NICHOLAS AND LEE BEGOVICH GW PAC Center for Gravitational-Wave Physics and Astronomy

CALIFORNIA STATE UNIVERSITY FULLERTON

Modeling binary black holes with nearly extremal spins using numerical relativity

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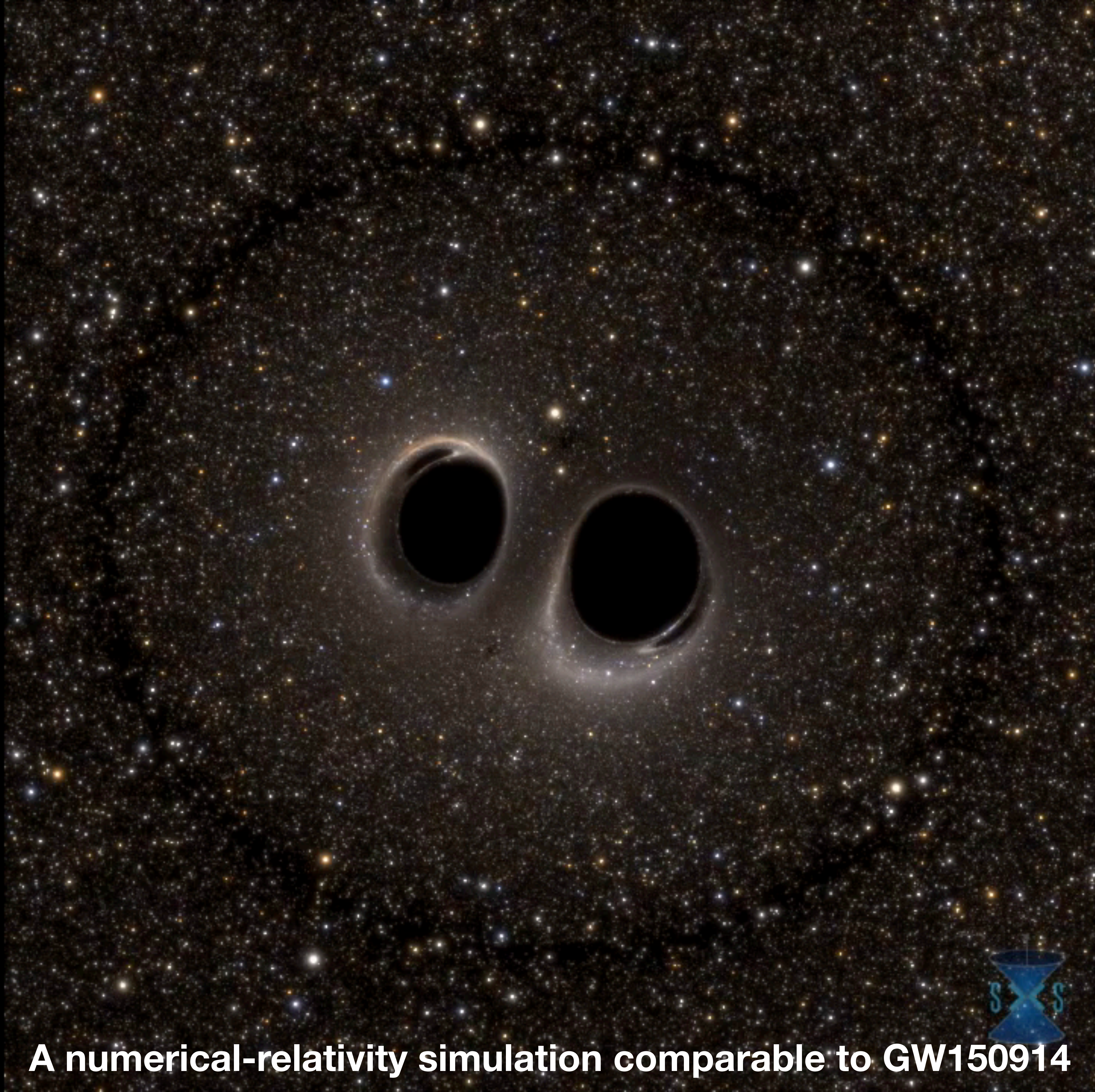
January 7, 2026
Extremal Black Holes and the Third Law of Black Hole Thermodynamics @ ICERM

What do binary black holes look like?

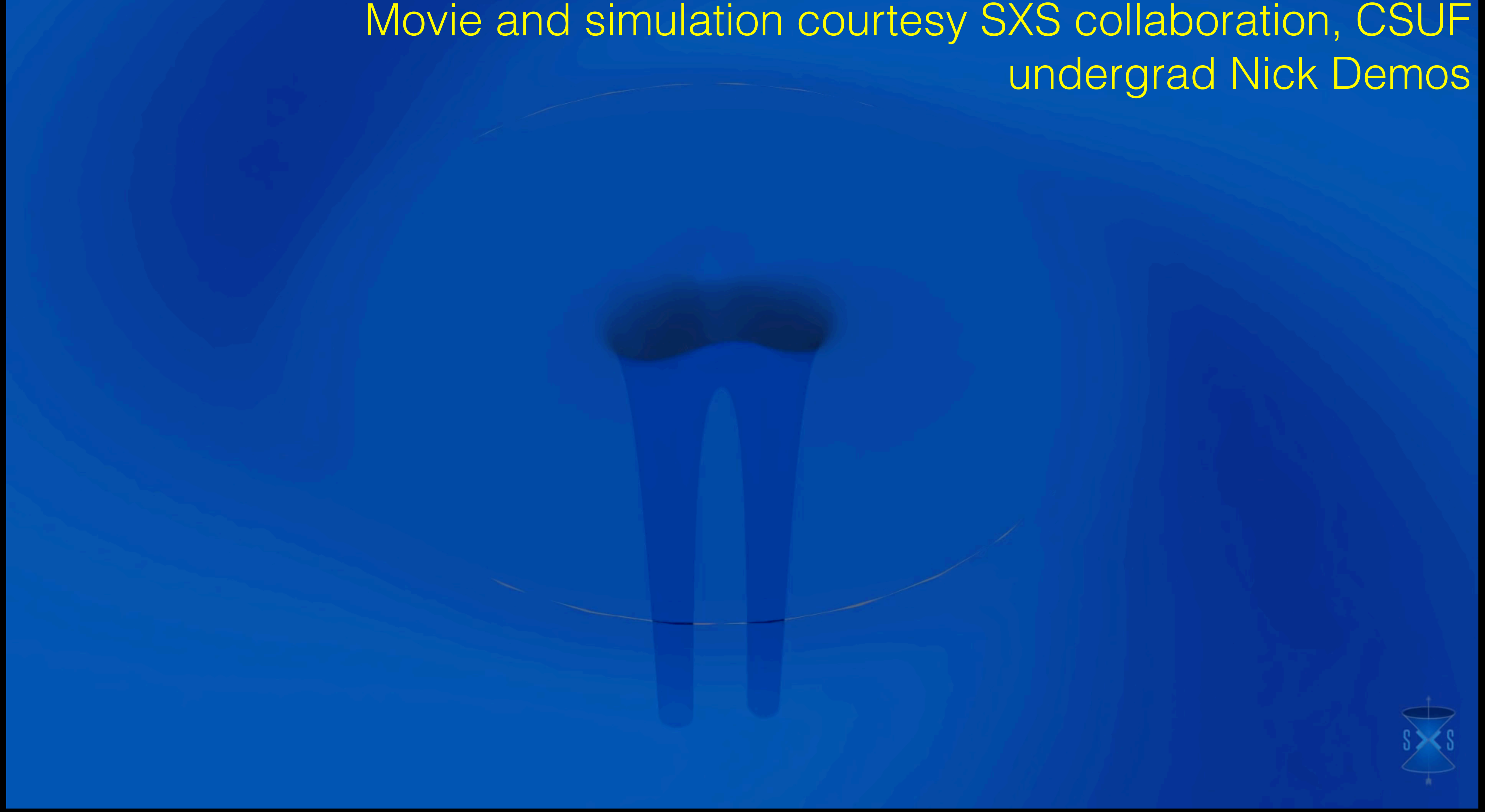
Calculation and movie by
SXS Lensing, CSUF undergraduates
Haroon Khan, Nick Demos

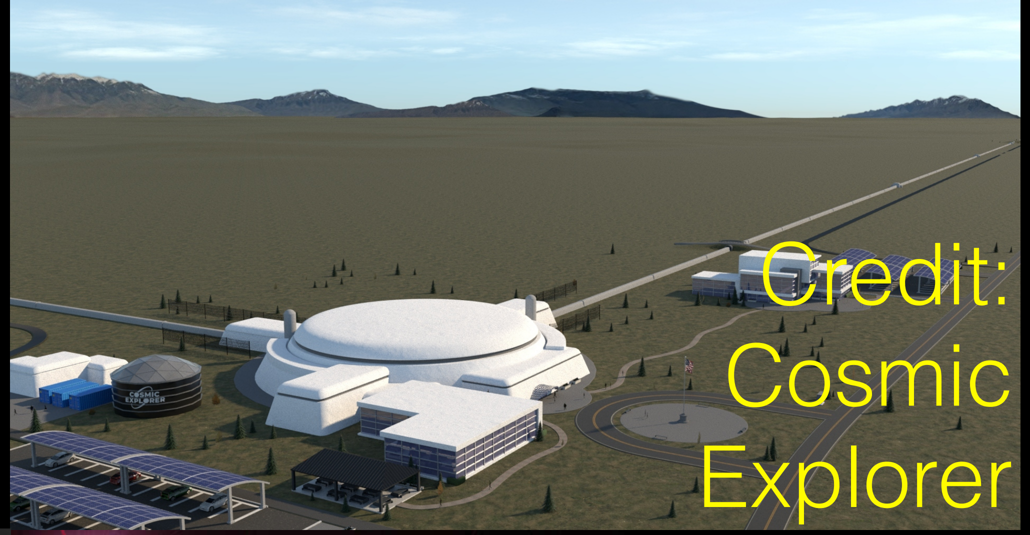


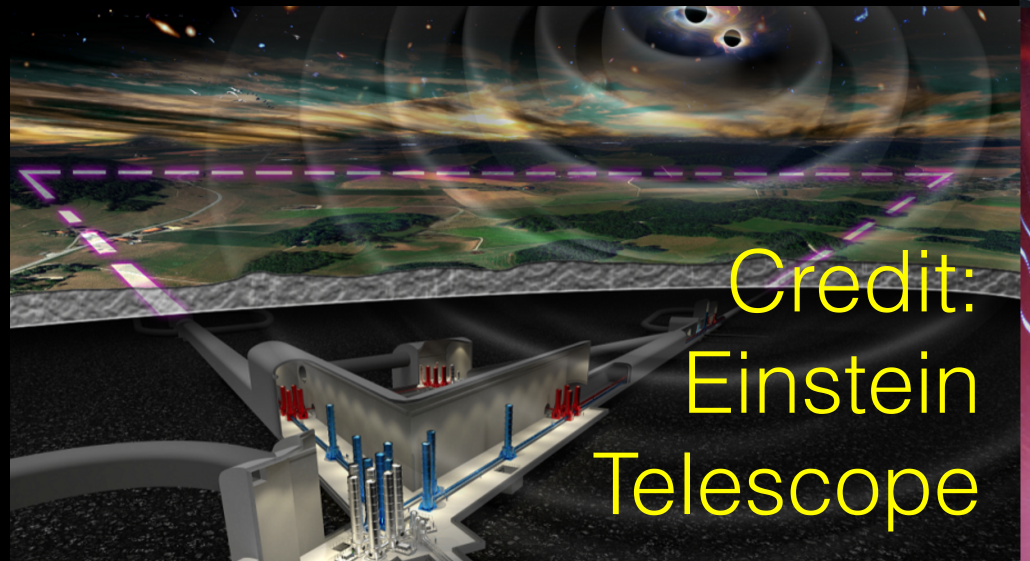
A numerical-relativity simulation comparable to GW150914

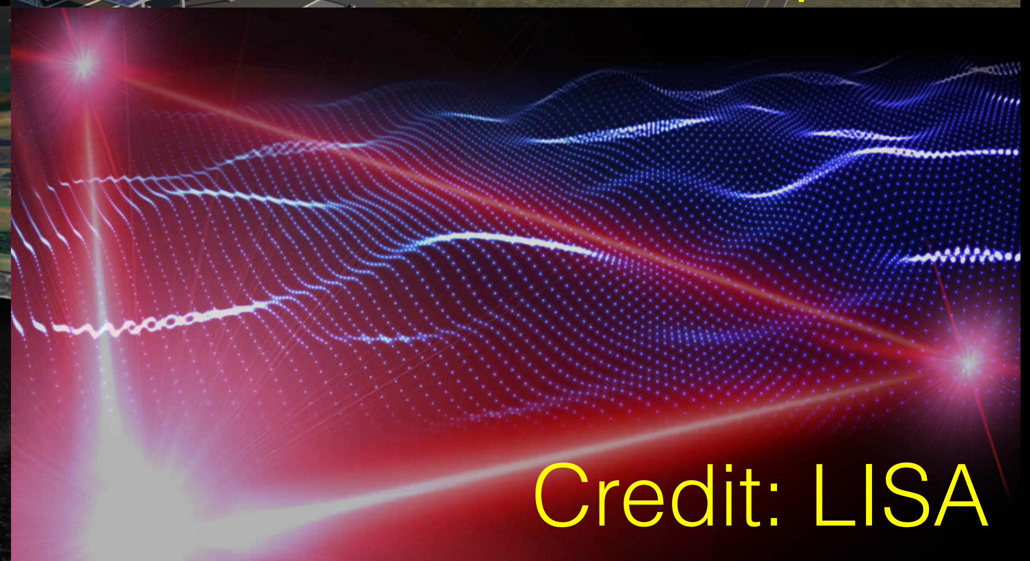
Binary black holes

- LIGO + Virgo
  - Hundreds of binary black holes so far, loudest signal-to-noise ratio (SNRs) ~ 80 (GW250114)
- Cosmic Explorer & Einstein Telescope & LISA
  - SNRs $\gtrsim 1000$, probe strongest gravity
  - Frequent detections, potential to observe rare black holes
  - Need accurate waveform models
    - Requires numerical relativity

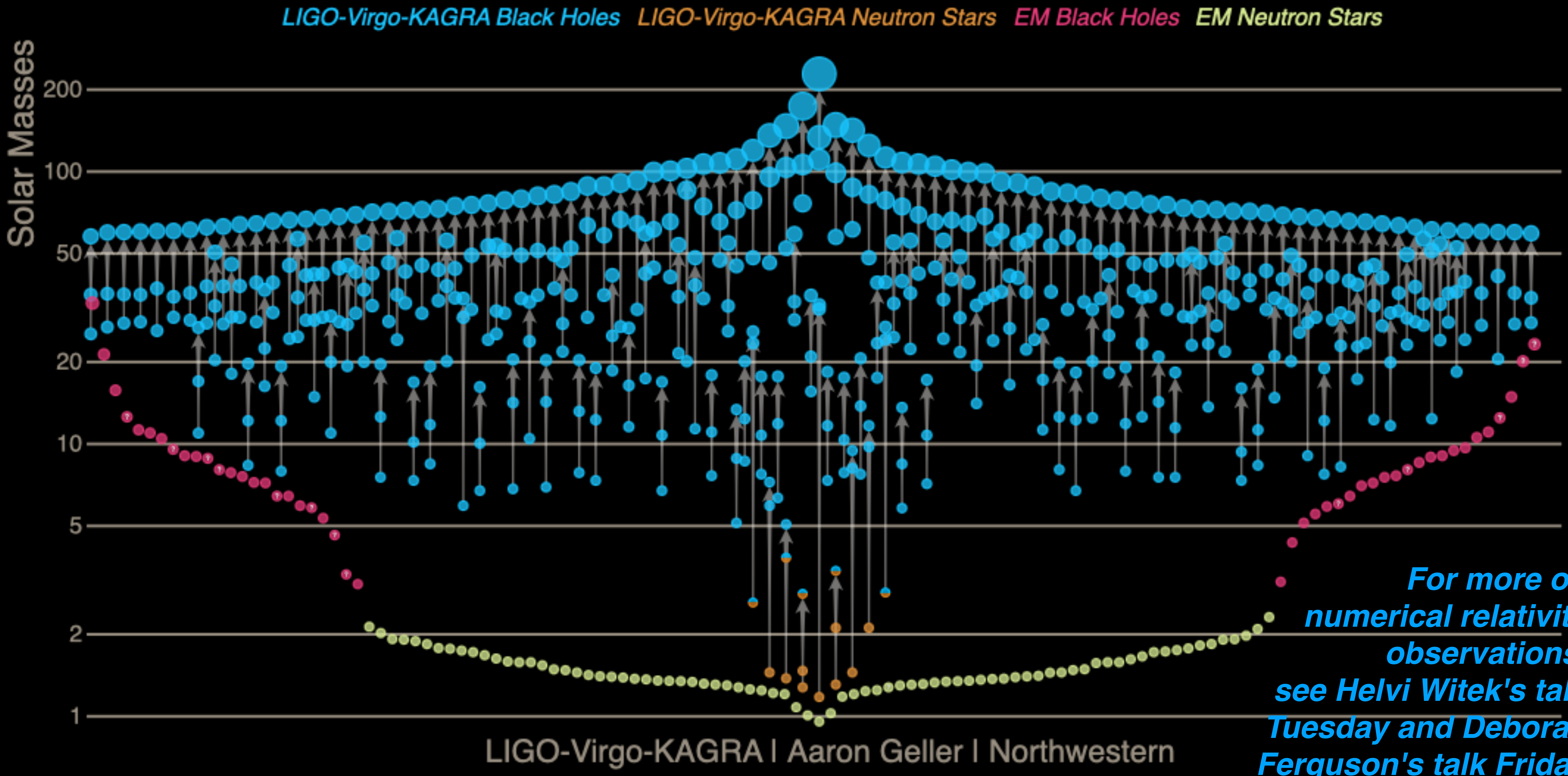








Masses in the Stellar Graveyard



For more on numerical relativity observations, see Helvi Witek's talk Tuesday and Deborah Ferguson's talk Friday

Simulating binary black holes

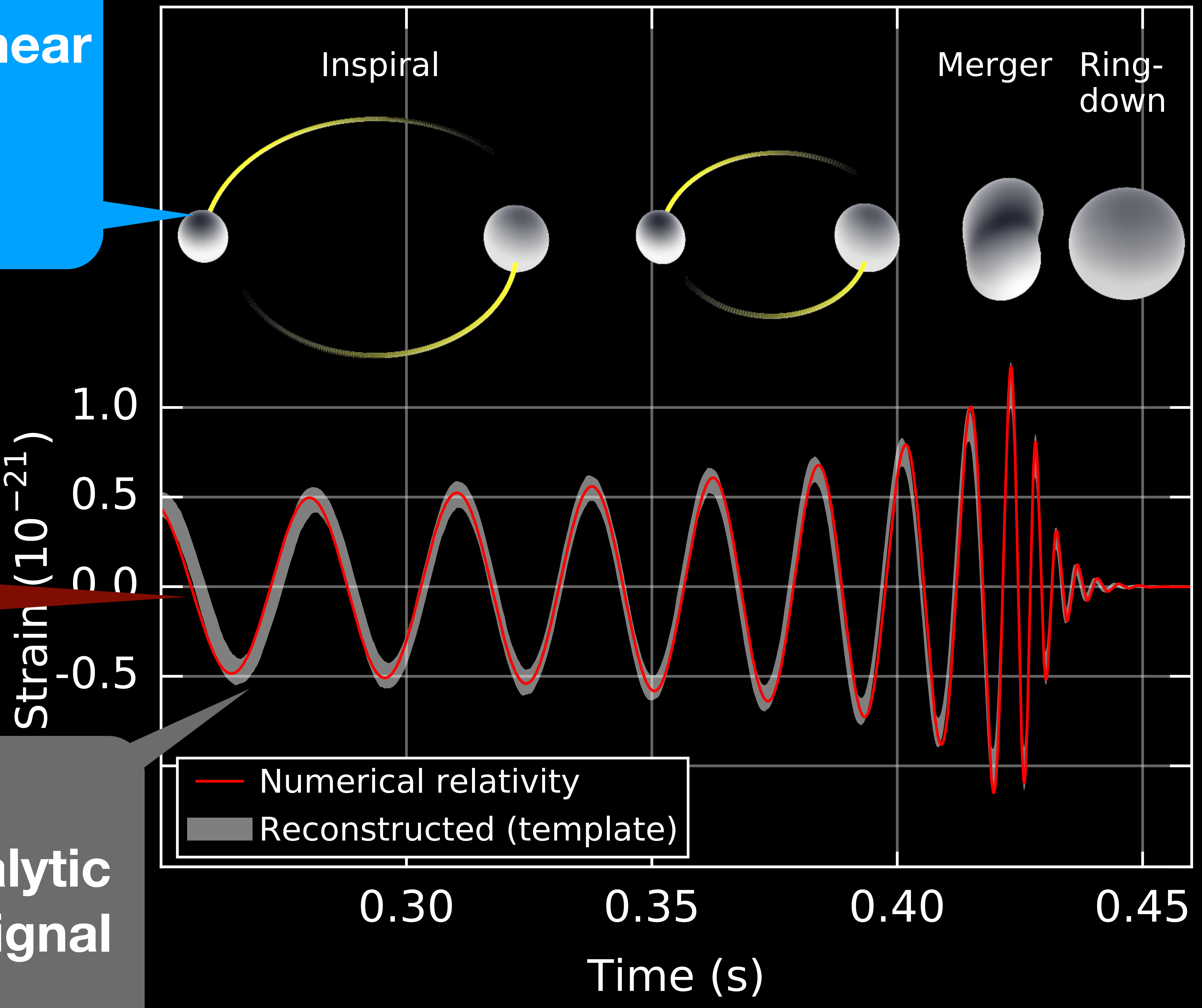
Abbott+ Phys. Rev. Lett. **116**, 061102 (2016) — LIGO GW150914

- Goal: modeling binary black holes and emitted gravitational waves
    - Need numerical relativity: solve Einstein's equations on computers
        - Near time of merger, all analytic approximations fail

Model warped spacetime nonlinear dynamics
Horizons, orbits, curvature, ...

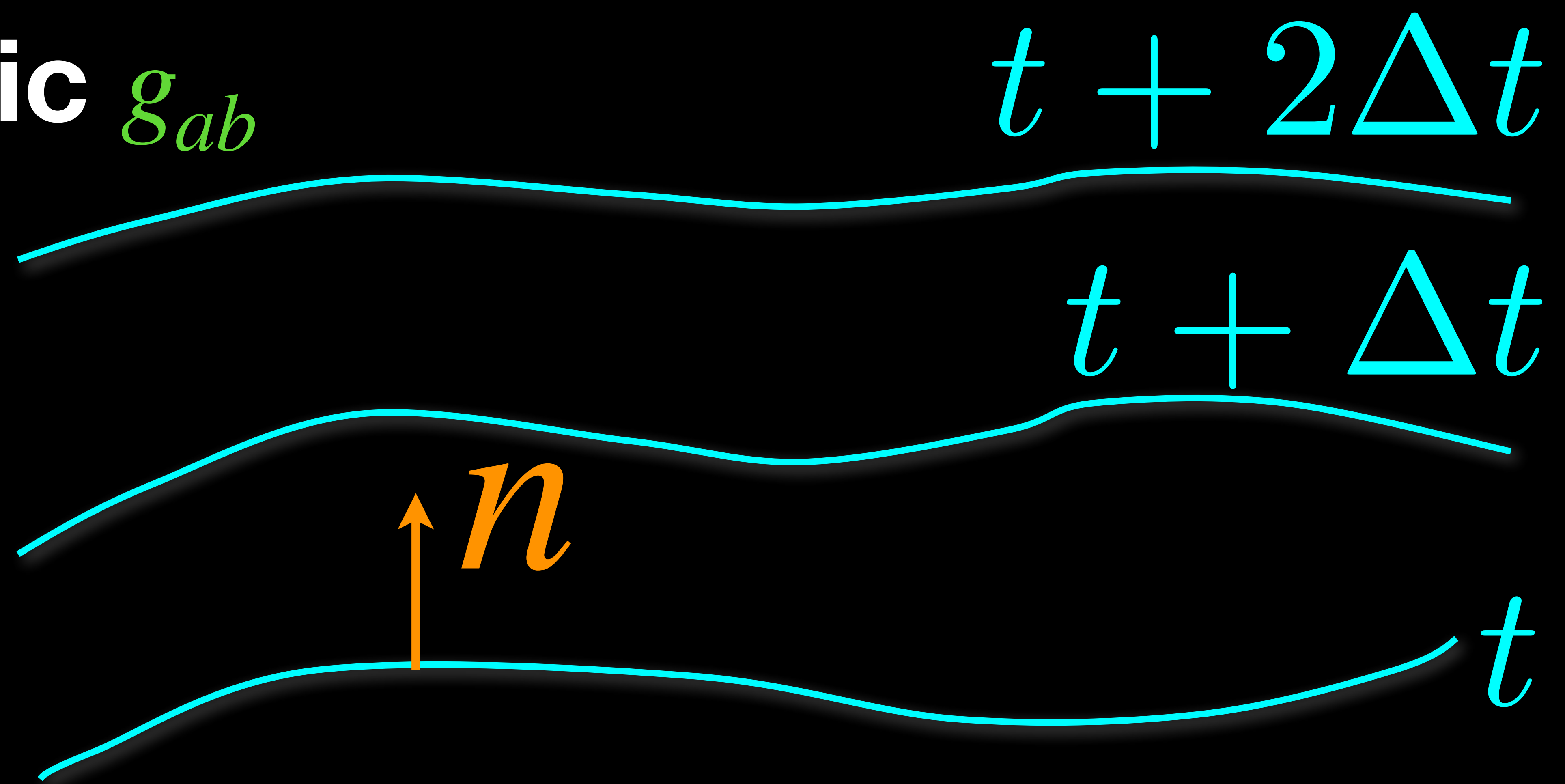
Model emitted gravitational waveforms

Tune & test approximate analytic waveforms for signal reconstruction



Solving Einstein's equations in vacuum

Goal: solve $G_{\mu\nu} = 0$ **for spacetime metric** g_{ab}



- Split spacetime into space + time
  - Constraint equations $G_{nj} = 0 \quad G_{nn} = 0$
    - Solve to create initial data like $\nabla \cdot E = 0, \nabla \cdot B = 0$
  - Evolution equations $G_{ij} = 0$
    - Constraints must stay satisfied

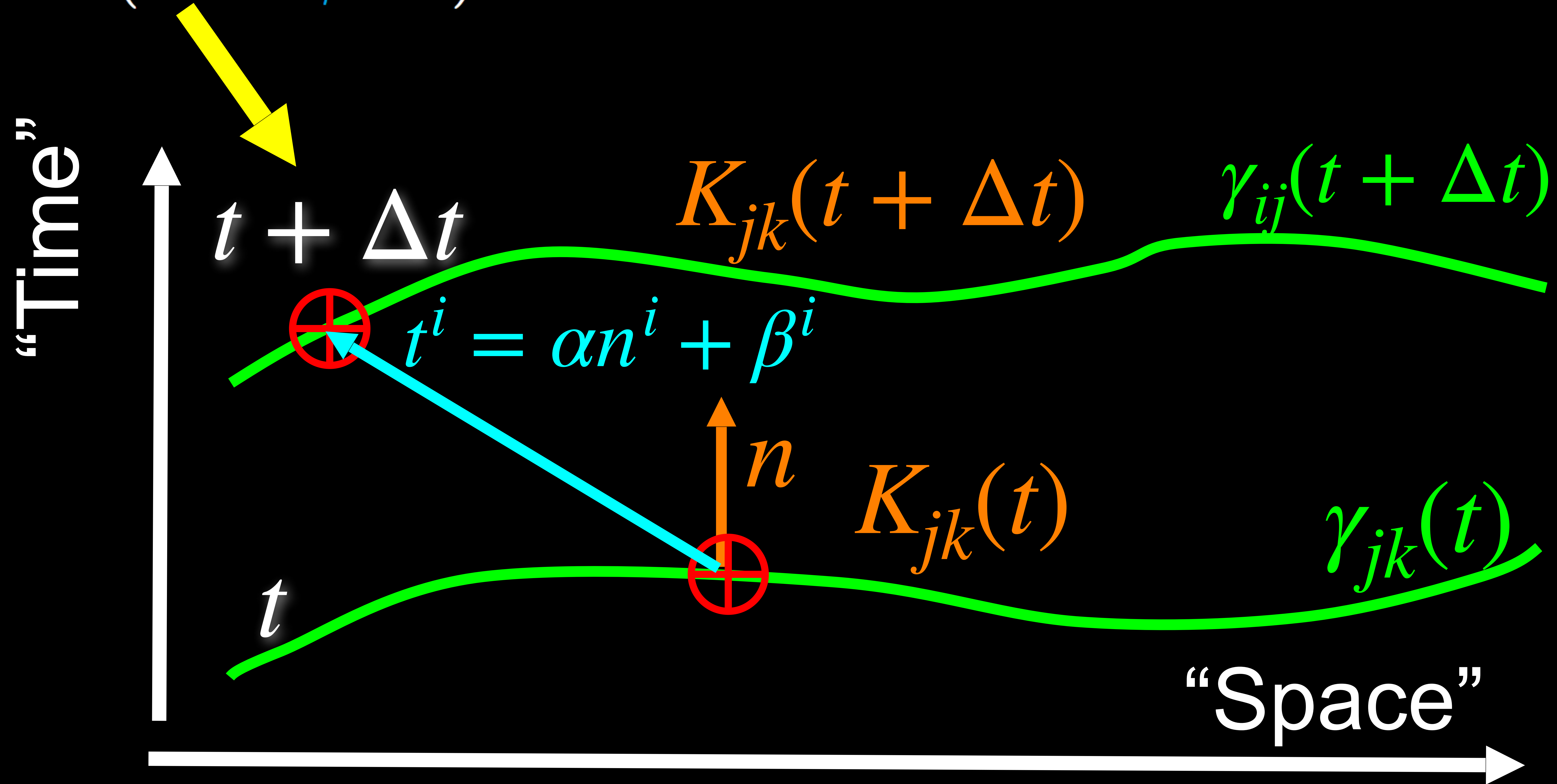
      like $\dfrac{\partial B}{\partial t} = -\nabla \times E,$

      $\dfrac{\partial E}{\partial t} = \nabla \times B$
    - Step 1: Step forward in time
    - Step 2: Repeat step 1 (a lot)

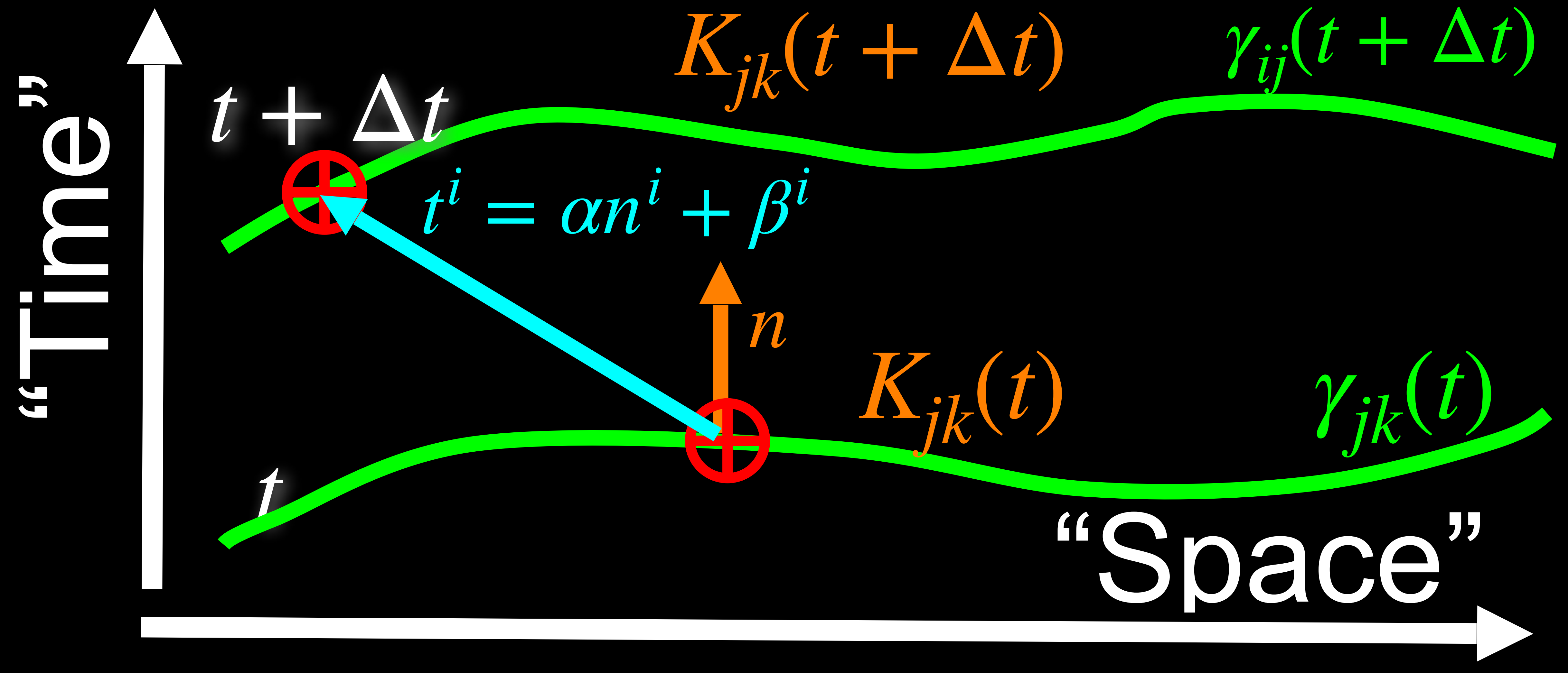
3+1 split

$$ds^2 = -\alpha^2 dt^2 + \gamma_{jk}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

γ_{jk}	spatial metric	slice geometry
K_{jk}	extrinsic curvature	$-1/2(d/dn)\gamma_{jk}$
$\alpha,\ \beta^i$	lapse, shift	slice coordinates



Initial data equations



$$\gamma_{jk} = \psi^4 \tilde{\gamma}_{jk}$$

- Goal: solve constraint equations for γ_{jk}, K_{jk}, α, β^i
- Strategy: conformal decomposition
  - Choose some things (like conformal metric $\tilde{\gamma}_{jk}$)
  - Solve $G_{nn} = 0, G_{nj} = 0$ for the rest
  - Make sure solution really is two black holes with the masses, spins you wanted
- Almost all numerical relativity calculations choose *conformally flat* $\tilde{\gamma}_{jk} = \delta_{jk}$,
  - $G_{nj} = 0$ has analytic solutions in terms of black-hole linear and angular momenta Bowen and York, Jr. (1980)

$$G_{nn} = 0 \rightarrow \nabla^2 \psi + \cdots = 0$$

Initial data equations

What if $\gamma_{ij} \neq \delta_{ij}$?

- One strategy: extended conformal thin sandwich + quaiequilibrium
  York (1999), York & Pfeiffer (20023), Cook (2002), Cook & Pfeiffer (2004)
  - Boundary conditions
    - Excision surface: is (or is slightly inside) marginally trapped surface (i.e. horizon); adjust black-hole spin
    - Outer boundary: asymptotically flat

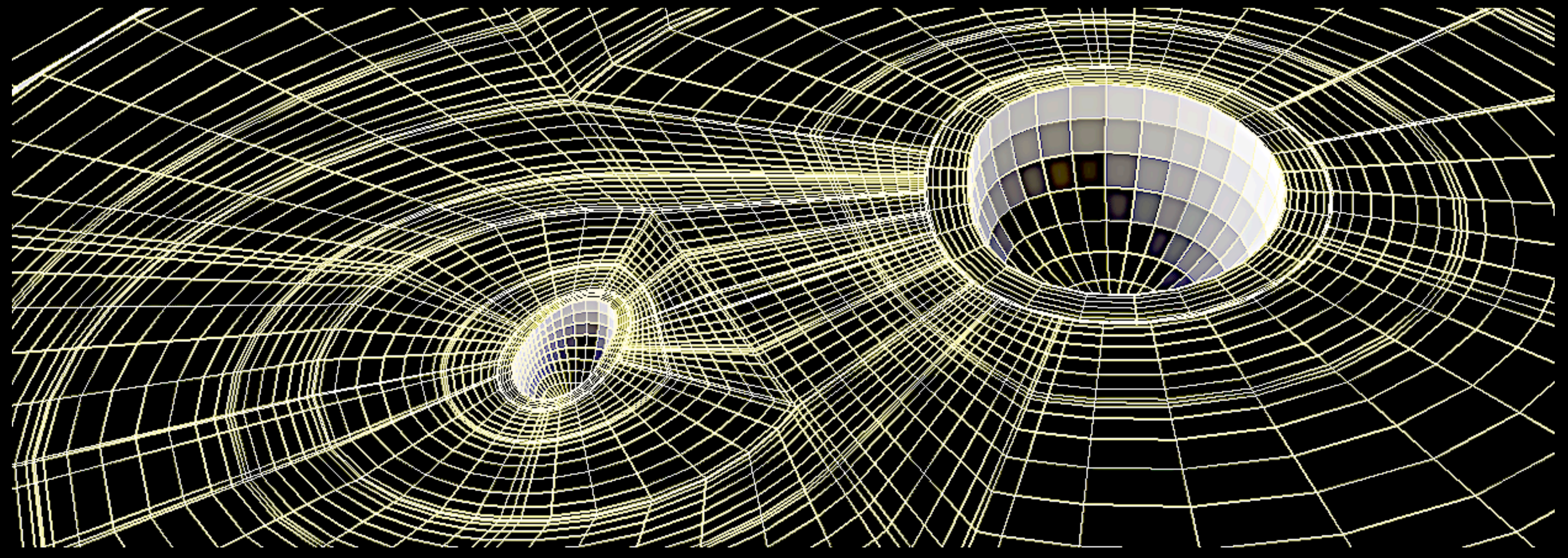
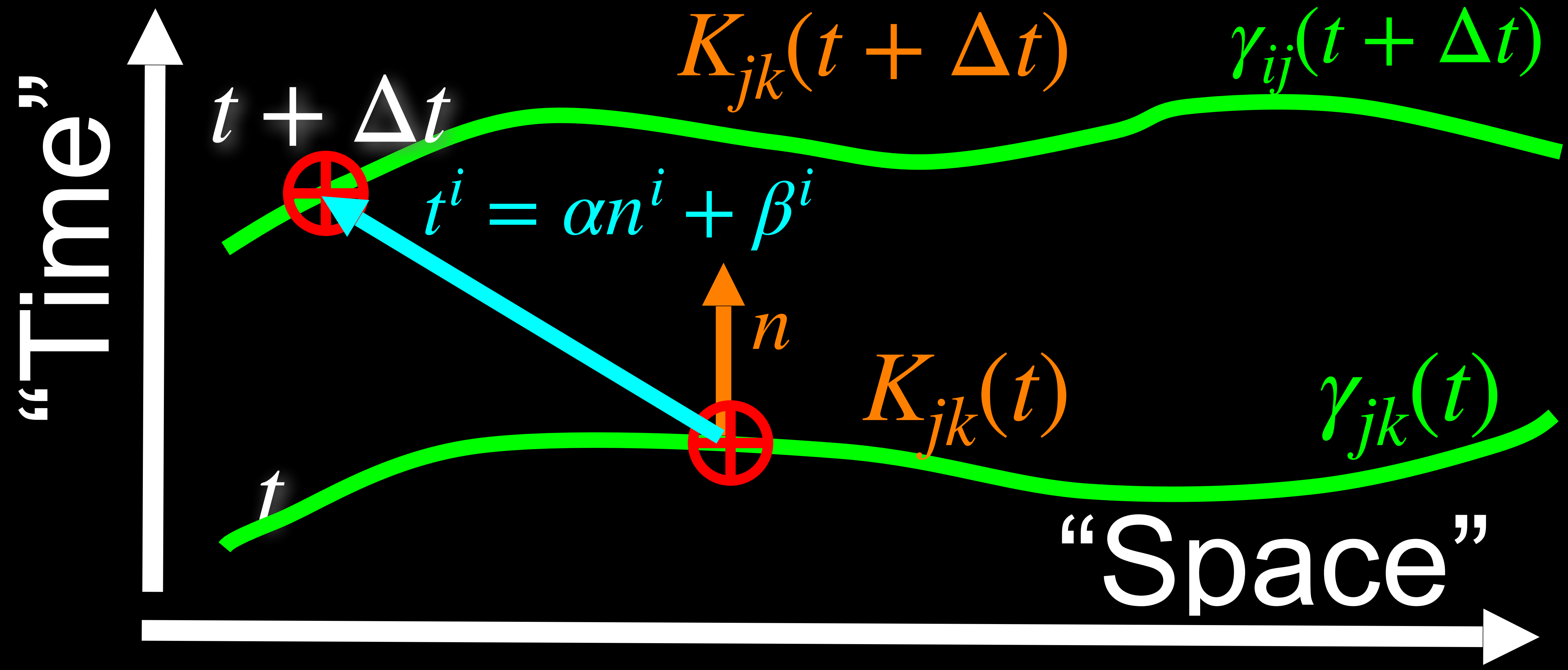


Image courtesy SXS Collaboration



$$\gamma_{jk} = \psi^4 \tilde{\gamma}_{jk}$$

$$G_{nn} = 0 \to \nabla^2 \psi + \cdots = 0$$

$$G_{nj} = 0 \to \nabla_i (\tilde{\mathbb{L}}\beta)^{ji} + \cdots = 0$$

$$\text{one of } G_{ij} = 0 \to \nabla^2 (\alpha\psi) + \cdots = 0$$

Einstein evolution equations $G_{ij} = 0$

An analogy

$$\frac{\partial^2 g}{\partial t^2} = \frac{\partial^2 g}{\partial x^2}$$

$$\Pi \equiv \frac{\partial g}{\partial t}$$

$$\Phi \equiv \frac{\partial g}{\partial x}$$

$$\frac{\partial g}{\partial t} = \Pi$$

$$\frac{\partial \Pi}{\partial t} = \frac{\partial \Phi}{\partial x}$$

$$\frac{\partial \Phi}{\partial t} = \frac{\partial \Pi}{\partial x}$$

Evolution equations

$$\mathscr{C} \equiv \Phi - \frac{\partial g}{\partial x} = 0$$

Constraint equation

Generalized harmonic formulation

$$
\begin{aligned}
\partial_t g_{ab} - (1+\gamma_1)\beta^k\partial_k g_{ab} &= -\alpha\Pi_{ab} - \gamma_1\beta^i\Phi_{iab}, \\
\partial_t \Pi_{ab} - \beta^k\partial_k\Pi_{ab} + \alpha\gamma^{ki}\partial_k\Phi_{iab} - \gamma_1\gamma_2\beta^k\partial_k g_{ab} & \\
= 2\alpha g^{cd}\left(\gamma^{ij}\Phi_{ica}\Phi_{jdb} - \Pi_{ca}\Pi_{db} - g^{ef}\Gamma_{ace}\Gamma_{bdf}\right) & \\
- 2\alpha\nabla_{(a}H_{b)} - \frac{1}{2}\alpha n^c n^d\Pi_{cd}\Pi_{ab} - \alpha n^c\Pi_{ci}\gamma^{ij}\Phi_{jab} & \\
+ \alpha\gamma_0\left(2\delta^c{}_{(a}n_{b)} - (1+\gamma_3)g_{ab}n^c\right)\mathcal{C}_c & \\
+ 2\gamma_4\alpha\Pi_{ab}n^c\mathcal{C}_c & \\
- \gamma_5\alpha n^c\mathcal{C}_c\left(\frac{\mathcal{C}_a\mathcal{C}_b - \frac{1}{2}g_{ab}\mathcal{C}_d\mathcal{C}^d}{\epsilon_5 + 2n^d\mathcal{C}_d n^e\mathcal{C}_e + \mathcal{C}_d\mathcal{C}^d}\right) & \\
- \gamma_1\gamma_2\beta^i\Phi_{iab} & \\
- 16\pi\alpha\left(T_{ab} - \frac{1}{2}g_{ab}T^c{}_c\right), & \\
\partial_t\Phi_{iab} - \beta^k\partial_k\Phi_{iab} + \alpha\partial_i\Pi_{ab} - \alpha\gamma_2\partial_i g_{ab} & \\
= \frac{1}{2}\alpha n^c n^d\Phi_{icd}\Pi_{ab} + \alpha\gamma^{jk}n^c\Phi_{ijc}\Phi_{kab} & \\
- \alpha\gamma_2\Phi_{iab}, &
\end{aligned}
$$

Evolution equations

$$C_a = H_a + g^{ij}\Phi_{ija} + t^b\Pi_{ba} - \frac{1}{2}g^i_a\psi^{bc}\Phi_{ibc} - \frac{1}{2}t_a\psi^{bc}\Pi_{bc}$$

$$H_a \equiv g_{ab}\partial^c\partial_c x^b$$

$$
\begin{aligned}
C_{ia} \equiv\ & g^{jk}\partial_j\Phi_{ika} - \frac{1}{2}g^j_a\psi^{cd}\partial_j\Phi_{icd} + t^b\partial_i\Pi_{ba} - \frac{1}{2}t_a\psi^{cd}\partial_i\Pi_{cd} \\
& + \partial_i H_a + \frac{1}{2}g^j_a\Phi_{jcd}\Phi_{ief}\psi^{ce}\psi^{df} + \frac{1}{2}g^{jk}\Phi_{jcd}\Phi_{ike}\psi^{cd}t^e t_a \\
& - g^{jk}g^{mn}\Phi_{jma}\Phi_{ikn} + \frac{1}{2}\Phi_{icd}\Pi_{be}t_a\left(\psi^{cb}\psi^{de} + \frac{1}{2}\psi^{be}t^c t^d\right) \\
& - \Phi_{icd}\Pi_{ba}t^c\left(\psi^{bd} + \frac{1}{2}t^b t^d\right) + \frac{1}{2}\gamma_2\left(t_a\psi^{cd} - 2\delta^c_a t^d\right)C_{icd}
\end{aligned}
$$

$$C_{iab} = \partial_i g_{ab} - \Phi_{iab}$$

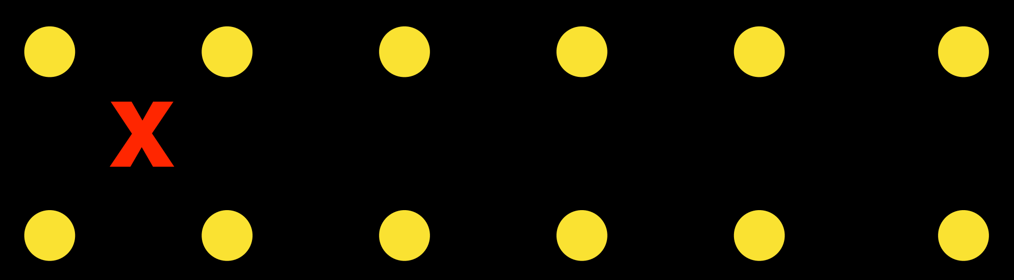
$$C_{ijab} = 2\partial_{[i}\Phi_{j]ab}$$

$$
\begin{aligned}
\mathcal{F}_a \equiv\ & \frac{1}{2}g^i_a\psi^{bc}\partial_i\Pi_{bc} - g^{ij}\partial_i\Pi_{ja} - g^{ij}t^b\partial_i\Phi_{jba} + \frac{1}{2}t_a\psi^{bc}g^{ij}\partial_i\Phi_{jbc} \\
& + t_a g^{ij}\partial_i H_j + g^i_a\Phi_{ijb}g^{jk}\Phi_{kcd}\psi^{bd}t^c - \frac{1}{2}g^i_a\Phi_{ijb}g^{jk}\Phi_{kcd}\psi^{cd}t^b \\
& - g^i_a t^b\partial_i H_b + g^{ij}\Phi_{icd}\Phi_{jba}\psi^{bc}t^d - \frac{1}{2}t_a g^{ij}g^{mn}\Phi_{imc}\Phi_{njd}\psi^{cd} \\
& - \frac{1}{4}t_a g^{ij}\Phi_{icd}\Phi_{jbe}\psi^{cb}\psi^{de} + \frac{1}{4}t_a\Pi_{cd}\Pi_{be}\psi^{cb}\psi^{de} - g^{ij}H_i\Pi_{ja} \\
& - t^b g^{ij}\Pi_{bi}\Pi_{ja} - \frac{1}{4}g^i_a\Phi_{icd}t^c t^d\Pi_{be}\psi^{be} + \frac{1}{2}t_a\Pi_{cd}\Pi_{be}\psi^{ce}t^d t^b \\
& + g^i_a\Phi_{icd}\Pi_{be}t^c t^b\psi^{de} - g^{ij}\Phi_{iba}t^b\Pi_{je}t^e - \frac{1}{2}g^{ij}\Phi_{icd}t^c t^d\Pi_{ja} \\
& - g^{ij}H_i\Phi_{jba}t^b + g^i_a\Phi_{icd}H_b\psi^{bc}t^d + \gamma_2\left(g^{id}\mathcal{C}_{ida} - \frac{1}{2}g^i_a\psi^{cd}\mathcal{C}_{icd}\right) \\
& + \frac{1}{2}t_a\Pi_{cd}\psi^{cd}H_b t^b - t_a g^{ij}\Phi_{ijc}H_d\psi^{cd} + \frac{1}{2}t_a g^{ij}H_i\Phi_{jcd}\psi^{cd} \\
& - 16\pi t^a T_{ab}
\end{aligned}
$$

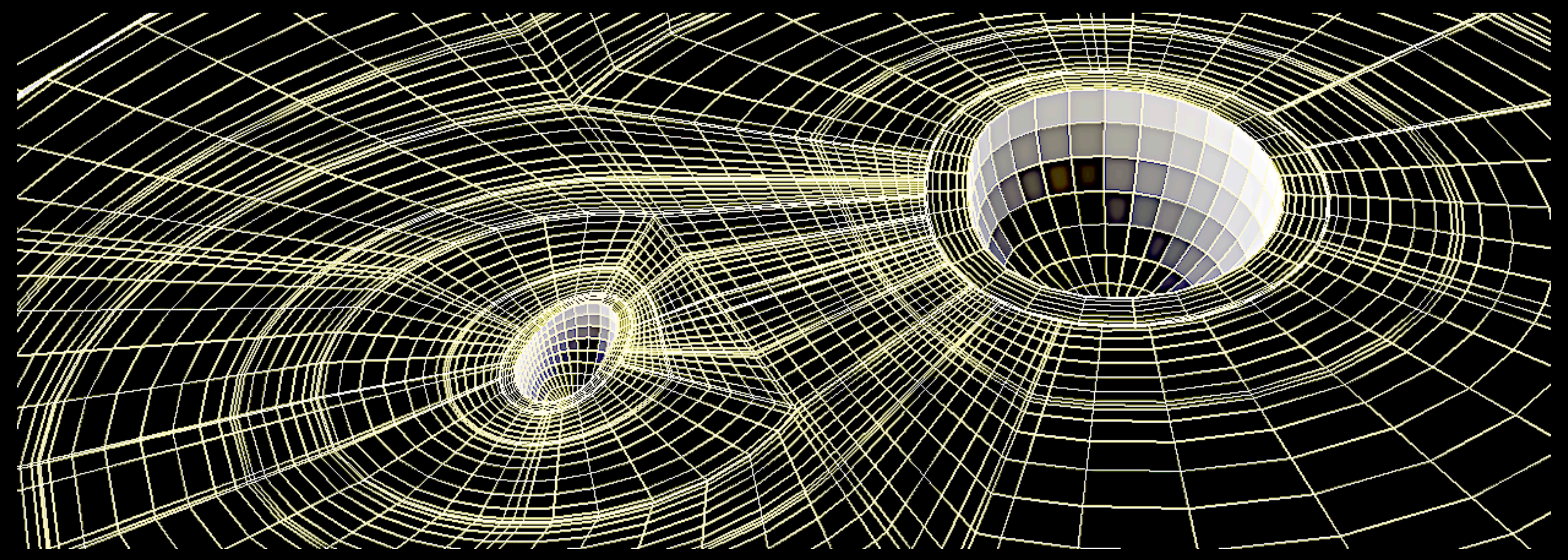
Constraint equations

Current-generation numerical relativity

Initial data	Puncture data	Quasiequilibrium with excision
	Pseudospectral	Pseudospectral
	(Usually) solve 1 elliptic eq.	Solve 4 or 5 elliptic eqs.
Evolution	BSSN/CCZ4 evolution eqs.	Generalized harmonic evolution eqs.
	High-order finite-difference	Pseudospectral
	Moving puncture	Excision
Codes	BAM, Hahndol, LazEv, Lean, Llama, MayaKranc, UIUC, Einstein Toolkit,...	**SpEC**



$$f'(x_i) \sim \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}}$$



$$f'(x_i) \sim \sum_n a_n b'(x_i)$$

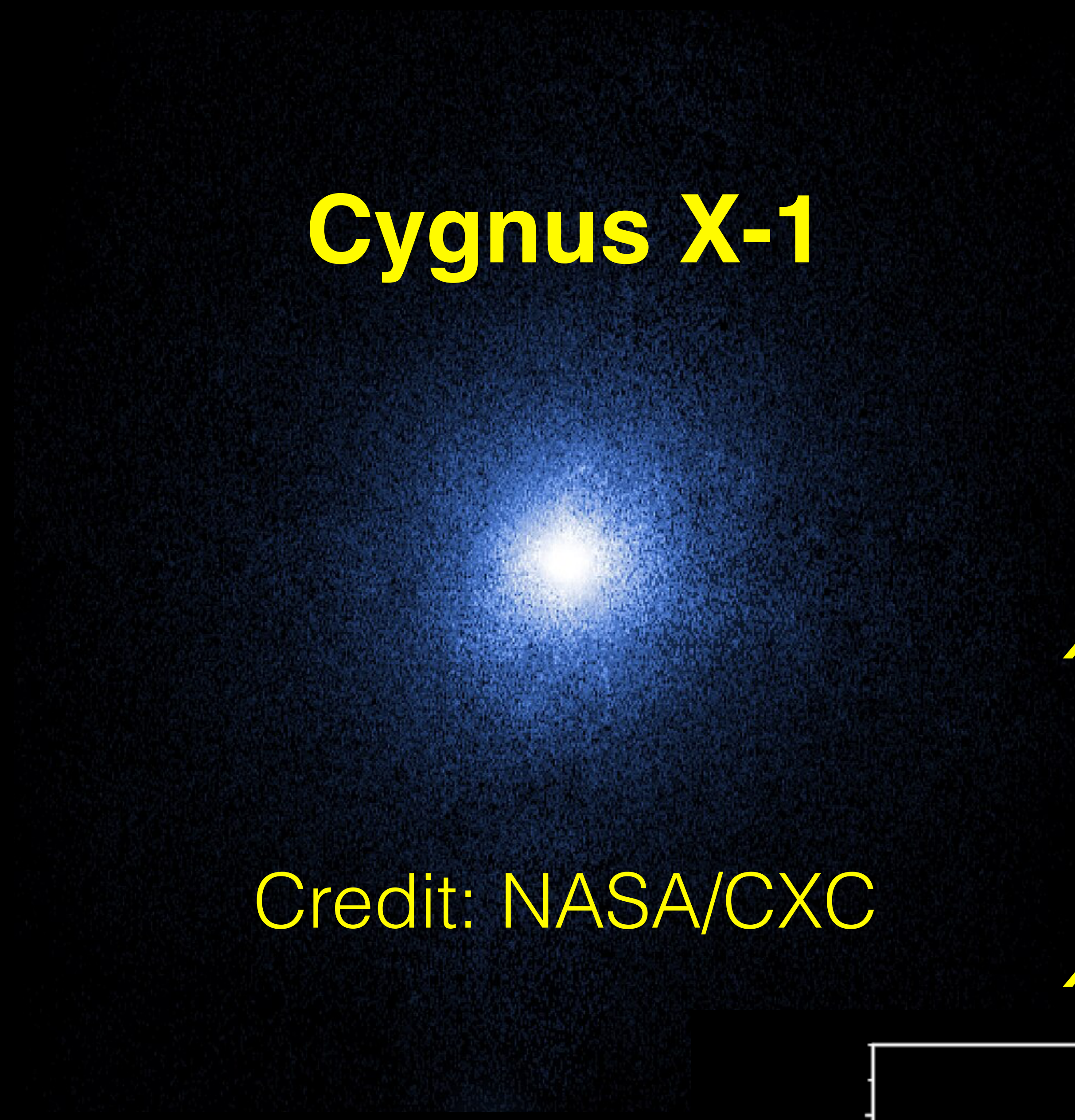
Advantages	Robust, open-source	Efficient

Review: e.g. Pfeiffer (2012)

Image courtesy SXS Collaboration

Do black holes spin?

- Extremal if $\chi = S/M^2 = 1$
    - S=spin angular momentum, M =mass, $G = c = 1$
- Electromagnetic waves
    - Some high-spin claims, but spin inference via uncertain accretion-disk models
- Gravitational waves
    - Spin only weakly affects waves
    - A few (of hundreds) show evidence for nonzero spin
    - **GW231123**: black holes have spins consistent with extremal: $\chi_1 = 0.9^{+0.10}_{-0.19}$ and $\chi = 0.8^{+0.2}_{-0.51}$

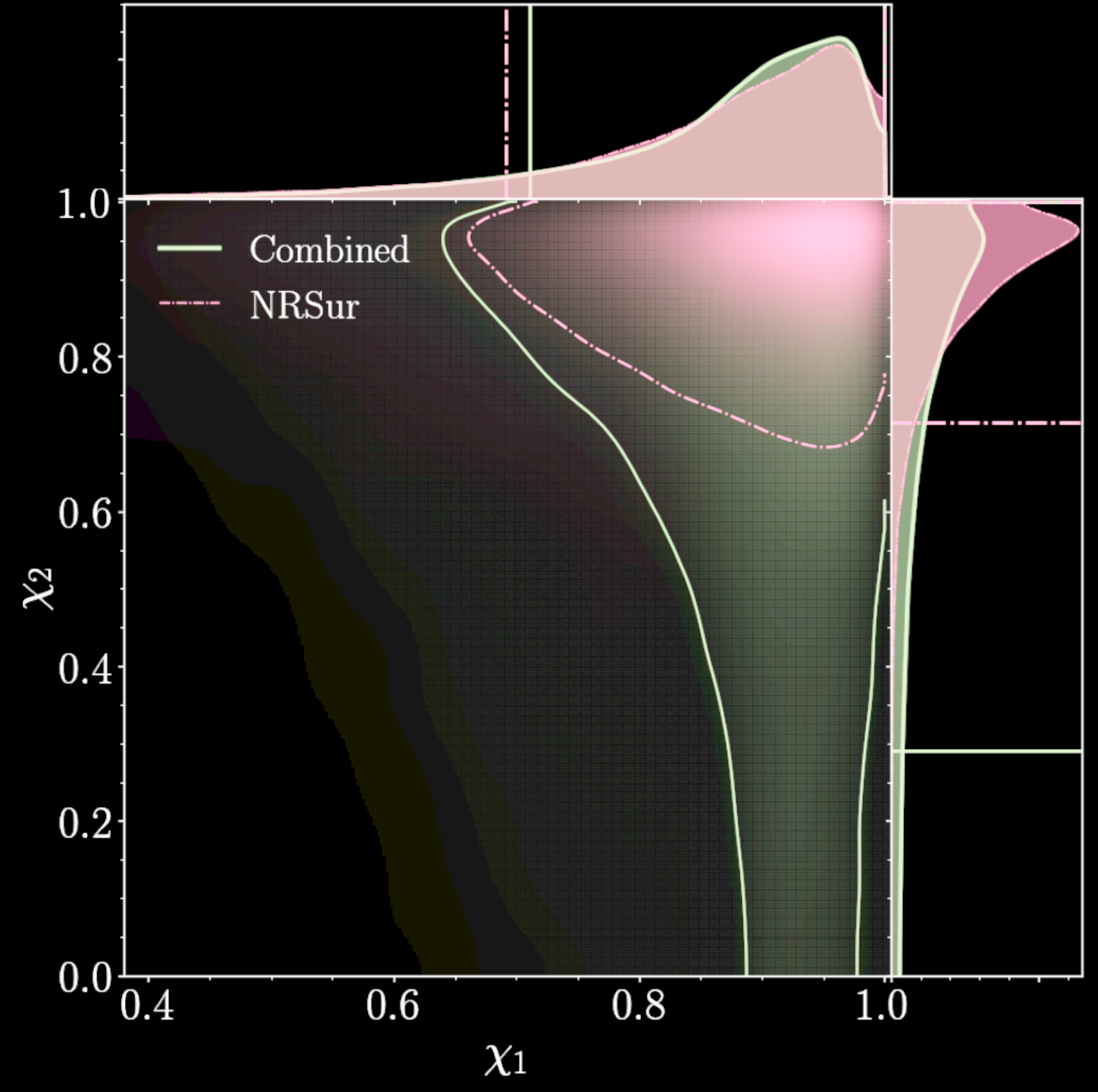


Credit: NASA/CXC

*Zdziarski+ ApJL **967** L9 (2024):*
"Standard"
$\chi = 0.985^{+006}_{-0.004}$

Fit inclination, color correct.
$\chi = 0.88^{+0.04}_{-0.01}$

"Nonstandard"
$\chi = 0.04^{+0.26}_{-0.04}$



*LIGO, Virgo, Kagra collaborations, Abac+ ApJL **993** L25 (2025)*
GW231123
spin posteriors

Extremality measures for numerical relativity

$$\chi \equiv \frac{S}{M^2}$$

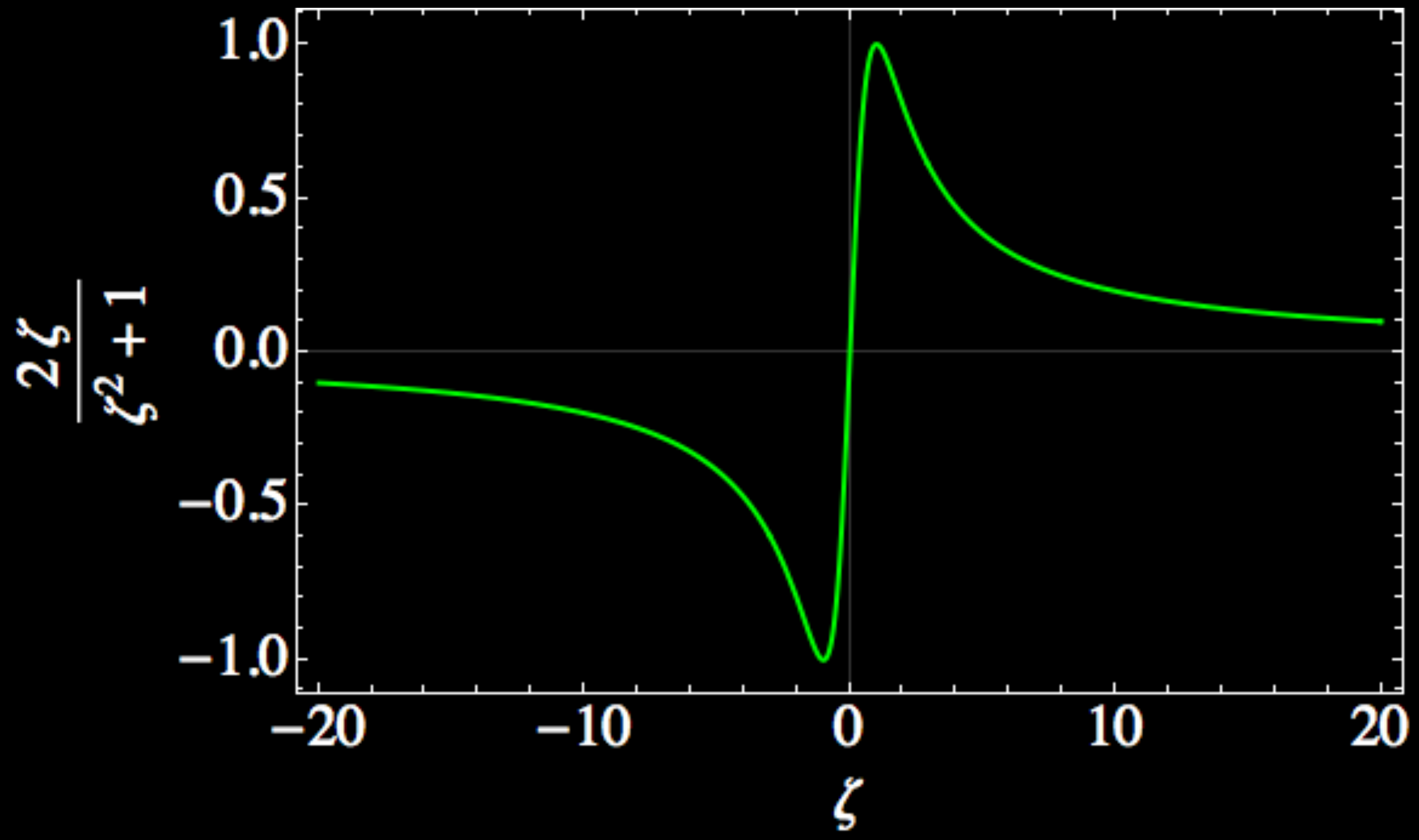
$$\zeta \equiv \frac{8\pi S}{A}$$

- Measure on (apparent) horizon *H*
- Horizon surface area $A \equiv \oint_{\mathcal{H}} dA$
- Mass *M*
  - Christodoulou mass

$$M^2 = \frac{A}{16\pi}\left(1+\zeta^2\right)$$

  - Spin limit obeyed by construction

$$\chi \equiv \frac{S}{M^2} = \frac{2\zeta}{1+\zeta^2}$$



Extremality & axisymmetry

$$\zeta \equiv \frac{8\pi S}{A}$$

$$A \equiv \oint_{\mathcal{H}} dA$$

- Spin angular momentum

$$S \equiv \frac{1}{8\pi} \oint_{\mathcal{H}} \omega_B \phi^B dA$$

$$\omega_B = \text{“angular momentum density”}$$
$$\phi^B = \text{“azimuthal vector about spin axis”}$$

- If axisymmetry…
  - ϕ^B is a rotational Killing vector, S conserved
  - Inequality $\zeta \leq 1$ proven under broad assumptions
    JL Jaramillo, M Reiris and S Dain, Phys. Rev. D **84** 121503 (2011)
    [review of black-hole inequalities proven: Dain Class. Quantum Grav. **29**, 073001 (2012)]

Extremality without axisymmetry

$$\zeta \equiv \frac{8\pi S}{A}$$

$$A \equiv \oint_{\mathcal{H}} dA$$

$$S \equiv \frac{1}{8\pi} \oint_{\mathcal{H}} \omega_B \phi^B dA$$

- Simulations not axisymmetric
  - Find best approx. symmetry
  - ϕ^B is approx. Killing vector

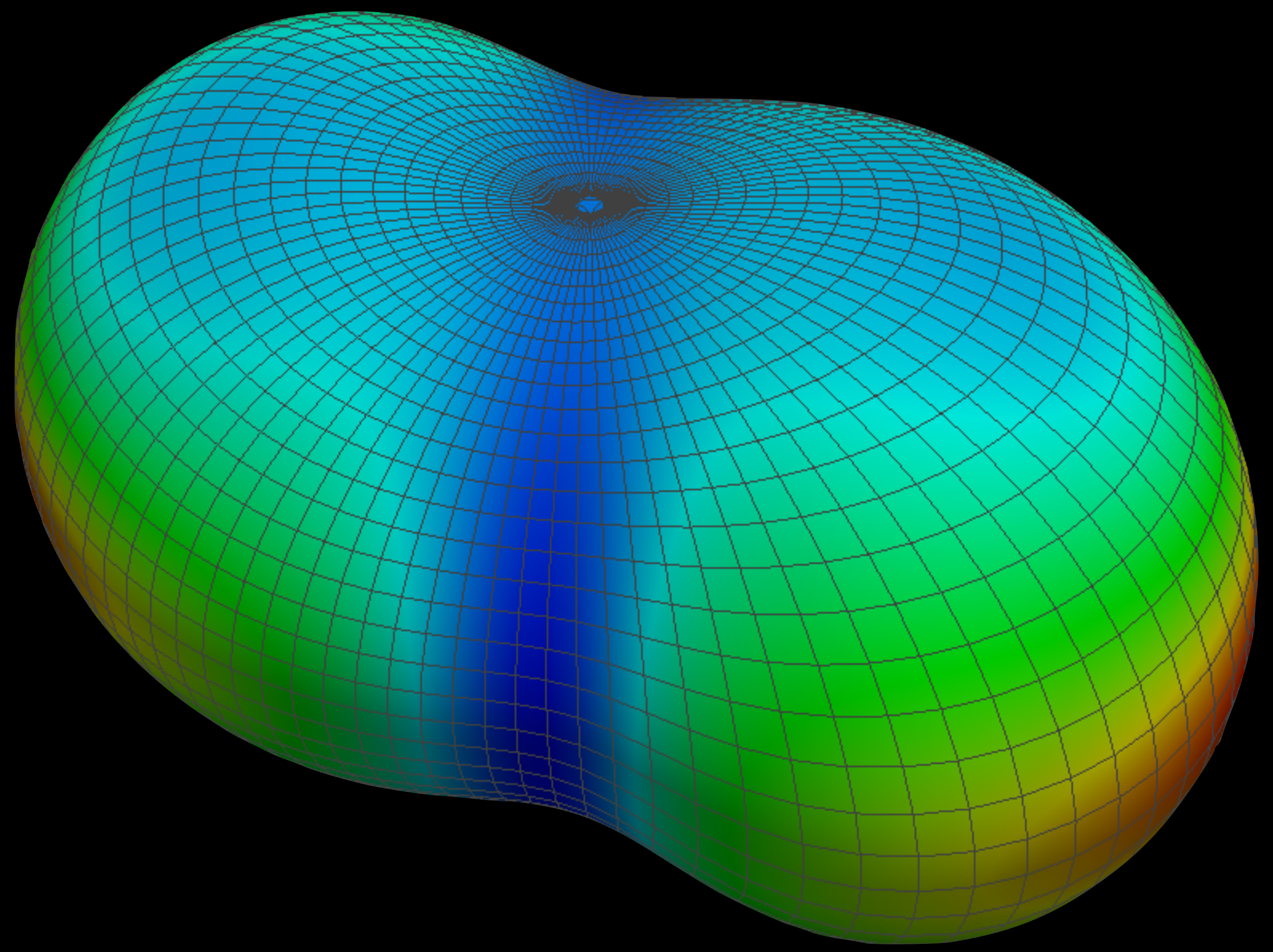
    Dreyer+, PRD **67**, 024018 (2003)
    Cook and Whiting, PRD **76,** 041501(R) (2007)
    Owen (2007), Ph.D. thesis.
    GL+ (2008), PRD **78**, 084017 (2008)
  - Depends on horizon's null normals $\vec{\ell}, \vec{n}$
  - Boost gauge invariant

$$\vec{\ell} \cdot \vec{n} = -1$$

If rescale normals… …*S* unchanged

$$\vec{\ell} \to e^a \vec{\ell} \qquad S \to S$$

$$\vec{n} \to e^{-a} \vec{n}$$



Common apparent horizon
just after two χ=0.994 holes merge
(color = intrinsic Ricci scalar curvature)

Extremality lower bound

$$\zeta \equiv \frac{8\pi S}{A}$$

$$A \equiv \oint_{\mathcal{H}} dA$$

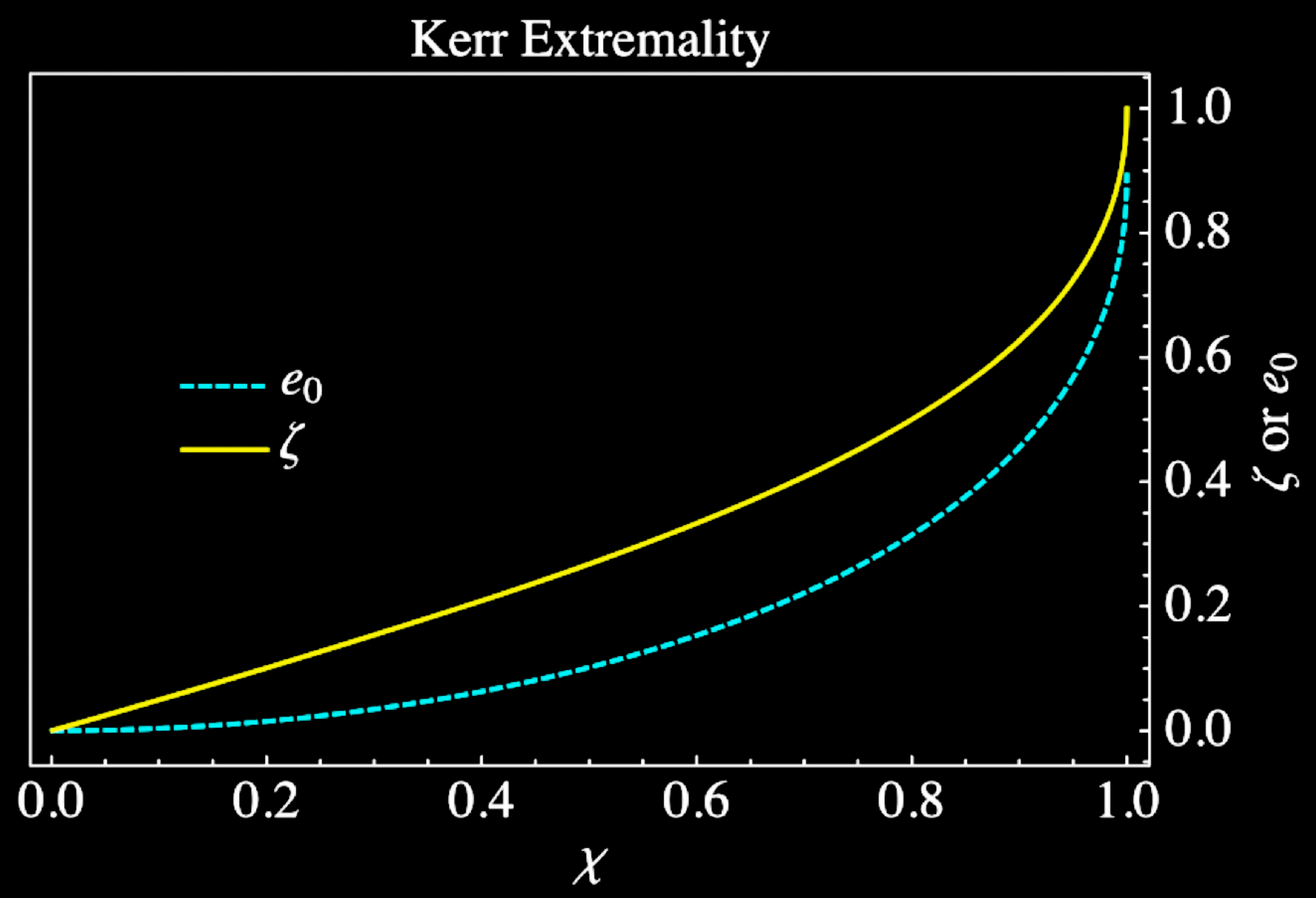
$$S \equiv \frac{1}{8\pi} \oint_{\mathcal{H}} \omega_B \phi^B dA$$

- Booth & Fairhurst extremality
  Booth & Fairhurst, PRD **77,** 084005 (2008)

$$e \equiv \frac{1}{4\pi} \oint_{\mathcal{H}} \omega_B \omega^B dA$$

- –No need for approximate symmetry
- –But: must choose scaling for horizon null normals
  - This talk: scale to minimize *e*

$$e_0 \equiv \min \left[\frac{1}{4\pi} \oint_{\mathcal{H}} \omega_B \omega^B dA \right]$$

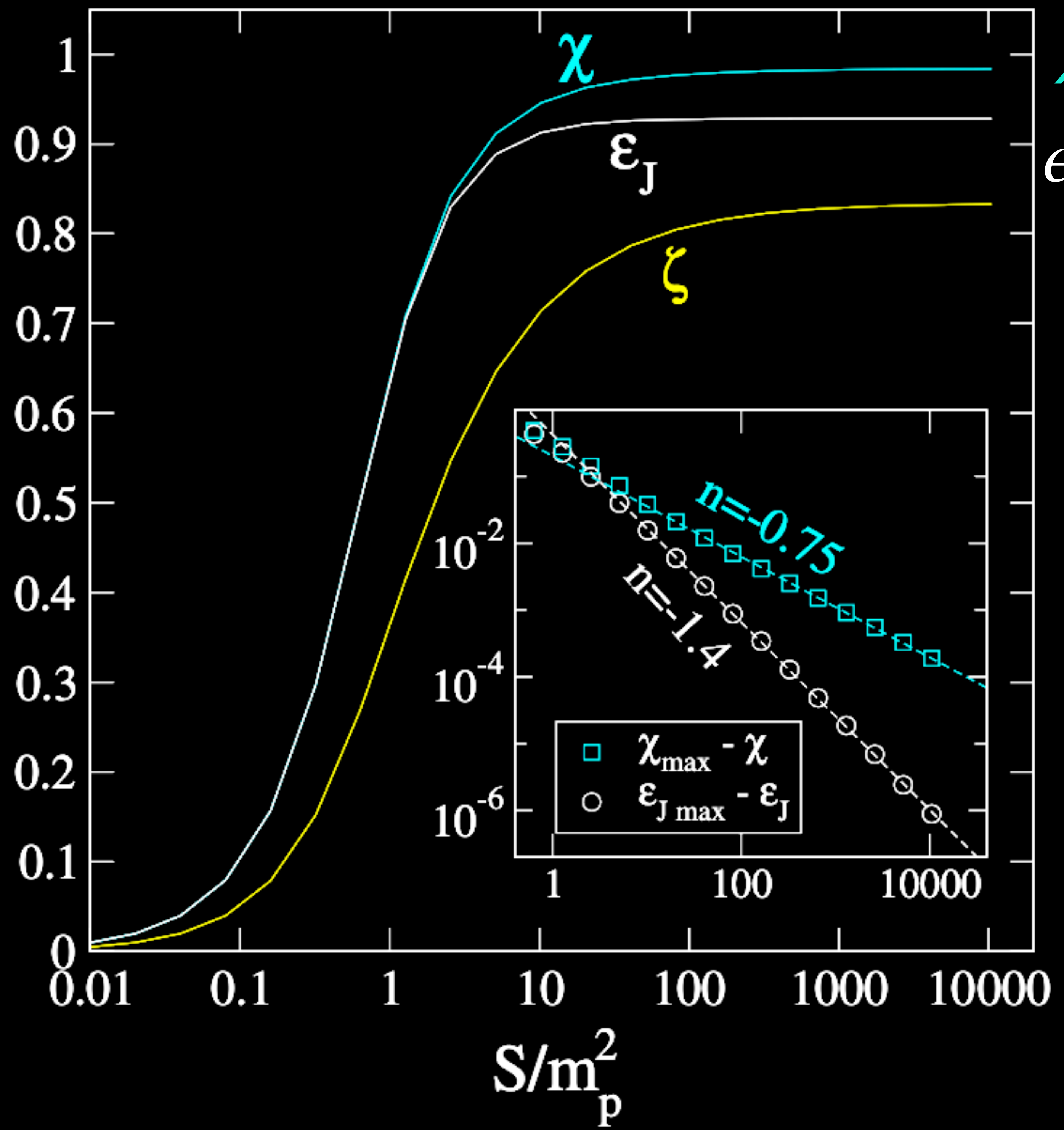


GL+ Class. Quantum Grav. **32**, 065007 (2015)

Challenges

Binary-black-hole initial data with nearly extremal spins

- Why conformally flat does't work for high spins
- ***Excision***: conformally curved (solve 5 coupled PDEs instead of 1) for $\chi \gtrsim 0.93$
  GL+ Phys. Rev. D **78**, 084017 (2008)
- ***Puncture***: conformally curved (solve 4 coupled PDEs instead of 1), coordinates compensate for horizon radius vanishing as $\chi \to 1$ for $\chi \gtrsim 0.93$
  Ruchlin+, Phys. Rev. D **95**, 024033 (2017)



$\chi_{\max} = 0.9837$

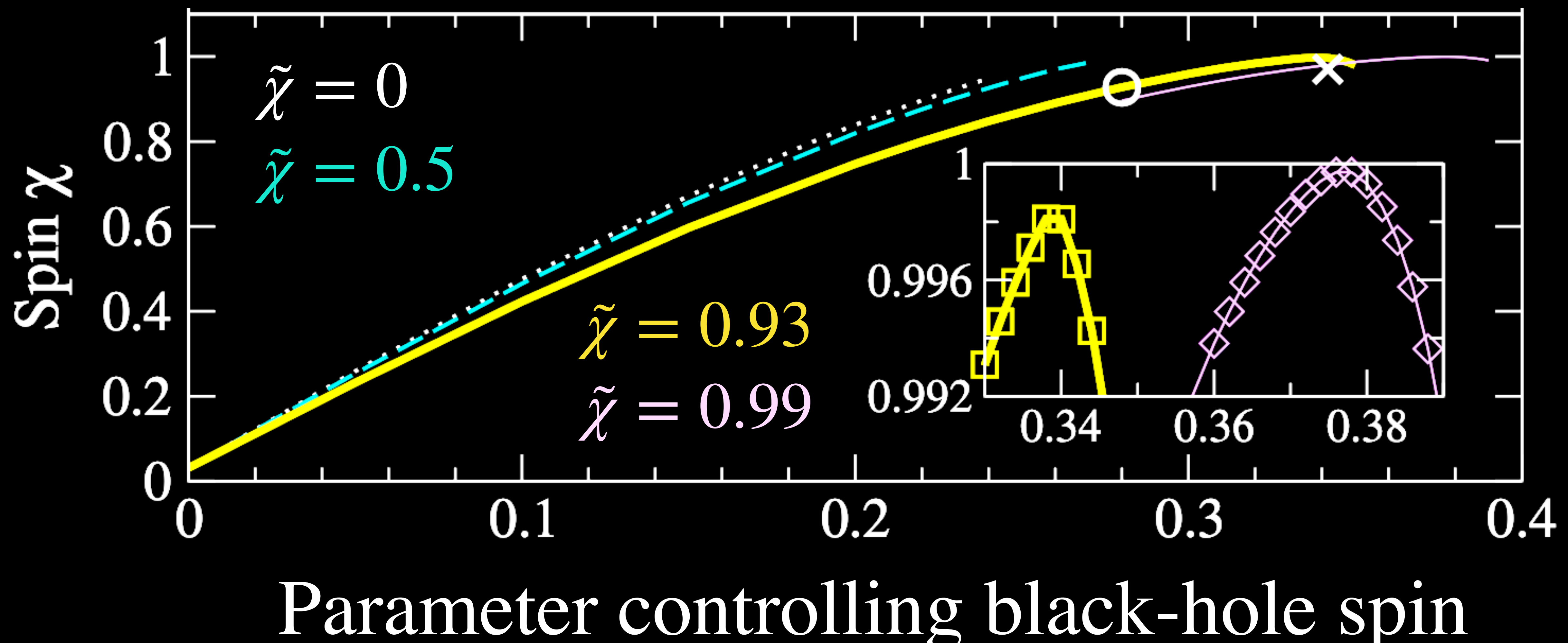
$\epsilon_{J\,\max} = 0.9282$

$= J_{\rm ADM}/E^2_{\rm ADM}$

$$g_{ij} = \psi^4 \delta_{ij}$$

Figures from GL+ Phys. Rev. D **78**, 084017 (2008)

$$g_{ij} = \psi^4\left(e^{-r_A^2/w^2}\tilde{g}_{ij}^{\rm Kerr-Schild\ A} + e^{-r_B^2/w^2}\tilde{g}_{ij}^{\rm Kerr-Schild\ B}\right)$$



Challenges

Evolving binary black holes with extremal spins

- Excision: need high resolution, excision delicate, no precession
  GL+ Phys. Rev. D **83**, 024010 (2011)
  GL+ Class. Quantum Grav. **29**, 045003 (2012)
  Scheel+, ... GL+ Class. Quantum Grav **32**, 105009 (2015)
  Boyle+ Class. Quantum Grav. **36**, 195006 (2019) (2019)
  - No boundary condition on excision surfaces
    - Well-posedness requires no incoming characteristic speeds (no incoming info) at excision boundaries
- Moving-puncture: in usual coordinates, horizon size vanishes for extremal black hole

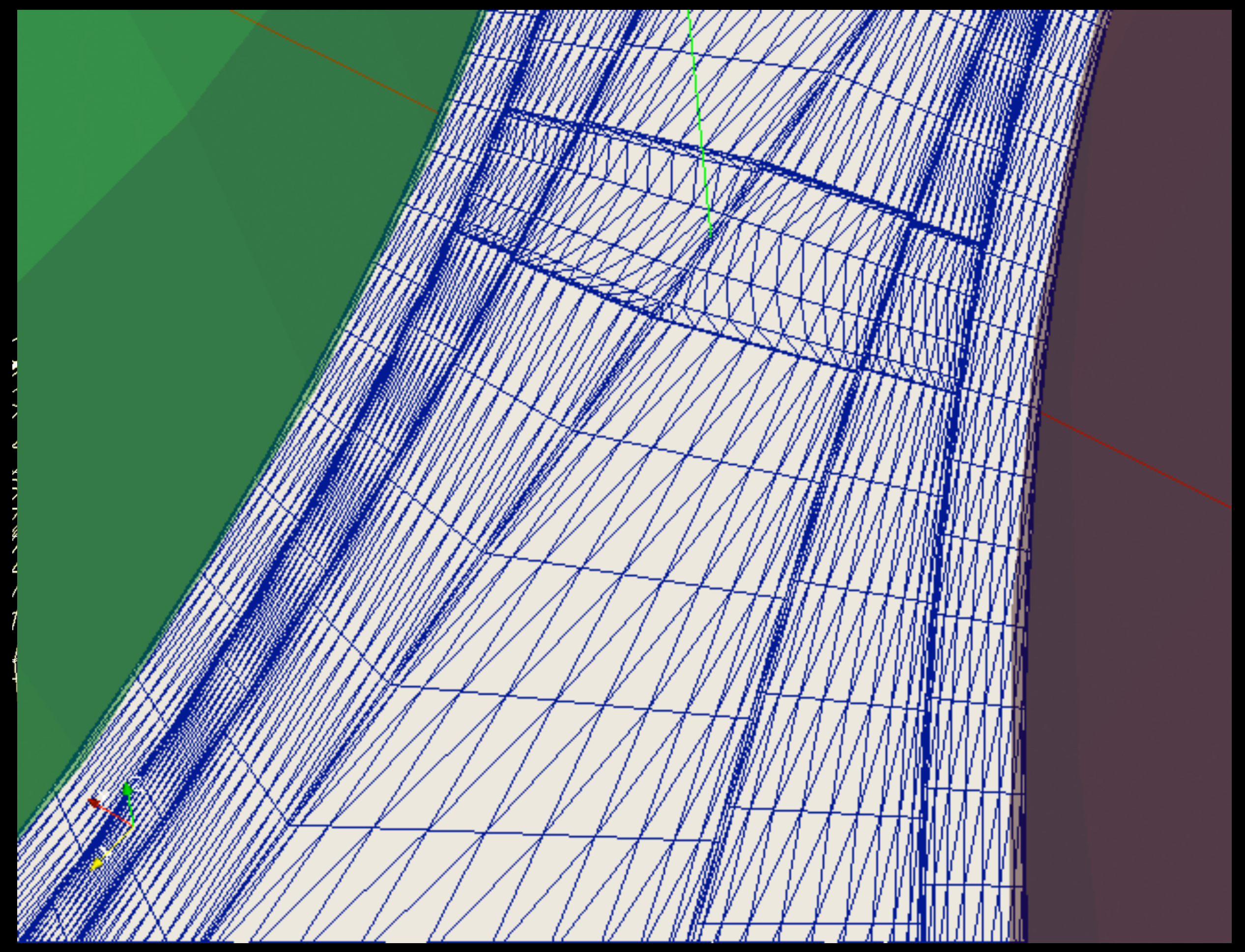
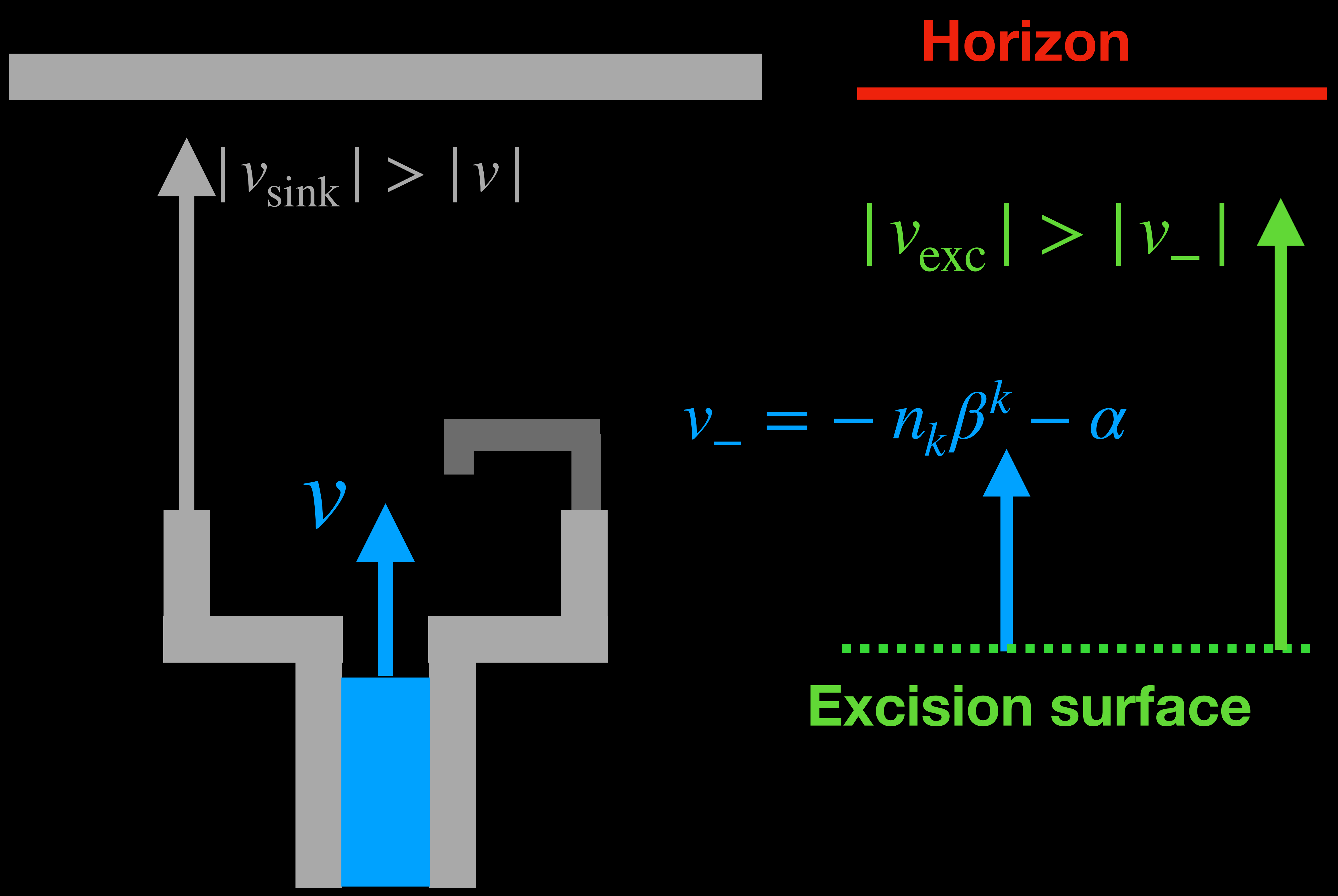


Figure courtesy Mark Scheel



An optimization

- Spherical Kerr Schild
  - New gauge that reduces resolution needed by keeping horizons coordinate spheres during inspiral
    Chen+ Phys. Rev. D **104,** 084046 (2021)
  - ~2x faster inspiral at same quality (constraint violation)

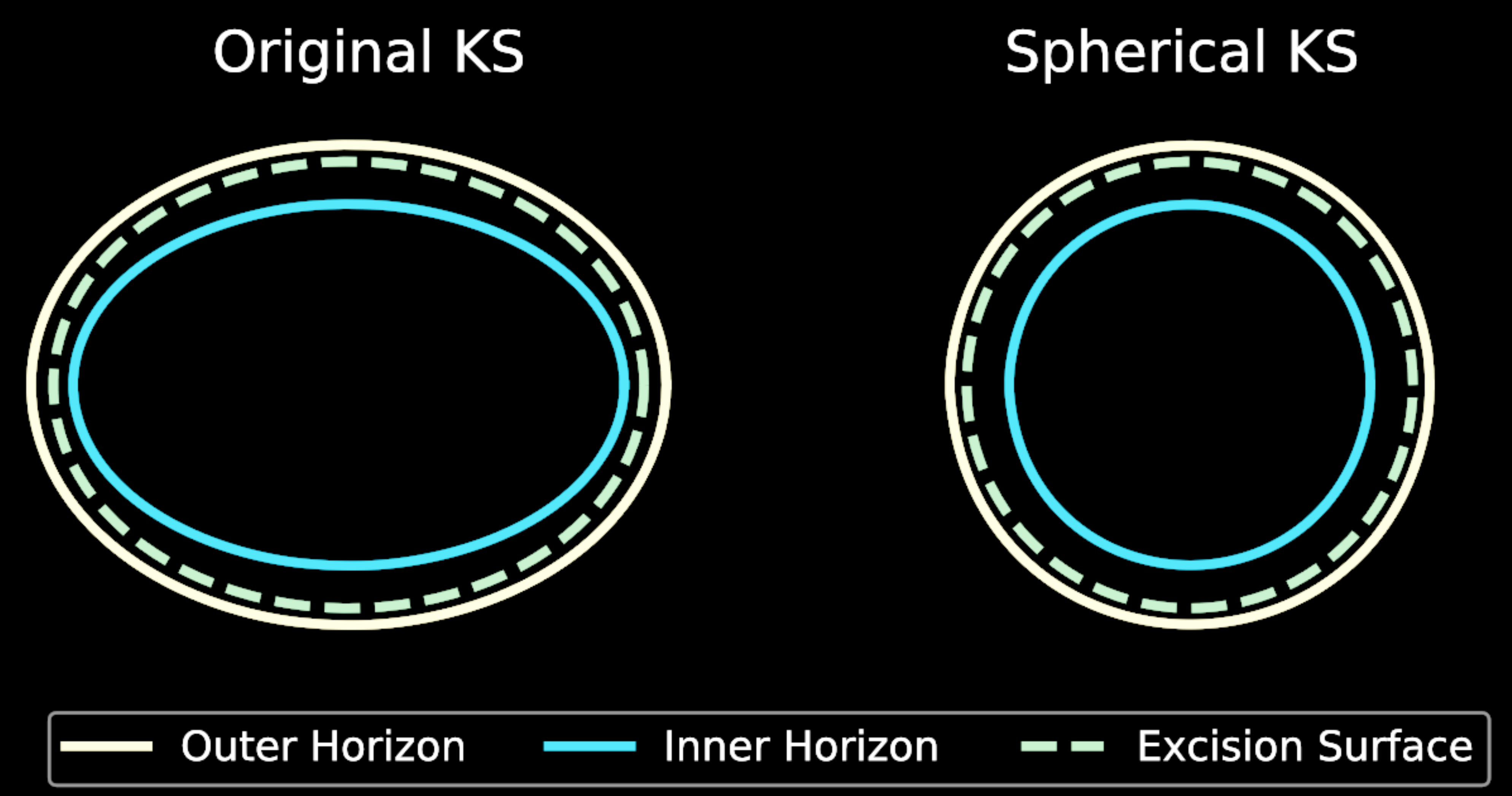
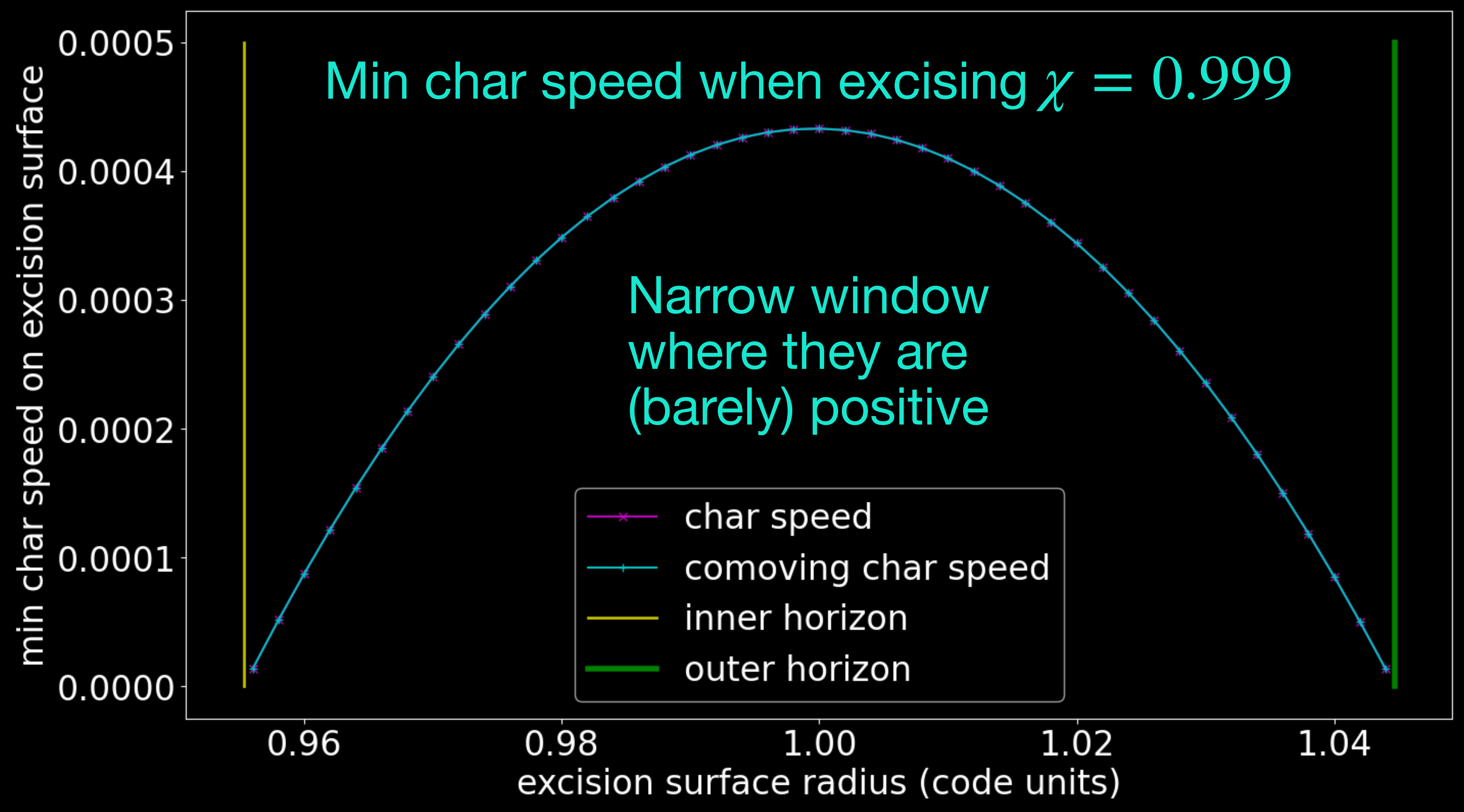
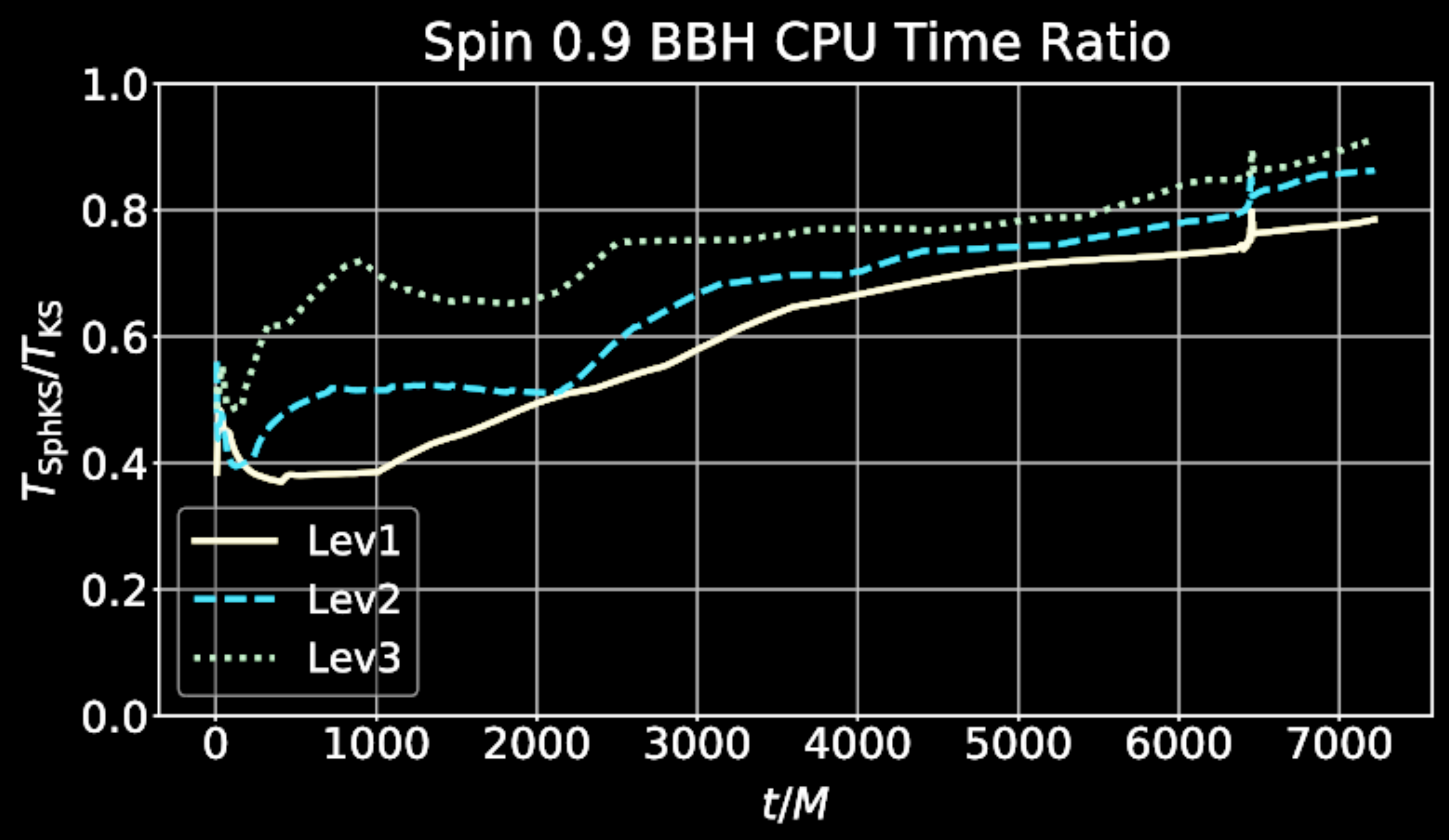


Figure & plot courtesy Chen+ Phys. Rev. D **104**, 084046 (2021)

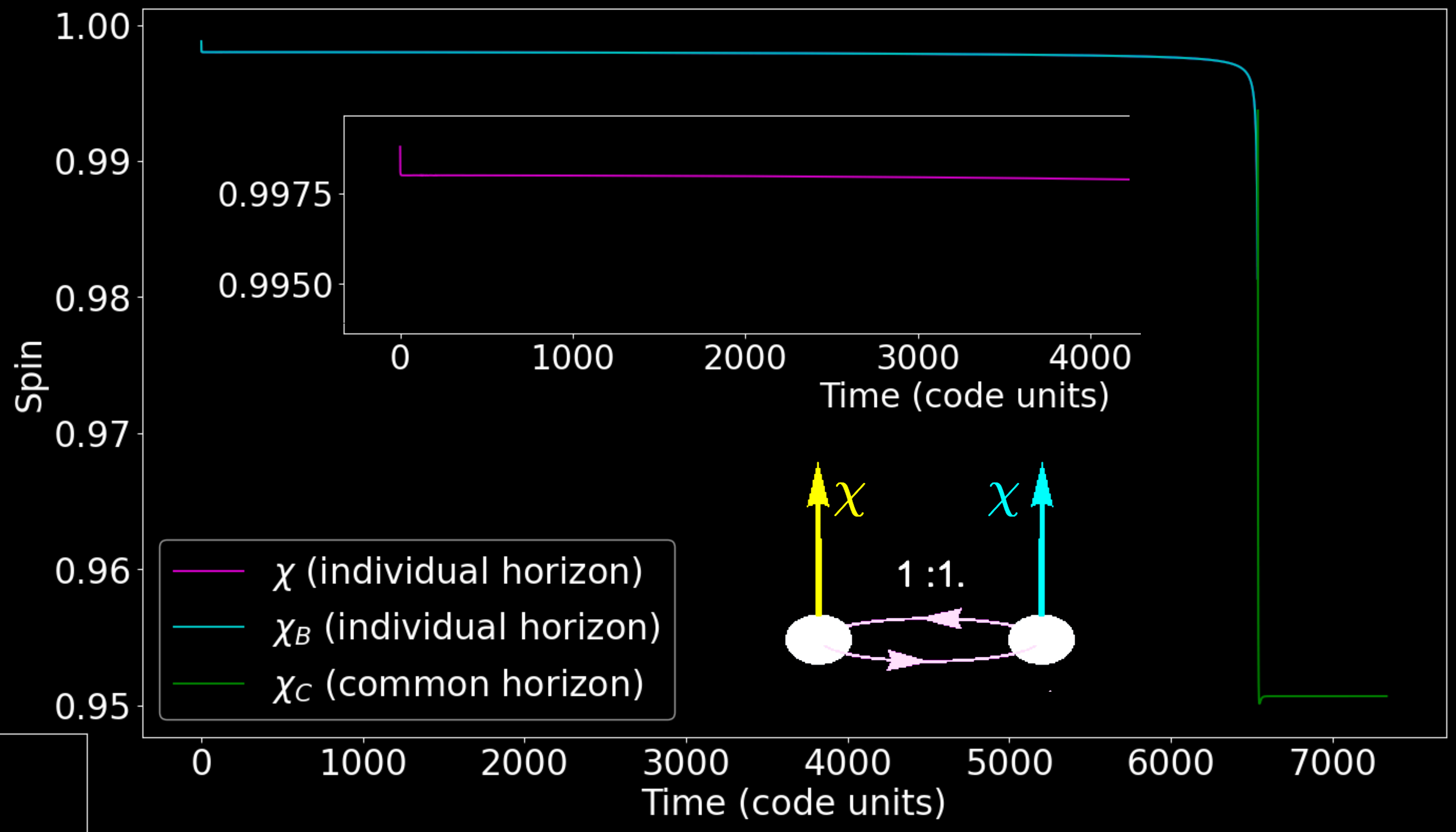


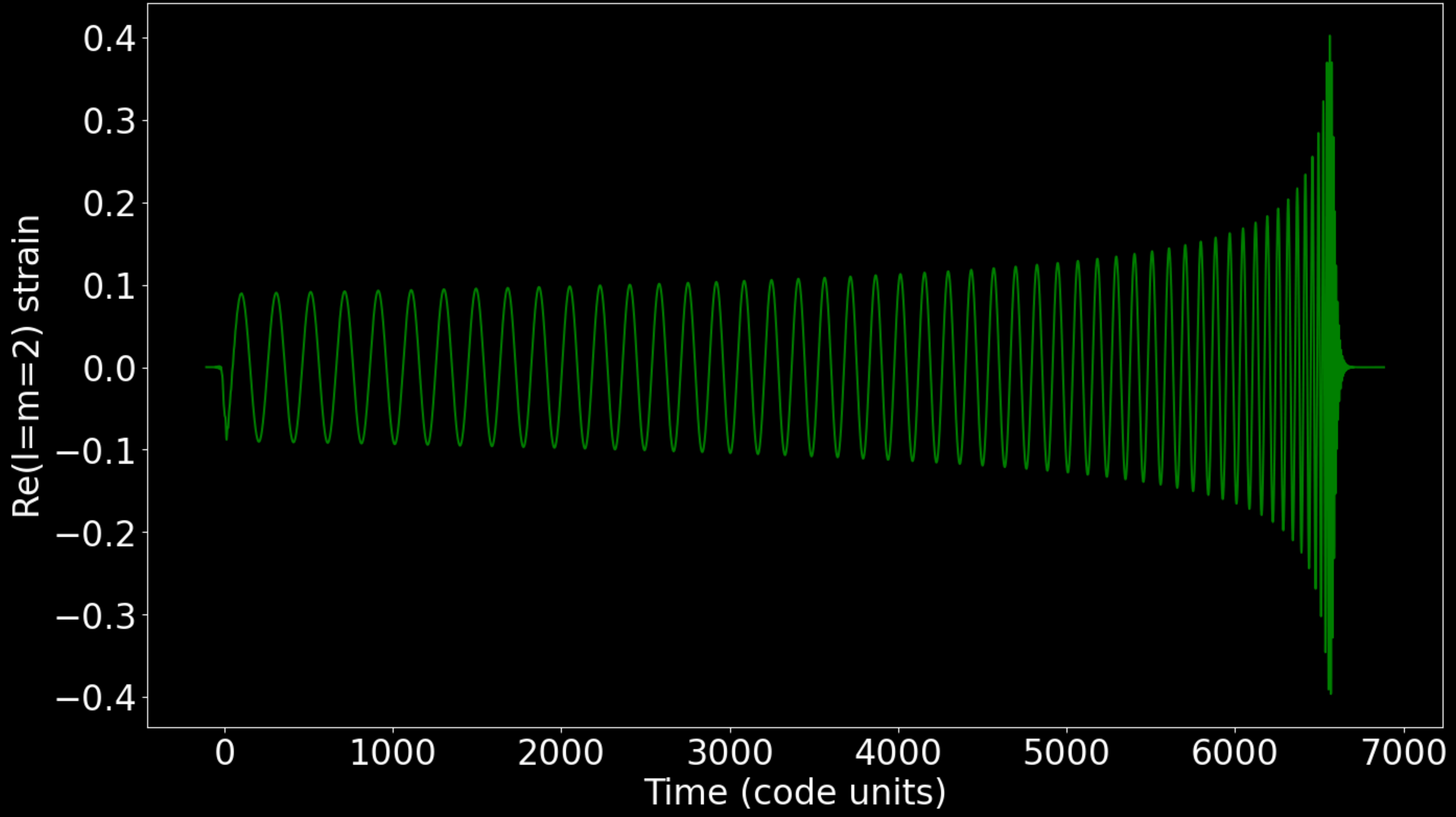


Highest BBH spin so far

- Excision: $\chi = 0.998$

Simulation by Matthew Giesler
Appears in Boyle+, ... GL+...
Class. Quantum Grav. **36**, 195006 (2019)

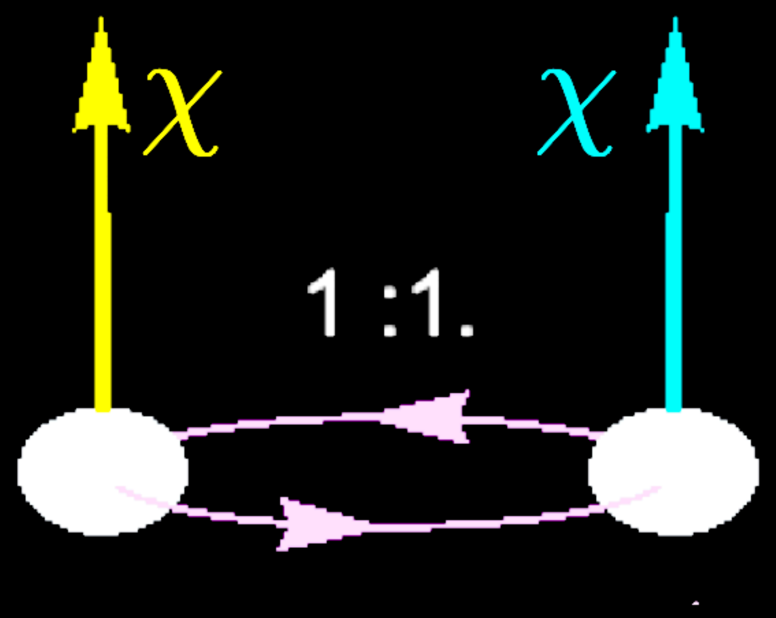




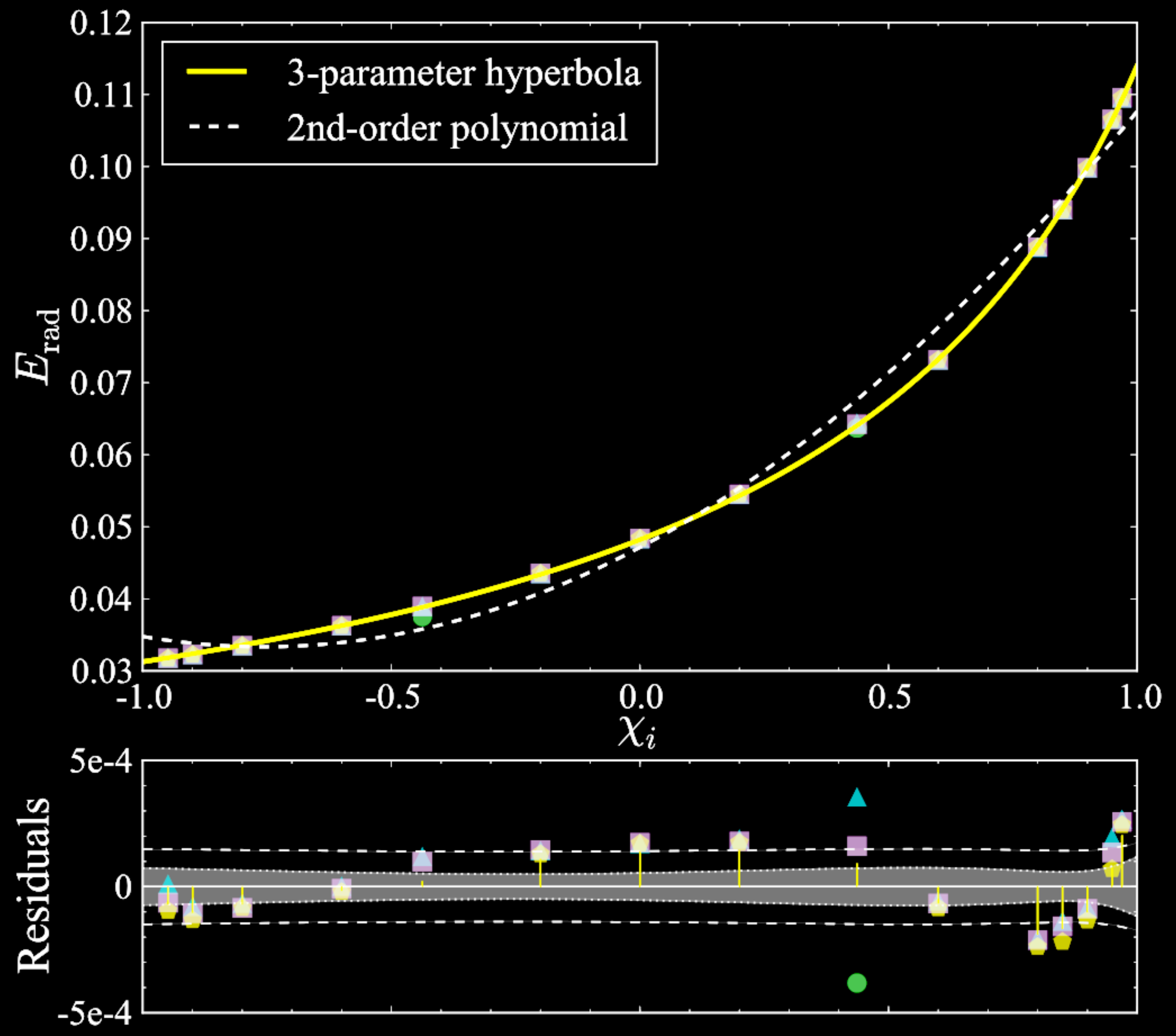
$\chi = 0.998$: last 25.7 orbits before merger

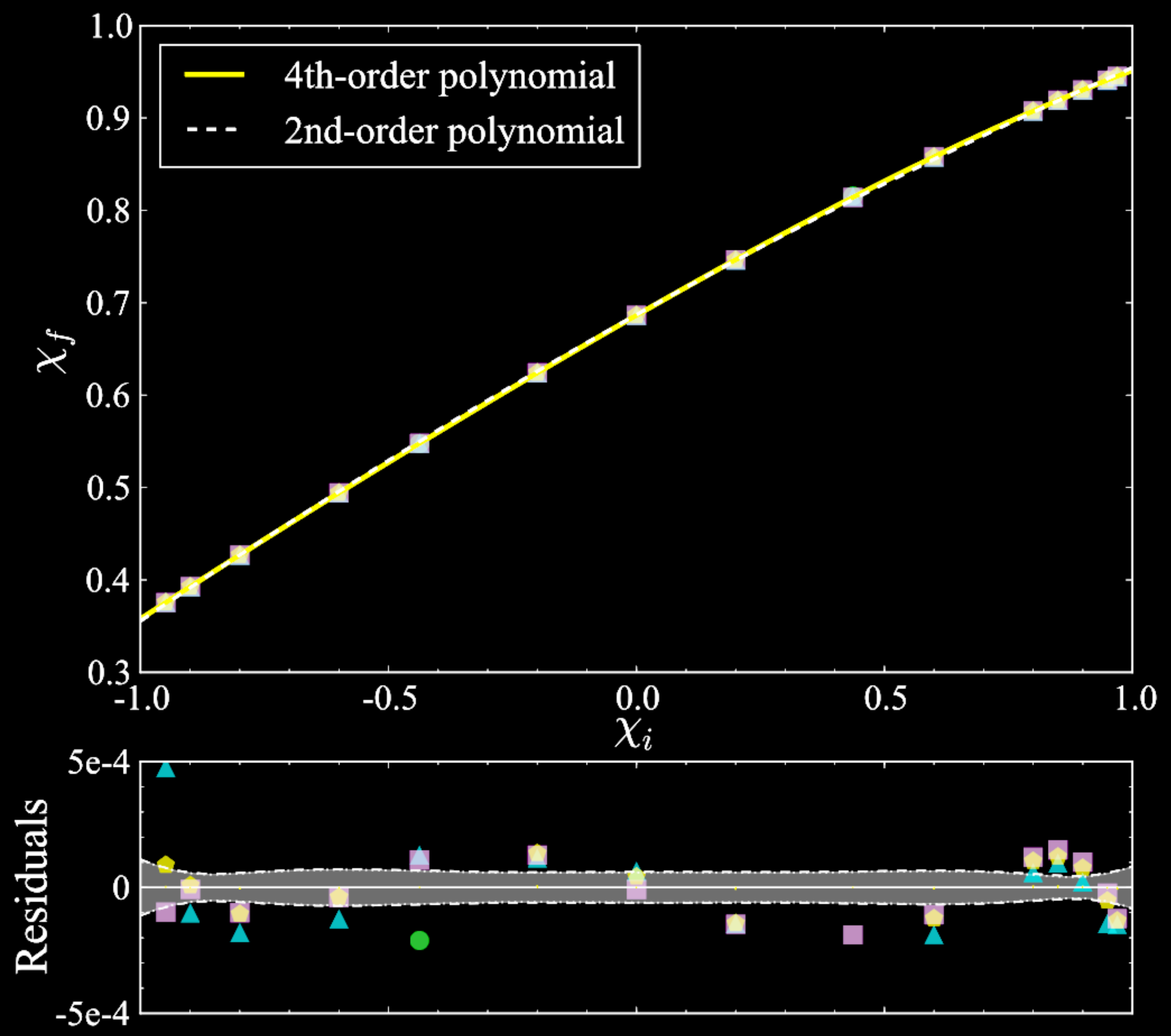
vs. $\chi = -0.97$, same initial separation:
12.2 orbits before merger

What kind of black hole is left behind?



- Fit final mass, spin for equal-mass, aligned-spin inspirals
  Hemberger+, ... GL, ...+ Phys. Rev. D **88**, 064014 (2013)





Do evolutions obey $\zeta \leq 1$?

$$\zeta \equiv \frac{8\pi S}{A}$$

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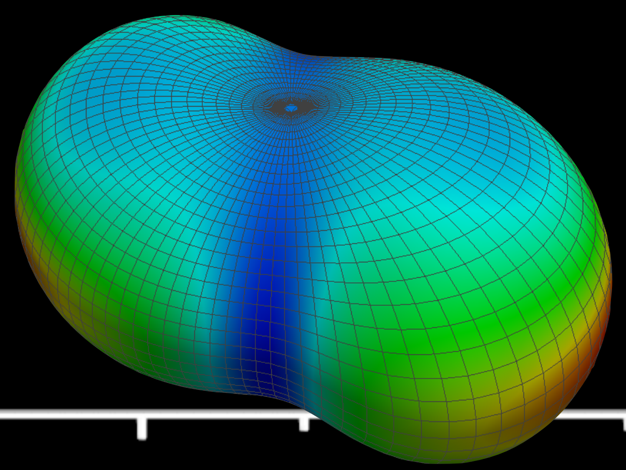
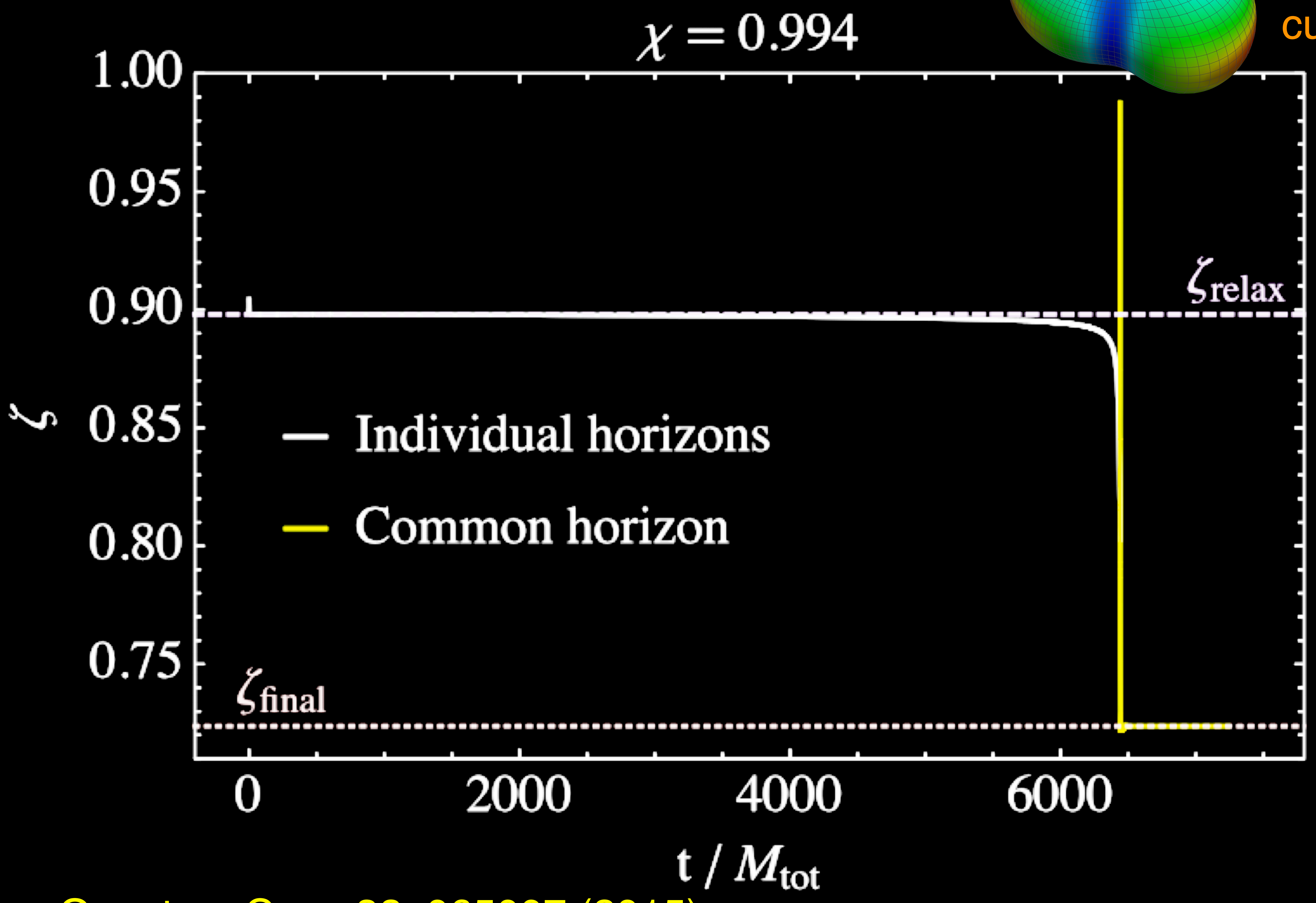


Color = horizon intrinsic Ricci scalar curvature

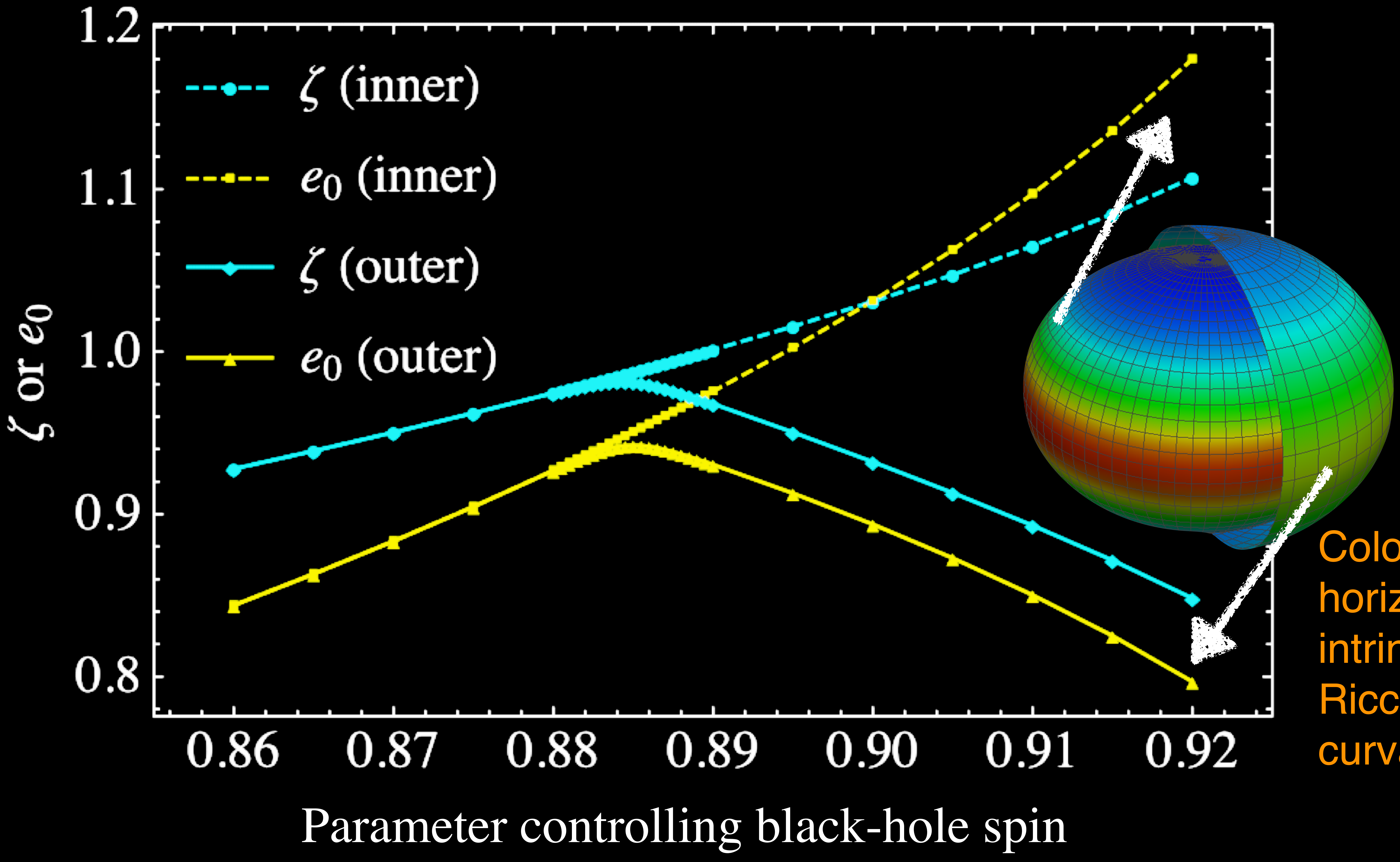


$$\chi \equiv \frac{S}{M^2}$$

GL+ Class. Quantum Grav. 32, 065007 (2015)

Can you generate initial data with $\zeta > 1$?

$$e_0 \equiv \min\left[\frac{1}{4\pi}\oint_{\mathcal{H}} \omega_B \omega^B dA\right]$$



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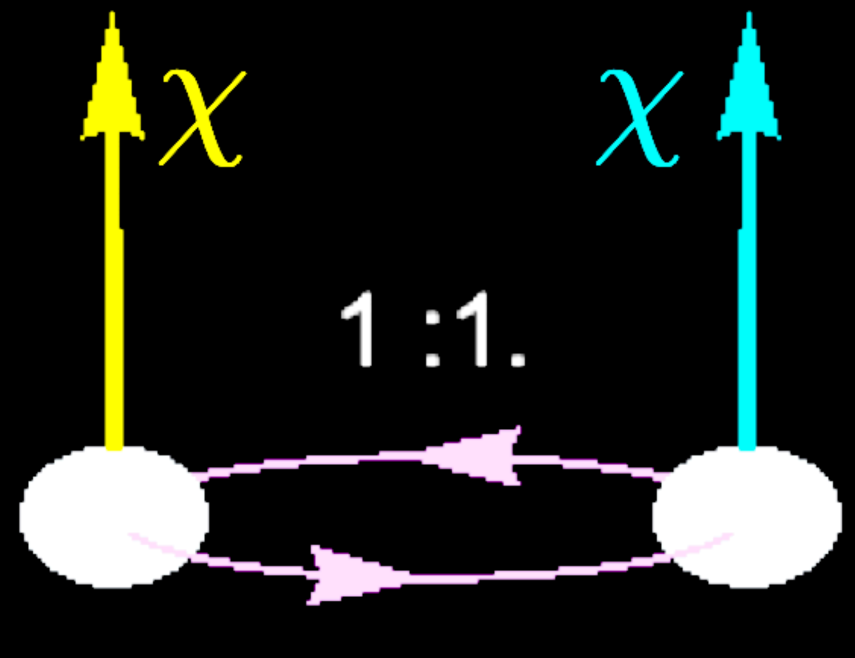


$$\chi \equiv \frac{S}{M^2}$$

GL+ Class. Quantum Grav. 32, 065007 (2015)

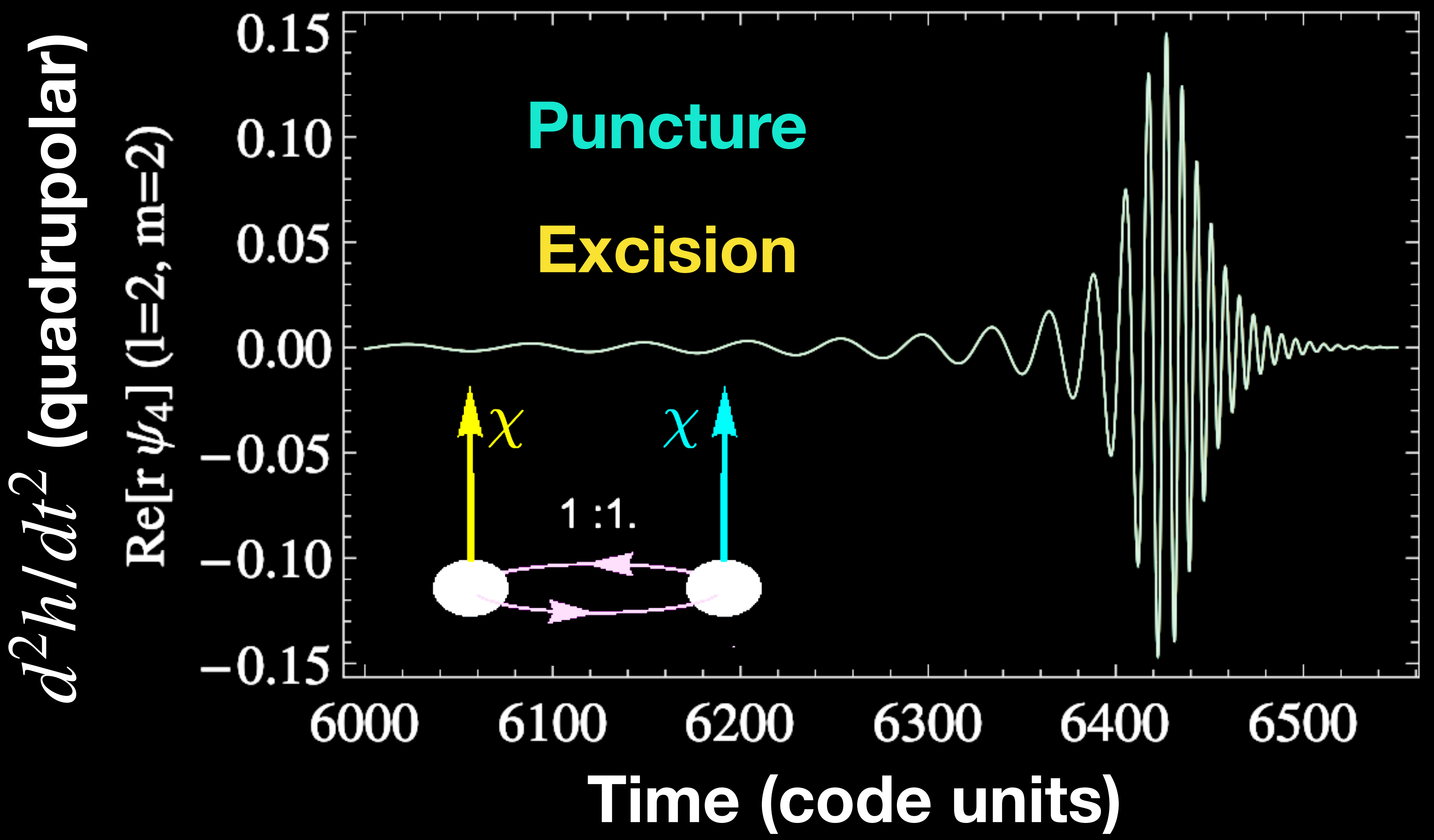
Puncture evolutions with nearly extremal spins

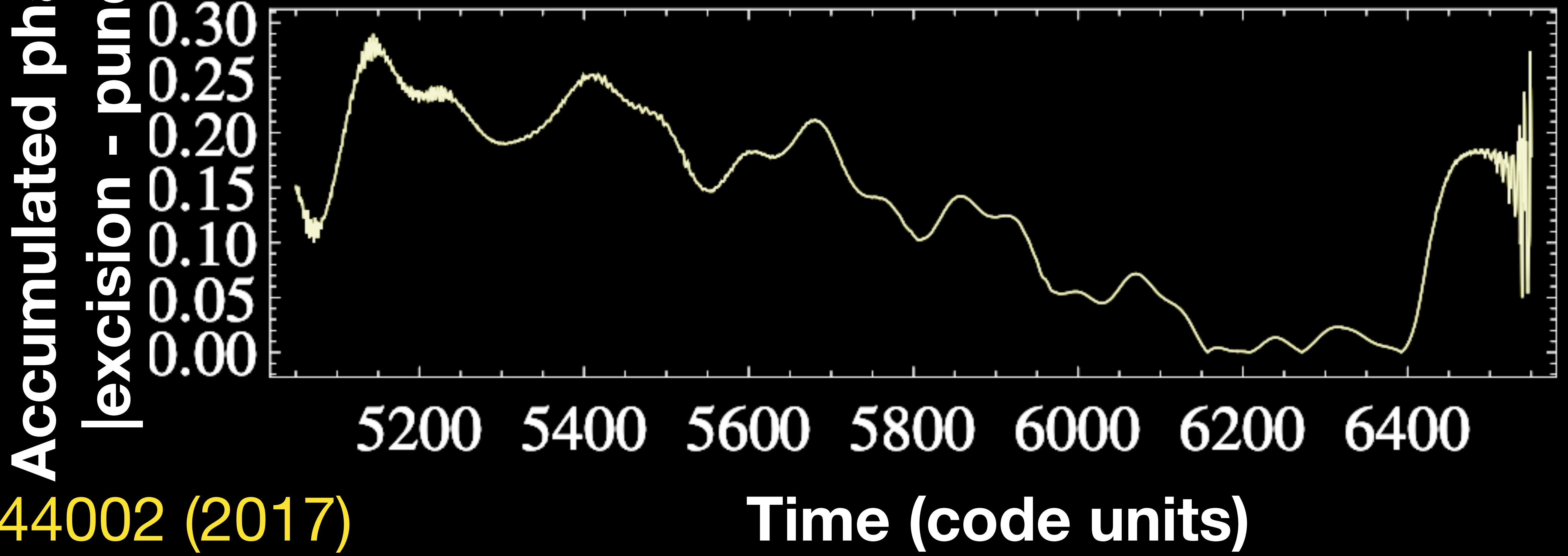
- Choose radial coordinate carefully, so extremal horizon radius not zero
  Liu+ (2009) — spins up to 0.95

- Puncture evolutions with $\chi = 0.95$
  Zlochower+ Phys. Rev. D **96,** 044002 (2017)
  — spins up to 0.99 (head-on) / 0.95 (inspiral)

- $\chi = 0.95$, 1.5:1 mass ratio: recoil > 500 km/s, largest for aligned-spin
  Healy+ Phys. Rev. D **97**, 104026 (2017)
  — very large recoil

- Initial data for charged, spinning black holes up to 90% extremity
  Mukherjee+ arXiv:2202.12133v1 (2022)



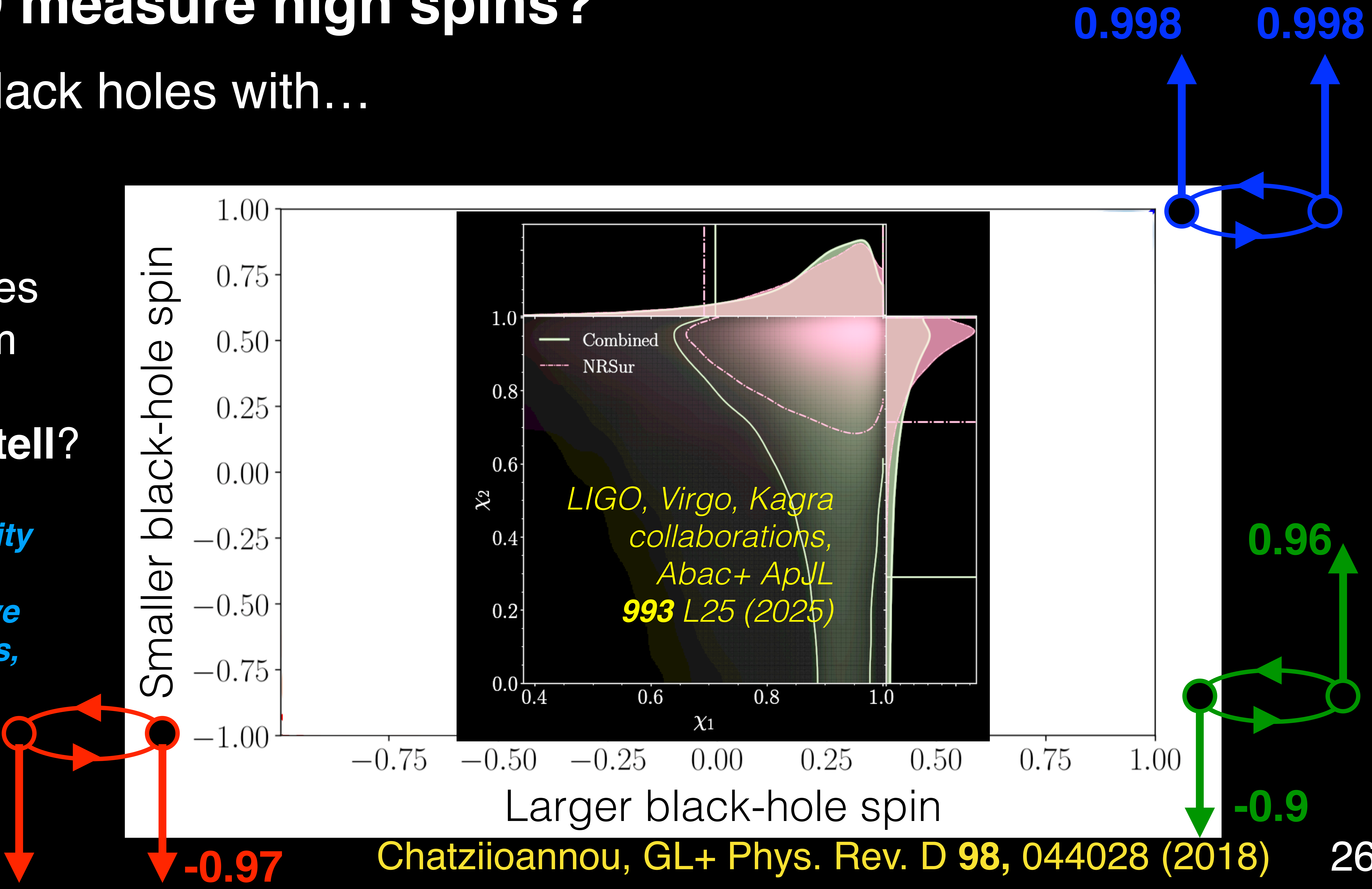


Figures courtesy Zlochower+ Phys. Rev. D **96,** 044002 (2017)

Can LIGO measure high spins?

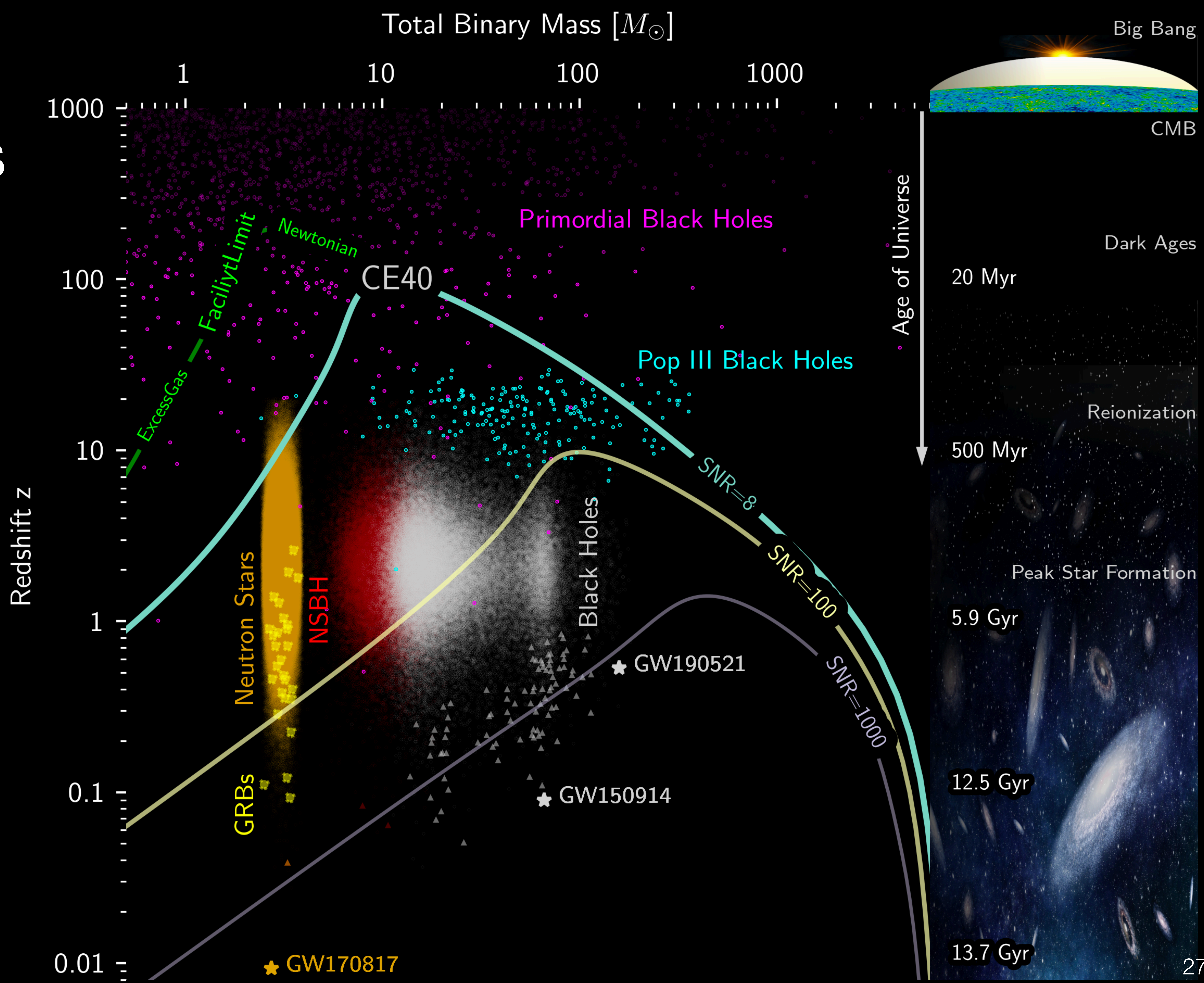
- Merging black holes with…
  - ≈ masses
  - spins ≈ 1
  - If LIGO sees waves from this binary, **could we tell**?

For more on numerical relativity & inferring gravitational-wave source properties, see Deborah Ferguson's talk Friday!



Chatziioannou, GL+ Phys. Rev. D **98,** 044028 (2018)

Cosmic Explorer's View into the Universe



Cosmic Explorer White Paper for NSF MSCAC ngGW , arXiv:2109.09882

Next-gen NR

- BBH catalogs for Cosmic Explorer, Einstein Telescope, LISA
  - Need much more accuracy (SNR > 1000)
    Pürrer & Haster (2020)
    Ferguson+ (2021), Jan+ (2024)
  - Novel codes aiming to achieve this
    AthenaK: Zhu+ (2024)

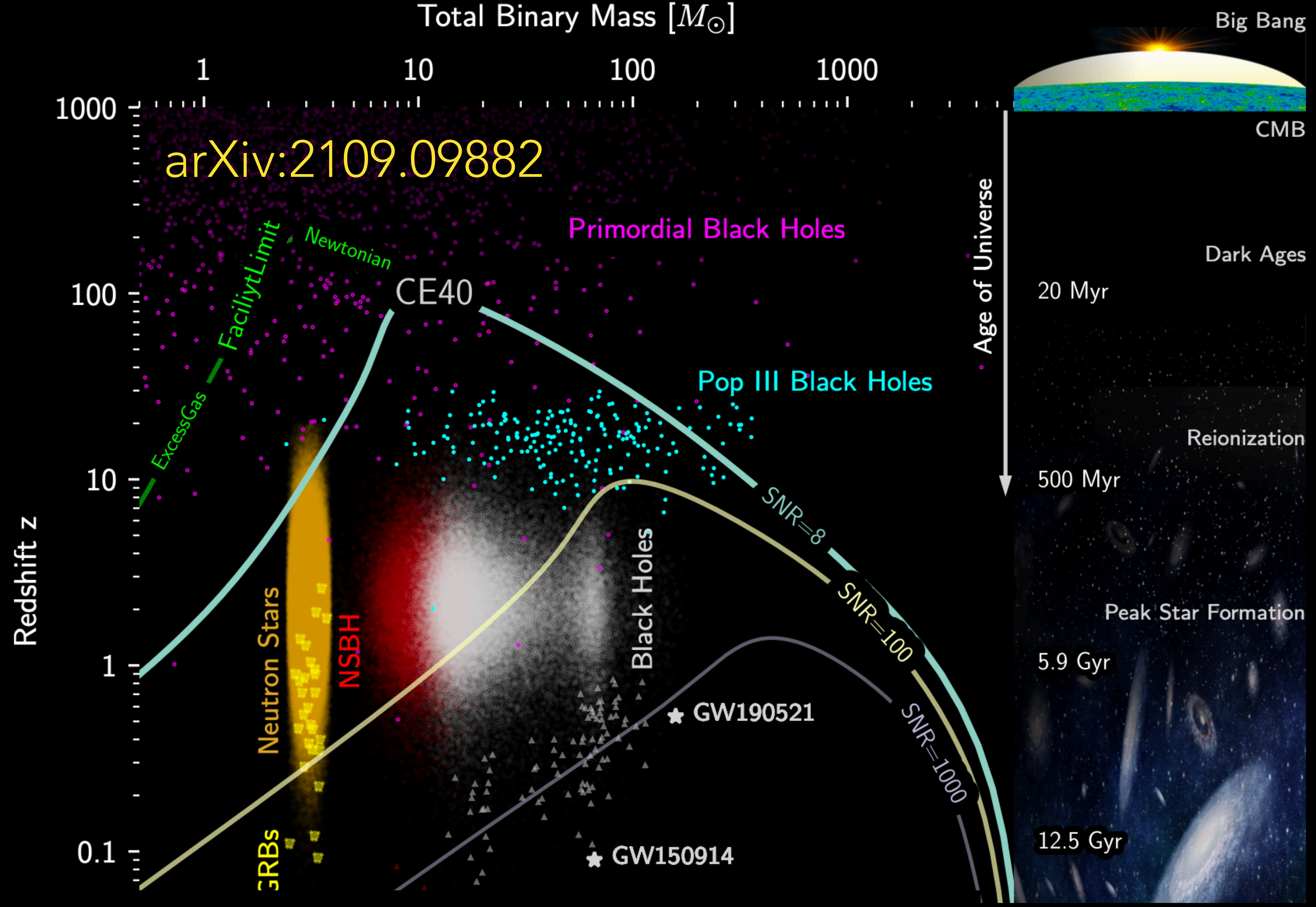
    NMesh: Adhikari+ (2025)

    Dendro-GR: Fernando+ (2023),

    AsterX: Kalinani+ (2024)

    SuperB: Tootle+ (2025)

    SpECTRE: GL, K Nelli+ (2025), successor to Spectral Einstein Code (SpE



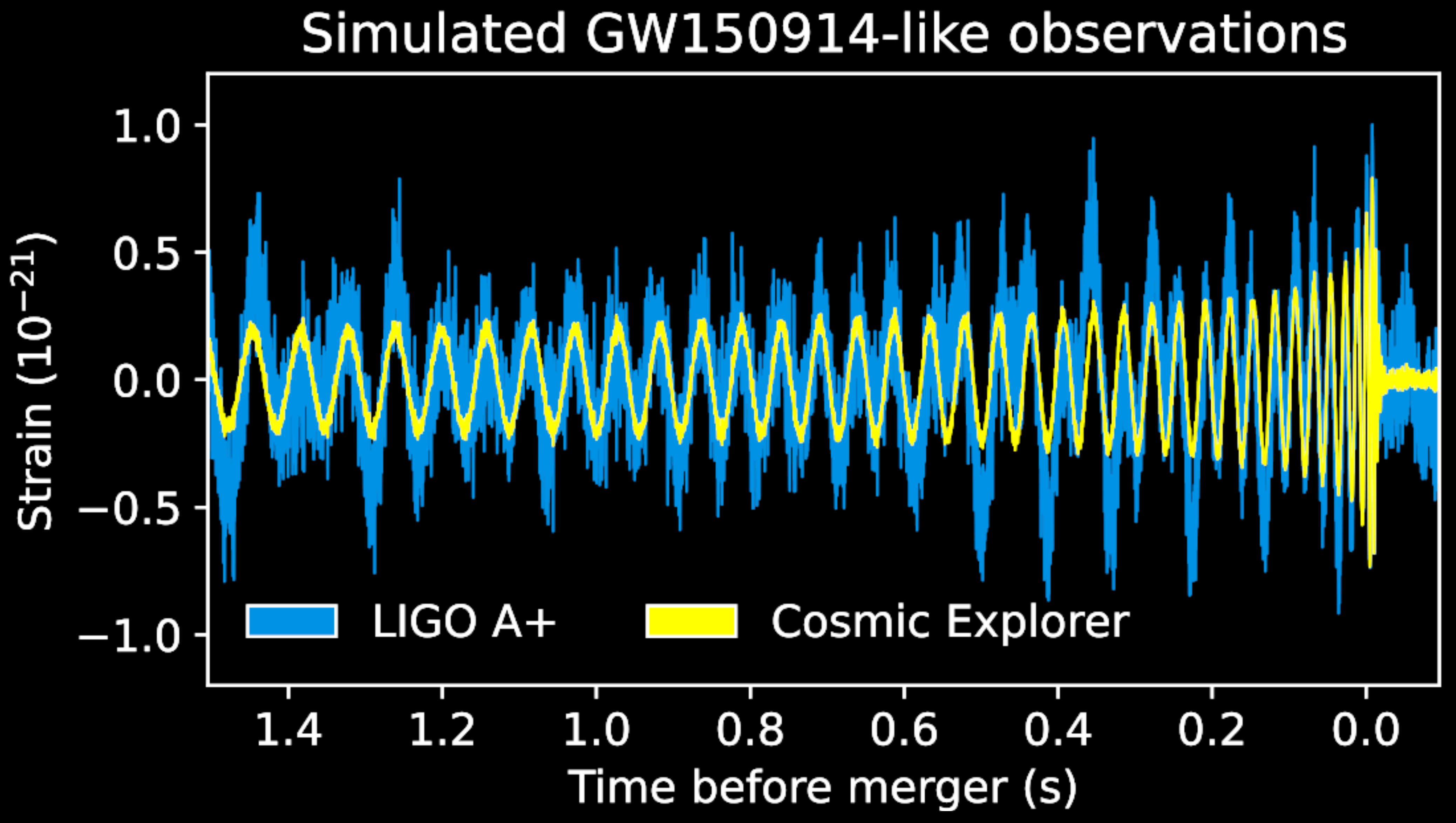


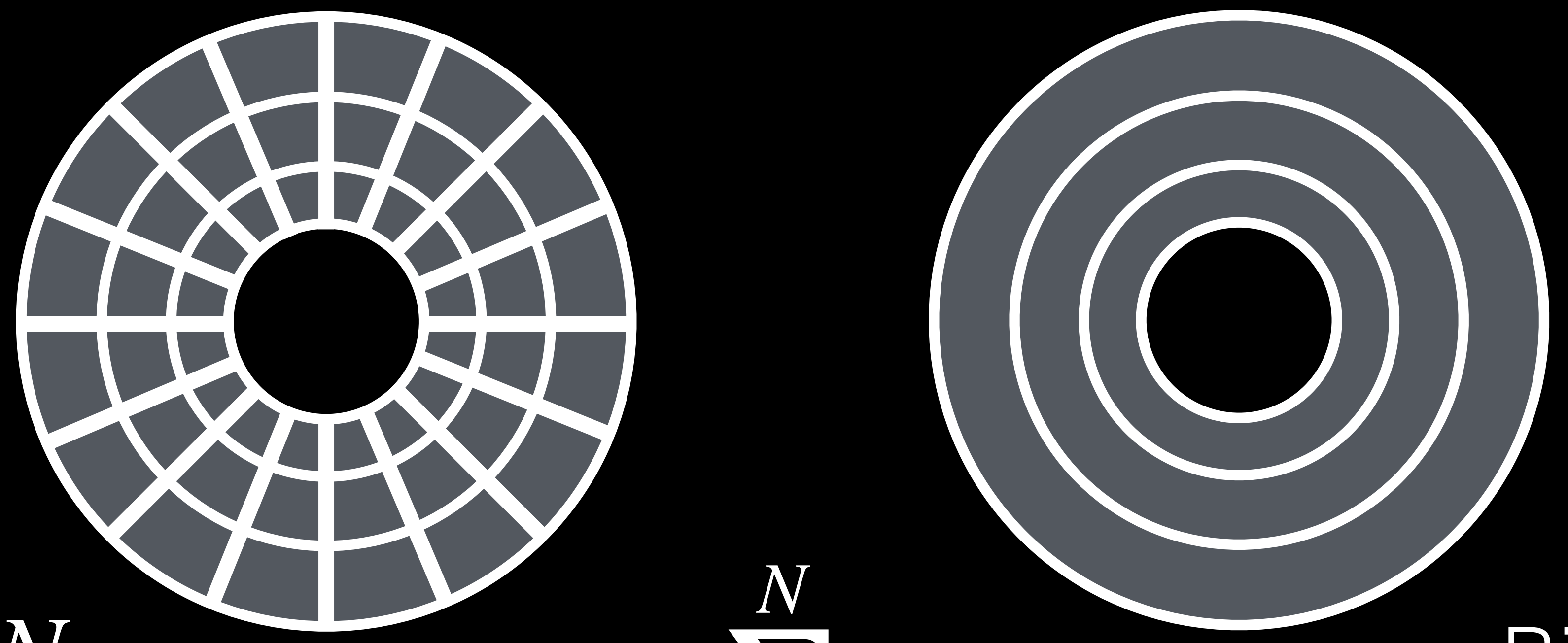
Figure courtesy Cosmic Explorer arXiv:2306.13745

SpECTRE methods

See GL, Kyle Nelli+, Class. Quantum Grav. **42,** 035001 (2025)

- Open-source code
- Discontinous-Galerkin method for initial data & evolution
- Local adaptive time stepping (4th-order Adams Moulton)
- Task-based parallelism via charm++ library
- Generalized-harmonic formulation, damped harmonic gauge, constraint-preserving boundary conditions
- Excision, maps & control system to match domain, horizons
  - Domain made entirely of deformed cubes
  - Scales to many more CPU cores — but many more points vs. spherical shells
- Waveforms via Cauchy Characteristic Evolution (CCE)
  Moxon+ Phys. Rev. D **107**, 064013 (2023)
  CCE: Bishop+ Phys. Rev. D **54**, 6153 (1996)

SpECTRE SpEC



Smaller N more cells

$$f(x) = \sum_{n=0}^{N} a_n \phi(x)$$

Bigger N fewer cells

Exponential convergence *when solution smooth*

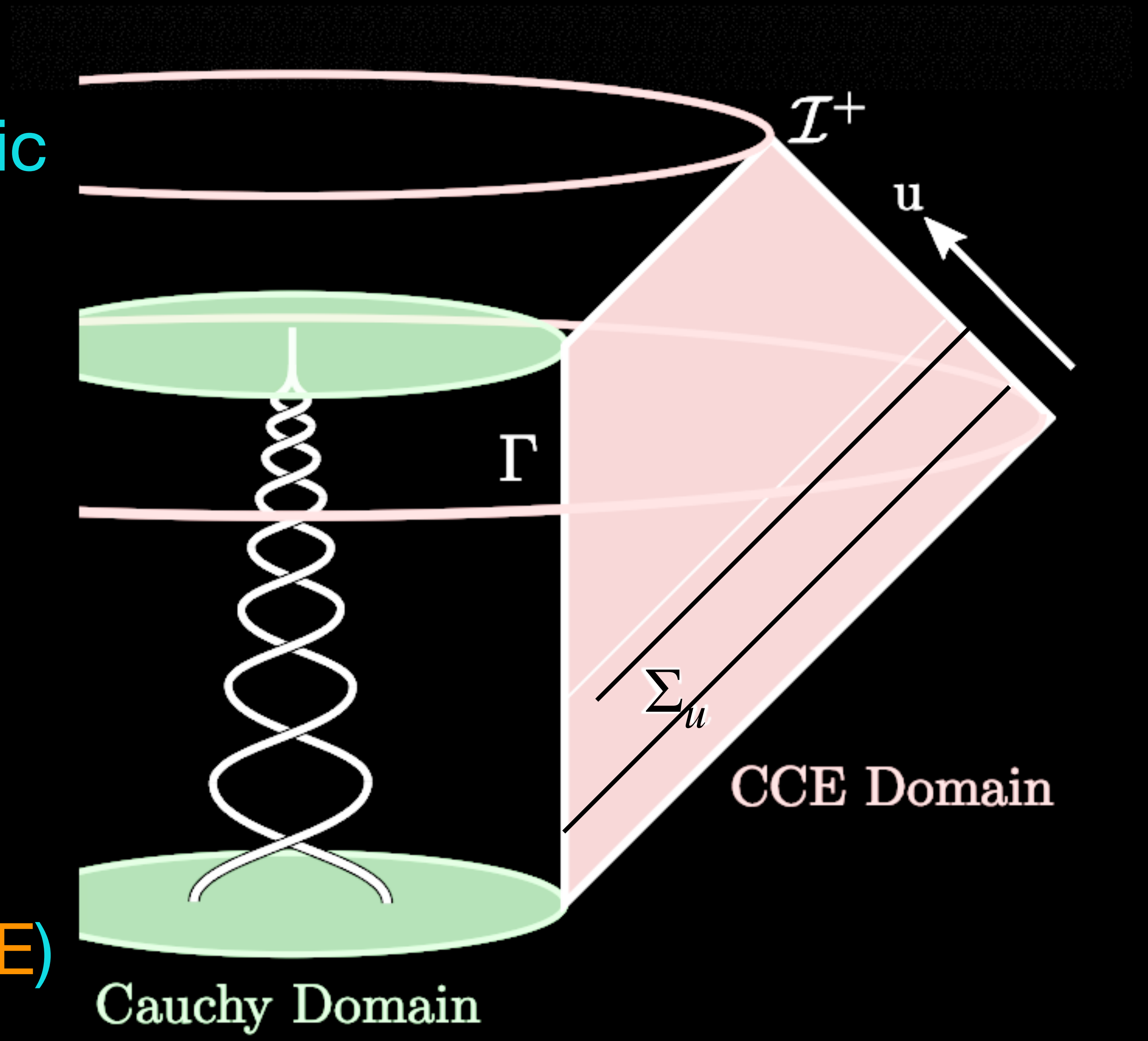
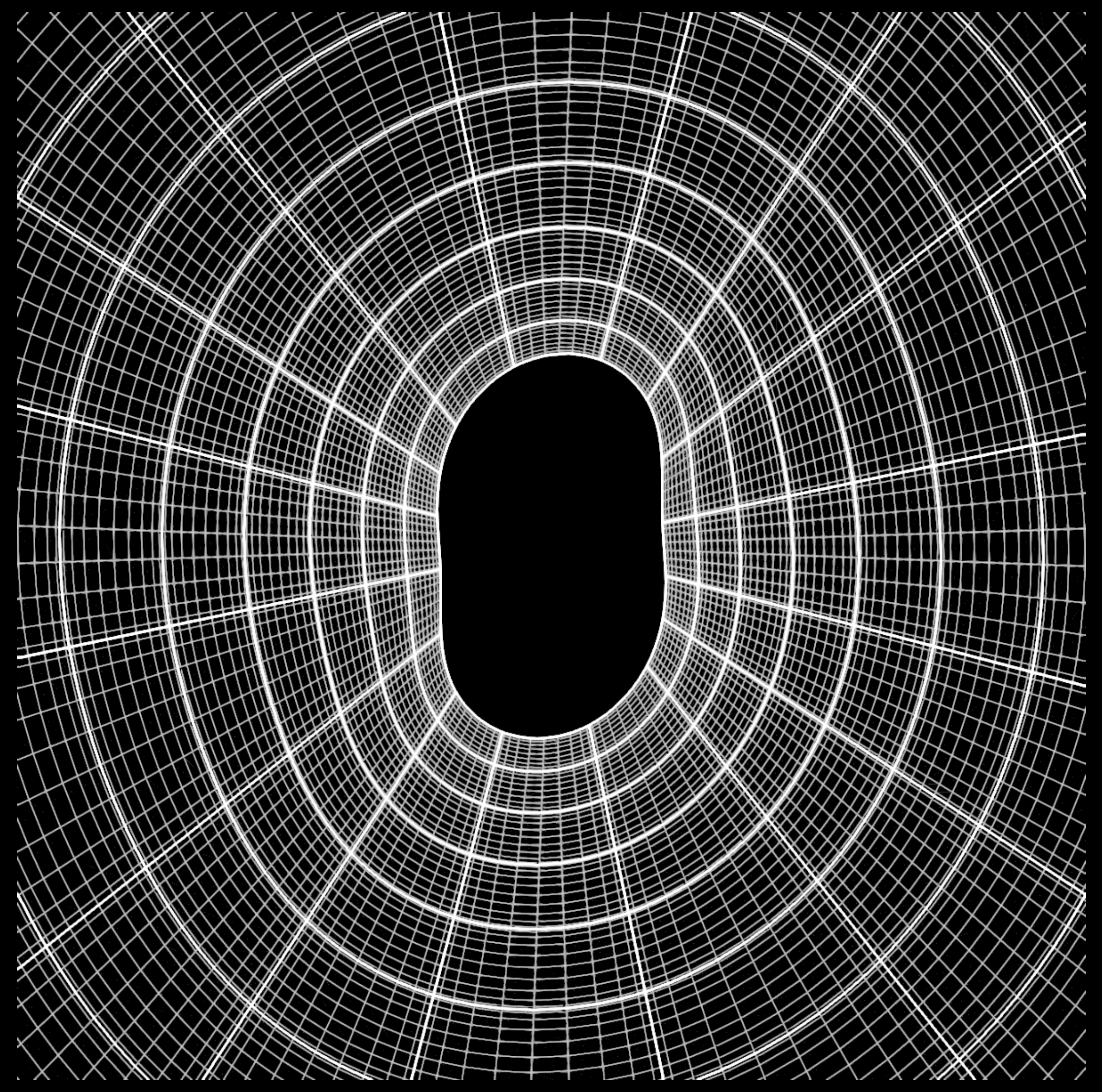


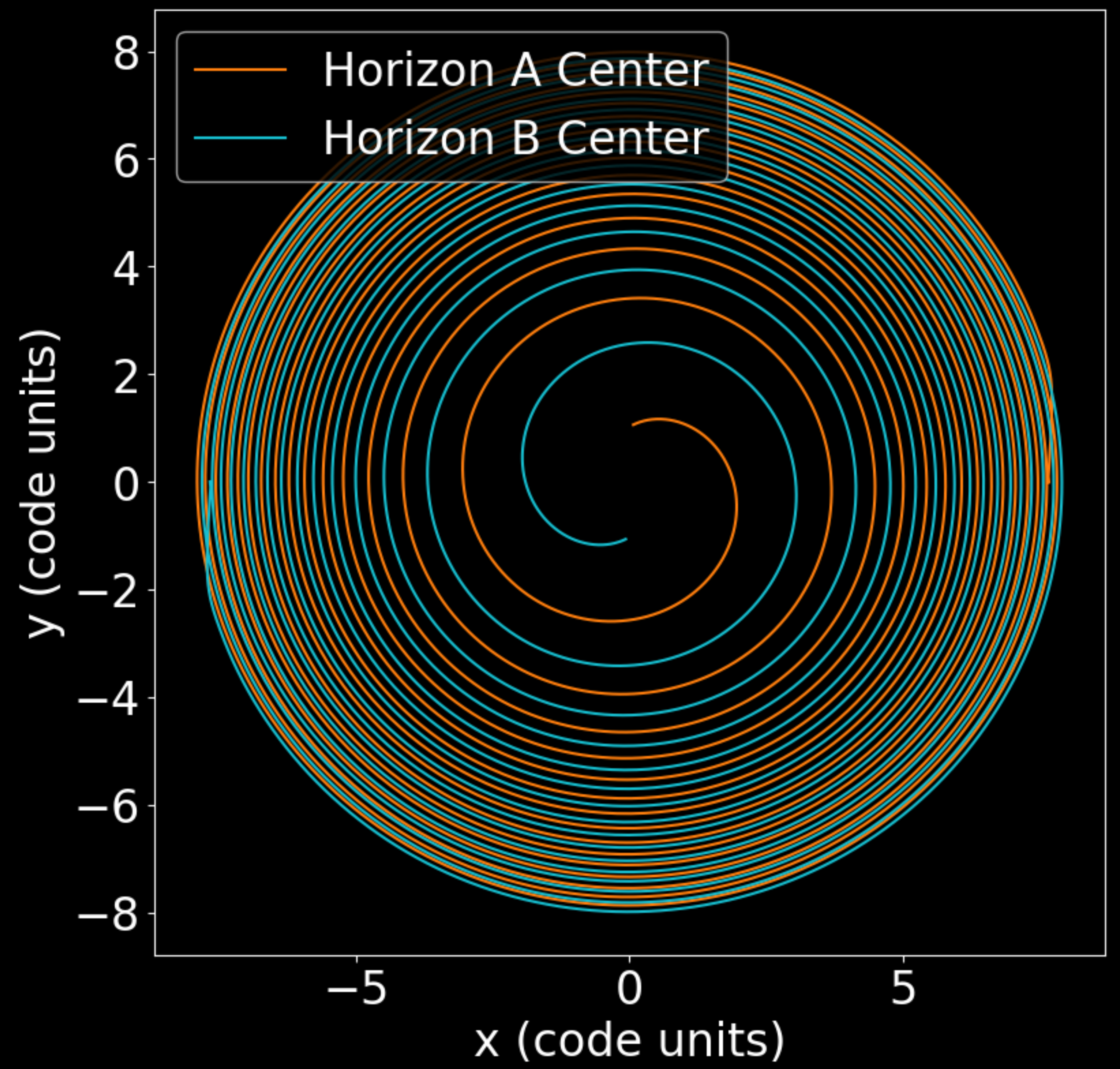
Figure courtesy Moxon+
Phys. Rev. D **107**, 064013 (2023)

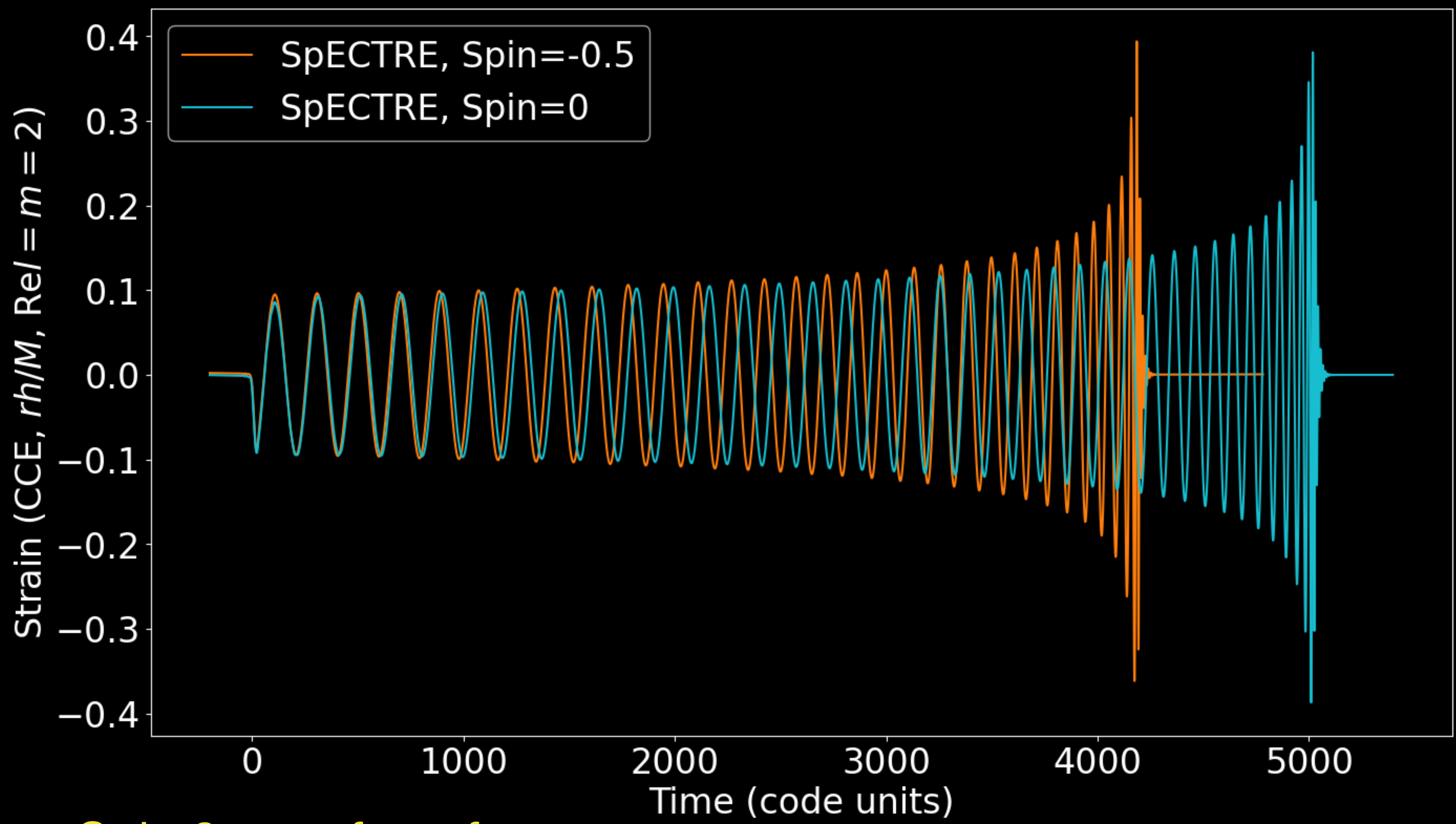


SpECTRE status

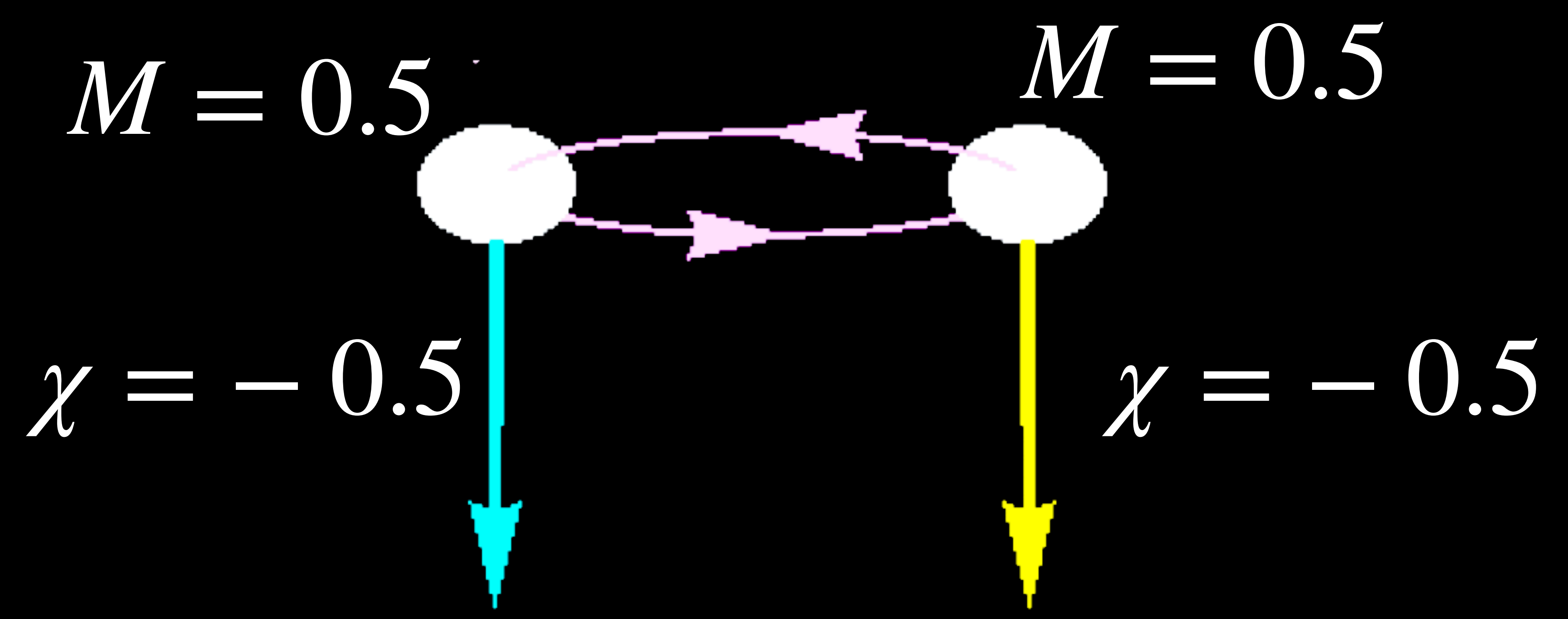
First spinning binary black hole

- ~15 orbits, equal-mas, spin=0.5 anti-aligned with orbital angular momentum





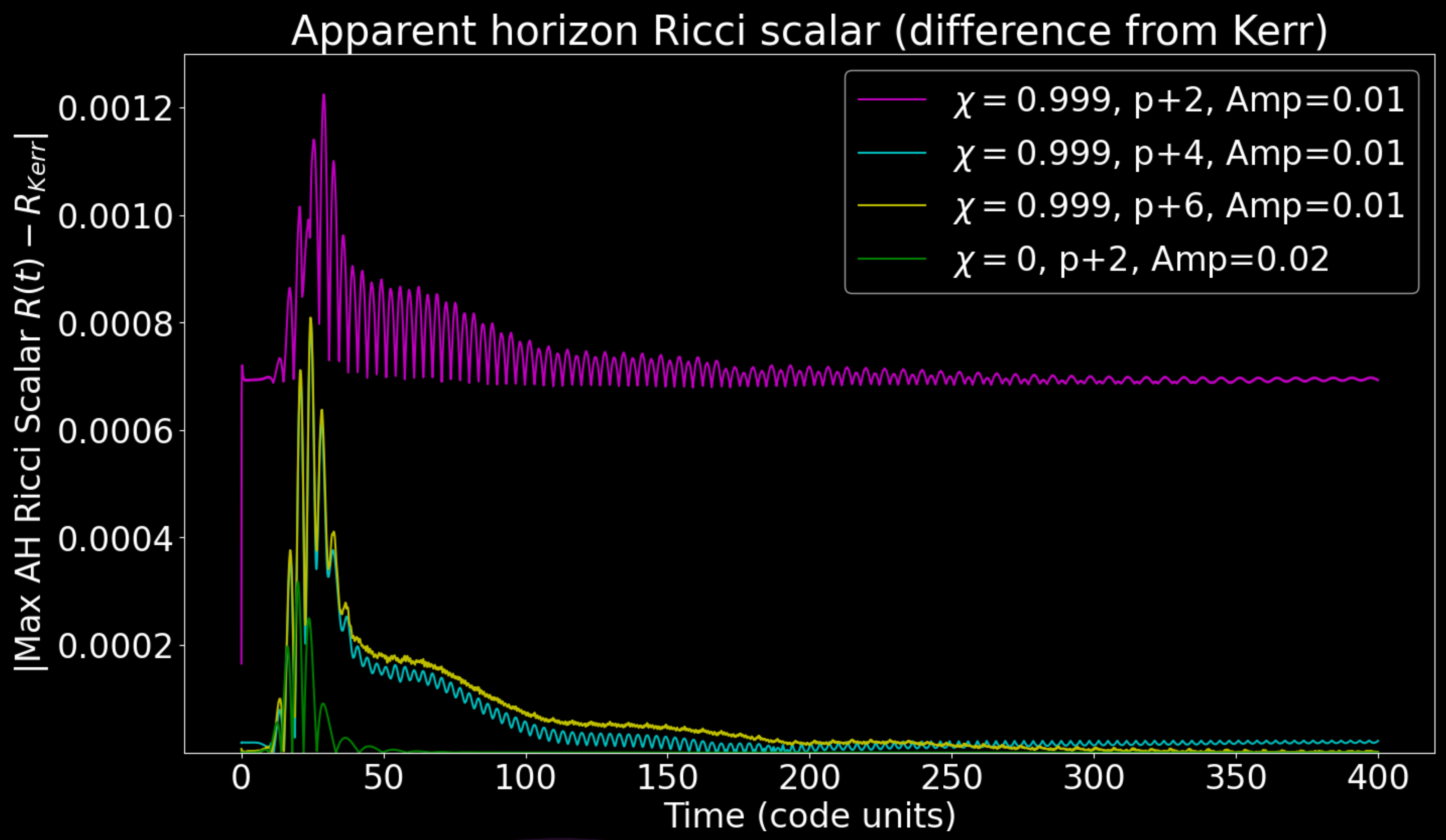
Spin 0 waveform from
GL, Kyle Nelli+, Class. Quantum Grav. **42,** 035001 (2025)

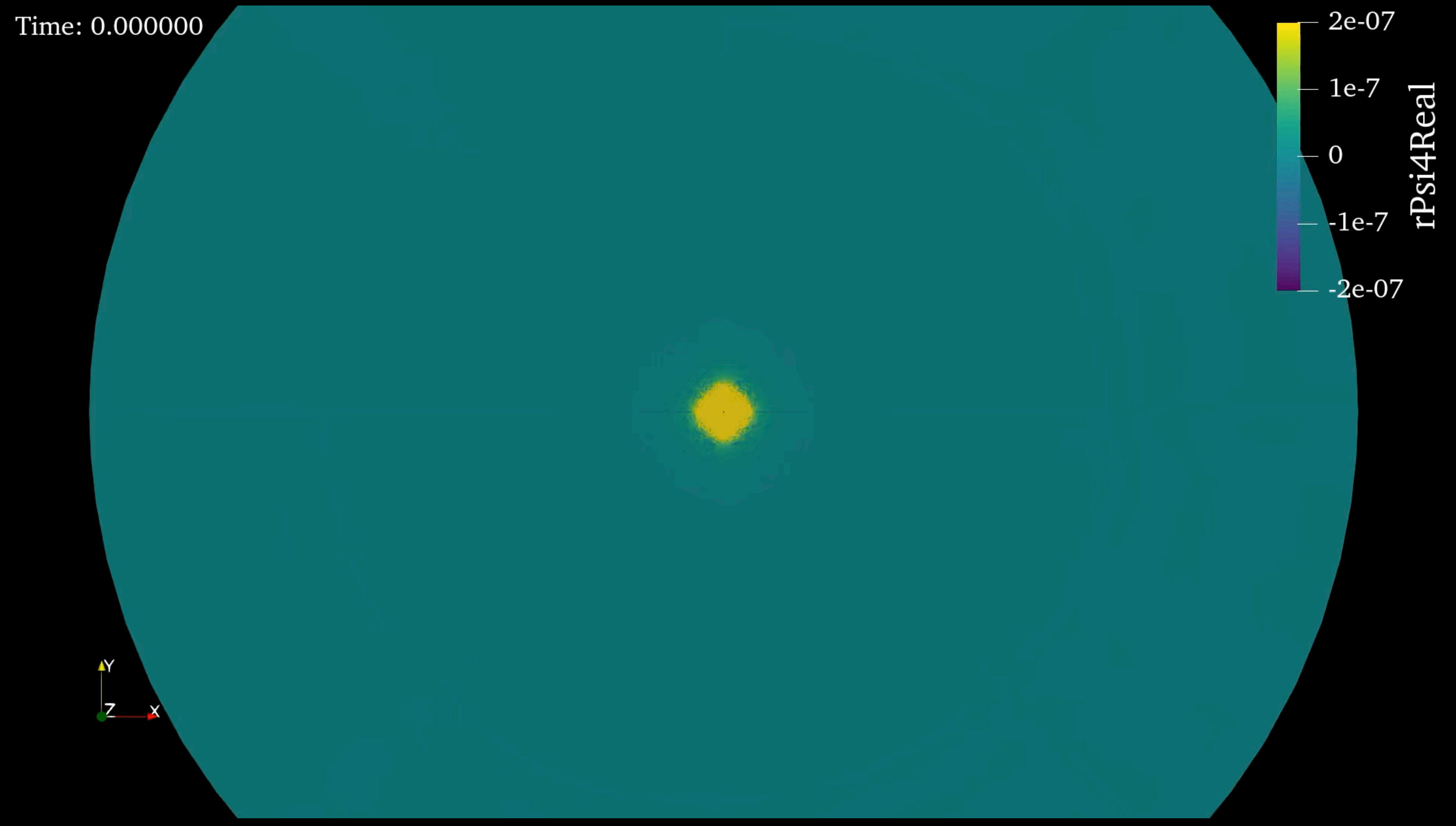


SpECTRE status

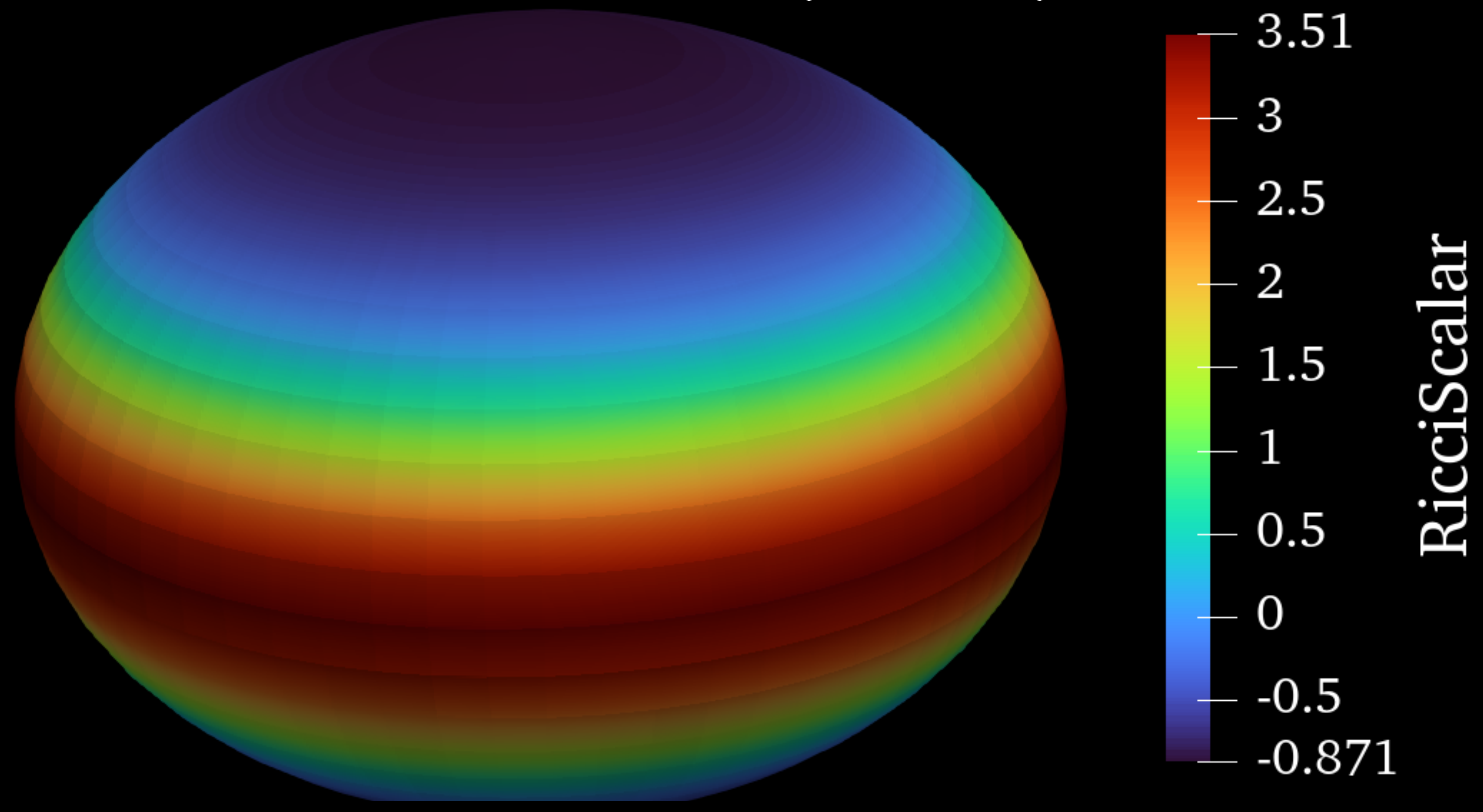
Perturbed black hole

- Initial data: solve constraints for $\chi = 0.999$ Kerr-Schild black hole + incoming $\ell = m = 2$ even Teukolsky wave
  Teukolsky, Phys. Rev. D **26**, 745 (1982)
  - Gaussian with width $w = 4M$ & amplitude $A = 0.01$ or 0.02 centered at $r_0 = 20M$





$r\Psi_4$ **(outgoing gravitational waves)**



Horizon curvature at time t=0 (SpECTRE p+4)

SpECTRE status

Perturbed black hole

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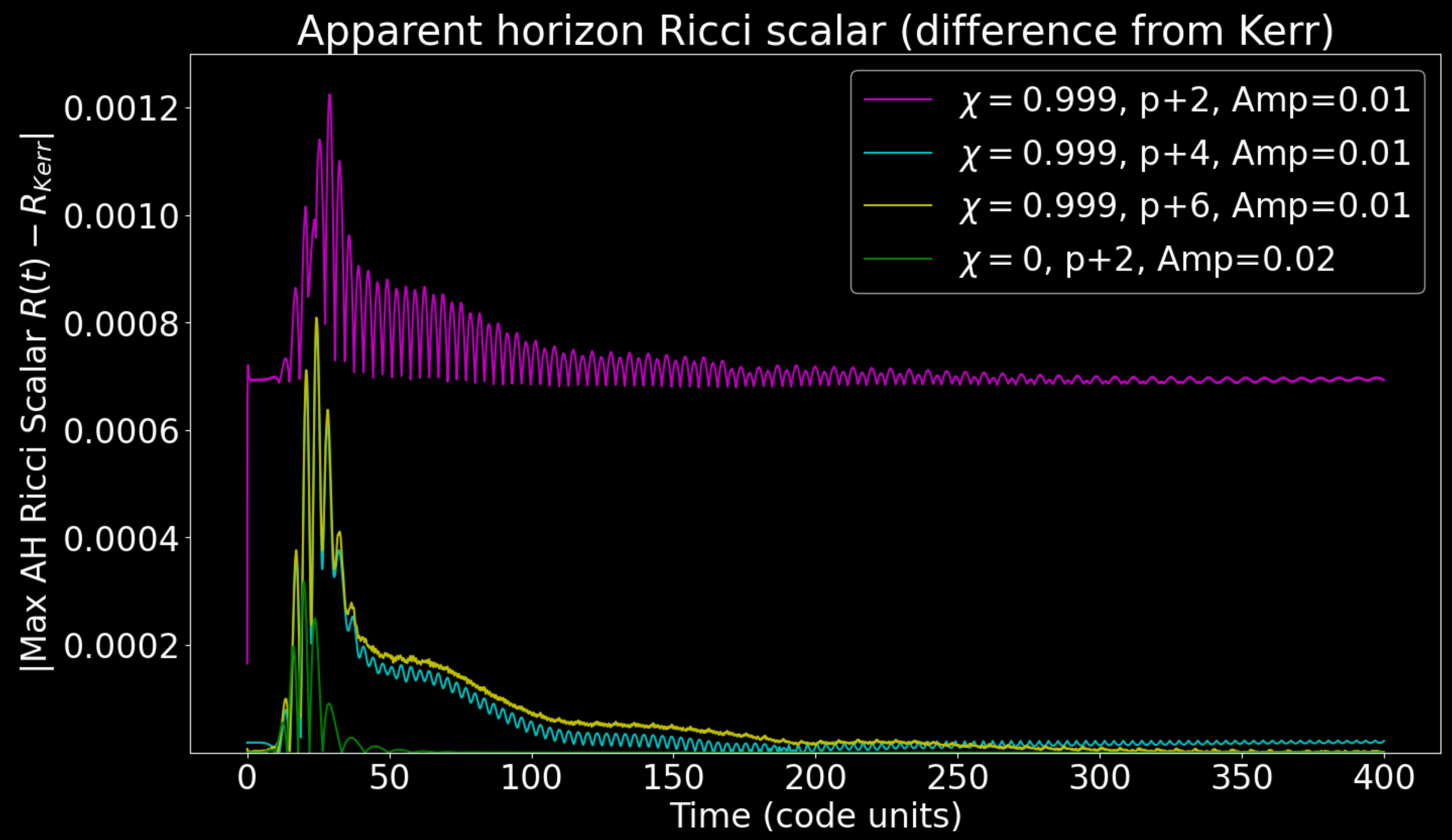
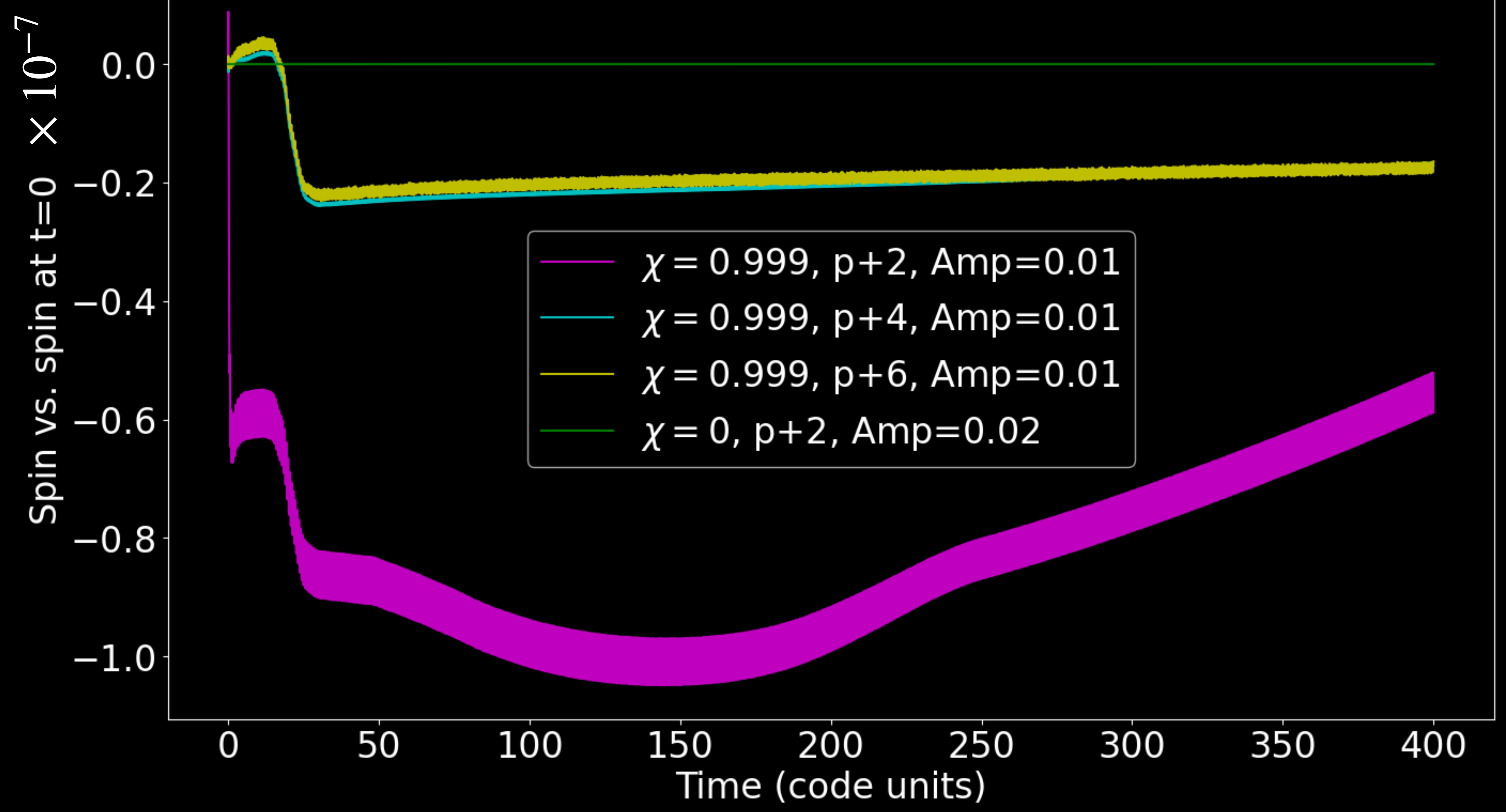
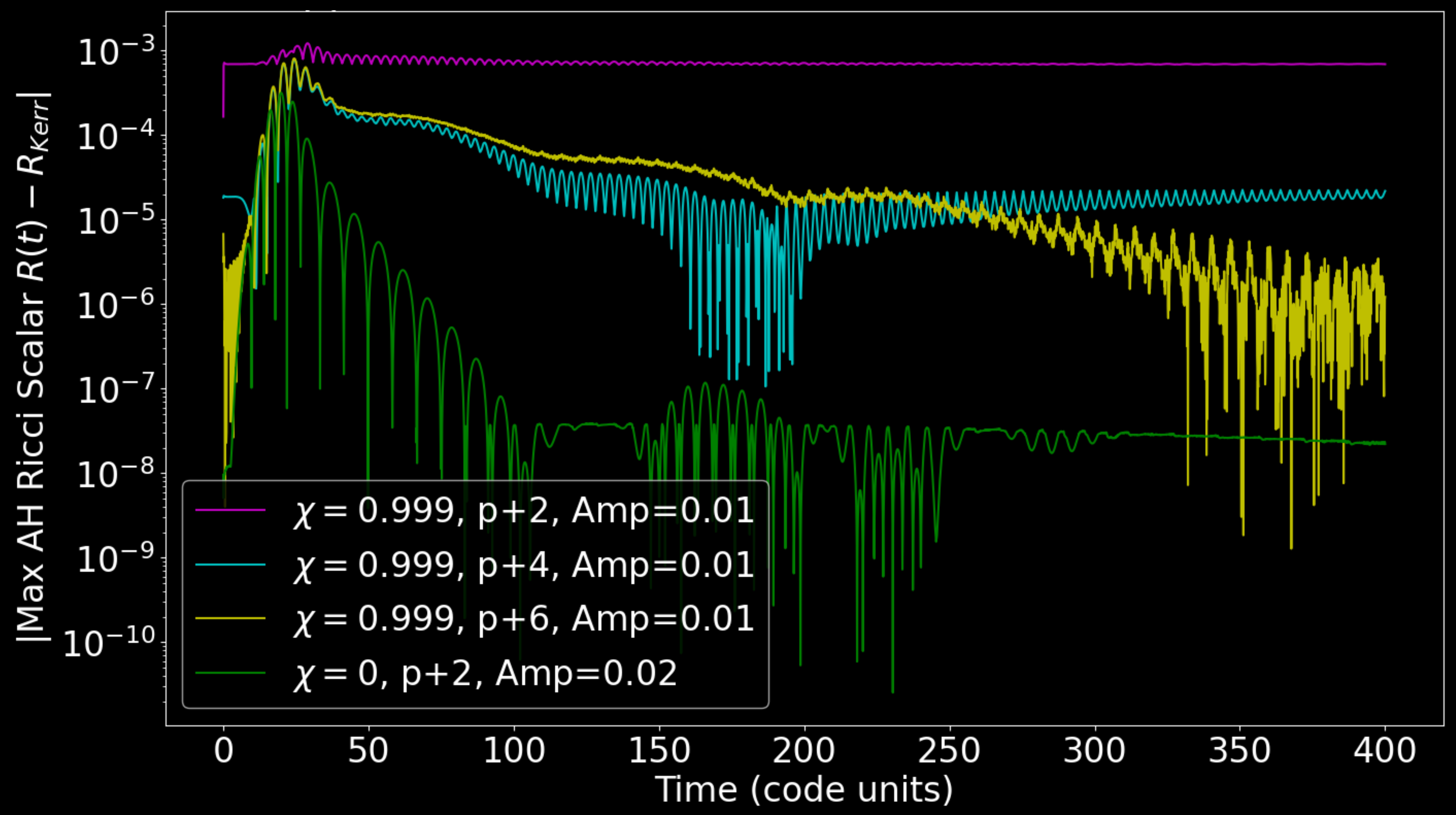
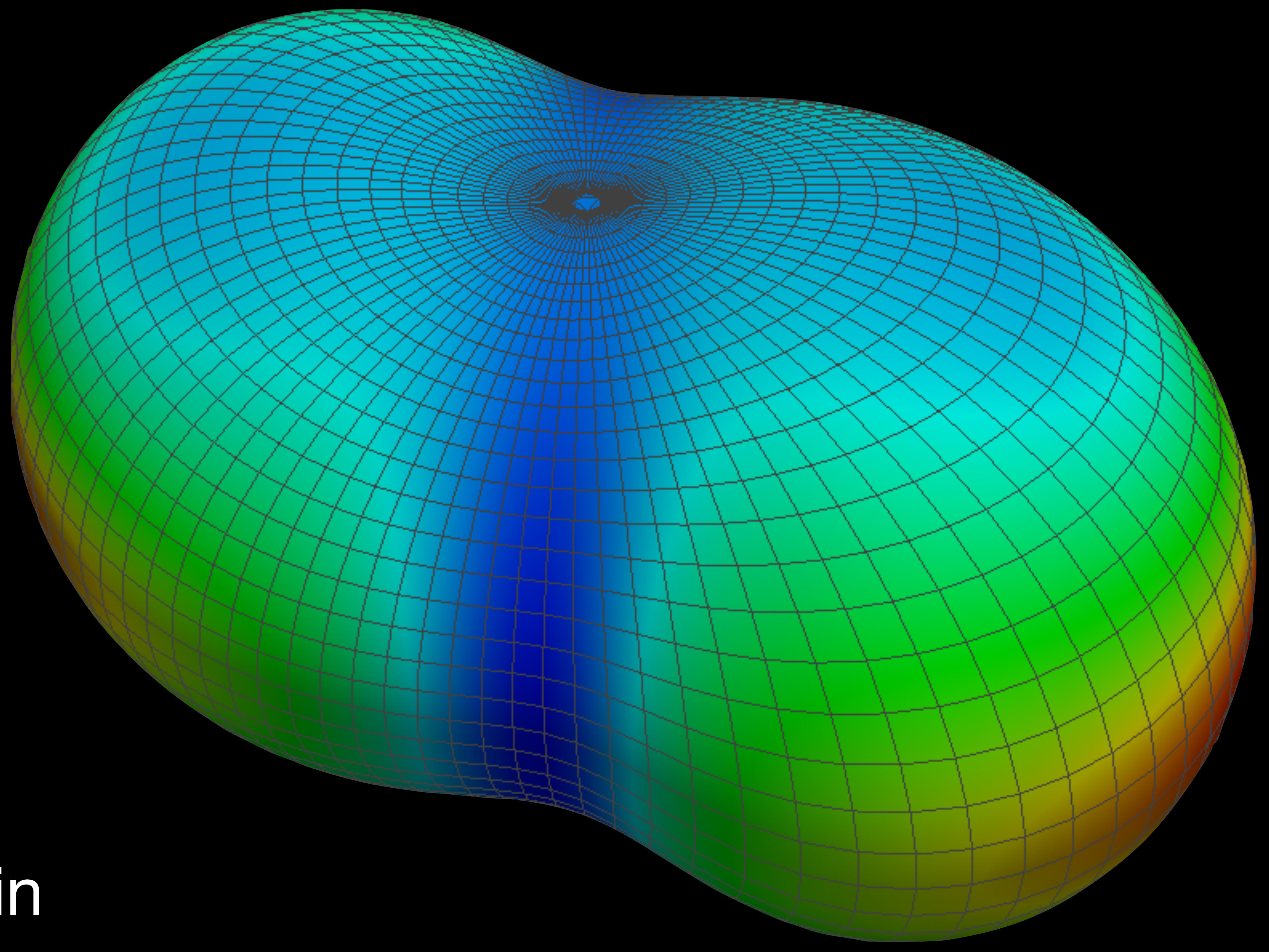






Conclusion



GL+ Class. Quantum Grav. 32, 065007 (2015)

- Possible but challenging to simulate merging black holes with nearly extremal spins with full 3+1 numerical relativity
- Future work
    - Evolve higher mass ratios with nearly-extremal spin
    - Precessing, nearly-extremal spins
    - Evolve nearly-extremal spins with next-generation codes
    - Achieve high accuracy future detectors will need
    - Perturbed nearly extremal black holes: improve accuracy, higher spin, higher amplitude