**Partial Differential Equations for Many Particles Systems**
**Poster Session Abstracts**
August 4, 2026
2:30-4:00pm

**Scattering map for the Vlasov-Poisson system with a repulsive harmonic potential**
Hyunwoo(Will) Kwon, Brown University

We consider the Vlasov--Poisson system with a repulsive harmonic potential and prove the (modified) scattering of solutions, as well as the existence of wave operators, in any spatial dimension $d\geq 2$. The main novelty of this work is the construction of the wave operators and the introduction of the lens transform for the Vlasov--Poisson system. Joint work with Wenrui Huang (Brown University)

**A Blob Method for Kalman-Wasserstein Gradient Flows**
Ritvik Teegavarapu, Brown University

Sampling from a target distribution is a fundamental problem in applied mathematics, arising naturally in Bayesian inference and uncertainty quantification. Classical methods such as Markov Chain Monte Carlo can converge prohibitively slowly for complex, high-dimensional targets, and typically require access to gradient information that may be unavailable when the forward model is a black-box numerical solver. The Ensemble Kalman Sampler (EKS) addresses both difficulties, as it is a gradient-free interacting particle algorithm in which particles evolve under an empirical covariance-preconditioned force, making the dynamics affine invariant and well-adapted to the geometry of the target. As the number of particles tends to infinity, the empirical distribution of the EKS converges formally to a nonlinear PDE (the mean-field limit) which can be interpreted as a gradient flow in the Kalman-Wasserstein metric. However, rigorous justification of this long-time limit has so far been confined to Gaussian or near-Gaussian targets, a restriction that precludes the majority of practical applications. We develop a regularized variant of the EKS mean-field PDE, obtained by replacing the potential and entropy terms with mollified counterparts parameterized by a smoothing scale. This covariance-modulated blob flow inherits the gradient flow structure of EKS while admitting a rigorous analysis for general targets. We prove the existence of weak solutions via a covariance-modulated JKO variational scheme, characterize the steady states and their bias, and prove the convergence of the mollified solutions to the EKS mean-field PDE as the smoothing vanishes. Numerical experiments on benchmark targets confirm that the resulting particle algorithm is competitive with EKS in practice.

**Fractal Opinions Among Interacting Agents**
Fei Cao, Amherst College

We investigate an opinion model consisting of a large group of interacting agents, whose opinions are represented as numbers in [-1,1]. At each update time, two random agents are selected, and the opinion of the first agent (the "listener") is updated based on the opinion of the second (the “persuader”). We derive the mean-field kinetic equation describing the large population limit of the model, and we provide several quantitative results establishing convergence to the unique equilibrium distribution. Surprisingly, in some range of the model parameters, the support of the equilibrium distribution exhibits a fractal structure. This provides a new mathematical description for the so-called opinion fragmentation phenomenon. This is a joint work with Roberto Cortez.

**Random Blob Methods for Diffusion**

Claire Murphy, UC Santa Barbara

Linear and nonlinear diffusion equations describe a range of phenomena of mathematical interest, including slow and fast diffusion, sandpile dynamics, height-constrained transport, and the two-dimensional Navier-Stokes equation. These equations are also of interest in sampling probability measures. These dynamics possess a 2-Wasserstein gradient flow structure, and applying blob methods to this structure yields a particle method for simulating solutions. To address the O(N^2) computational bottleneck of existing particle methods, we consider stochastic alternatives. We compare the random batch method with a new approach, which we call the random multirate method. Both of these random methods build on classical stochastic methods in the optimization literature, with many similarities to stochastic gradient descent and random coordinate descent. We find that in the tradeoff between computational complexity and accuracy, the random multirate method has the best performance. We prove that the random multirate method converges to the underlying ODE system at a rate of O(kΔt), matching forward Euler, and show by example that this rate is theoretically sharp. We observe even better rates of convergence for the random multirate method when applied to ODEs arising from blob methods.

**Asymptotic Convergence of Nonconvex Wasserstein Gradient Flows**

Seunghoon Jeong, POSTECH

Over the past two decades, Wasserstein gradient flows in the optimal transport framework have received significant attention due to their wide-ranging applications. A central example is the McKean--Vlasov equation (also known as a granular media equation) which describes the macroscopic evolution of a large system of interacting particles under diffusion, confinement, and nonlocal interaction.

However, a general theory for non-convex energy functionals remains largely open. In my poster, I will discuss the theory of asymptotic convergence of the McKean--Vlasov equation
[
\partial_t \mu_t
= \operatorname{div}\left(
\nabla \mu_t

* \mu_t \nabla \left[ V + W \ast \mu_t \right]
\right)
]
for non-convex functionals associated with (V) and (W) on (\mathcal{P}_2(\mathbb{T}^n)), by applying Łojasiewicz--Simon theory in its tangent space. As a fundamental application, I will present the asymptotic behaviour of the parabolic--elliptic Keller--Segel system
[
\begin{cases}
\partial_t \rho
= \nabla!\cdot!\big(\nabla \rho - \chi, \rho, \nabla c\big)
* \nabla!\cdot!\big(\rho,\nabla V\big),[0.2em]
(-\Delta + \alpha)c = \rho - \overline{\rho}, \qquad \alpha\ge 0.
\end{cases}
]

**Relaxation Dynamics of the Continuum Kuramoto Model with Non-Integrable Kernels**
Valeriia Zhidkova, University of Mannheim

We study the asymptotic behavior of the continuum Kuramoto model with a fractional Laplacian-type kernel. For this, we construct global weak solutions via a two-parameter regularization procedure using a kernel truncation with fractional dissipation. Using a priori uniform estimates derived in fractional Sobolev spaces, we employ compactness arguments to construct global weak solutions to the singular continuum Kuramoto model. Furthermore, we also establish an exponential relaxation toward the initial phase average under suitable assumptions on initial data and system parameters. These findings provide a rigorous characterization of the existence of solutions and the emergent dynamics of Kuramoto ensembles under physically important strongly singular interactions, including power-law singular kernels and Coulomb-type kernels.

**Inference of interacting kernel in the mean-field regime**
Peiyi Chen, UW-Madison

We study the problem of reconstructing interaction kernels in systems of interacting agents from macroscopic measurements when posed as an optimization problem. The reconstruction procedure depends on the formulation of the forward model, which may be given either by a finite-dimensional coupled ODE system tracking individual agent trajectories or by a mean-field PDE describing the evolution of the agent density. We investigate the similarities and differences between these two formulations in the mean-field regime. While the first variation derived from the particle system does not provide an unbiased estimator of the first variation associated with the limiting PDE, we prove that, under mild assumptions, the two are close in a weak sense with a convergence rate O(N^-1/2). This rate is further confirmed by numerical evidences.

**Uniform-in-Time Propagation of Chaos for Consensus-Based Optimization**
Dohyeon Kim, California Institute of Technology

Consensus-Based Optimization (CBO) is a class of derivative-free global optimization algorithms governed by interacting stochastic particle systems. These systems are characterized by a non-local structure where particles are attracted toward a weighted consensus point via a Laplace-principle-based approximation of the global minimizer. This mechanism enables global optimization without gradient information and poses mathematical challenges for rigorous mean-field analysis, necessitating new approaches. In this poster, we establish uniform-in-time propagation of chaos for CBO of both first-order CBO and its kinetic (second-order) extension. By leveraging a novel stability estimate for the weighted mean and quantitative concentration inequalities, we prove that the microscopic particle system tracks the mean-field limit with the classical Monte Carlo rate over infinite time horizons. Notably, for CBO with anisotropic noise, the error prefactor is independent of the problem dimension. We also present results on the almost sure exponential convergence of the microscopic system. Then we conclude by discussing the extension to the kinetic regime, focusing on the analytical challenges of establishing long-time stability when velocity dynamics are introduced. These are joint works with [1] Nicolai Gerber, Franca Hoffmann, and Urbain Vaes, and [2] Seung-Yeal Ha and Franca Hoffmann.

**A variational approach to an anisotropic nonlocal porous medium equation with drift**
Ricardo Guimaraes, University of Oxford

We analyze nonlocal porous-medium type equations driven by anisotropic Riesz-type repulsive kernels and an external confining potential. The pressure is related to the density by convolution with a kernel of the form

$$
W(x)=|x|^{-s}\Omega\left(\frac{x}{|x|}\right),\qquad d-2<s<d,
$$
for an angular profile $\Omega$. We prove existence of solutions and a gradient flow structure for a suitable interaction energy in the Wasserstein metric. We also prove a Talagrand-type transport inequality, characterize bounded continuous steady states, establish $L^p$, $L^\infty$, and H\"older estimates for solutions, and show asymptotic convergence to equilibrium in H\"older norms.

### Swarm-Based Inertial Methods (SBIMs) for Optimization

Qiyu Wu, University of South Carolina

We introduce a new class of swarm-based inertial methods (SBIMs) for global minimization, formulated as coupled dissipative inertial dynamical systems derived from the generalized Onsager principle. The framework identifies the friction operator and the potential energy scaling as the key mechanisms controlling relaxation over the objective landscape. Within this framework, we propose an underdamped inertial dynamics whose damping incorporates both gradient and Hessian information, allowing the agents to adapt their acceleration and dissipation to the local trajectory and curvature. Under suitable conditions, we prove energy dissipation for the continuous system and obtain the objective gap estimate $O(1/\delta(t))$, which becomes $O(1/t^2)$ under an additional lower-bound condition. We also construct structure-preserving discretizations that preserve discrete energy dissipation and yield the corresponding estimate $O(1/\delta_k)$, reducing to $O(1/(k(k+1)))$ under the discrete analogue of this condition. Numerical experiments support the theory on convex problems and show faster objective gap decay than the worst-case bound. On nonconvex benchmarks, the proposed methods achieve high success rates and more stable energy decay than swarm-based gradient descent and Nesterov methods.

### Structure Preserving Detailed-Balance Master-Equation Discretizations for Fokker Planck Equations

Satish Chandran, University of California Riverside

We propose a variational–Markov framework that builds detailed-balance master-equation discretizations of Fokker–Planck equations directly from the energy–dissipation law, rather than by discretizing the drift–diffusion flux as in standard PDE-based schemes. The discrete energy–dissipation law fixes the net interface flux edge by edge, while detailed balance fixes the ratio of forward and backward jump rates. Once an interface reconstruction is chosen, both transition rates are uniquely determined, giving a unified design principle for structure-preserving discretizations. A logarithmic-mean reconstruction naturally yields the classical Scharfetter–Gummel/Wang–Peskin–Elston rates, which standard PDE-based discretizations can only reach through special exponential-fitting constructions. The resulting semi-discrete master equations conserve mass, preserve positivity, satisfy detailed balance, and dissipate a discrete free energy. For the linear fixed-potential case we further establish unconditional positivity and entropy decay at the fully discrete level; for nonlinear mobilities and nonlocal free energies we give structure-preserving IMEX schemes and identify conditions under which discrete energy decay is retained. Numerical experiments confirm convergence order, long-time equilibrium recovery, free-energy decay, robustness for discontinuous potentials, and nonlocal extensions.

### Uniform-in-diffusivity mixing by shear flows: stochastic and dynamical perspectives.

Kunhui Luan, University of South Carolina

We study the mixing of passive scalars by parallel shear flows in the presence of weak molecular diffusion. In particular, we recover the sharp uniform-in-diffusivity mixing rates for shear flows with finitely many critical points, a result recently established by D. Albritton and R. Beekie. Our approach is based on the stochastic representation formula of the associated advection-diffusion equation. This probabilistic framework yields two short proofs. The first generalizes the estimate of oscillatory integrals from the zero-diffusivity case and establishes the optimal mixing rate under the weakest possible regularity assumption, answering the question regarding the sharp regularity posed in Section 4 of Albritton & Beekie. The second adopts a dynamical systems perspective and provides a proof of shear flow mixing in a different topology, which, to our knowledge, is new even in the zero-diffusivity setting. This is joint work with Kyle L. Liss.

### Existence and Long Time Behavior of Martingale $L^\infty$ Solution to the Stochastic Isentropic Euler Equation with Linear Damping in 1D

Rongyi (Amy) Dai, UC Berkeley

We study the one-dimensional isentropic compressible Euler equations with linear (frictional) damping, subject to multiplicative, white-in-time stochastic forcing. The system is posed on a bounded interval with $L\infty$ initial data and Dirichlet boundary conditions imposed on the momentum. We establish the global-in-time existence of $L\infty$ martingale solutions that satisfy an appropriate entropy inequality. Then, we analyze the long-time behavior of these solutions and show that, under suitable assumptions on the noise, they converge almost surely and exponentially fast to a constant steady state of the system. The limiting density is well-approximated by the asymptotic solution of the deterministic porous medium equation, while the momentum exhibits the asymptotic behavior predicted by Darcy's law.

### Homoenergetic Landau equation: Well-posedness vs blow-up

Francisco Unai Caja Lopez, University of Texas at Austin

We study homoenergetic solutions of the spatially inhomogeneous Landau equation, whose main feature is that their density and temperature are spatially homogeneous, although they may evolve in time. We establish two main results. First, homoenergetic flows remain smooth for as long as their deformation matrix remains regular. Second, for a particular initial deformation matrix, we observe that solutions have hydrodynamic quantities which blow up in finite time even though the function itself remains bounded and smooth. To our knowledge, in the very soft potential regime, these results provide the first examples of both global-in-time, spatially inhomogeneous classical solutions away from equilibrium, and finite-time hydrodynamic singularity formation for the Landau equation.

### Convergence rate for Fluctuations of mean field interacting diffusion and application to 2D viscous Vortex model

Paul Nikolaev, TU Berlin

We prove a sharp quantitative version of this central limit theorem for mean field limit systems: for smooth observables of the fluctuation field, the finite-system average differs from its Gaussian limit at the optimal rate of order $1/N^{1/2}$ in the number of particles.The approach compares the two dynamics through their generators on a suitable infinite dimensional Hilbert space, which keeps it flexible. In particular, the same rate holds well beyond smooth interactions. It covers genuinely singular cases of physical interest such as the 2D viscous vortex model (Biot–Savart) or the Coulomb model.