Mathematical AI for Bioscience

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Fostering Cross-Disciplinary Collaboration in Biology, Medicine, and **Computational Science** ICERM, Jul 28 - Aug 1, 2025

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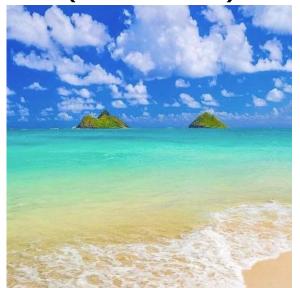
Mathematical Foundations of AI and Data Topological Aspects

Algebraic topology for point cloud data (discrete)



Forbes

Differential topology for data on manifolds (continuum)



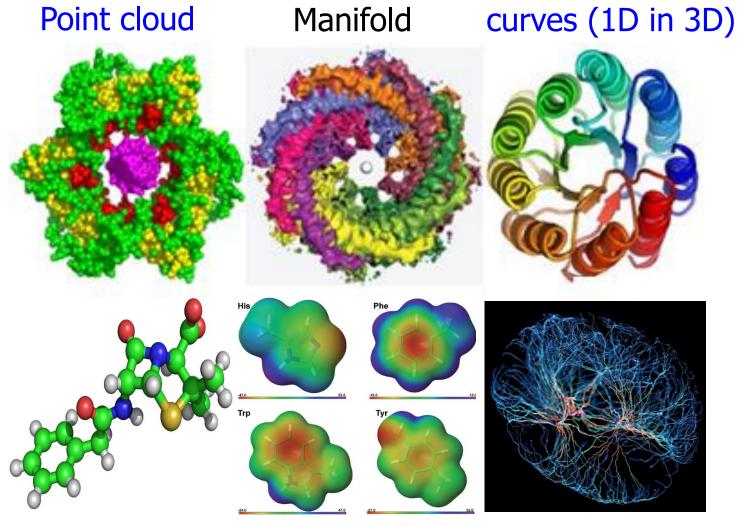
2024 Microsoft

Geometric topology for 1D curves (1D in 3D)



Lookphotos

Mathematical foundations of AI and bioscience



Algebraic topology/ Commutative algebra

Differential geometry Geometric topology

Topology



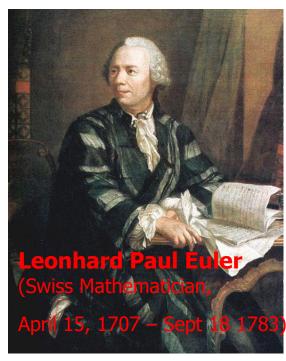




Double Torus



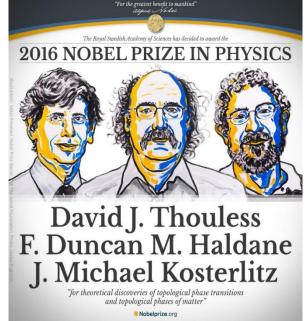
Augustin-Louis Cauchy, Ludwig Schläfli, Johann Benedict Listing, Bernhard Riemann, and Enrico Betti







Möbius Strips (1858)



Mathematics of Topological Data Analysis (TDA)

Foundations

- Homology (most popular)
- Homotopy (hardly applied)
- Laplacian (Wang et al.)
- Dirac (Maroulas and others)
- Sheaf theory (Yegnesh)
- Mayer topology (Shen et al)
- Interaction (Liu et al)



Algebraic Topology

Simplicial complexes (most popular) Cellular complex

Path complexes (directed, GLMY)

Directed flag complexes

Hypergraphs

Hyperdigraphs,

Superhypergraph,

. . .

Geometric Topology

Knots (closed or open)
Links (closed or open)
Tangles (closed or open)
Braids

• • •

Differential Topology

Differential forms on manifolds without boundary

Differential forms on manifolds with boundary

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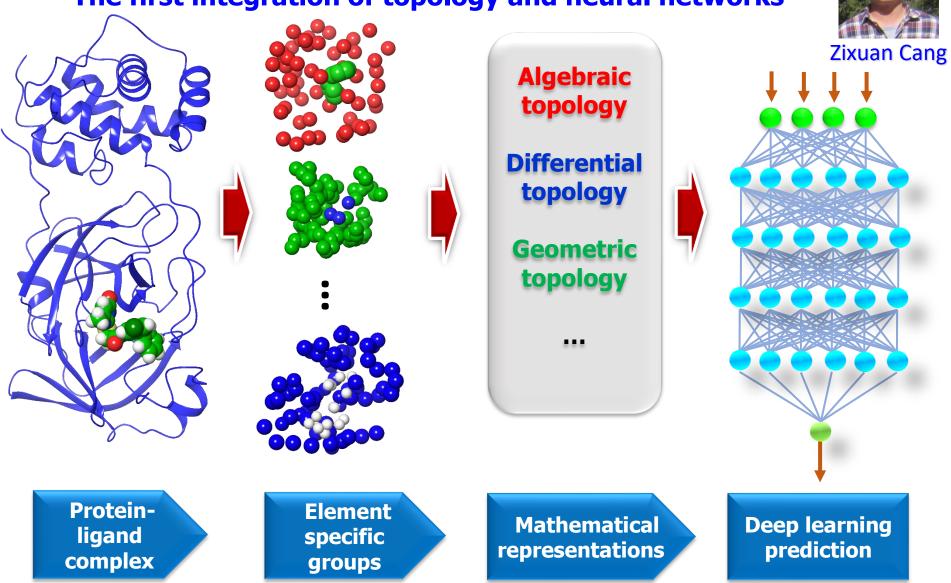
Machine Learning Aspects of TDA

Logistic regression Random forest Support vector machine **Ensemble methods** Transfer learning **Active learning** Convolutional neural network Deep neural network Nature language processing Recurrent neural network Graph neural network Long-short term memory Generative Al **ChatGPT Transformer** Autoencoder Manifold learning **Graph learning** Geometric learning Correlated clustering and projection **PCA UMAP** t-SNE Topological deep learning Multiscale Laplacian learning

TopologyNet: Cang & Wei, PLOS Comput. Biol. 2017.

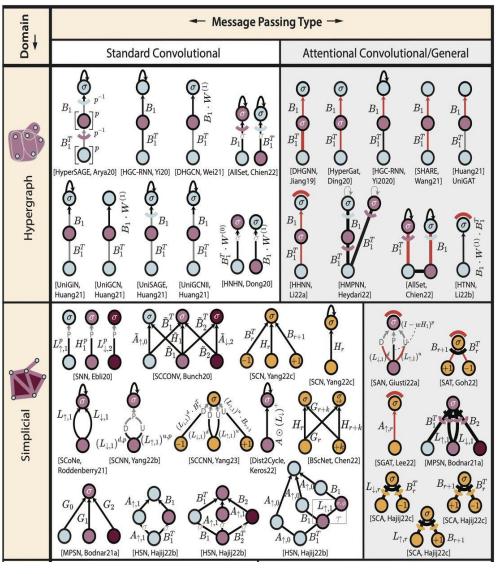
Topological Deep Learning (TDL)

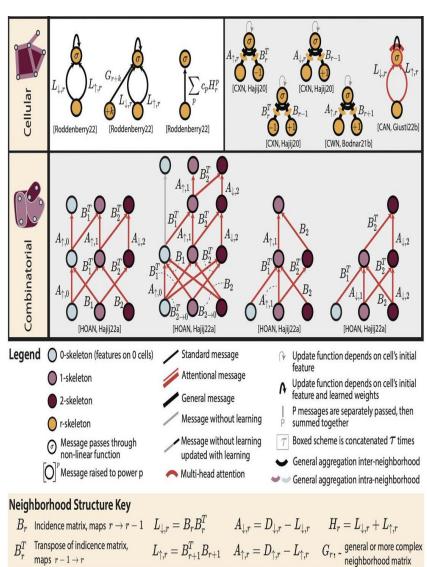
The first integration of topology and neural networks



Cang & Wei, PLOS Comput. Biol. 2017

Topological Neural Networks (Computer Science Aspects of TDL)





Topological invariants: Betti numbers

 β_0 is the number of connected components.

 β_1 is the number of tunnels or circles.

 β_2 is the number of cavities or voids.

 $\beta_2 = 1$

 $\beta_{\gamma} = 0$ $\beta_{\gamma} = 0$

Point Sphere Circle Limitation **Torus** L. Vieira $\beta_0 = 1$ $\beta_0 = 1$ $\beta_0 = 1$ $\beta_0 = 1$ $\beta_1 = 0$ $\beta_1 = 1$ $\beta_1 = 0$

 $\beta_1 = 2$

 $\beta_2 = 1$



Crane and Segerman

Algebraic Topology (Persistent homology)

Simplexes:

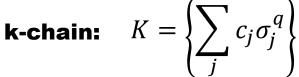




2-simplex *0*-simplex 1-simplex

3-simplex

Frosini and Nandi (1999), Robins (1999), Edelsbrunner, Letscher and Zomorodian (2002), Zomorodian and Carlsson (2005), Edelsbrunner and Harer, (2007) Kaczynski, Mischaikow and Mrozek (2004), Ghrist (2008), ...





Boundary operator:

$$\partial_q \sigma^q = \sum_{i=0}^{\infty} (-1)^j \left\{ v_0, v_1, \dots, \widehat{v_j}, \dots, v_k \right\}$$

$$j=0$$
Cycle group: $7 =$

group: $Z_a = \operatorname{Ker} \partial_a$

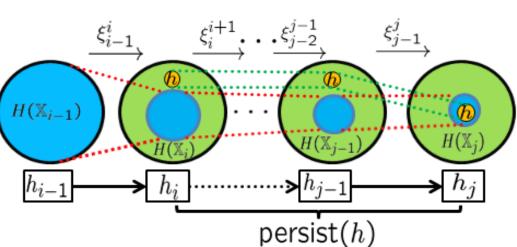
Boundary group: $B_q = \operatorname{Im} \partial_{q+1}$

Homology group: $H_q = Z_q/B_q$

Betti number: $\beta_a = \operatorname{Rank}(H_a)$

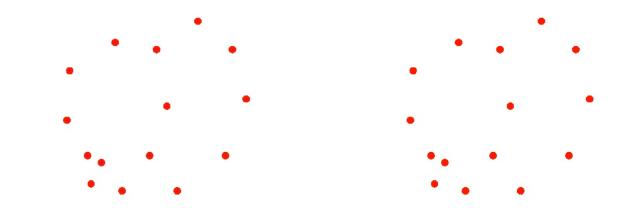
Xia, Wei, IJNMBE, 2014; Xia, Feng, Tong, Wei, JCC, 2015

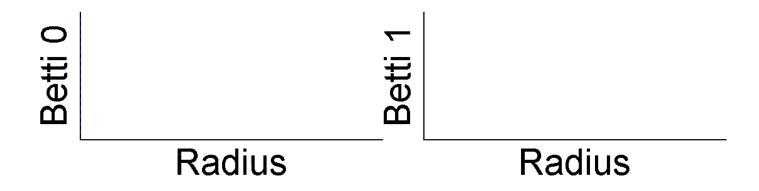
Filtration:



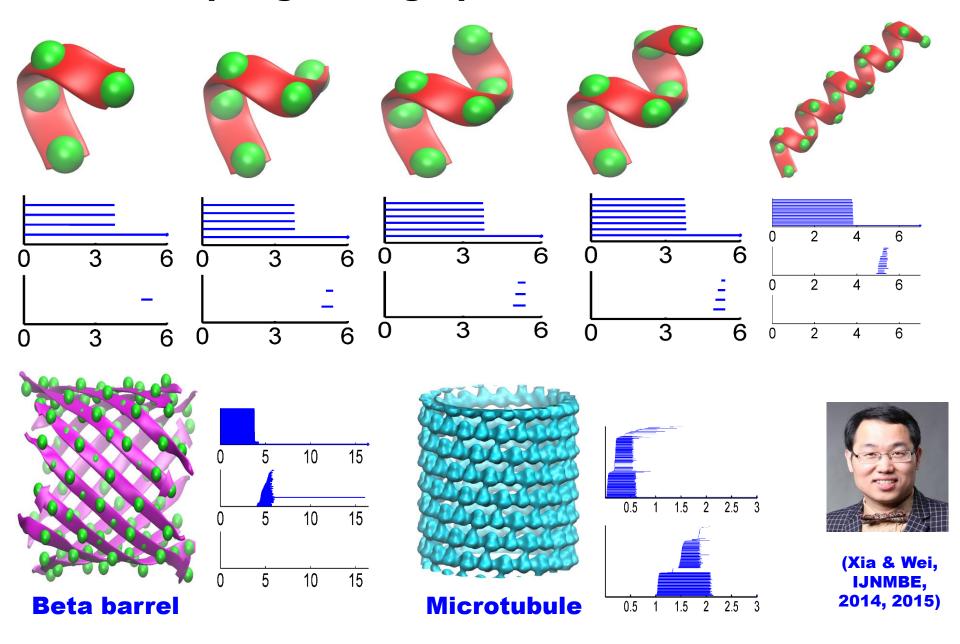
Topological data analysis

Vietoris-Rips complexes, persistent homology and topological fingerprint (Xia, Wei, 2014)





Topological fingerprints of biomolecules



Limitations of Persistent Homology associated with simplicial complex

- It cannot handle heterogeneous information (i.e., different type of objects in the data).
- It is qualitative rather than quantitative (e.g., a 5-member ring is counted the same as a 6-member ring).
- It cannot describe non-topological changes (i.e., homotopic shape evolution over filtration).
- It is incapable of dealing with directed networks and digraphs (polarization, regulation, & control issues).
- It is unable to characterize structured data (e.g., hypergraphs, directed networks).

We address these limitations with new topological methods

Combinatorial Laplacian (topological Laplacian)









0-simplex 1-simplex

2-simplex 3-simplex

•
$$K$$
-chain: $K = \left\{ \sum_{j} w_{j} \sigma_{j}^{q} \right\}$

(Eckmann 1944; Goldberg 2002; Horak, Jost, 2013; Serrano, Gomze, 2019,...)

- Chain group: $C_q(K, \mathbb{Z}_2)$
- Boundary operator: $\partial_q : C_q(K) \to C_{q-1}(K)$

$$\partial_q \sigma^q = \sum_{i=0}^q (-1)^j \left\{ v_0, v_1, \dots, \widehat{v_j}, \dots, v_q \right\}$$

- Adjoint boundary operator: $\partial_q^*: C_{q-1}(K) \to C_q(K)$
- q-combinatorial Laplacian operator: $\Delta_q = \partial_{q+1} \partial_{q+1}^* + \partial_q^* \partial_q$
- q-combinatorial Laplacian matrix: $\mathcal{L}_q = \mathcal{B}_{q+1}\mathcal{B}_{q+1}^T + \mathcal{B}_q^T\mathcal{B}_q$
- Betti numbers:

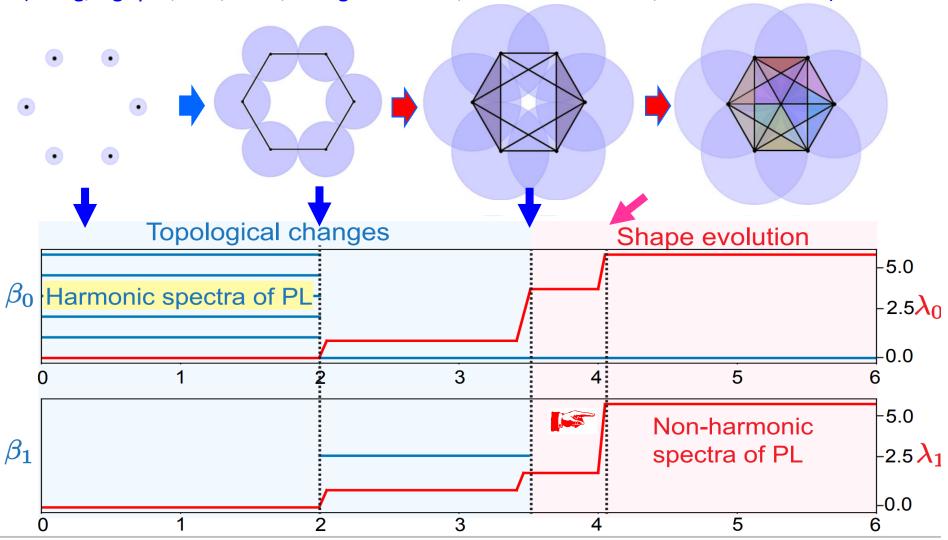
$$\beta_q = \dim(\mathcal{L}_q(K)) - \operatorname{rank}(\mathcal{L}_q(K)) = \# \text{ of zero eigenvalues of } \mathcal{L}_q(K)$$

Persistent (Combinatorial) Laplacians

$$\mathcal{L}_{q}^{t+p} = \mathcal{B}_{q+1}^{t+p} \left(\mathcal{B}_{q+1}^{t+p} \right)^{T} + \left(\mathcal{B}_{q}^{t+p} \right)^{T} \mathcal{B}_{q}^{t+p}$$



(Wang, Nguyen, Wei, 2019; Meng et al. 2021; Memoli et al. 2022; Liu and Wu 2023)



Alternative: Persistent Dirac by Maroulas and coworkers, Xia and coworkers

More in our toolbox for TDA



Evolutionary homology

Zixuan Cang, Munch, & Wei, J. Appl. Comput. Topology, 2020



Persistent sheaf Laplacians

Xiaoqi Wei, & Wei, Foundation of Data Science, 2025



Persistent path Laplacians

Rui Wang, & Wei, Foundation of Data Science, 2023



Persistent hyperdigraph Laplacians

Dong Chen, Liu, Wu, & Wei, Foundation of Data Science, 2024



Persistent Mayer Dirac

Suwayyid &Wei, J. Physics: Complexity, 2025



Persistent interaction topology

Jian Liu, Chen, & Wei, FoDS 2025



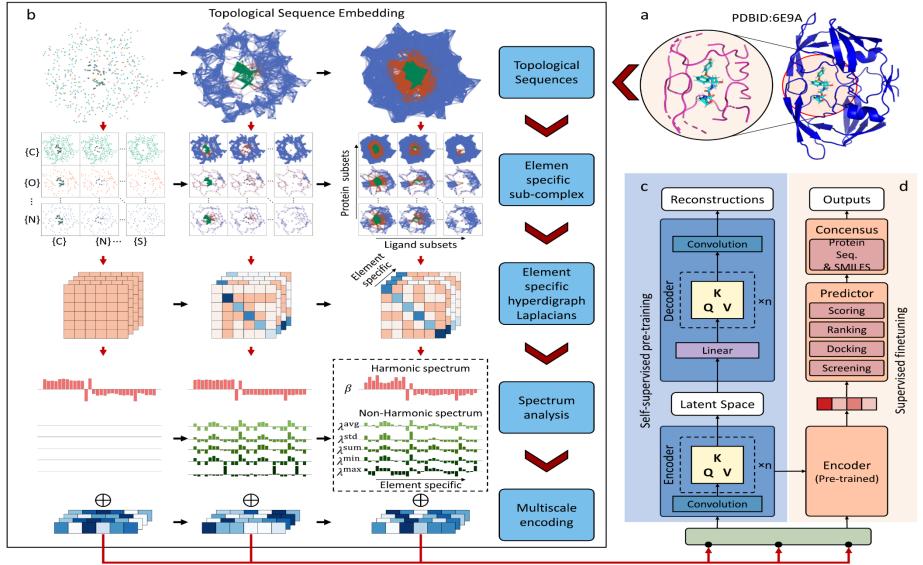
Persistent Mayer topology

Li Shen, Jian Liu, & Wei, Foundation of Data Science, 2024

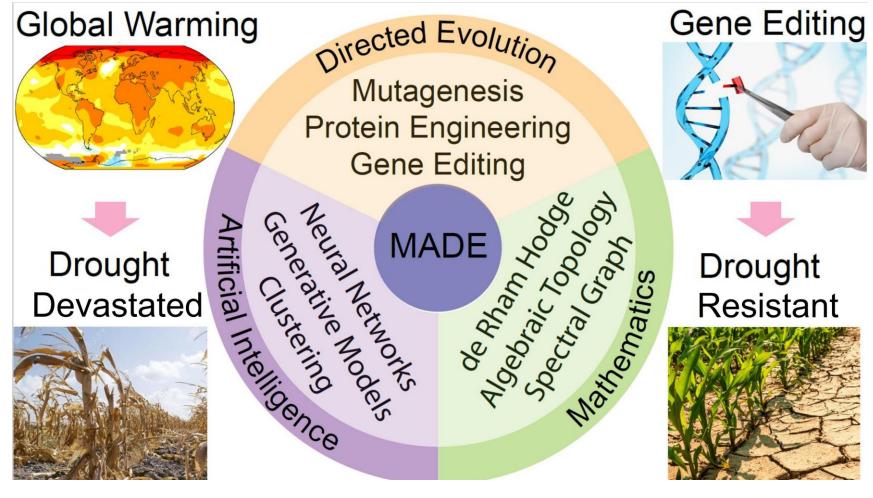
Topology-enabled Transformers (3D to 1D)

Chen, Liu, and Wei, Nature Machine Intelligence, 2024





Mathematics-Assisted Directed Evolution (MADE)





Impact of climate change on food security and global health

Y. Qiu, Hu, Wei, Nature Computational Science, 1(12) 809, 2021.

Y. Qiu, Wei, Nature Computational Science, 3, 149, 2023.

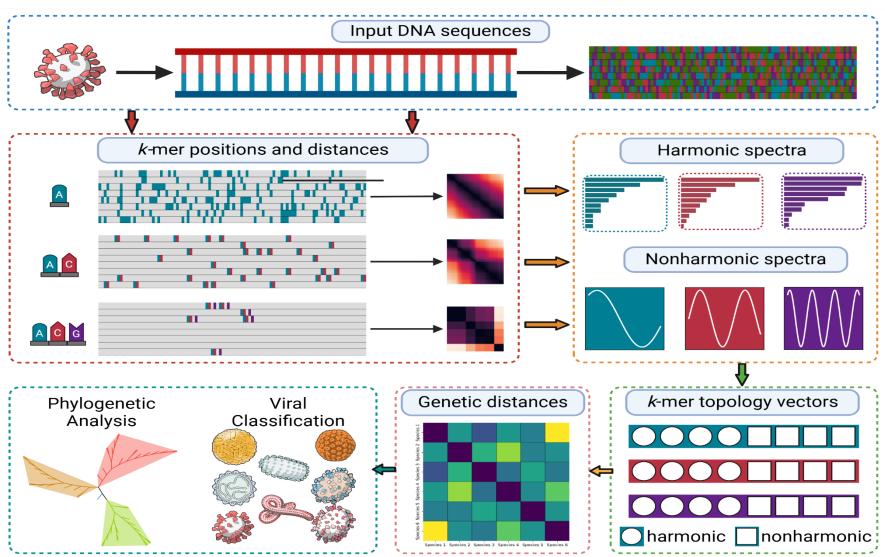
Revealing the Shape of Genome Space via K-mer Topology

Yuta Hozumi and GW Wei

arXiv:2412.20202, 2024



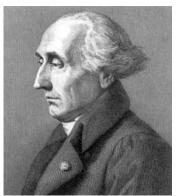
Yuta. Hozumi



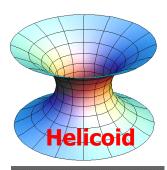
Differential geometry



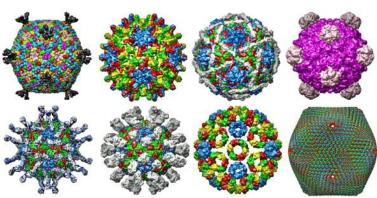
Leonhard P. Euler (Swiss Mathematician, April 15, 1707 – Sept 18 1783



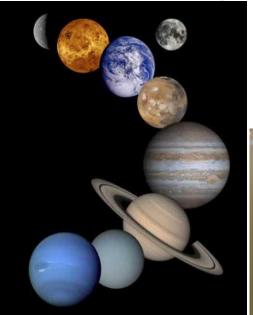
Joseph L. Lagrange (Italian Mathematician, January 25 1736 – April 10, 1813)







Viral morphology



Minimal Surfaces

A way to minimize energy and maximize stability





Man-made life, Mycoplasma mycoides

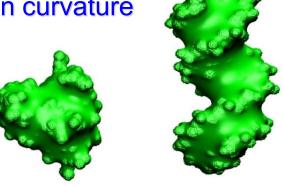
Differential geometry (DG) based multiscale model

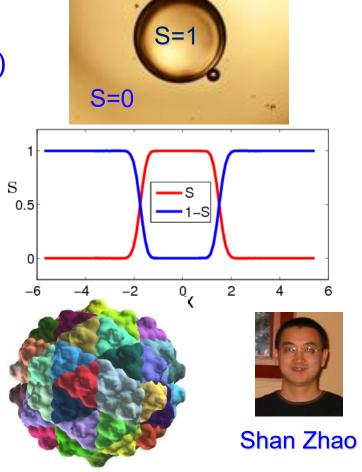
$$G = \int \gamma[\text{area}] d\mathbf{r}$$
 area = $|\nabla S|$

where G is the surface energy, gamma (γ) is the surface tension, and S is a surface characteristic function:

Generalized Laplace-Beltrami flow:

$$\frac{\partial S}{\partial t} = |\nabla S| \left[\nabla \cdot \frac{\gamma \nabla S}{|\nabla S|} \right]$$
Mean curvature





DG-based Poisson-Boltzmann Nernst Planck model

(Bates, Wei, Zhao, 2006; JCC, 2008; Wei et al., SIAM Review 2012)

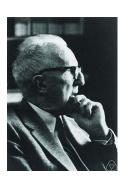
Differential topology



George de Rham 1903-1990



William Hodge 1903-1973



Marston Morse 1892-1977



Heinz Hopf 1894-1971



Henri Poincaré 1854-1912

- De Rham cohomology
- Hodge decomposition
- Morse theory

. . .

- Discrete exterior calculus
- Finite element exterior calculus



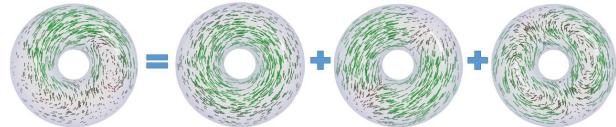
Crane and Segerman

Differential Topology (De Rham-Hodge theory)

Hodge decomposition:

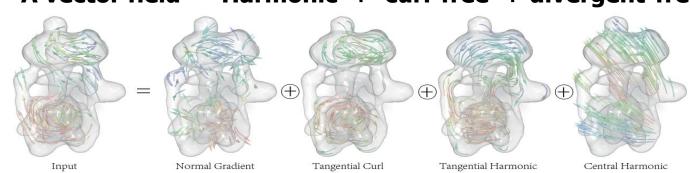


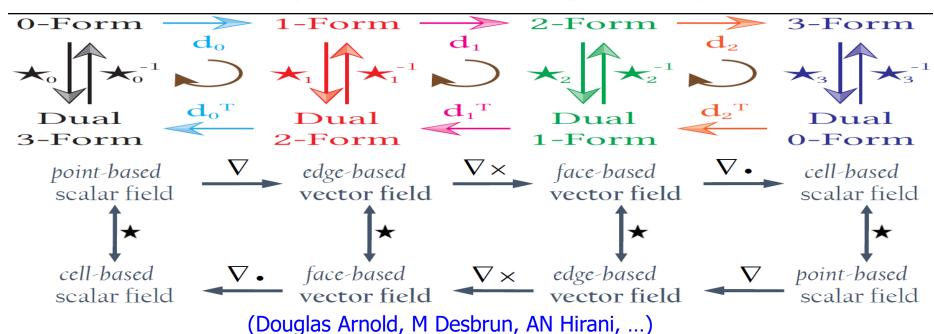
(Zhao, Wang, Chen, Tong & Wei, BMB, 2020)



A vector field = Harmonic + curl-free + divergent-free

Cryo-EM data:





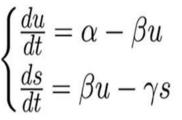
Hodge Decomposition of Single-Cell RNA Velocity

Su, Tong and Wei J. Chem. Inf. Model. 2024, 64, 8, 3558-3568



Dr. Zhe Su

Data preprocessing and manifold generation



RNA velocity model





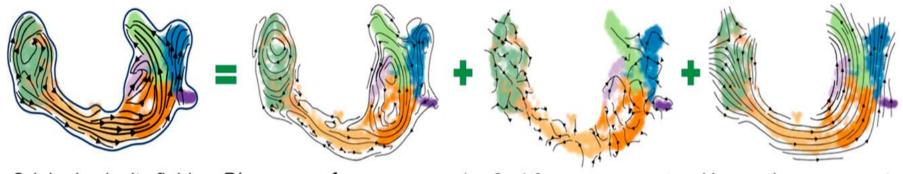
Manifold generation



Vector field on a manifold

RNA velocity in a 2D representation

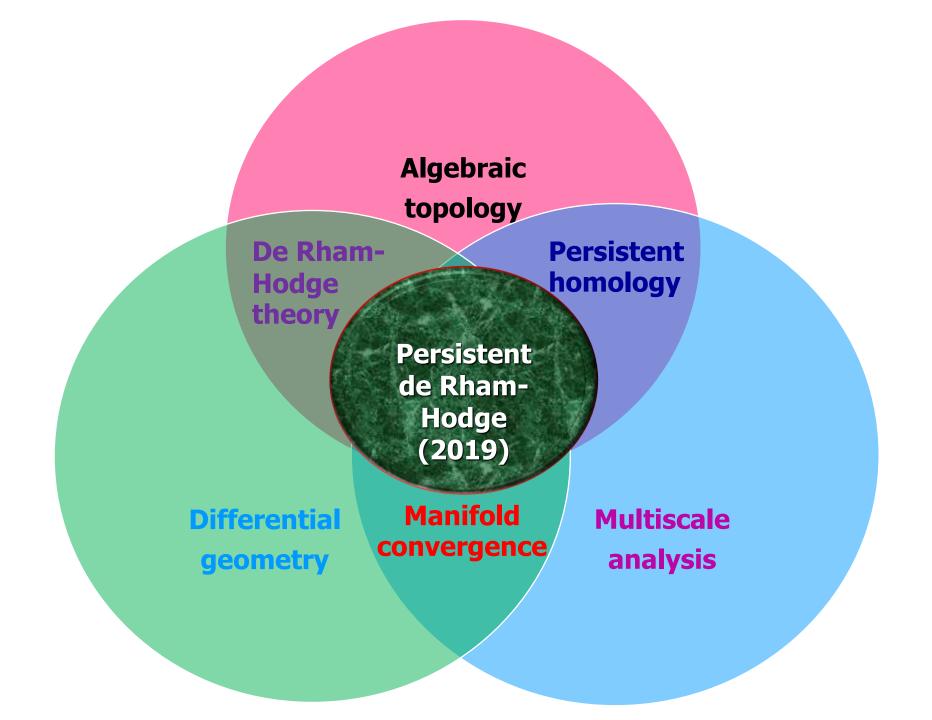
Hodge decomposition of velocity field



Original velocity field

Divergence-free component Curl-free component

Harmonic component



Evolutionary de Rham-Hodge (Differential Topology)

Zenghouyi Chime Bells (433 BC in China)

Ribando-Gros, et al. SIAM Review 2024



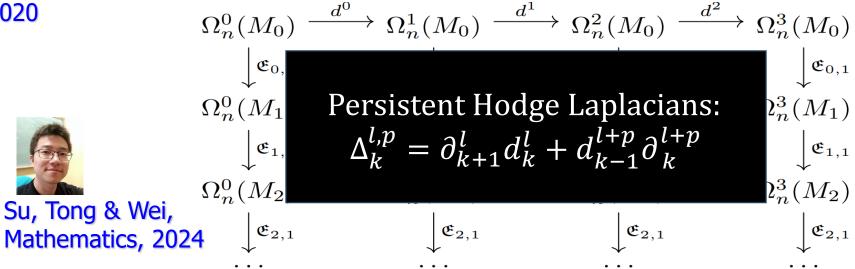
Chen, Zhao, Tong & Wei, DCDS-B, 2020

Su, Tong & Wei,

Manifold evolution

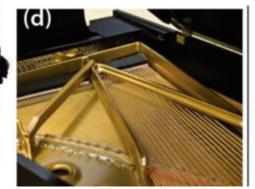
$$M_0 \xrightarrow{\mathfrak{I}_{0,1}} M_1 \xrightarrow{\mathfrak{I}_{1,2}} M_2 \xrightarrow{\mathfrak{I}_{2,3}} \cdots \xrightarrow{\mathfrak{I}_{n-1,n}} M_n \xrightarrow{\mathfrak{I}_{n,n+1}} M$$

Filtration-induced de Rham complexes:



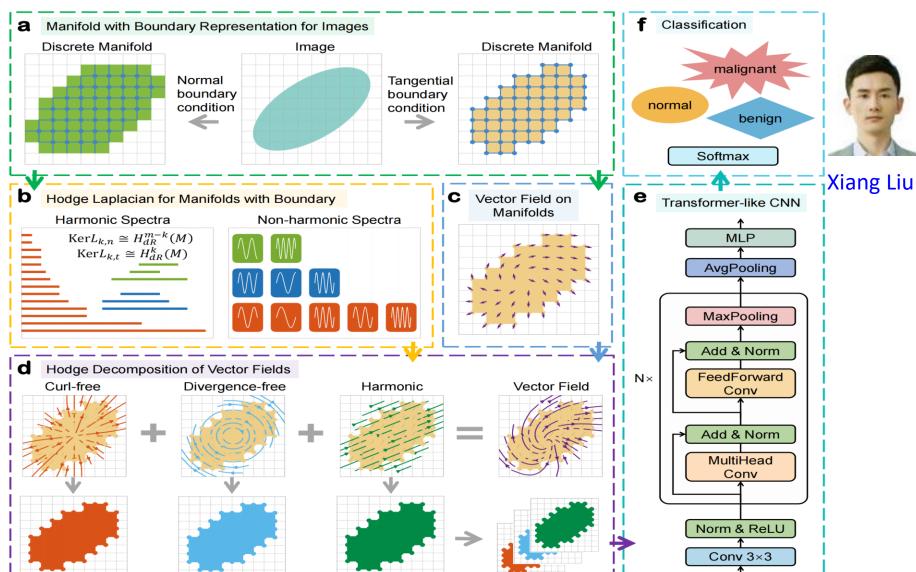




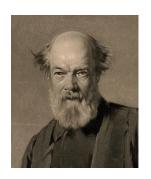


Medical image classification using differential topology

Liu, Wang, Song, Su, & Wei, Nat Comm., Under rev.



Geometric topology



Peter Tait 1831-1901



Kurt Reidemeister 1893-1971



Vaughan Jones 1952-2020



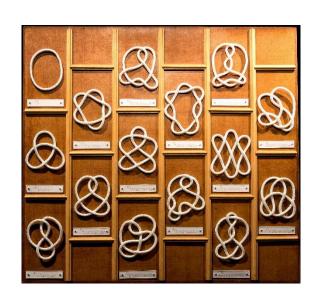
Mikhail Khovanov 1972-



Henri Poincaré 1854-1912



Hassler Whitney 1907-1989



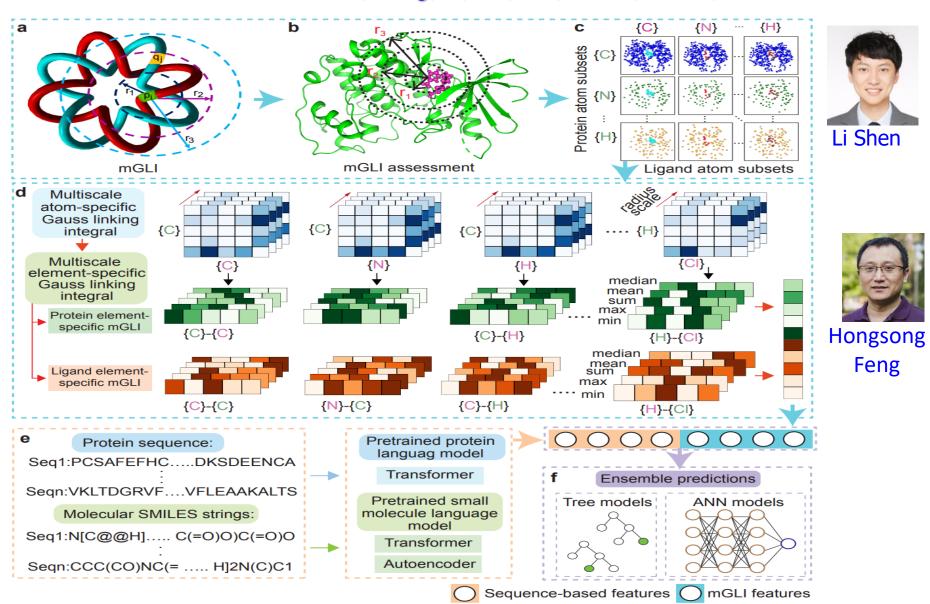
Matemateca



Book of Kells

Multiscale Gauss link integral

Shen, Feng, Li, Lei, Wu, & Wei, PNAS, 2024



E Panagiotou, LH Kauffman,...

Geometric Topology (1D curve in 3-space)

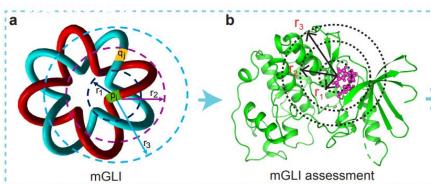






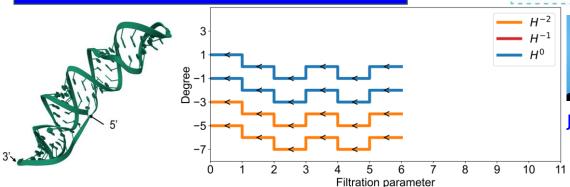
H Feng

Multiscale Gauss link integral



Shen, Feng, Li, Lei, Wu, & Wei, PNAS, 2024

Persistent Khovanov homology





Jian Liu



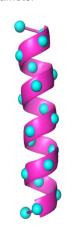
Li Shen

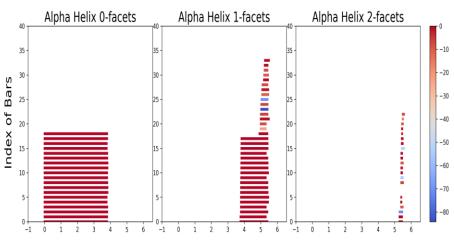
Shen et al. AIMS Mathematics, 2024

Persistent Jones Polynomial



Ruzhi Song, Fengling Li, Jie Wu, Fengchun Lei, and GW Wei, Mathematics 2025





Commutative Algebra



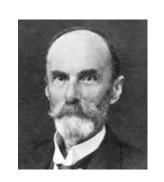
Wolfgang Krull (1899 – 1971)



Oscar Zariski (1899 – 1986)



Melvin Hochster (1943 --)



Francis Macaulay (1862 –1937)



Richard Stanley (1944-)

Persistent Commutative Algebra (Suwayyid and Wei, FoDS, 2025)

- > Introduce multiscale algebraic invariants to data analysis
- ➤ Integrate commutative algebra, algebraic topology, algebraic geometry, and combinatorics with machine learning.
- Offer a new rational learning platform.

Stanley-Reisner rings and ideals



Given a field k, define the polynomial ring:

$$S = k[x_1, x_2, ..., x_n]$$
 on the vertex set V .

Faisal Suwayyid & Wei, Foundations of Data Science, 2025.

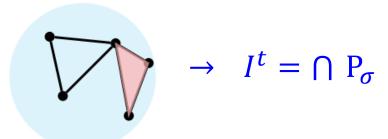
• A simplicial complex Δ^t induces the **Stanley–Reisner ideal** (i.e., **non-faces** of Δ^t):

$$I^{t}(\Delta^{t}) \coloneqq \left\langle x_{i_{1}}, \dots, x_{i_{r}} | \left\{ x_{i_{1}}, \dots, x_{i_{r}} \right\} \notin \Delta^{t} \right\rangle$$

• This ideal can also be expressed by **minimal facet ideals** P_{σ} :

$$I^t = \bigcap_{\sigma \in \mathcal{F}(\Delta^t)} P_{\sigma}$$
, where, $P_{\sigma} \coloneqq (x_i | x_i \notin \sigma)$

with $\mathcal{F}(\Delta^t)$ denoting the facets (maximal faces) of Δ^t .



Persistent Facet Ideals



• For a fixed dimension *i*, define the set

$$\mathcal{P}_i^t = \{ P_{\sigma} | \dim(\sigma) = i \}.$$

Faisal Suwayyid & Wei, Foundations of Data Science, 2025.

• The persistent prime ideals (or persistent facet ideals) of dimension i between two-time steps t and t' are given by the intersection:

$$\mathcal{P}_{i}^{t,t'} \coloneqq \mathcal{P}_{i}^{t} \cap \mathcal{P}_{i}^{t'}$$

This captures the common i-dimensional combinatorial features that persist across the filtration levels.

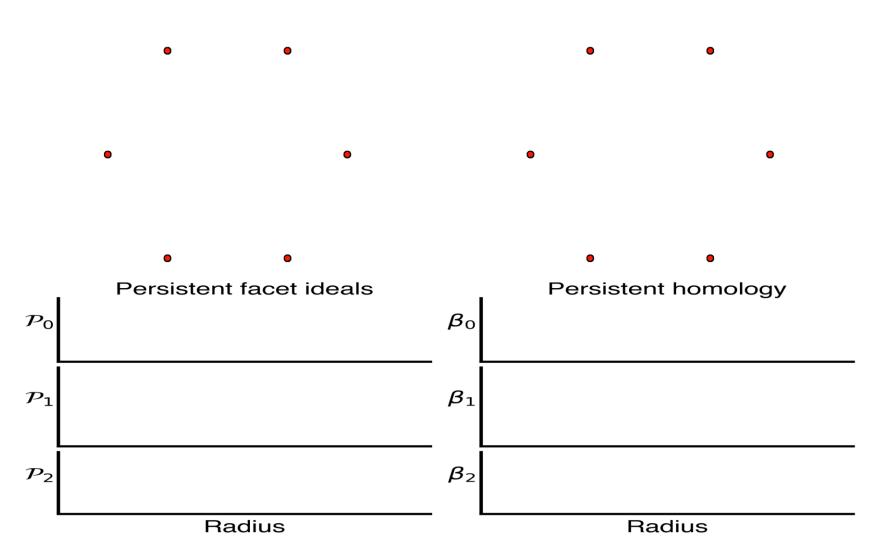
• The facet persistence Betti number $\beta_i^{t,t'} \coloneqq |\mathcal{P}_i^t \cap \mathcal{P}_i^{t'}|$ counts the number of such persistent prime ideals, offering an algebraic measure of the topological stability in dimension i.

Comparison between Persistent Facet Ideals and Persistent Homology



Feng et al., JCIM, 2025

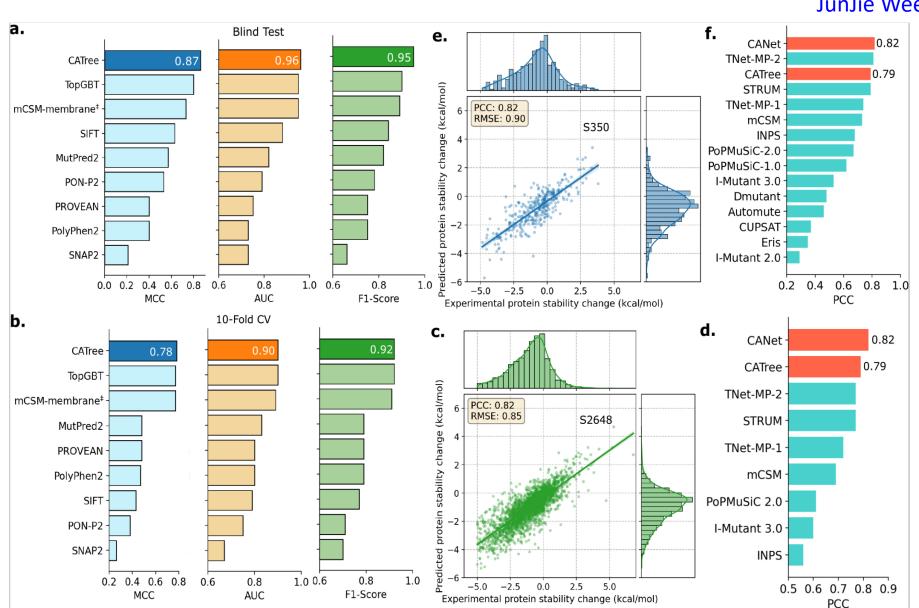
Hongsong Feng



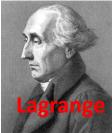
Commutative Algebra Tree/Net Protein mutations

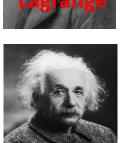


JunJie Wee

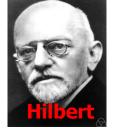




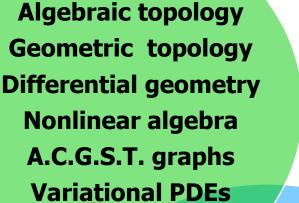


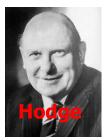






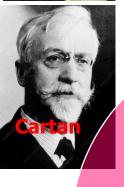












Sequence data
Structure data
Omics data
Biophysics
Bioinformatics
Systems biology
Systems pharmacology

Datadriven biological discovery

Machine learning
Deep learning
Manifold learning
Transformer
Autoencoder
Generative AI
Large language
model

Drug Design Data Resource (D3R) Grand Challenges

- Funded in part by National Institute of General Medical Sciences
- Hosted at the University of California, San Diego
- Annually 2015-2019

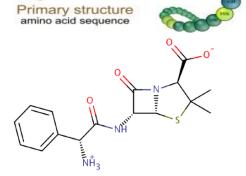




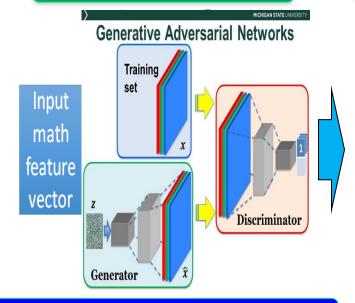


Drug Design Data Resource (D3R) Grand Challenge

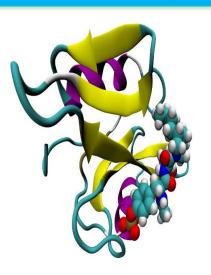
Given data Glu Phe Gly As Arg



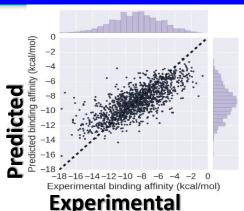
Math based GAN

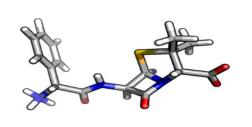


Predicted complex

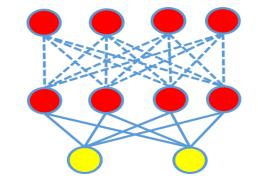


Final predictions to be compared with experiments









(Nguyen et al, JCAMD, 2018)

D3R Grand Challenge 4 (2018-2019)

Pose Predictions

BACE Stage 1A

Pose Predictions (Partials)



BACE Stage 1B Pose Prediction (Partials)



Affinity Predictions

Cathepsin Stage 1

Combined Ligand and Structure Based Scoring

Ligand Based Scoring (No participation)

Structure Based Scoring

Free Energy Set





BACE Stage 1

Combined Ligand and Structure (No participation) Combined Ligand and Structure

Ligand Based Scoring(Partials) (No participation) Ligand Based Scoring(No participation)

Structure Based Scoring(Partials)(No participation)Structure Based Scoring (Partials)

Free Energy Set (No participation)

Our performance in D3R Grand Challenges, worldwide competitions in computer-aided drug design, 2016-2019.

D3R Grand Challenge 3 (2017-2018)

Pose Prediction

Cathepsin Stage 1A Pose Predictions (partials)

Cathepsin Stage 1

Scoring (partials) Free Energy Set

VEGFR2

Scoring (partials)

JAK2 SC3 Scorina

Free Energy Set Z

Active / Inactive Classification VEGFR2

Scoring (partials)

JAK2 SC3 Scoring

Free Energy Set 🍊 Affinity Rankings for Cocrystalized Ligands

Cathepsin Stage 1

Scoring (partials) Free Energy Set

Cathepsin Stage 1B Pose Prediction

Affinity Rankings excluding Kds $> 10 \mu M$ Cathepsin Stage 2

Scoring (partials) Free Energy Set JAK2 SC2

Scoring (partials) TIE2

Scorina Free Energy Set 2

JAK2 SC2

Scoring (partials) TIE2

Scoring (partials) Free Energy Set 1

Cathepsin Stage 2 2/2 Scoring (partials) Free Energy Set

BACE Stage 2

Free Energy Set

(Nguyen et al, JCAMD, 2018)







p38-a

Scorina ABL1

Scoring (partials) 👗

p38-a

Scoring (partials)

ABL1

Scoring (partials)

We made many people upset at AI/ML/DL as noticed by Pedro Ballester (ICL).

AlphaFold started to change people mind about AI/ML/DL in 2018.

D3R Grand Challenge 2 (2016-2017)

Given: Farnesoid X receptor (FXR) and 102 ligands Tasks: Dock 102 ligands to FXR, and predict their poses,

binding free energies and energy ranking

Stage 1 Stage 2 Pose Predictions (partials)

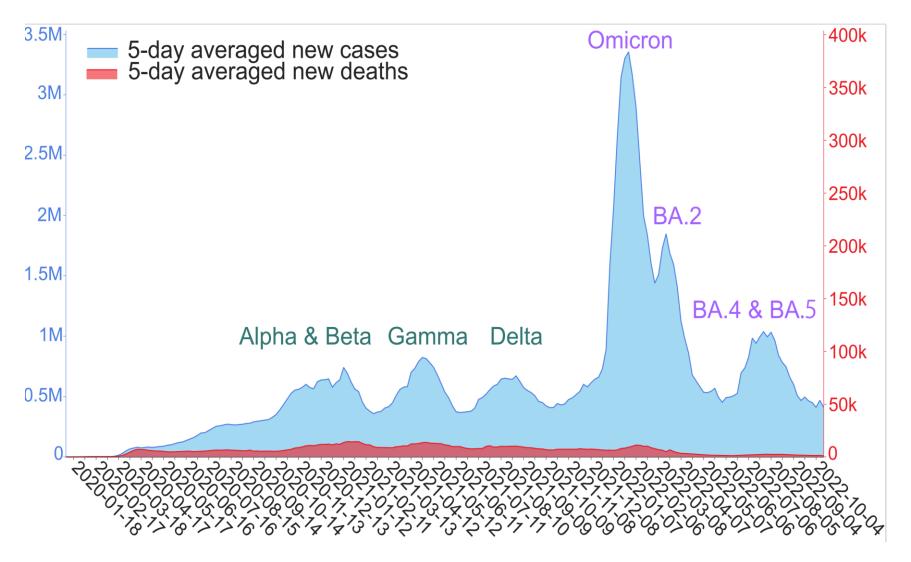
Scoring (partials) Free Energy Set 1 (partials) Free Energy Set 2 (partials) Scoring (partials)

Free Energy Set 1 (partials) Free Energy Set 2 (partials)



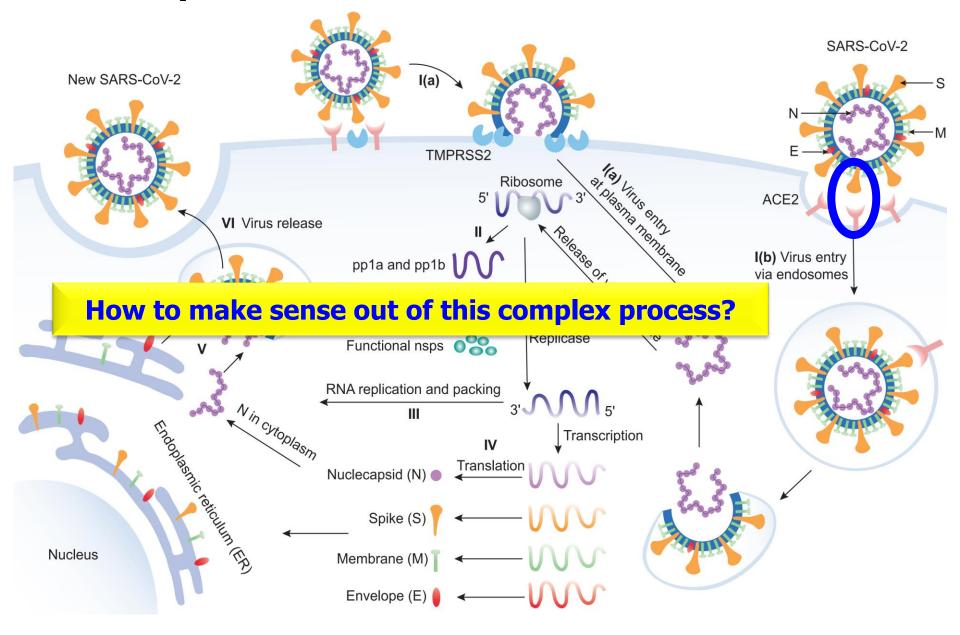


Evolution of SARS-CoV-2 variants



What are the evolutionary mechanisms?

Life cycle of SARS-CoV-2 in a host cell

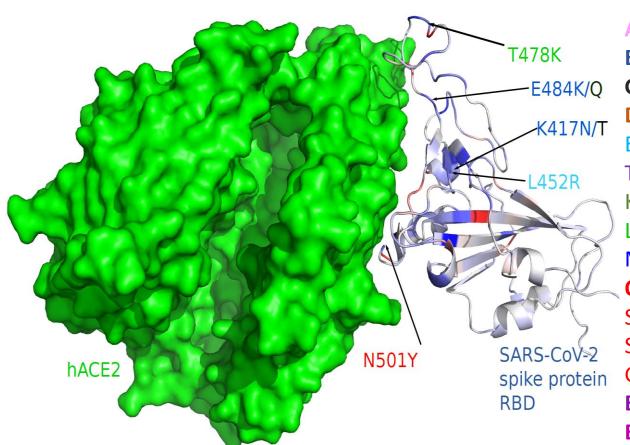




Mutations Strengthened SARS- Dr Jiahui Chen CoV-2 Infectivity

We predicted prevailing SARS-CoV-2 variants to occur at residues 452 and 501 using T-deep mutational scanning

Jiahui Chen¹, Rui Wang¹, Menglun Wang¹ and Guo-Wei Wei^{1,2,3}



Alpha: N501Y

Beta: K417N, E484K, N501Y

Gamma: K417T, E484K, N501Y

Delta: L452R, T478K

Epsilon: L452R

Theta: E484K, N501Y

Kappa: L452R, E484Q

Lambda: L452Q, F490S

Mu: R346K,E484K,N501Y

Omicron: G339D, S371L, S373P,

S375F, K417N, N440K, G446S,

S477N, T478K, <u>E484A</u>, Q493R,

G496S, Q498R, N501Y, Y505H;

BA.2.12.1: Omicron + L452Q;

BA.4/BA.5: Omicron + **L452R**

We discovered the mechanism of viral transmission and evolution

89) of all mutations on the RBD, which potentially increases the complexity of antiviral drug and vaccine development. This global analysis indicates that mutations on the RBD strengthen the binding of S protein and ACE2, leading to more infectious SARS-CoV-2.

We hypothesize that natural selection favors those mutations that enhance the viral transmission and if our predictions are correct, the predicted infectivity strengthening mutations will outpace predicted infectivity weakening mutations over time. Figure 3 illustrates the increase in the frequency of each

strengthening mutations occurred. It is interesting to note that overall, infectivity-strengthening mutations grow faster than infectivity-weakening mutations, which also reveals that SARS-CoV-2 subtypes having infectivity-strengthening mutations are able to infect more people. Specifically, frequencies of S477N, N439K, V483A, and V367F are higher than those of other mutations, indicating these mutations have a stronger transmission capacity.

The SARS-CoV-2 genotypes are clustered into six clusters or subtypes based on their single nucleotide

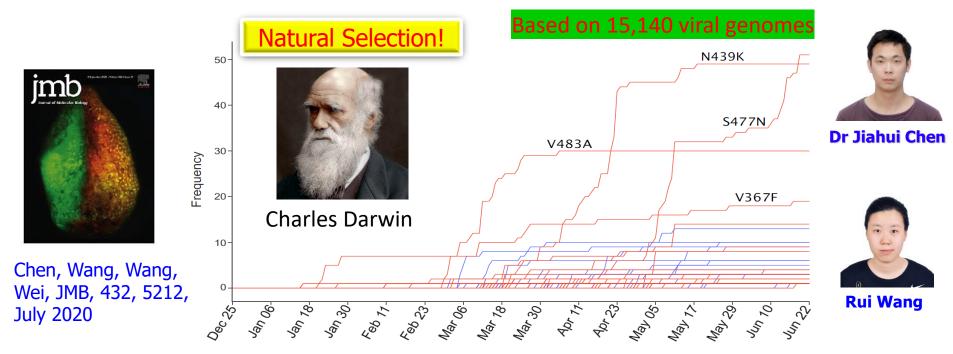


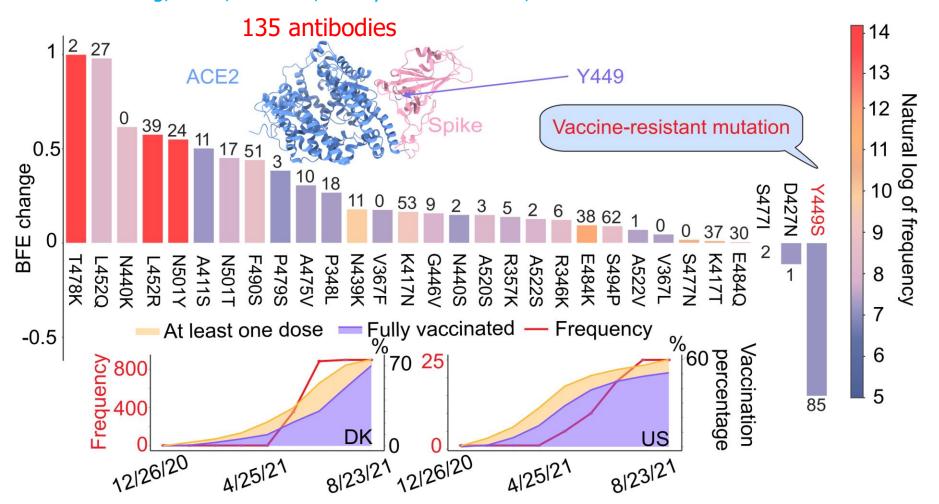
Figure 3. The time evolution of 89 SARS-CoV-2 S protein RBD mutations. The red lines represent the mutations that strengthen the infectivity of SARS-CoV-2 (i.e., $\Delta\Delta G$ is positive), and the blue lines represent the mutations that weaken the infectivity of SARS-CoV-2 (i.e., $\Delta\Delta G$ is negative). Many mutations overlap their trajectories. Here, the collection date of each genome sequence that deposited in GISAID is applied.

We discovered vaccine-breakthrough mechanism

By genotyping 2,298,349 viral genomes isolated from patients



Wang, Chen, and Wei, J. Phys. Chem. Letter, 12. 11850-11857 2021



Evolution mechanisms --- Natural selection via two complementary transmission pathways: Infectivity strengthening and vaccine breakthrough

Omicron BA.2 (B.1.1.529.2): high potential to becoming the next

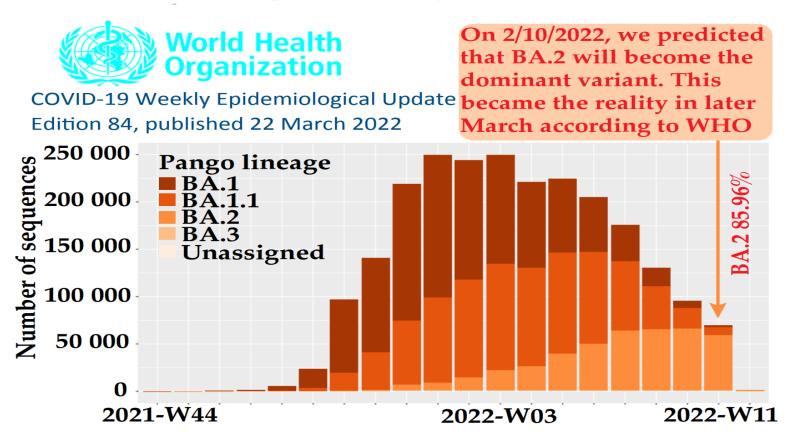
dominating variant

Jiahui Chen¹ and Guo-Wei Wei^{1,3,4*}

¹ Department of Mathematics,
Michigan State University, MI 48824, USA.



Dr Jiahui Chen



This was confirmed by WHO on March 22, 2022!
All other predictions were confirmed within 50 days

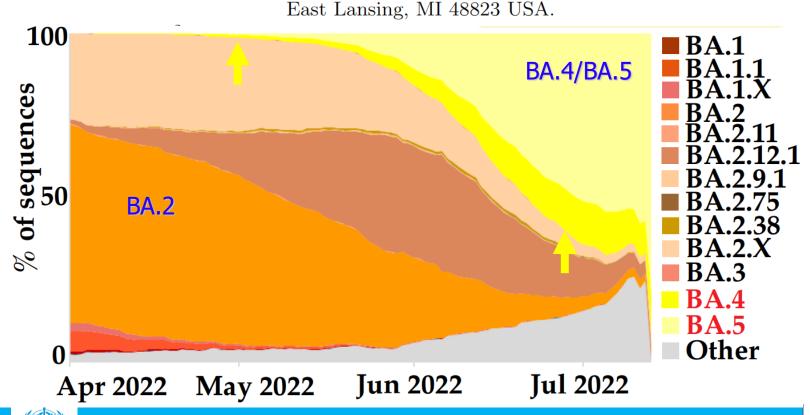
Chen, Wei, J. Phys. Chem. Lett., 13, 2840-3849, 2022.

Persistent Laplacian projected Omicron BA.4 and BA.5 to become new dominating variants

Jiahui Chen¹, Yuchi Qiu¹, Rui Wang¹, and Guo-Wei Wei^{1,2,3*}

¹ Department of Mathematics,
Michigan State University, MI 48824, USA.

Dr Jiahui Chen

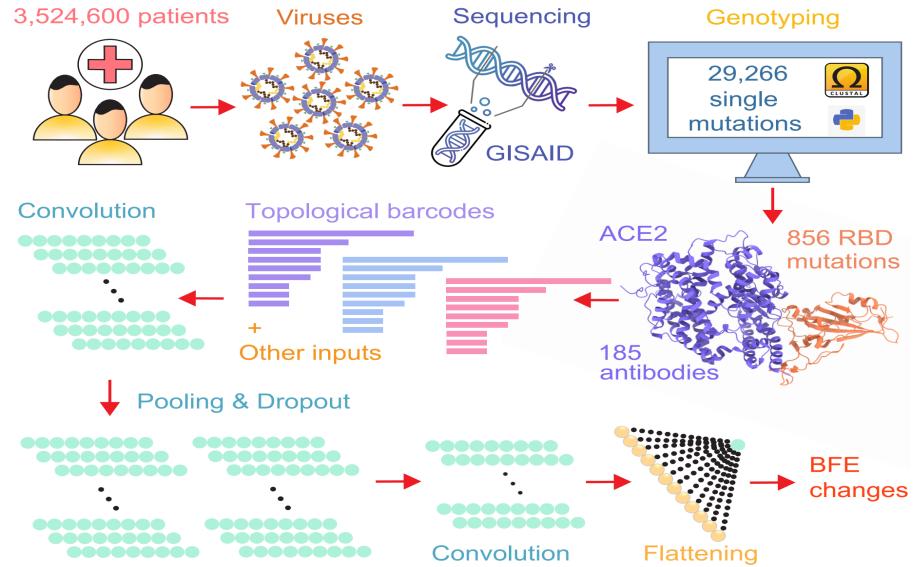


World Health Organization

This was confirmed by WHO in early July (WHO weekly update release number 101)

Jiahui Chen, Qiu, Wang, and Wei, Computers in Biology and Medicine, 151, 106262 (2023)

Genome-Math-AI-experimental data modeling of proteinprotein binding affinity changes following mutations



(Wei, Nature Machine Intell. (2019); Wang et al. Nature Machine Intell. 2020; Chen et al. Nature, Commun., 2021; Qiu et al. Nature Comput. Sci. 2021)



