The interior of black holes in gravitational collapse

Maxime Van de Moortel

Extremal Black Holes and the Third Law of Black Hole Thermodynamics, ICERM Workshop, Brown University, January 2026

- The breakdown of weak null singularities inside black holes
- The coexistence of null and spacelike singularities inside spherically symmetric black holes
- Asymptotically flat black holes with a singular Cauchy horizon and a spacelike singularity

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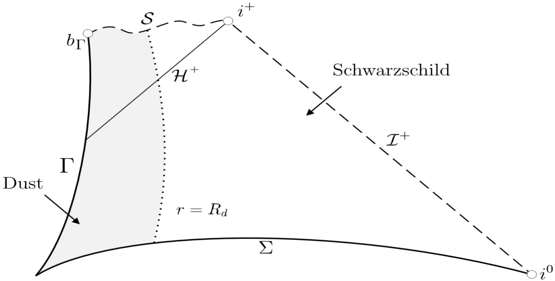
1. Foreword: a new model of gravitational collapse
2. Weak singularity at a dynamical Cauchy horizon
3. The breakdown of the Cauchy horizon in gravitational collapse
4. Coexisting null and spacelike singularities inside black holes

Gravitational collapse: the Oppenheimer–Snyder solution

- Collapse of homogeneous ball of dust of radius $R_d > 0$ (O–S, 1939).
- **Spacelike singularity** $\mathcal{S} = \{r = 0\}$: tidally destructive.
- Tidally compressive for $r < R_d$: (all) Jacobi fields $\vec{J}$ tend to $\vec{0}$ at $\mathcal{S}$.
- Tidally stretching for $r > R_d$: (some) Jacobi fields $|\vec{J}|$ tend to ∞ at $\mathcal{S}$.



- Spherical collapse of a scalar field: Christodoulou 90's, Choptuik 1993.

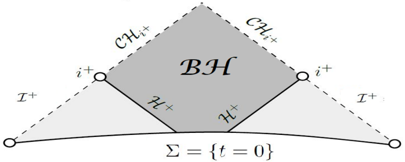
Kerr/Reissner–Nordström black hole's Cauchy horizons

Stationary (not "gravitational collapse") solution in (electro)-vacuum.

$$g_K = -\left(\frac{1-\frac{2M}{r}+\frac{a^2\cos^2(\theta)}{r^2}}{1+\frac{a^2\cos^2(\theta)}{r^2}}\right)dt^2 + \left(\frac{1-\frac{2M}{r}+\frac{a^2}{r^2}}{1+\frac{a^2\cos^2(\theta)}{r^2}}\right)^{-1}dr^2 - g_{t\psi}dtd\psi + g_{|\mathbb{S}^2}.$$

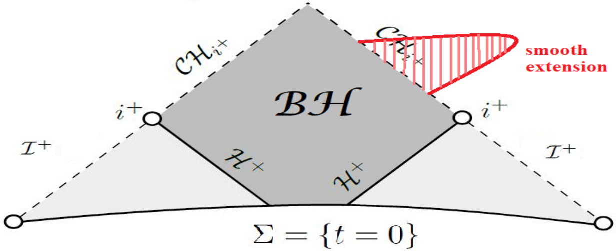
- Globally hyperbolic development of Kerr initial data $\Sigma = \{t=0\}$ terminates at (smooth) **Cauchy horizon** $\mathcal{CH}_{i^+} = \{r = r_-(M,a)\}$.
- Not globally hyperbolic beyond $\mathcal{CH}_{i^+}$: no "timelike singularity".

Maximal Globally Hyperbolic Development of Kerr spacetime



Kerr/Reissner–Nordström black hole's Cauchy horizons

- Globally hyperbolic development of Kerr initial data $\Sigma = \{t = 0\}$ terminates at (smooth) **Cauchy horizon** $\mathcal{CH}_{i^+} = \{r = r_-(M, a)\}$.
- $\mathcal{M}$ has (infinitely-many) smooth extensions: **determinism defeated**.
- Cannot explain fate of in-falling observers from their initial conditions.
- Reissner–Nordström black hole, too, has a Cauchy horizon $\mathcal{CH}_{i^+}$.

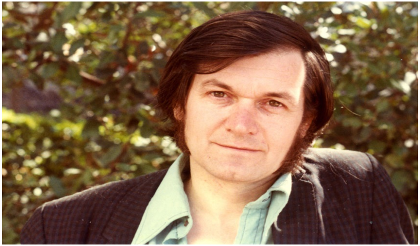


Determinism and Strong Cosmic Censorship Conjecture

- What does it mean for General Relativity to be a deterministic theory?

Conjecture (Strong Cosmic Censorship. Penrose 69', Christodoulou 99')

The (maximal) globally hyperbolic development (M, g) *of a generic solution to the Einstein equations is inextendible (in some regularity class).*



- Hope: for a generic black hole, inextendibility caused by a **singularity**.
- Belief: **instability of Cauchy horizons** and spacelike singularity.

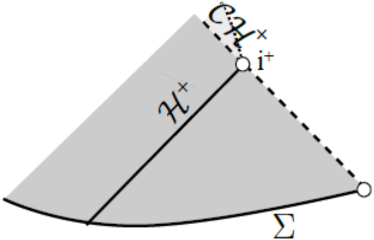
Stability of Cauchy horizons in dynamical black holes

- Myth debunked: Cauchy horizons are actually **stable** to perturbations.

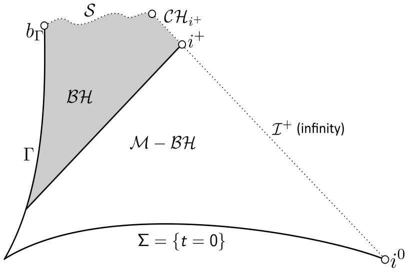
Theorem (Kerr Cauchy horizon stability, Dafermos-Luk 2017)

Small vacuum perturbations of Kerr admit a small piece of a (null) Cauchy horizon $\mathcal{CH}^+$ at which tidal deformations are finite.

- Proved in vacuum, but generalizes with Maxwell/scalar field matter.
- Is the terminal boundary even singular? Strong Cosmic Censorship?



An arguably more realistic model of gravitational collapse



- Model: Einstein–Maxwell-(charged)-scalar-field in spherical symmetry.

Theorem (AF BH with a singular CH and a spacelike singularity, VdM 25')

There exist gravitational collapse (asymptotically flat) solutions with

- *A (weak) null singularity: the Cauchy horizon $\mathcal{CH}_{i^+}$ (mass inflation).*
- *A (strong) spacelike singularity $\mathcal{S}$ (tidal destruction).*

- Outside spherical symmetry: same picture is conjectured (numerics).

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Einstein-Maxwell-charged-scalar-field in spherical symmetry

$$Ric_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \mathbb{T}^{EM}_{\mu\nu} + \mathbb{T}^{KG}_{\mu\nu}, \qquad \text{(Einstein)}$$

$$\nabla^{\mu} F_{\mu\nu} = q_0 \; Im(\phi \overline{D_{\nu}\phi}), \text{ where } q_0 \neq 0, \qquad \text{(Maxwell)}$$

$$F = dA, \text{ and } D_{\mu} = \nabla_{\mu} + iq_0 A_{\mu},$$

$$g^{\mu\nu} D_{\mu} D_{\nu} \phi = 0. \qquad \text{(Charged scalar field)}$$

- Electromagnetism: emulates effect of angular momentum (Wheeler).
- If $\phi \equiv 0$, obtain the Reissner–Nordström metric

$$g_{RN} = -(1 - \frac{2M}{r} + \frac{e^2}{r^2}) dt^2 + (1 - \frac{2M}{r} + \frac{e^2}{r^2})^{-1} dr^2 + r^2 (d\theta^2 + \sin(\theta)^2 d\psi^2).$$

- **Model of spherical gravitational collapse**: charged matter $q_0 \neq 0$ allows for a center $\Gamma = \{r = 0\}$. $q_0 = 0$ case: uncharged scalar field.
- Question: interior of a **generic black hole** for this model?

Blueshift instability: weakly singular Cauchy horizon

- Consider black holes approaching Reissner–Nordström exterior, i.e., $\phi_{\mathcal{H}^+}(v) \to 0$ as $v \to +\infty$ (**generic** decay rates à la Price's law).

Theorem (Weak singularity at dynamical Cauchy horizon, VdM 18', Kehle-VdM 24')

The Reissner–Nordström Cauchy horizon $\mathcal{CH}_{i^+}$ is stable, but due to blueshift instability, a weak singularity forms dynamically at $\mathcal{CH}_{i^+}$:

- *The metric g and scalar field ϕ are continuously extendible.*
- *The scalar field energy is infinite at $\mathcal{CH}_{i^+}$: $\int_{\mathcal{U}} |\nabla_g \phi|^2 = \infty$.*
- *The metric g has infinite curvature at $\mathcal{CH}_{i^+}$ (C^2-inextendible).*

- Indicates Strong Cosmic Censorship is true in spherical symmetry (as understood in the sense of C^2 or C^1-inextendibility, but not C^0).
- Uncharged case $q_0 = 0$: Dafermos 03', Luk–Oh 17', Gautam 24'. SCC for two-ended black holes in spherical symmetry and mass inflation.

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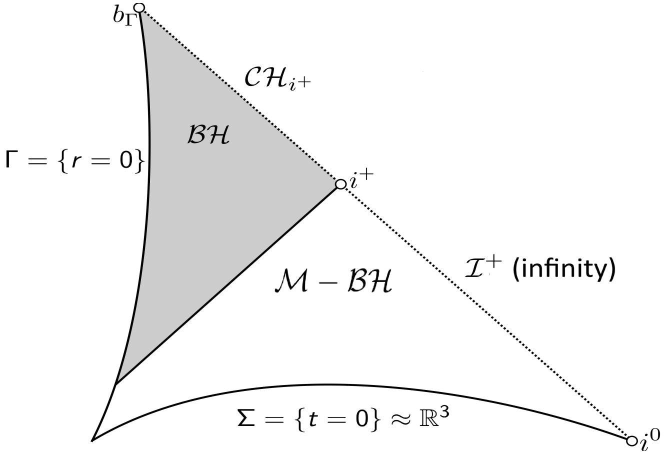
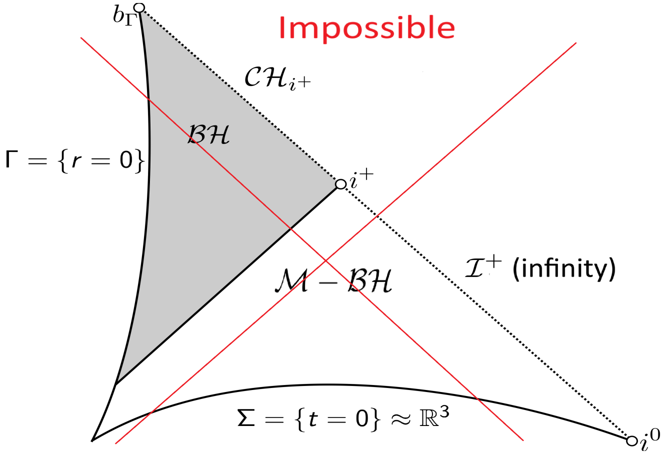
Global collapse dynamics of the Cauchy horizon

- Can the Cauchy horizon close-off the space-time and reach the center $\Gamma \subset \{r = 0\}$, i.e., **is the following Penrose diagram possible** ?



Global collapse dynamics of the Cauchy horizon

- Can the Cauchy horizon close-off the space-time and reach the center $\Gamma \subset \{r = 0\}$, i.e., is the following Penrose diagram possible ? **No!**

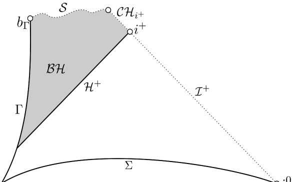


The breakdown of weak null singularities inside black holes

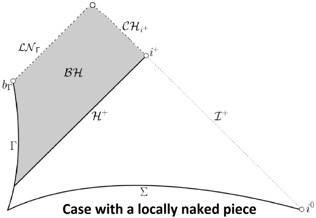
- Gravitational collapse assumes a regular center $\Gamma = \{r = 0\}$.

Theorem (Breakdown of the weakly singular Cauchy horizon, VdM. 23')

In addition, assume the existence of a regular center $\Gamma = \{r = 0\}$*. Then, the weakly singular Cauchy horizon* $\mathcal{CH}_{i^+}$ *cannot close-off the spacetime at the center* Γ*. Moreover, either there exists a strong singularity* $\mathcal{S} = \{r = 0\}$*, or there exists a locally naked singularity* $\mathcal{LN}_\Gamma$*.*



Case with a strong singularity



Case with a locally naked piece

- The locally naked singularity $\mathcal{LN}_\Gamma$ is conjecturally non-generic. (See Christodoulou's 1999 proof of Weak Cosmic Censorship for $F = 0$).

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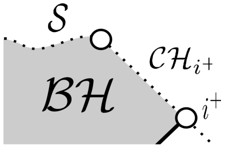
Quantitative description of the singularity near $\mathcal{CH}_{i^+}$

- Assuming no locally naked singularity and scalar field decay: for $s > 1$, $|D_v\phi|(u_0, v) \approx v^{-s}$, $|\Im(\bar{\phi} D_v \phi)|(u_0, v) \ll v^{-s}$, $|D_v^2\phi|(u_0, v) \lesssim v^{-s-1}$.

Theorem (Coexistence of weak-null & strong-spacelike singularities, VdM.)

Then, near $\mathcal{CH}_{i^+}$, the boundary consists of the (weakly singular) null Cauchy horizon $\mathcal{CH}_{i^+}$ and a spacelike singularity $\mathcal{S} = \{r = 0\}$ with infinite tidal compression (Jacobi fields tend to $\vec{0}$), degenerating near $\mathcal{CH}_{i^+} \cap \mathcal{S}$.



- 1st quantitative example of **weak/strong singularities transition**.
- Dynamical phenomenon: $\mathcal{CH}_{i^+}$'s singularity causes existence of $\mathcal{S}$.
- Quantitative description as a *Kasner model* with variable exponents.

Quantitative description of the spacelike singularity $\mathcal{S}$

- Kasner Einstein-scalar-field solution: exponents (p_1, p_2, p_3, q)

$$g_K = -dt^2 + \sum_{i=1}^{3} t^{2p_i} dx_i^2, \ \sum_{i=1}^{3} p_i = 1,$$

$$\phi = q \log(t), \ \sum_{i=1}^{3} p_i^2 = 1 - q^2.$$

- Coexistence: generalized Kasner metric with x-dependent exponents

$$g \approx -dt^2 + t^{2(1-2p(t,x))} dx^2 + t^{2p(t,x)}(d\theta^2 + \sin^2(\theta) d\varphi^2), \ x(u,v) \approx v^{2(1-s)}$$

- Kasner exponents **degenerate** to (trivial value) $(1,0,0)$ at $\mathcal{CH}_{i^+} \cap \mathcal{S}$.
- Universal behavior (decay rate s-independent): $\phi(u_{\mathcal{CH}_{i^+}}, v) \approx \sqrt{v}$.

$$(1-2p(u,v), p(u,v), p(u,v)) \text{ and } p(u,v) \approx v^{-1} \text{ as } v \to +\infty.$$

Quantitative description of the spacelike singularity $\mathcal{S}$

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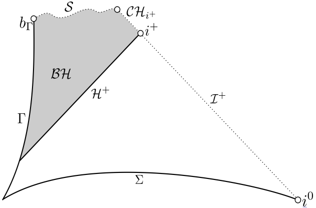
New global gravitational collapse solutions (examples)

Theorem (Gravitational collapse with null and spacelike singularities, VdM.)

There exist black hole spacetime solutions with a regular center Γ *(one-ended asymptotically flat) such that the terminal boundary is*

- *a weakly singular (null) Cauchy horizon* $\mathcal{CH}_{i^+}$ *(mass inflation).*
- *a strong singularity* $\mathcal{S} = \{r = 0\}$*, spacelike and tidally contractive near* $\mathcal{S} \cap \mathcal{CH}_{i^+}$*, and asymptotically Kasner, degenerating at* $\mathcal{S} \cap \mathcal{CH}_{i^+}$*.*

- Near the center Γ, $\mathcal{S}$ is a FRLW spacelike singularity (isotropic).
- Spacelike/characteristic gluing: small BH mass and extremality ratio.



Future possible directions

Extremal case (charged scalar field, spherical symmetry)

- No mass inflation at extremal $\mathcal{CH}_{i^+}$ (Gajic–Luk 19', for small BH charge).
- Cauchy horizon closing-off the spacetime? Spacelike singularity?
- Late-time asymptotics in the black hole exterior?

Outside spherical symmetry (Einstein-scalar-field, sub-extremal BH)

- Stability of Cauchy horizon $\mathcal{CH}_{i^+}$ for Kerr perturbations (Dafermos–Luk 17' extends to Einstein-scalar-field).
- Weak singularity expected at $\mathcal{CH}_{i^+}$ (Luk 13', Sbierski 23', Gurriaran 25').
- Null/spacelike coexistence observed numerically (Chesler 19'); scalar field stabilizes the spacelike singularity, even outside symmetry.

Outside spherical symmetry (Einstein vacuum, sub-extremal BH)

- BKL instability of spacelike singularities: chaotic dynamics?

Thank you for your attention!

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