

ICERM SageMath trimmed

July 7, 2026

1 SageMath and homotopy theory

- Simplicial complexes, simplicial sets, semisimplicial sets (a.k.a. Δ -complexes), cubical complexes
- The Steenrod algebra
- Chain complexes
- A bit of homological algebra

1.1 Topological spaces

```
[ ]: K = simplicial_complexes.RealProjectiveSpace(6)
K
```

```
[ ]: %time H = K.cohomology(base_ring=GF(2), algorithm='dhs')'
```

```
[ ]: H
```

```
[ ]: K.f_vector()
```

```
[ ]: RP4 = simplicial_complexes.RealProjectiveSpace(4)
```

```
[ ]: HRP4 = RP4.cohomology_ring(GF(2))
```

```
[ ]: HRP4
```

```
[ ]: HRP4.basis(1)
```

```
[ ]: x = HRP4.basis(1)[(1,0)]
```

```
[ ]: x.Sq(2)
```

```
[ ]: HRP4.gens()
```

```
[ ]: simplicial_complexes.
```

```
[ ]:
```

```
[ ]: X = simplicial_sets.RealProjectiveSpace(70)
```

```
[ ]: %time X.cohomology(base_ring=GF(2))
```

```
[ ]: X
```

```
[ ]: simplicial_sets.
```

```
[ ]:
```

1.2 Constructing simplicial sets

(look at the reference manual, https://doc.sagemath.org/html/en/reference/topology/sage/topology/simplicial_sets.html)

```
[ ]: Bsigma3 = simplicial_sets.ClassifyingSpace(groups.permutation.Symmetric(3))
```

```
[ ]: Bsigma3
```

```
[ ]: Bsigma3.n_skeleton(4).f_vector()
```

```
[ ]:
```

```
[ ]:
```

1.3 The Steenrod algebra

```
[ ]: A = SteenrodAlgebra()
```

```
[ ]: A
```

```
[ ]: a = Sq(10)
a.coproduct()
```

```
[ ]: pretty_print(a.coproduct())
```

```
[ ]: latex(a.coproduct())
```

```
[ ]: A = SteenrodAlgebra(basis='serre-cartan')
```

```
[ ]: A
```

```
[ ]: A.Sq(0,2)
```

```
[ ]: A.Sq(4) * A.Sq(3)
```

```
[ ]: A1 = SteenrodAlgebra(p=2, profile=(2,1,0))
```

```
[ ]: A1
```

```
[ ]: A1.top_class()
```

```
[ ]: A1.dimension()
```

```
[ ]: [(n, A1.basis(n)) for n in range(0, 7)]
```

```
[ ]:
```

```
[ ]: from sage.modules.fp_graded.steenrod.module import SteenrodFPModule  
k = SteenrodFPModule(A1, [0], [[Sq(1)], [Sq(2)]])
```

```
[ ]: k
```

```
[ ]: k.generator_degrees()
```

```
[ ]: k.resolution(4)
```

```
[ ]: A1.rename("A(1)")
```

```
[ ]: k.resolution(4)
```