Nonlinear Waves: From Theoretical and Computational Advances to Experimental Observations
Poster Session Abstracts

April 27, 2026
3:00-4:15pm

Optimal error bounds on the exponential integrator for dispersive equations with highly concentrated potential

Chushan Wang, University of Chicago

We study a one-dimensional linear dispersive equation of differential order \kappa \geq 2 with concentrated potential of extension \varepsilon with 0<\varepsilon \ll 1, featuring a competition between weak dispersion of strength \varepsilon^\alpha (0≤\alpha≤\kappa) and localization induced by the concentrated potential. We first obtain precise regularity estimates of the exact solution in terms of \varepsilon. We then apply a natural first-order exponential integrator with step size τ to discretize the equation, and establish an optimal error bound of the form O_{L^\infty}(\tau \varepsilon^\betta) (up to logarithmic factors in \tau and \varepsilon). Salient features of the result are: (i) error bounds are not only uniform in \varepsilon but improve as \varepsilon \rightarrow 0; and (ii) no restriction on \tau in terms of \varepsilon. The analysis combines iterated Duhamel's expansions and a transformation that exploits cancellations in oscillatory phases that cannot be obtained directly from regularity estimates of the exact solution. We also show that other classical numerical schemes, such as Lie or centered splitting schemes and low regularity integrators, fail to display optimal rates of convergence. Extensive numerical results are presented and confirm the theoretical error estimates.

Spectral Precursors of Discrete Rogue-Wave Cascades in Near-Integrable Lattices

Jimmie Adriazola, Arizona State University

We investigate the emergence of extreme focusing events in discrete nonlinear Schrödinger lattices near the Ablowitz-Ladik limit, with particular emphasis on higher-order rogue waves and multi-generation cascade patterns generated by Thomas-Fermi (TF) quench data. Our working premise is that these striking lattice events are not merely transient numerical phenomena, but are organized by geometry in the nonlinear Lax spectrum. In the integrable periodic theory, rogue waves arise from spectral degeneracies at band endpoints on modulationally unstable arcs. Here we explore the possibility that related degeneracies continue to shape strongly inhomogeneous lattice dynamics and provide a spectral organizing mechanism for discrete cascade formation.

The project develops along two complementary directions. On the discrete side, we study TF-type initial states on the Ablowitz-Ladik lattice and compute nonlinear Floquet or Lax spectral portraits, using periodic extensions of the data to identify branch crossings, arc reconnections, and other signatures that may act as precursors to extreme focusing events. In parallel, we examine the associated continuum and semiclassical picture generated by TF quench data, using the Madelung transform and Whitham modulation to probe dispersionless focusing, gradient-catastrophe-type behavior, and the onset of oscillatory structure. The broader goal is to understand whether discrete rogue-wave cascades can be interpreted through a common framework that links continuum modulation dynamics with nonlinear spectral geometry in the lattice setting.

Pulses, waves, and mesas in a mass-conserved reaction diffusion system

Jack Hughes, Concordia University

The transition between random walk and directional motion is one of the intriguing phenomena observed in eukaryotic cells. Typical theoretical studies distinguish between the two phenomena and focus either on dissipative or gradient flow models, respectively. Using a 3 variable dissipative reaction-diffusion system with mass conservation, we show how pulses, waves, mesas, and fronts generically organize about high codimension bifurcations. Specifically, we demonstrate the novelty of mass conservation, which enters as a persistent large-scale mode possibly leading to a long wavelength bifurcation. Following the biological interest, we address the bistability of travelling wave and mesa solution branches that emerge from a novel codimension-2 long wavelength/ finite wavenumber Hopf bifurcation.

Global well-posedness and scattering for the Dirac-Klein-Gordon system

Vitor Borges, UC San Diego

We prove global well-posedness and scattering for the Dirac-Klein-Gordon system with small and low regularity initial data.

Fractal Hasimoto Surfaces

Jiri Minarcik, Carnegie Mellon University

The vortex filament equation describes the evolution of vortex filaments in an ideal fluid. As the filament evolves, it traces a two-dimensional trajectory surface, which we call a Hasimoto surface. We study the geometry of these surfaces for polygonal solutions. For smooth initial curves they remain smooth, while polygons with sharp corners evolve through explicit self-similar solutions that recreate polygonal configurations at rational times. For regular polygons, point trajectories exhibit multifractal spectra, as shown by Banica, Eceizabarrena, Nahmod, and Vega. The corresponding Hasimoto surfaces develop cascades of singular spikes and romanesco-like fractal patterns, which we illustrate through numerical visualizations.

A KP model for waves over Topography

David Andrade, Universidad del Rosario

In this poster I will present a recently derived KP equation for weakly nonlinear, dispersive and transversal waves, propagating over a ridge-like bathymetry. The model uses the conformal mapping to handle the variable geometry of the flow domain and, by means of standard asymptotic techniques, the KP model is obtained.

Long-wave scattering on balanced quantum graphs

Binan Gu, Worcester Polytechnic Institute

In mathematics and physics, a quantum graph is a metric graph where a partial differential equation is posed on each edge, together with compatibility conditions at the vertices. The present quantum-graphs problem applies to surface water waves in a channel, or river network. Most mathematical research on quantum graphs considers flow on graphs. The goal here is to study wave reflection-and-transmission at a vertex, where converging and diverging junctions are constrained by the Neumann-Kirchhoff vertex conditions. Different vertex configurations lead to different scattering matrices. It is shown that the scattering matrices here formulated have features similar to those for the time-independent Laplacian in quantum graphs. A less explored case refers to balanced quantum graphs. This is a purely geometric property, where different weights are attributed to the edges. In quantum graphs, balanced star graphs lead to the notion of transparent graphs where no reflection is observed at each vertex. Here it is shown that the geometrical properties for balanced graphs are necessary, but not sufficient for obtaining a transparent graph. Numerical simulations illustrate when a balanced junction is reflectionless or not. There is a dynamic condition for a balanced star graph to be transparent: synchronization. The present scattering-matrix formulation indicates why synchronization is needed and also yields a very efficient and accurate Lagrangian numerical method. In many cases, the numerical wave propagation is exact.

An Integrable Parity-Time Symmetric Oligomer

Frank Smuts, University of Cape Town

A parity-time-symmetric oligomer is a finite-size discrete nonlinear Schrodinger equation subject to balanced gain and loss of energy. These equations may model networks of optic fibres [1], or Bose-Einstein condensates confined to multiple potential wells [2]. One may intuitively assume that these systems are nonintegrable, owing to the presence of gain and loss terms. To the contrary - at least in the two-site case (or dimer case) there exists a four-parameter class of Hamiltonian integrable oligomers [3]. Two natural questions arise:
1) Are there any integrable oligomers with more than two interacting sites?
2) Do these integrable dimers possess a Lax pair?
To gain insight into both questions, we construct a prototypical integrable PT-symmetric oligomer, as a nonlocal reduction of the Ablowitz-Ladik equation with gain-loss boundary terms. The oligomer admits a Lax pair and remains Hamiltonian integrable for arbitrarily many interacting sites.
The linearized oligomer exhibits symmetry breaking when the gain loss coefficient is raised past a critical value. The fully nonlinear system is, of course, much more complicated. We solve the two-site case in terms of elementary functions, and show that the system conspires to suppress symmetry-broken orbits.
[1] S. Jensen, IEEE Journ. Quant. Electronics, Volume 18, p. 1580 – 1583. (1982)
[2] G. J. Milburn, J. Corney, E. M. Wright, D. F. Walls, Phys. Rev. A, Volume 55, p. 4318-4325. (1997)
[3] I. V. Barashenkov, D.E. Pelinovksy, and P. Dubard, J. Phys. A: Math. Theor., Volume 48, 325201 (2015).

Physics-Informed Learning and Spectral Analysis for Nonlinear PDE Dynamics

Wenjian Liu, City University of New York

Coherent structures and localized patterns play a fundamental role in the dynamics of nonlinear partial differential equations. This poster considers the use of spectral and physics-informed data-driven methods for the analysis of nonlinear and stochastically driven PDEs, with particular emphasis on the identification of dynamically significant modes, reduced descriptions of solution behavior, and qualitative questions of stability and long-time evolution. Motivated by recent work on spectral analysis for parabolic equations with stochastic inputs, as well as related problems in nonlinear dynamics on sparse graphs and trees, we examine how learned low-dimensional representations can complement more classical analytical and computational approaches. The emphasis is on mathematically structured procedures, rather than black-box prediction, with the goal of providing interpretable tools for studying coherent behavior, regime transitions, and emergent structure in complex nonlinear systems.

Wedge problems and dispersive shock waves in the two-dimensional Toda lattice

Marco Calabrese, University of Massachusetts Amherst

We study the formation and interaction of dispersive shock waves in the two dimensional Toda lattice with wedge-type initial data. While the local wave structure along each side of the wedge is governed by an oblique line soliton whose profile is inherited from the one dimensional Toda lattice, the global dynamics is genuinely two dimensional and it can give rise to Mach stem formation. Using analytical arguments based on finite gap theory and Whitham modulation theory, together with numerical simulations, we characterize how the initial discontinuity and the wedge geometry determine the emerging wave structures. In particular, we quantify the amplitude of the leading structures and analyze how two dimensional effects govern the interaction of the wave fronts.

Oscillations in a scalar differential equation coupled to a diffusive field
Merlin Pelz, University of Minnesota, Twin Cities

We study the emergence of periodic oscillations through a Hopf bifurcation in a scalar diffusion equation on the half line coupled to a dynamic boundary condition. Our results quantify the effect of delay through the buffering in the diffusive field on boundary kinetics, drawing a parallel to the emergence of oscillations in delay equations. Technically, the Hopf bifurcation occurs in the presence of essential spectrum induced by the diffusive field, preventing a simple approach via center-manifold reduction. The results are motivated by observations in biological systems where dynamic boundary conditions arise when modeling surface dynamics coupled to bulk diffusion.

The closest point method for solving surface PDEs with general boundary condition
Tony Wong, UCLA

We generalize the closest point method (CPM) to solve surface partial differential equations with general boundary conditions. The proposed method provides a unified framework for treating a broad class of inhomogeneous Neumann and Robin boundary conditions within the CPM framework. This work encourages numerical exploration of nonlinear dynamics on complicated geometry.

Auto-Adaptive PINNs
Kevin Buck, Indiana University

We propose an adaptive sampling method for the training of Physics Informed Neural Networks (PINNs) which allows for sampling based on an arbitrary problem-specific heuristic which may depend on the network and its gradients. In particular we focus our analysis on the Allen-Cahn equations, attempting to accurately resolve the characteristic interfacial regions using a PINN without any post-hoc resampling. In experiments, we show the effectiveness of these methods over residual-adaptive frameworks.

Curvature and Confinement: Vegetation on Undulating Terrain
Timothy Roberts, University of Chicago

Vegetation in arid environments must overcome scarcity of resources, primarily of water, to survive. Yet we find vegetation in very arid environments all over the world. Early phenomenological models by Klausmeier [Klausmeier, Science (1999)] demonstrated that vegetation on sloped terrain is able to survive in arid conditions by breaking up into densely vegetated stripes along the elevation contours of the slope. Bare soil regions between the stripes collect water which flows downhill before being caught and captured by the vegetation stripe immediately following. The formation of vegetation patterns can then be seen as a mechanism for developing resilience against aridity. While there are multiple models of this type in use today, they are typically studied on idealized 1-dimensional slopes. In this talk, I will present new results on the formation of patterns on 2-dimensional undulating slopes in a phenomenological model similar to Klausmeier's. We investigate links between the topography and the selected patterns, and how the formation of patterns affect the resilience of vegetation as water becomes more scarce. This is a joint work with Punit Gandhi and Mary Silber.

Noise-Induced Waves in Neural Field Equations

James MacLaurin, New Jersey Institute of Technology

We study the dynamics of waves, oscillations, and other spatio-temporal patterns in stochastic evolution systems, including SPDEs and stochastic integral equations. Representing a given pattern as a smooth, stable invariant manifold of the deterministic dynamics, we reduce the stochastic dynamics to a finite dimensional SDE on this manifold using the isochronal phase. The isochronal phase is defined by mapping a neighborhood of the manifold onto the manifold itself, analogous to the isochronal phase defined for finite-dimensional oscillators by A.T. Winfree and J. Guckenheimer. We then determine a probability measure that indicates the average position of the stochastic perturbation of the pattern/wave as it wanders over the manifold. It is proved that this probability measure is accurate on time-scales greater than , but less than , where is the amplitude of the stochastic perturbation. Moreover, using this measure, we determine the expected velocity of the difference between the deterministic and stochastic motion on the manifold.

Dynamics of Peakon Solutions: ODE Systems and New Properties

Yonghong Chen, School of Mathematical and Statistical Sciences, University of Texas Rio Grande Valley

We study peakon solutions for a class of nonlinear partial differential equations with polynomial nonlinearities of various orders. The equations are formulated in a convolution form, which provides a convenient framework for analysis. By employing a peakon ansatz, we derive explicit finite-dimensional ODE systems governing the evolution of peakon amplitudes and positions.