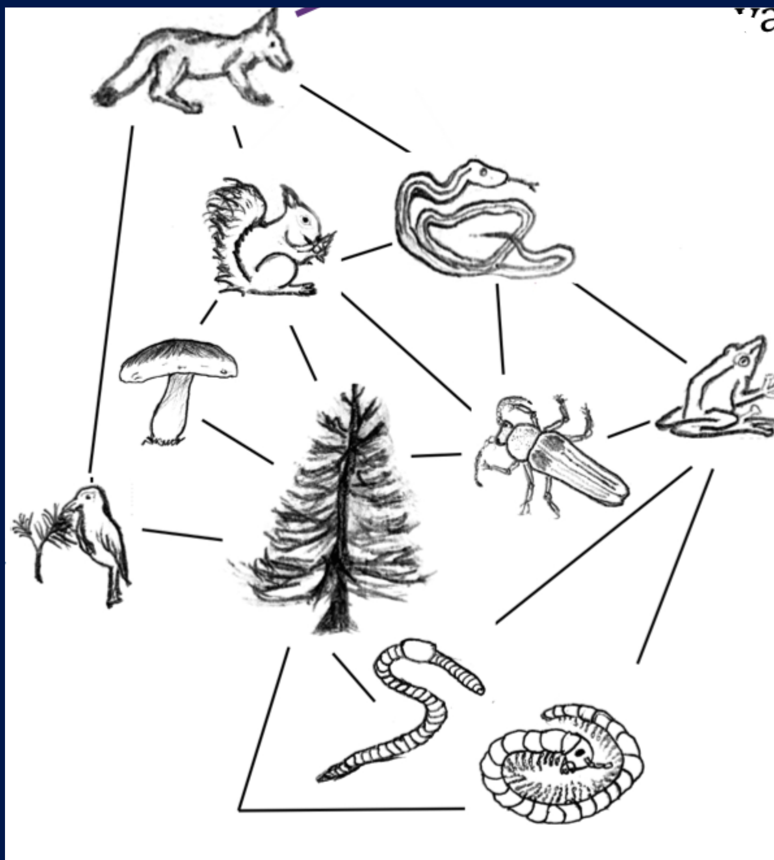


Forward Model → Theory → empirical systems

parameterize from

- time series
- experimental manipulations

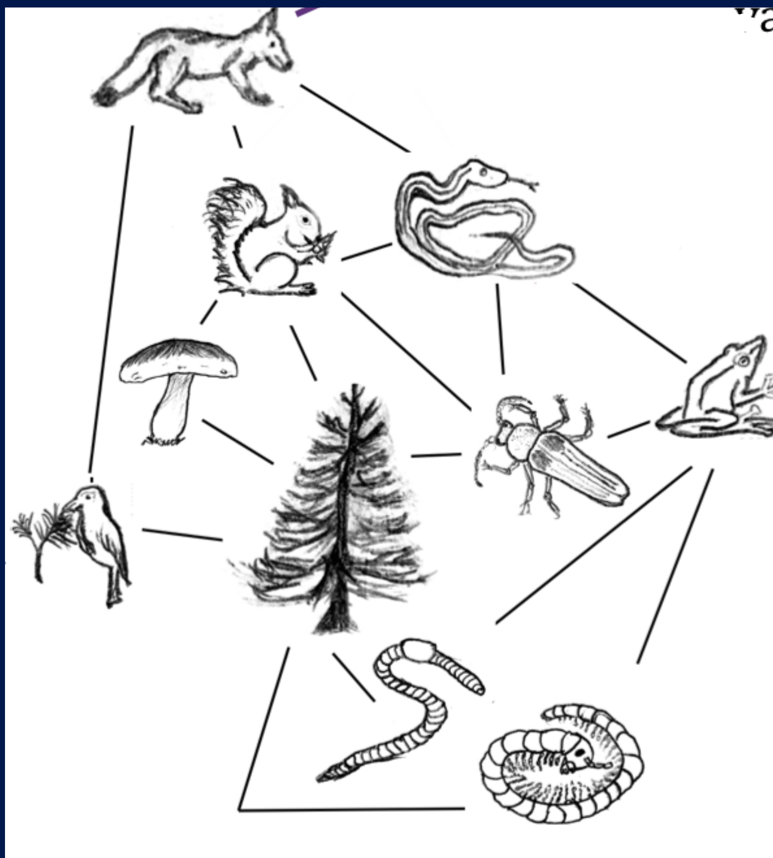
e.g. Godoy & Levine 2014

Jacob Levine's talk

(original)

Motivation: Coexistence Theory

Question: when & how do species coexist?



Forward Model → Theory → empirical systems

parameterize from

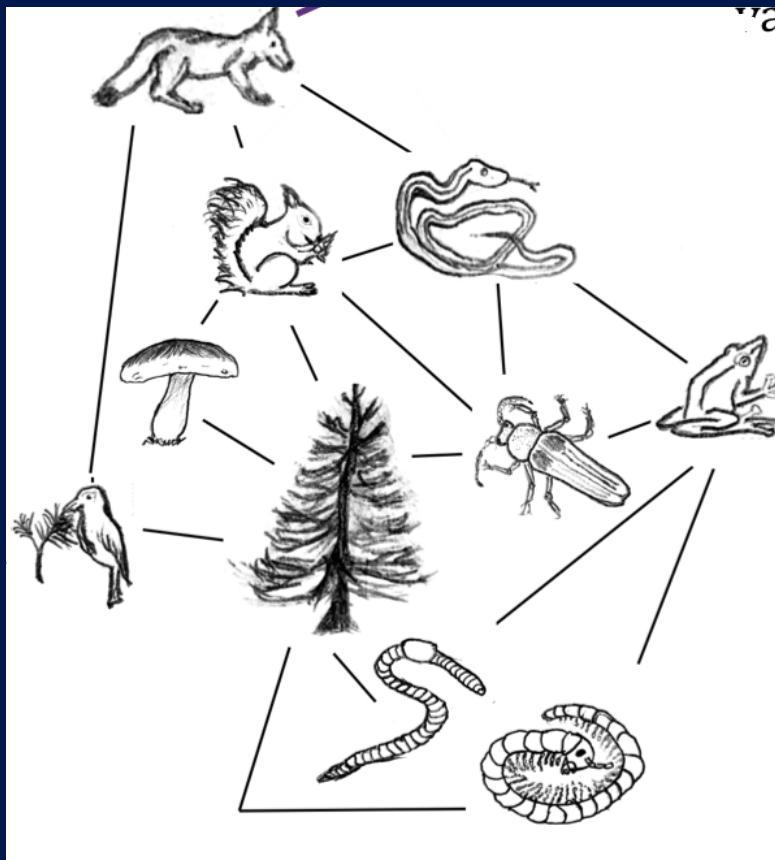
- time series
- experimental manipulations

→ SMTPB connect video
Modern Coexistence Theory

(original)

Motivation: Coexistence Theory

Question: when & how do species coexist?



Forward Model

Theory

empirical systems

parameterize from

- time series
- experimental manipulations

Labor intensive & short time scales

(original)

Motivation: Coexistence Theory

Question: when & how do species coexist?

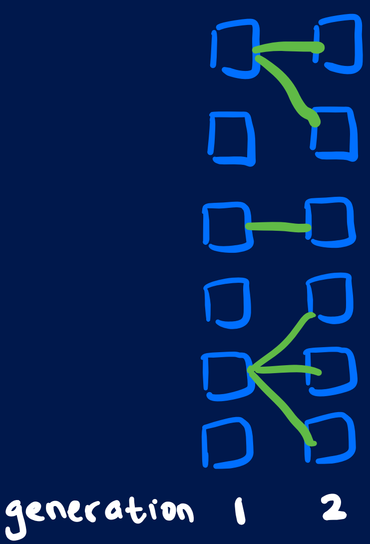
Question: Can genetic data help understand coexistence?

- over longer timescales
- more feasibly

The Coalescent

- a powerful framework for inferring population history from genealogical trees

- central idea: population history leaves a distinguishing pattern on tree (and genetics)



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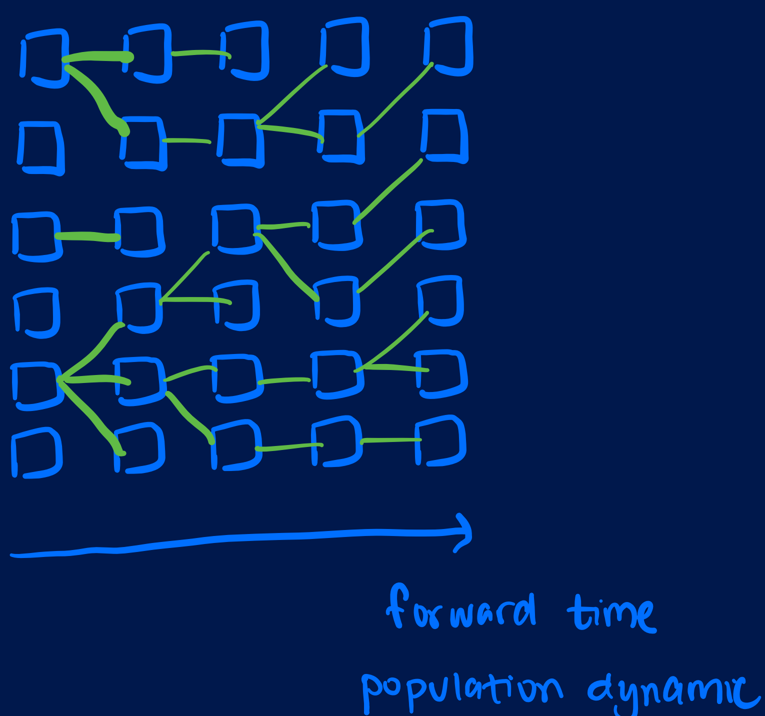

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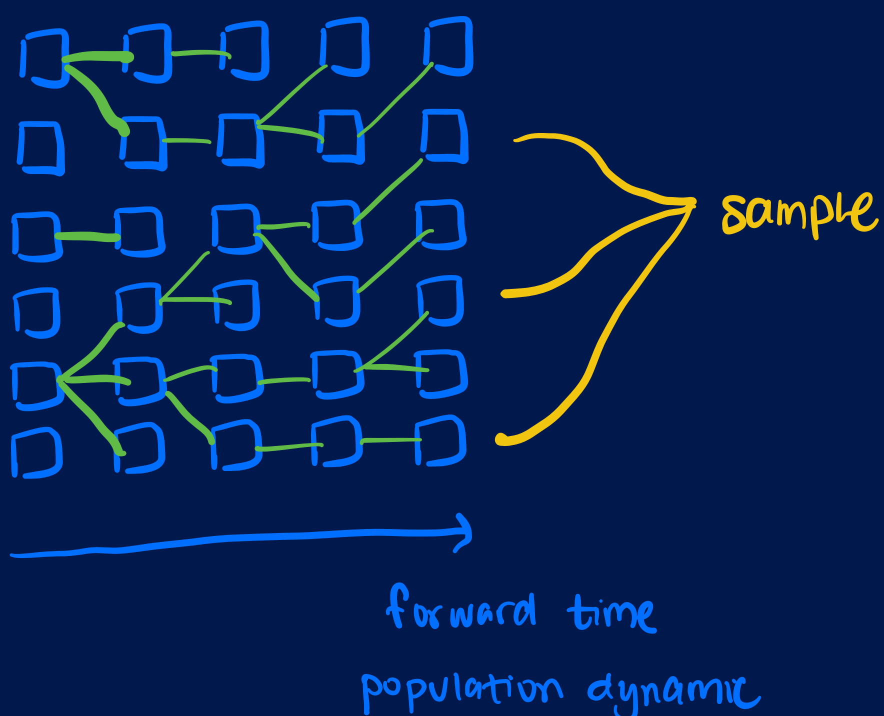

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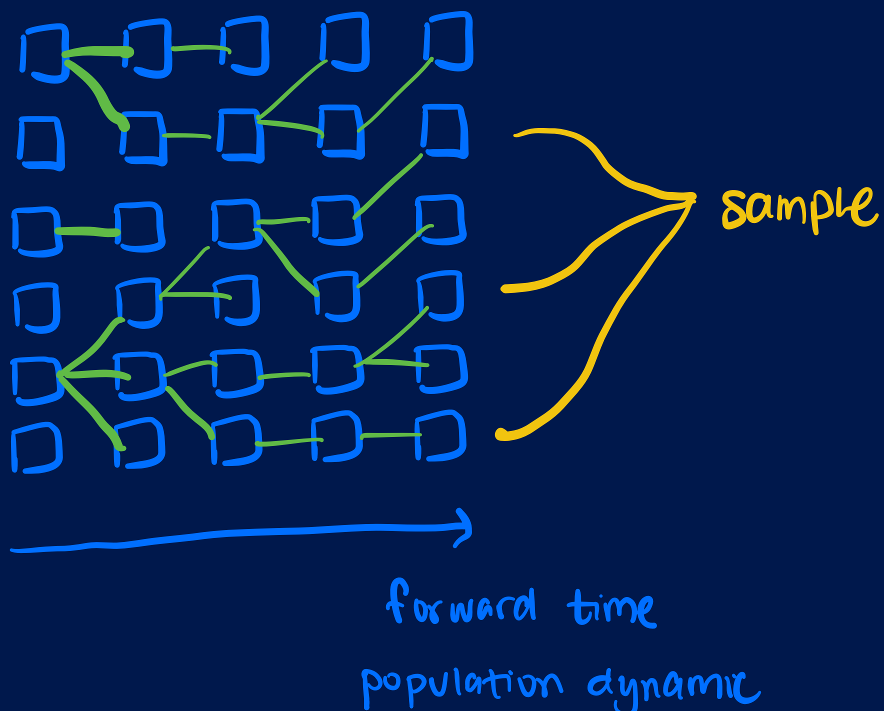

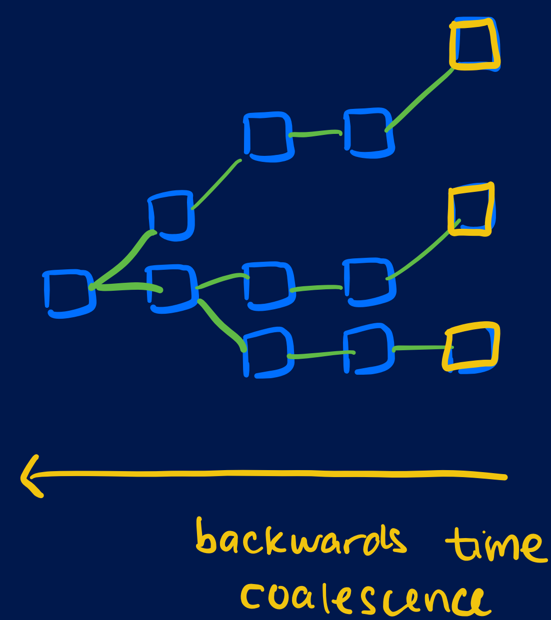

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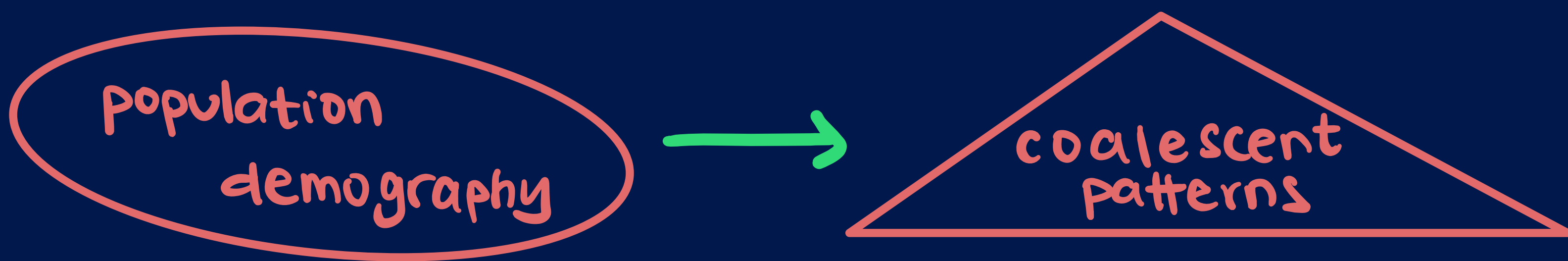

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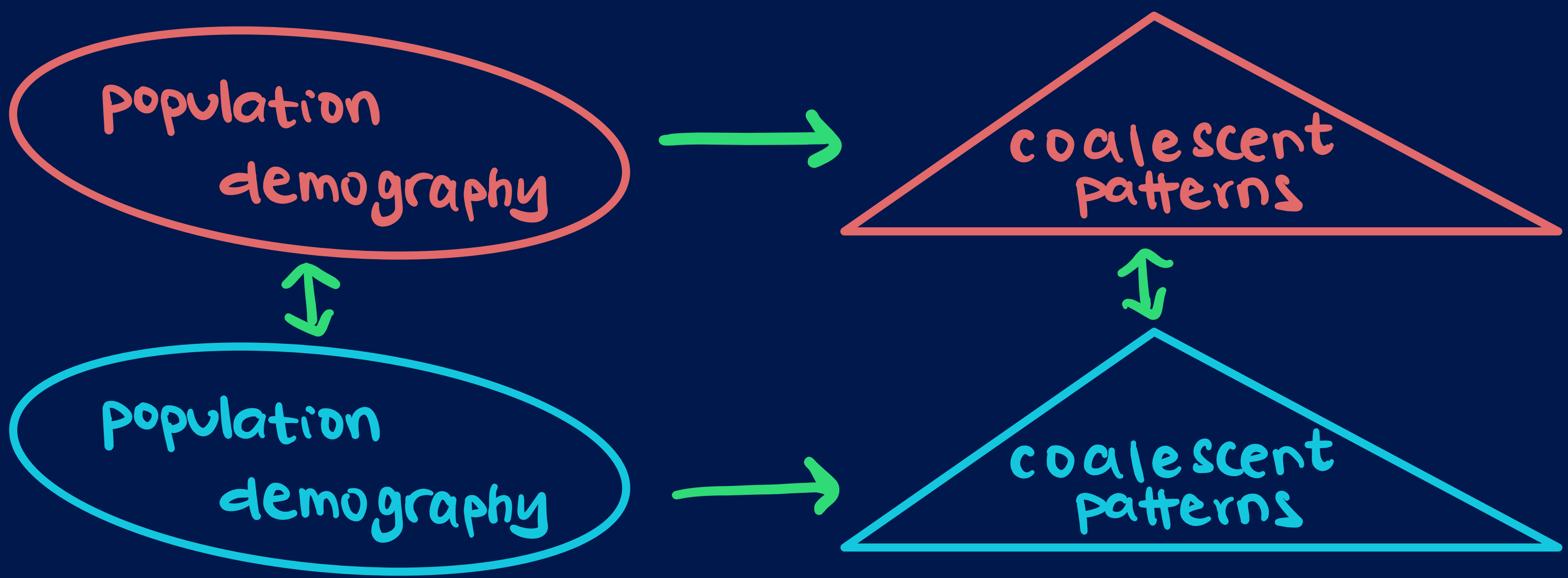
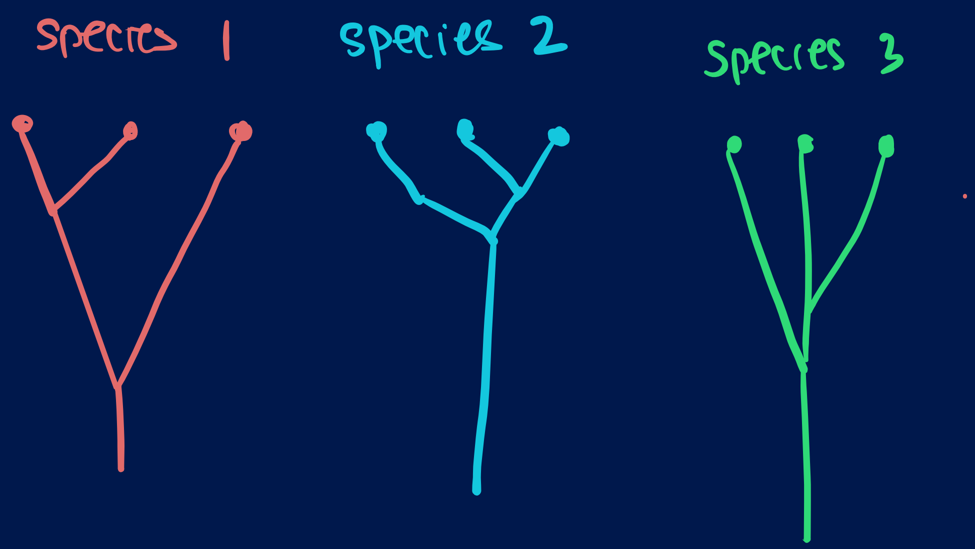


The Coalescent

What can joint genealogies tell us about histories of communities?



Goal:

understand patterns of joint genealogies

Patterns of Joint Genealogies

Start with two species at equilibrium

species 1: • constant (effective) pop. size N_1

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↳ standard k_1-coalescent

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Patterns of Joint Genealogies

Start with two species at equilibrium

species 1: • constant (effective) pop. size N_1

• sampled K_1 individuals

↳ standard K_1-coalescent

species 2: • constant (effective) pop. size $N_2 = \frac{1}{\lambda} N_1$

• sampled K_2 individuals

↳ K_2-coalescent w/ rate λ

Patterns of Joint Genealogies

Start with two species at equilibrium

Patterns of Joint Genealogies

Start with two species at equilibrium

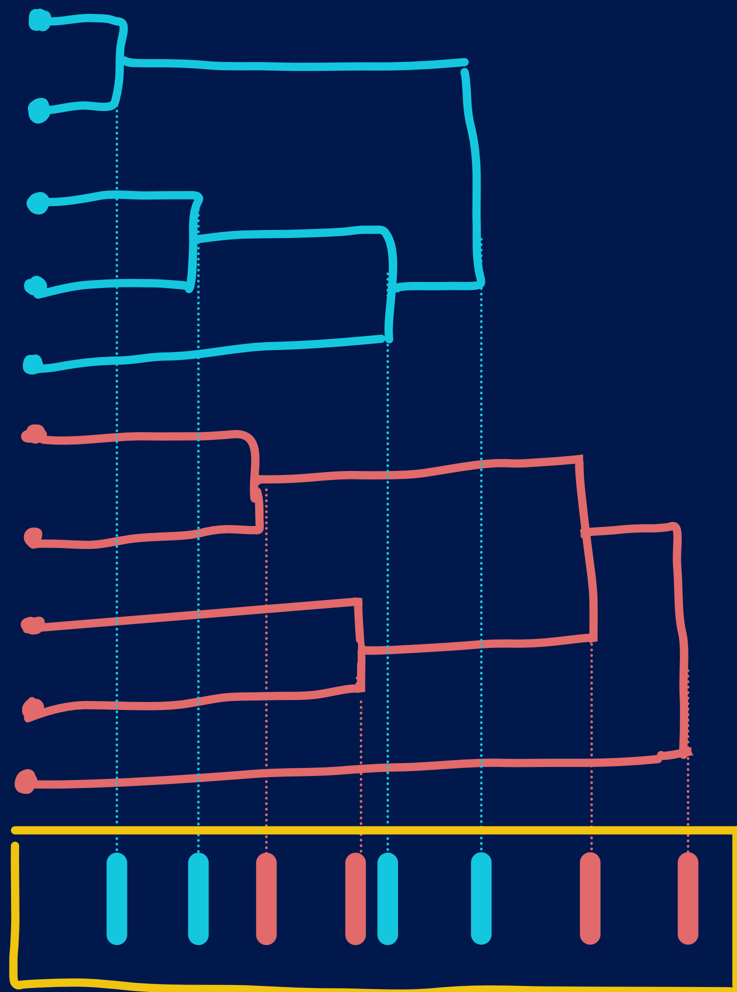
Object of Interest is
Ordering of Coalescent Events

Patterns of Joint Genealogies

Start with two species at equilibrium

species 1

species 2



Object of Interest is
Ordering of Coalescent Events

Main Result is a Law of Large Numbers type result on this ordering

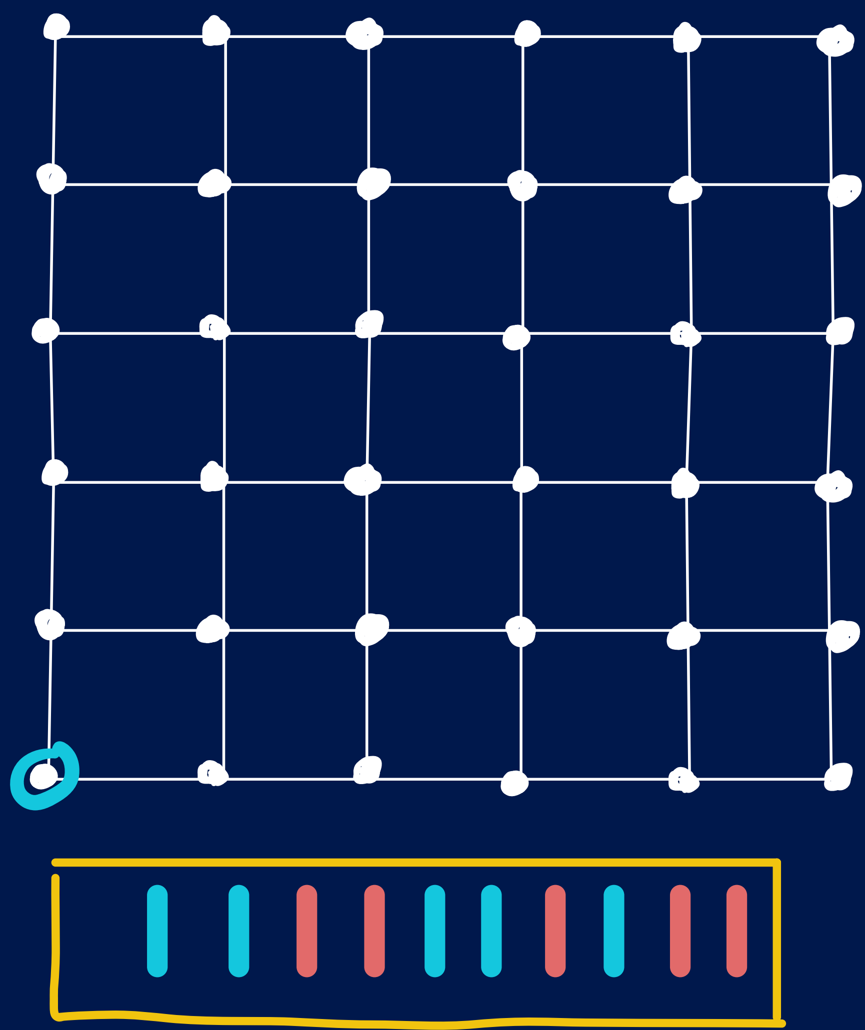
Ordering of Coalescent Events $\longrightarrow$ Percolation on Lattice

Ordering of Coalescent Events $\rightarrow$ Percolation on Lattice

$(k_1 = k_2 =: k)$

- $k-1$ lattice

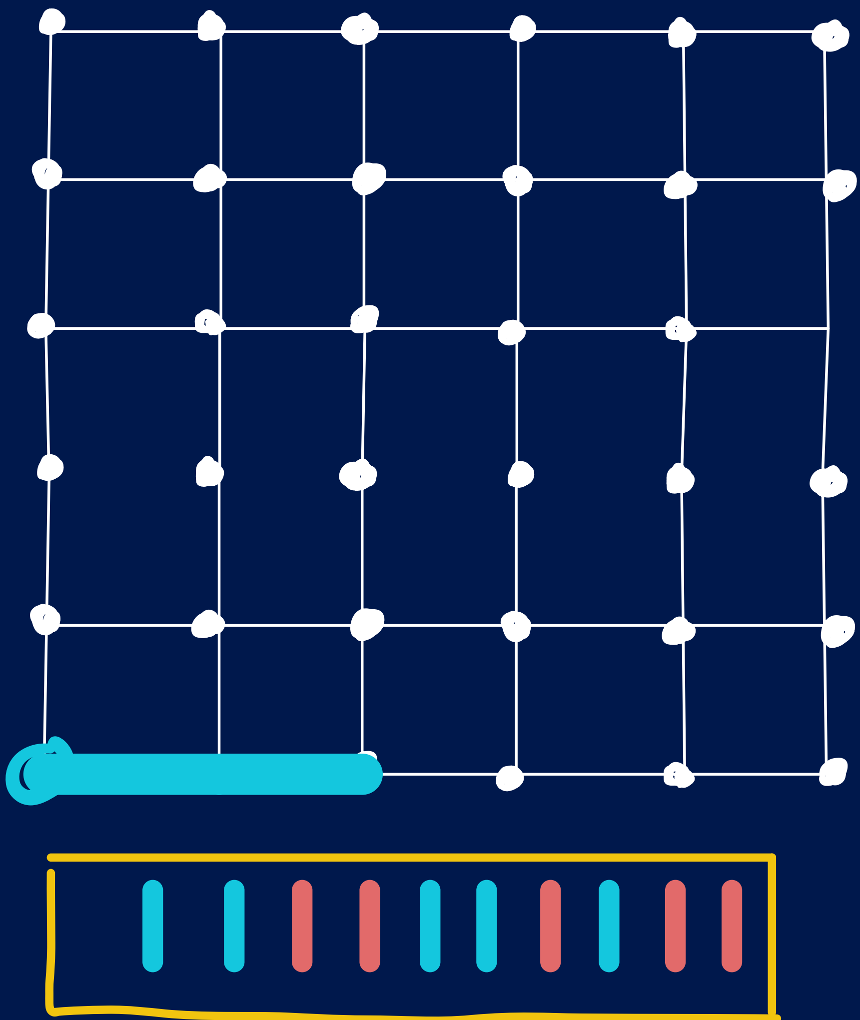
- Start at $(0,0)$



Ordering of Coalescent Events $\longrightarrow$ Percolation on Lattice

$(k_1 = k_2 =: k)$

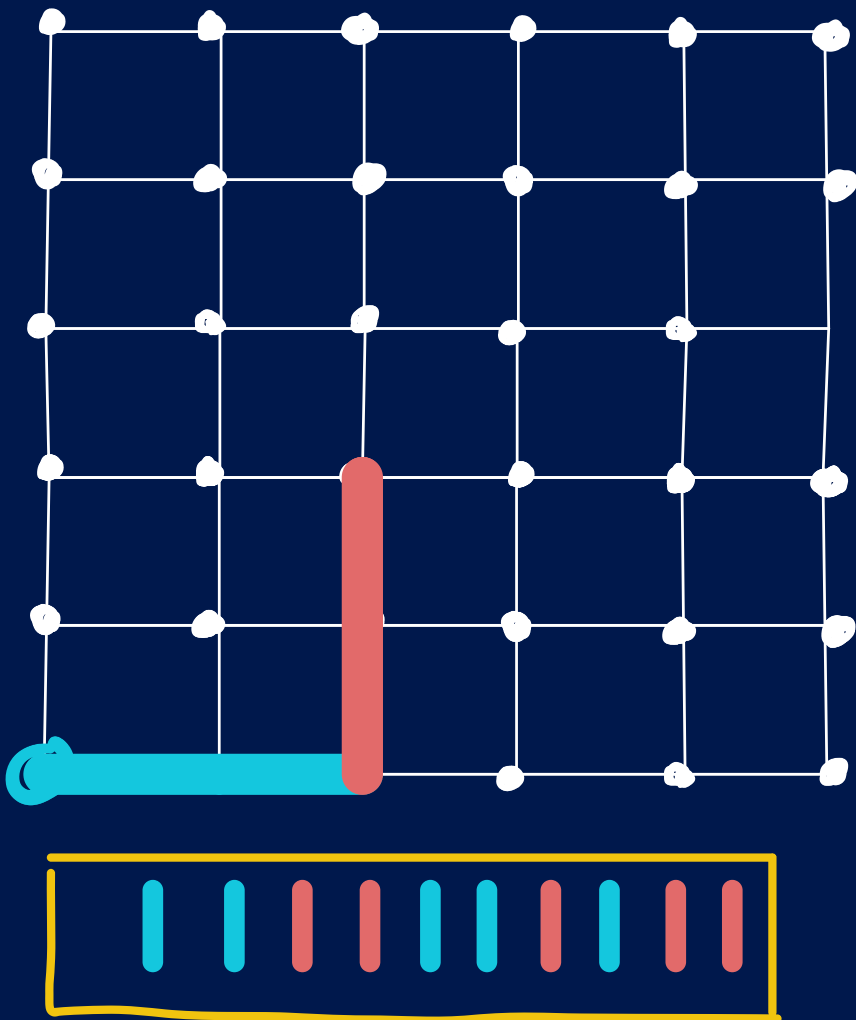
- $k-1$ lattice
- start at $(0,0)$
- for sp 1 coal. move right $\longrightarrow$



Ordering of Coalescent Events → Percolation on Lattice

$(k_1 = k_2 =: k)$

- $k-1$ lattice
- start at $(0,0)$
- for sp 1 coal. move right →
- for sp 2 coal. move up ↑



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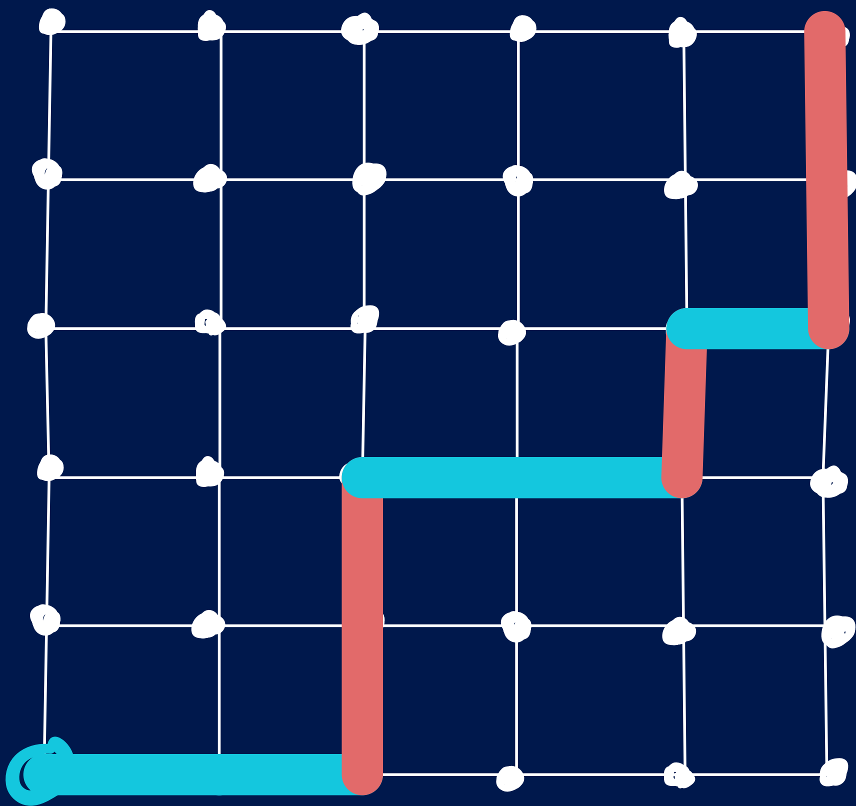

↖ visualize ordering as path

Ordering of Coalescent Events → Percolation on Lattice

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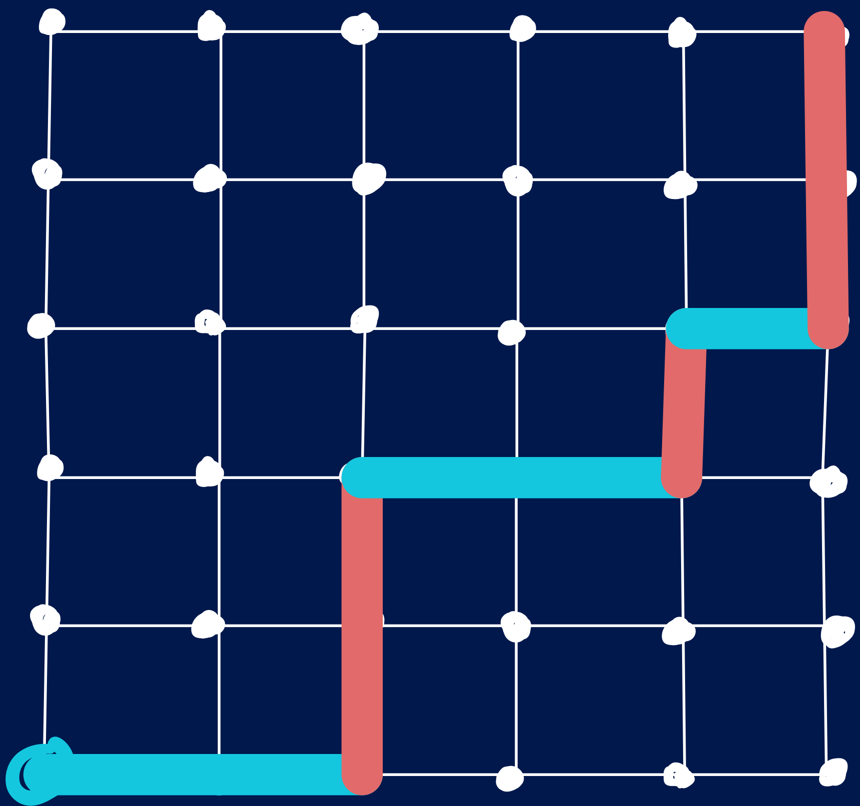


Note: does not explicitly depend on time

Simulations

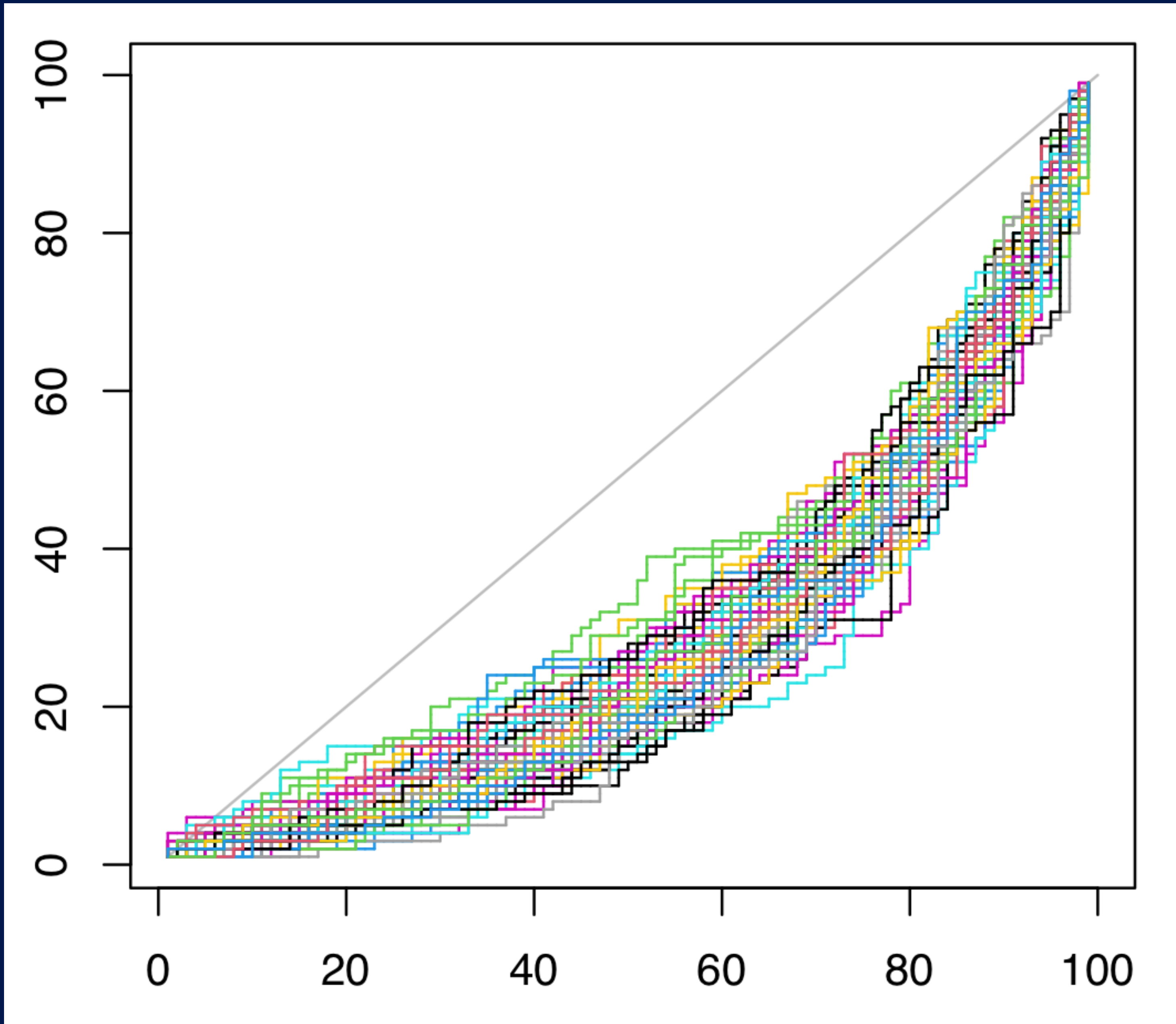
- $K = 100$
- $\frac{1}{\gamma} = 4$

recall:

- $N_2 = \frac{1}{2} N_1$

→ sp 1 coalesces

↑ sp 2 coalesces



representing 100 tree pairs

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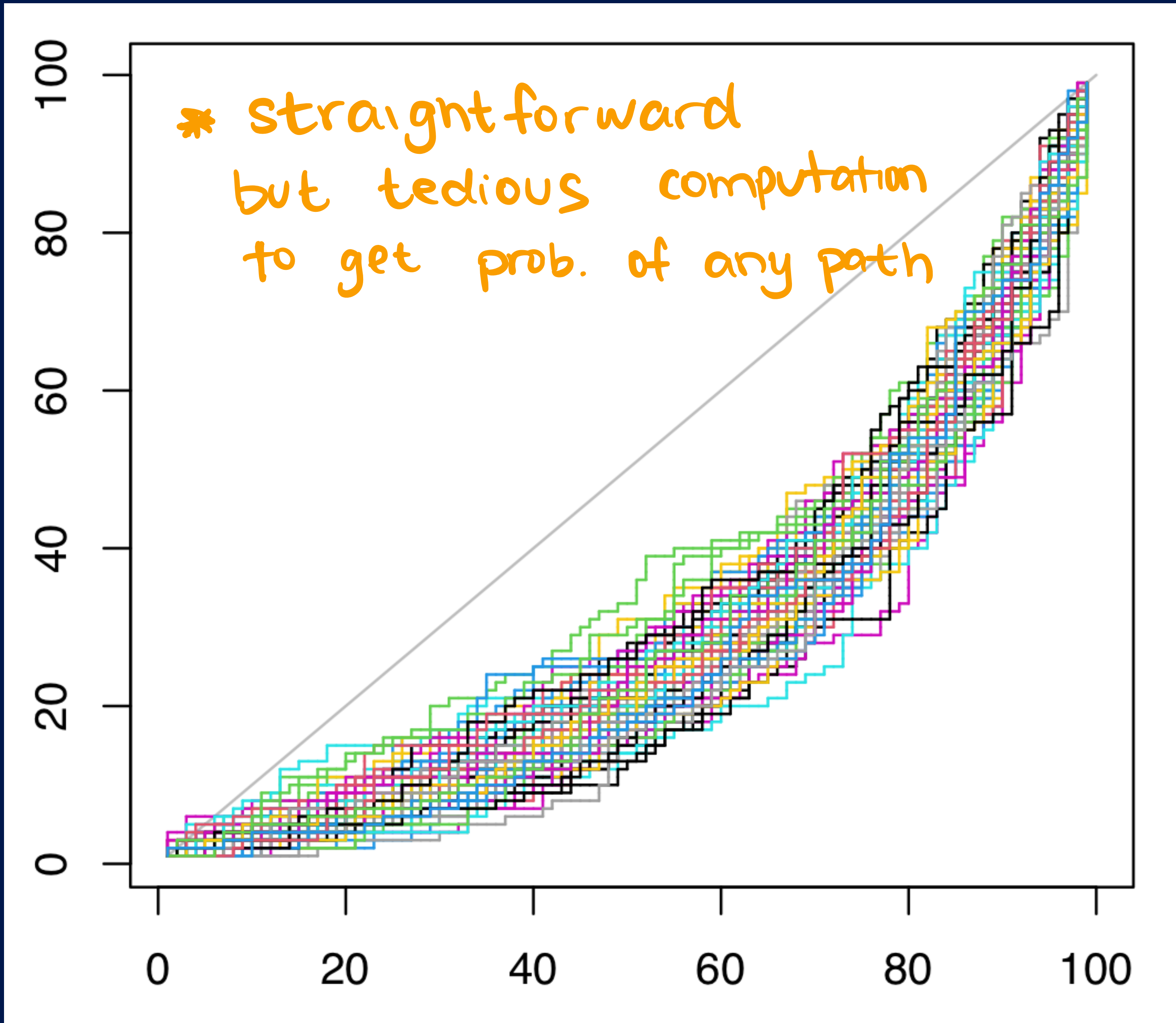

representing 100 tree pairs

Main LLN Result

Rough Statement: (avoiding technicalities)

as $k \to \infty$, (large # of samples)

paths (orderings) converge to solution of ODE

$$\frac{dy}{dx} = \frac{(1-y)^2 \gamma}{(1-x)^2}$$

(when paths rescaled to be in $[0,1]^2$)

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(when paths rescaled to be in $[0,1]^2$)

explicitly solvable: separation of variables

Main LLN Result

Rough Statement: (avoiding technicalities)

as $k \to \infty$, (large # of samples)

paths (orderings) converge to solution of ODE

$$\frac{dy}{dx} = \frac{(1-y)^2 \lambda}{(1-x)^2}$$

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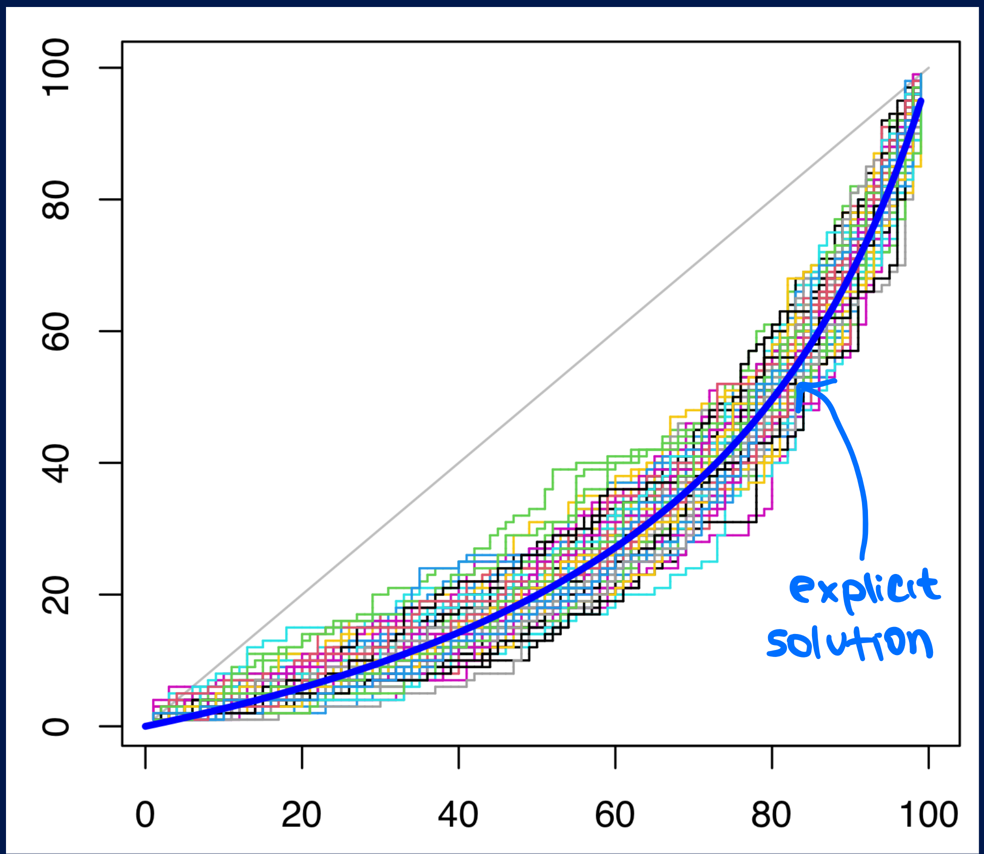
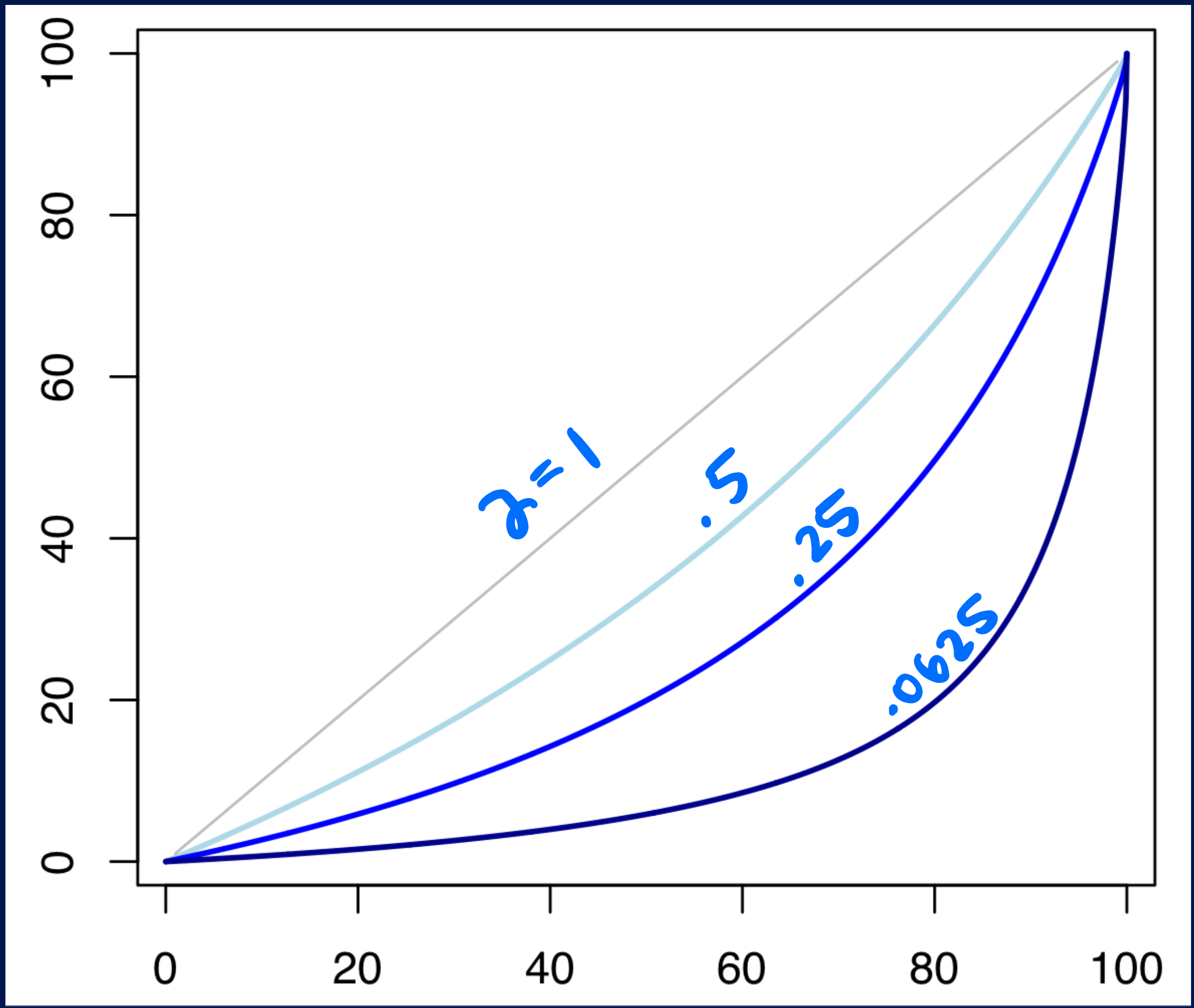
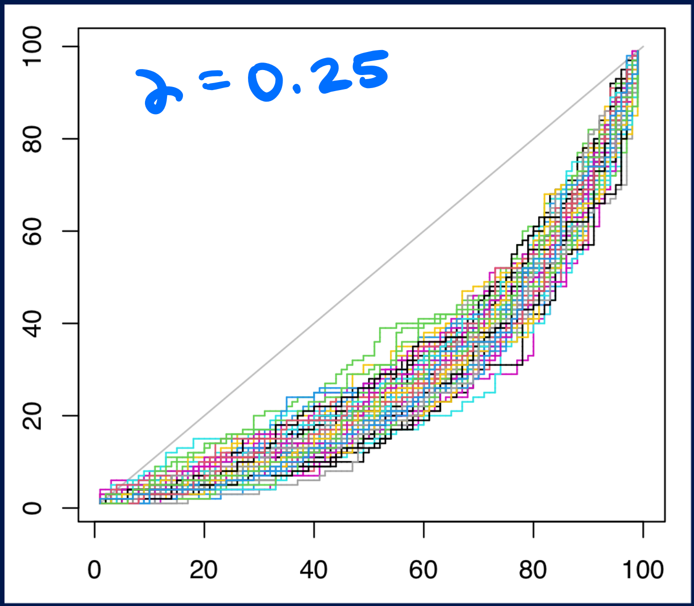
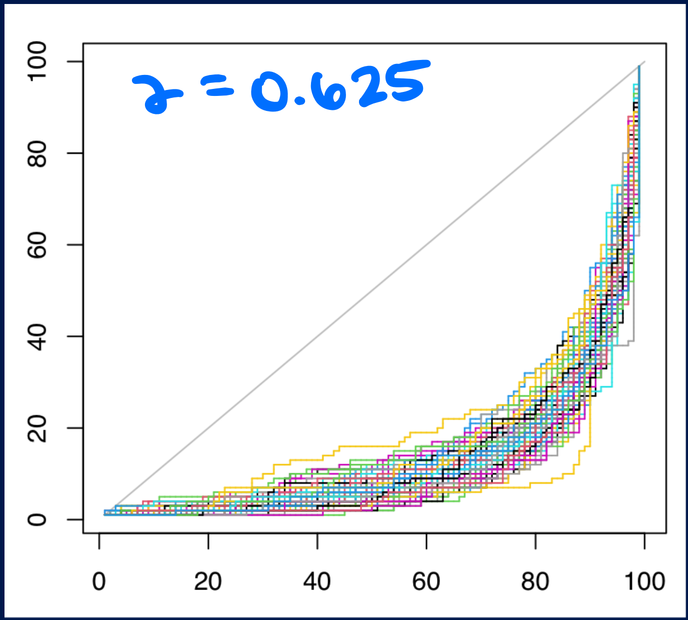


$k = 100$ $\lambda = 0.25$

Implications: Ordering → Ratio of Pop. Sizes



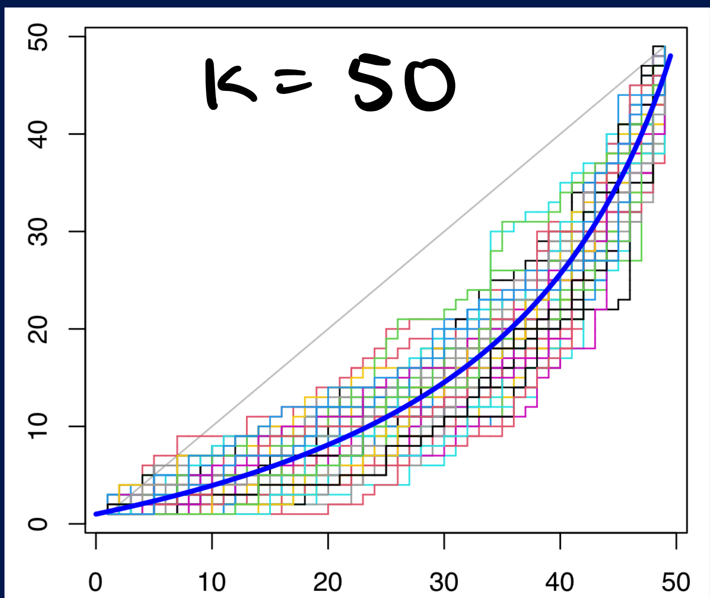


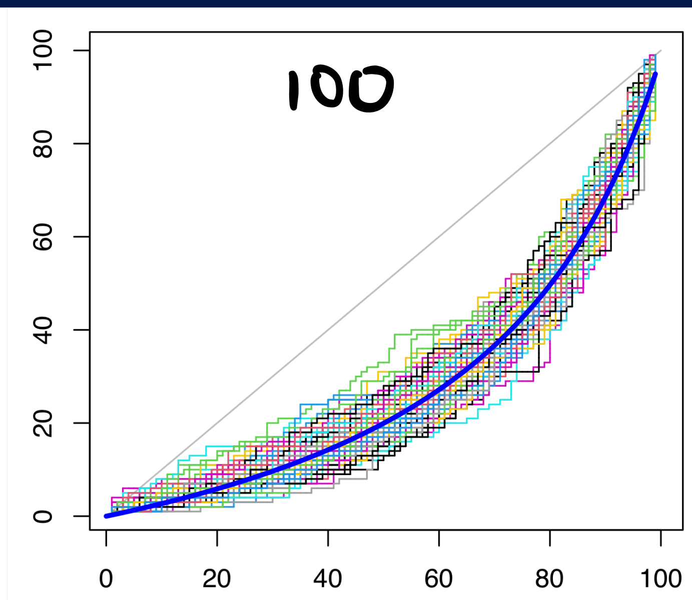


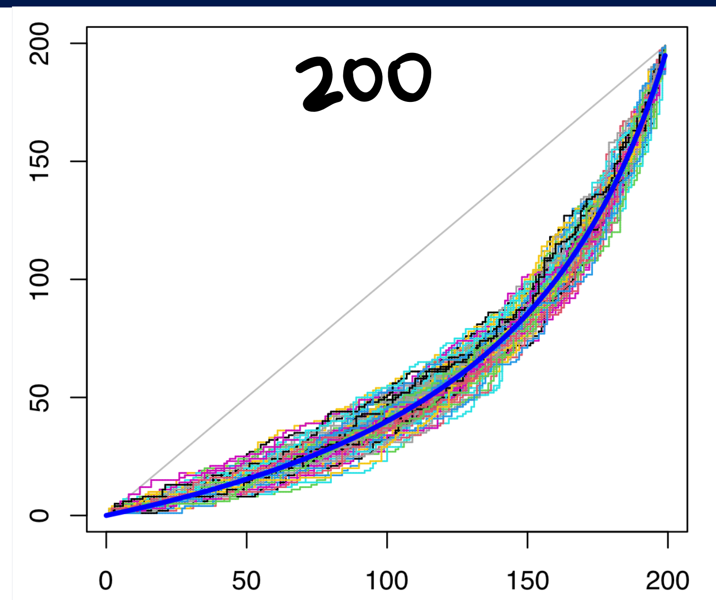
inference via minimizing "error"
(ML)

Ongoing Work

(1) Fluctuations (Central Limit Theorem)







Ongoing Work

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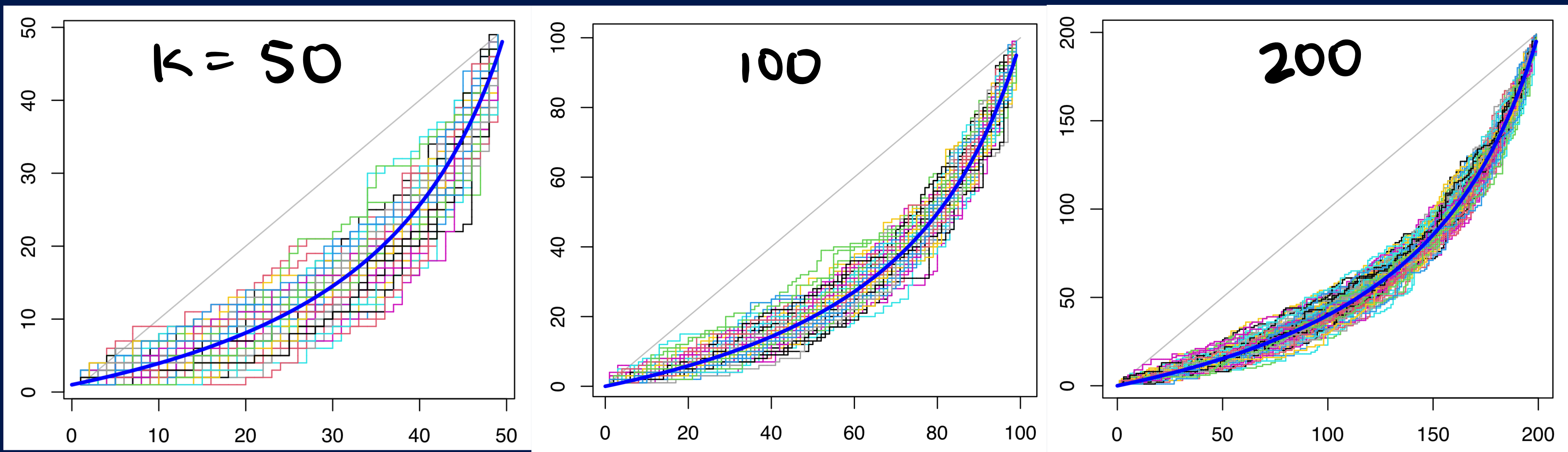


(2) Dynamic 2 (Changing Pop Ratio)

Beyond Constant Population Sizes

(dynamic λ)

$\leftarrow$ ratio of pop. sizes

Same Idea:

as $k \to \infty$,

paths converge to
solution of ODE

$$\frac{dy}{dx} = \frac{(1-y)^2 \lambda(x,y)}{(1-x)^2} \; ?$$

* to be worked on

Beyond Constant Population Sizes

(dynamic λ)

← ratio of pop. sizes

Same Idea:

as $k \to \infty$,

paths converge to
Solution of ODE

$$\frac{dy}{dx} = \frac{(1-y)^2}{(1-x)^2}\lambda(x,y)\ ?$$

* to be worked on

Test:

(a) sudden shift
$\lambda_1 \to \lambda_2$

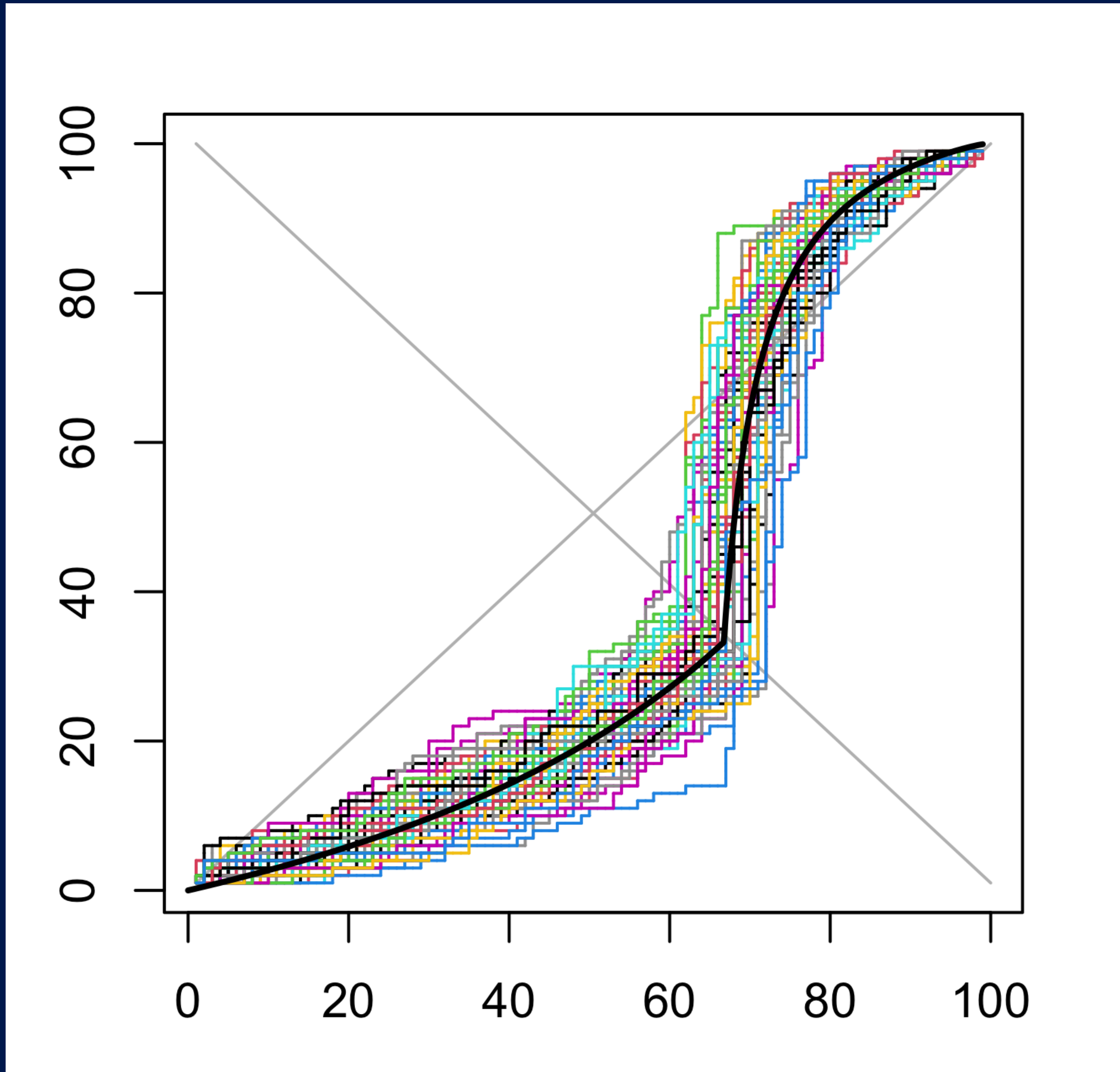
(b) continuously changing λ

Beyond Constant Population Sizes

Test:

(a) sudden shift

$\lambda_1 \to \lambda_2$



$\lambda_1 = 0.25$ $\lambda_2 = 4$

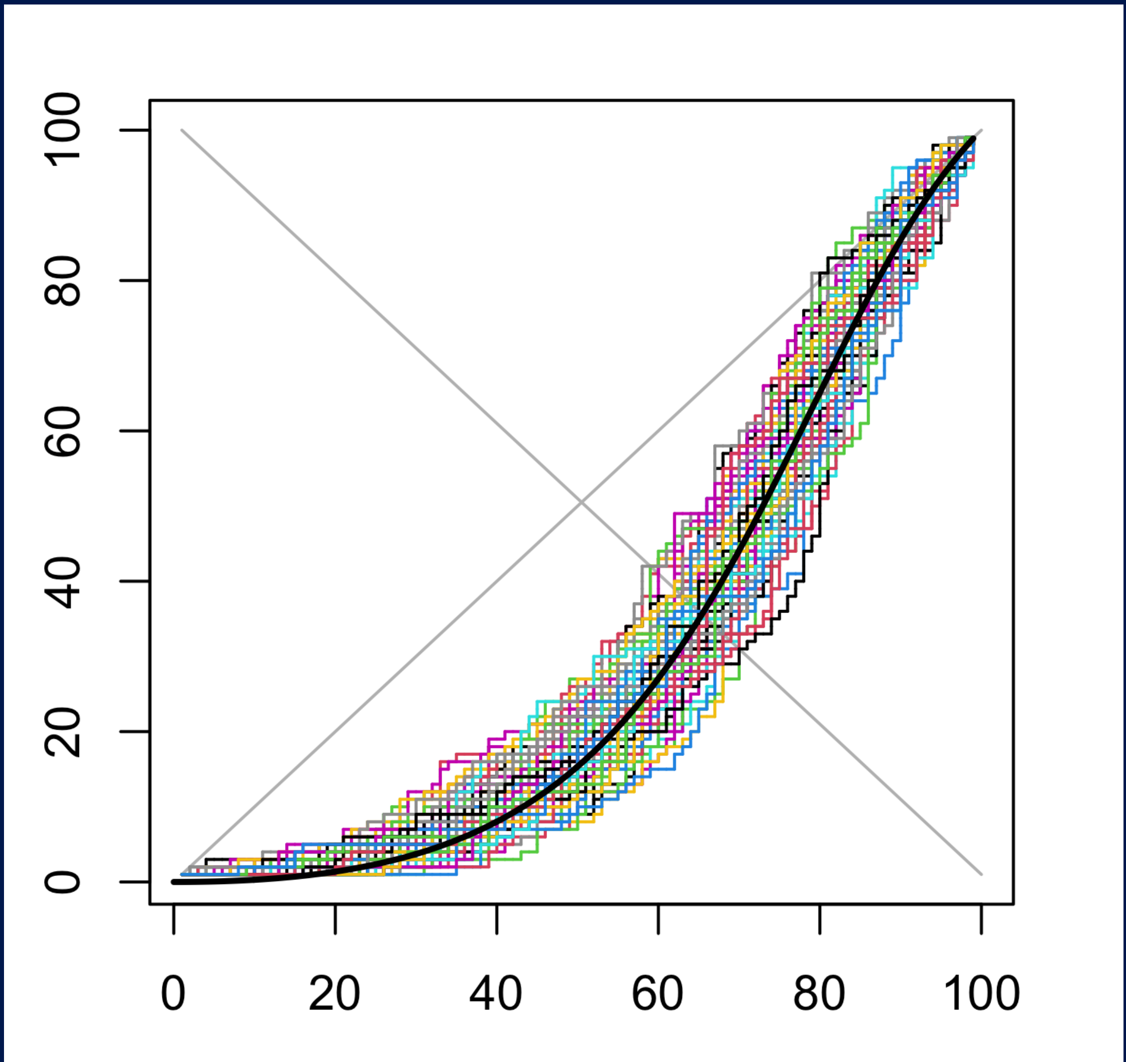
switch at $i+j=K$

Beyond Constant Population Sizes

Test:

(b) continuously changing λ

(linearly from $\frac{1}{k}$ to 1)



* caveat: time scale of λ dynamics different then time scale of generations

Open Area

computational stats

math

joint coalescent

community ecology

Open Area

challenging inference problem

computational stats

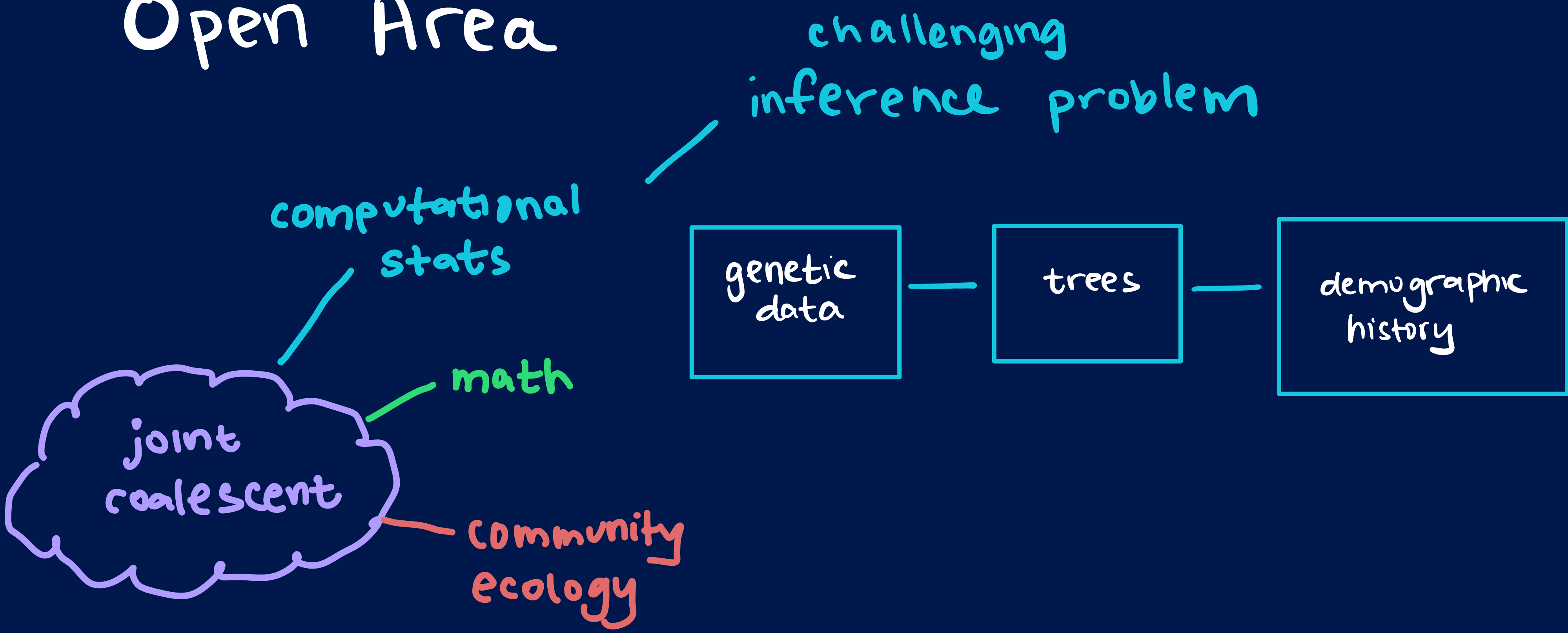
joint coalescent

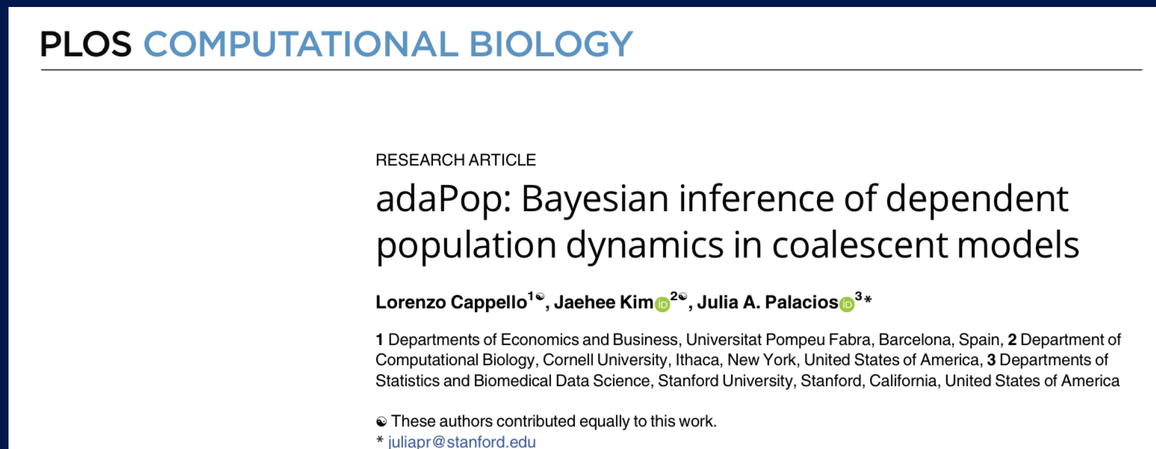
math

community ecology

genetic data — trees — demographic history

Open Area




PLOS COMPUTATIONAL BIOLOGY

RESEARCH ARTICLE

adaPop: Bayesian inference of dependent population dynamics in coalescent models

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Open Area

inference problem

computational stats

joint coalescent

math — finding right models representations, scalings

ecology

coalescent

community ecology

Open Area

joint coalescent

computational stats

inference problem

math

finding right models representations, scalings

community ecology

empirical systems

e.g. paleoecology

OR

fast generation species

Thank You

SMTPB

Society for Modeling & Theory in Population Biology

National Institute for Theory and Mathematics in Biology

Funded by the U.S. National Science Foundation and the Simons Foundation

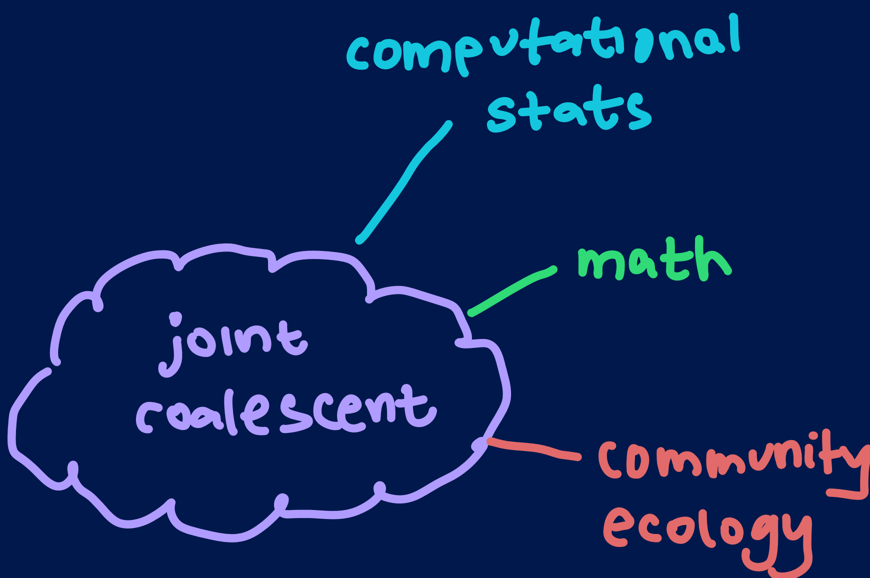
SMTPB Diffusions

ICERM

Thank You

QUESTIONS
OR
THOUGHTS
WELCOME

References



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