Critical Phenomena in Gravitational Collapse

Thomas Baumgarte

Department of Physics and Astronomy
Bowdoin College

January 5, 2026

Critical Phenomena in Gravitational Collapse

Things that we know in spherical symmetry...

Thomas Baumgarte

Department of Physics and Astronomy
Bowdoin College

January 5, 2026

Critical Phenomena in Gravitational Collapse

Things that we know in spherical symmetry...

... and things that we don't know in the absence of spherical symmetry

Thomas Baumgarte

Department of Physics and Astronomy
Bowdoin College

January 5, 2026

Things that we know in spherical symmetry...

VOLUME 70, NUMBER 1 PHYSICAL REVIEW LETTERS 4 JANUARY 1993

Universality and Scaling in Gravitational Collapse of a Massless Scalar Field

Matthew W. Choptuik
Center for Relativity, University of Texas at Austin, Austin, Texas 78712-1081
(Received 22 September 1992)

I summarize results from a numerical study of spherically symmetric collapse of a massless scalar field. I consider families of solutions, $\mathcal{S}[p]$, with the property that a critical parameter value, $p^\star$, separates solutions containing black holes from those which do not. I present evidence in support of conjectures that (1) the strong-field evolution in the $p \to p^\star$ limit is universal and generates structure on arbitrarily small spatiotemporal scales and (2) the masses of black holes which form satisfy a power law $M_{\rm BH} \propto |p - p^\star|^\gamma$, where $\gamma \approx 0.37$ is a universal exponent.

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\phi = 0$$

coupled to Einstein's equations

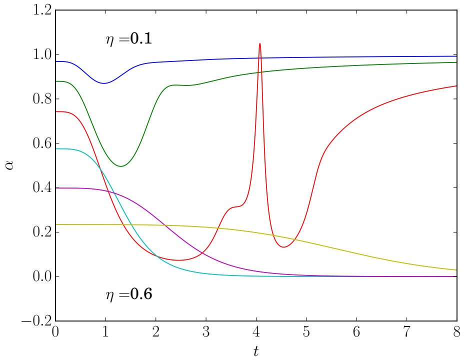
- Initial data

$$\phi = \eta \exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.3 < \eta_* < 0.4$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\,\phi = 0$$

coupled to Einstein's equations

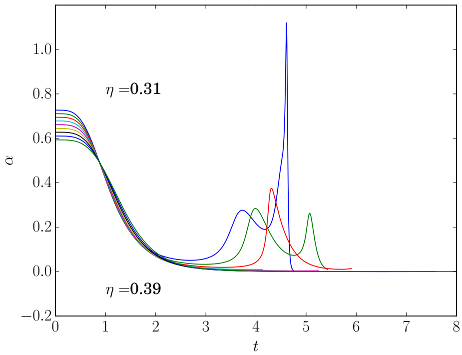
- Initial data

$$\phi = \eta\,\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.30 < \eta_* < 0.31$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\phi = 0$$

coupled to Einstein's equations

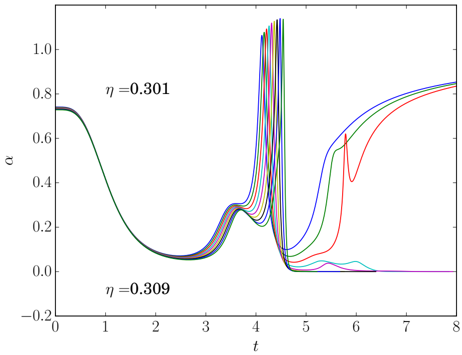
- Initial data

$$\phi = \eta\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.303 < \eta_* < 0.304$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\phi = 0$$

coupled to Einstein's equations

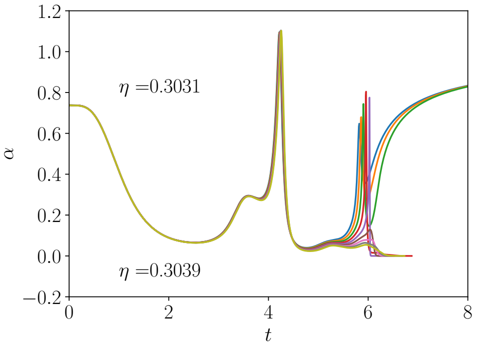
- Initial data

$$\phi = \eta\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.3033 < \eta_* < 0.3034$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\phi = 0$$

coupled to Einstein's equations

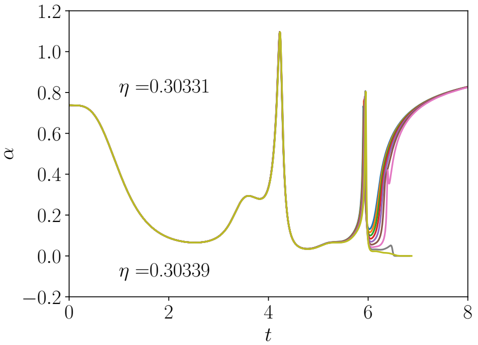
- Initial data

$$\phi = \eta\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.30337 < \eta_* < 0.30338$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\,\phi = 0$$

coupled to Einstein's equations

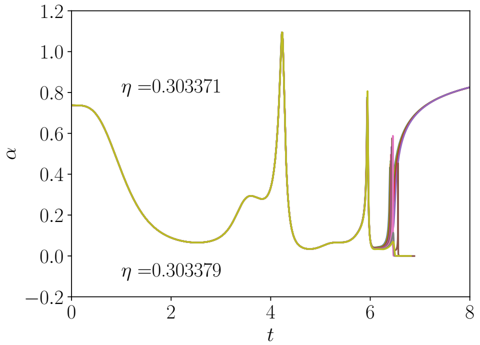
- Initial data

$$\phi = \eta\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.303375 < \eta_* < 0.303376$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\phi = 0$$

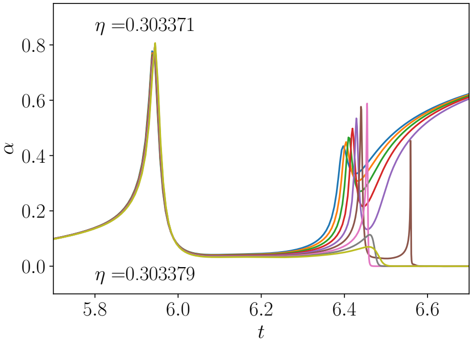
  coupled to Einstein's equations
- Initial data

$$\phi = \eta\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.303375 < \eta_* < 0.303376$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\phi = 0$$

coupled to Einstein's equations

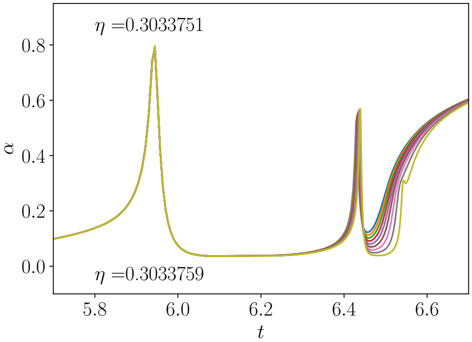
- Initial data

$$\phi = \eta\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.3033759 < \eta_* < 0.3033760$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\,\phi = 0$$

coupled to Einstein's equations

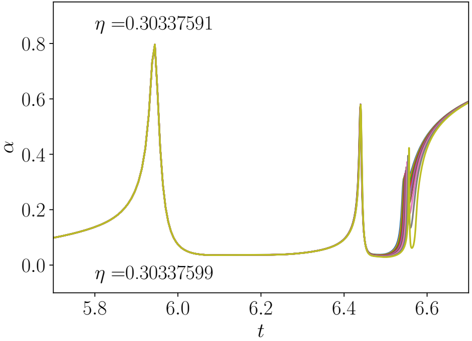
- Initial data

$$\phi = \eta \exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.30337599 < \eta_* < 0.30337600$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\,\phi = 0$$

coupled to Einstein's equations

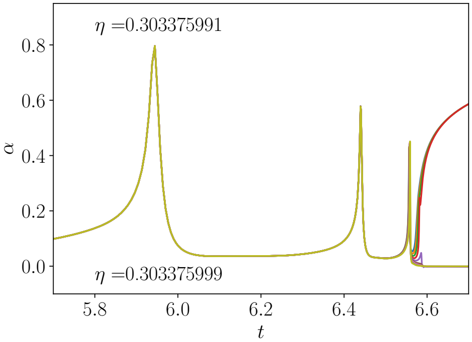
- Initial data

$$\phi = \eta\,\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.303375994 < \eta_* < 0.303375995$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\,\phi = 0$$

coupled to Einstein's equations

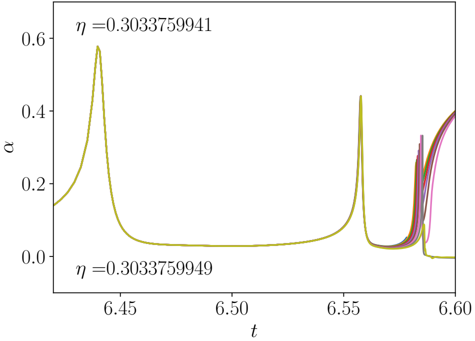
- Initial data

$$\phi = \eta\,\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.3033759947 < \eta_* < 0.3033759948$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\,\phi = 0$$

coupled to Einstein's equations

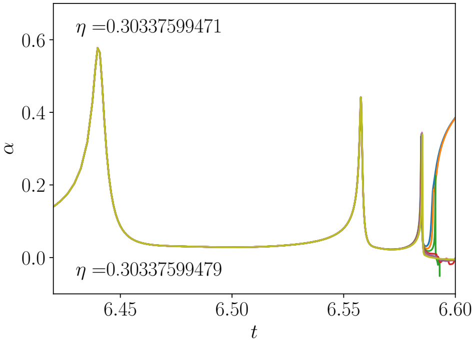
- Initial data

$$\phi = \eta\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.30337599472 < \eta_* < 0.30337599473$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\,\phi = 0$$

coupled to Einstein's equations

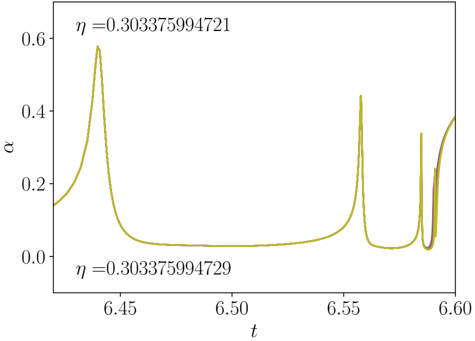
- Initial data

$$\phi = \eta\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.303375994729 < \eta_* < 0.303375994730$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\,\phi = 0$$

coupled to Einstein's equations

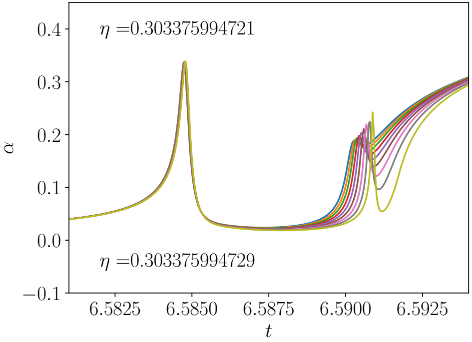
- Initial data

$$\phi = \eta\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.303375994729 < \eta_* < 0.303375994730$$

A numerical experiment...

- Consider massless scalar field

$$\Box\phi \equiv g^{ab}\nabla_a\nabla_b\,\phi = 0$$

coupled to Einstein's equations

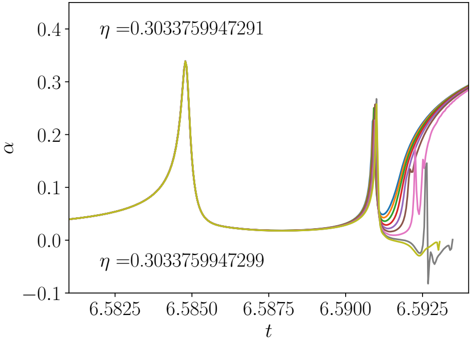
- Initial data

$$\phi = \eta\,\exp(-R^2/R_0^2)$$

- evolve for different amplitudes η...
- Have *critical parameter* η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \to \text{flat space}$$

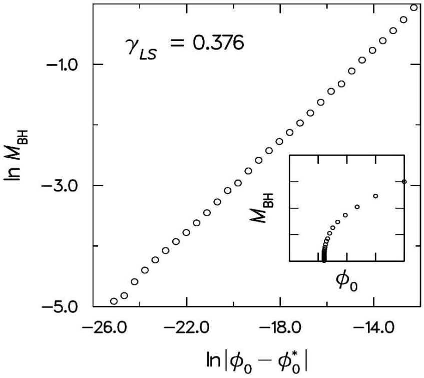
$$\eta > \eta_* \qquad \alpha \to 0 \qquad \to \text{black hole}$$



$$0.3033759947297 < \eta_* < 0.3033759947298$$

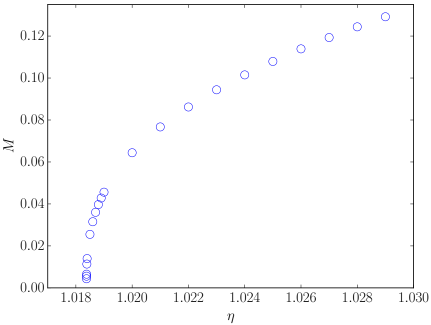
Scaling

- For supercritical data, mass M of forming black holes satisfies *power-law scaling*
$$M \simeq (\eta - \eta_*)^\gamma$$
with *critical exponent* $\gamma \simeq 0.37$
- For subcritical data, maximum attained curvature satisfies similar scaling law with same exponent [Garfinkle & Duncan, 1998]

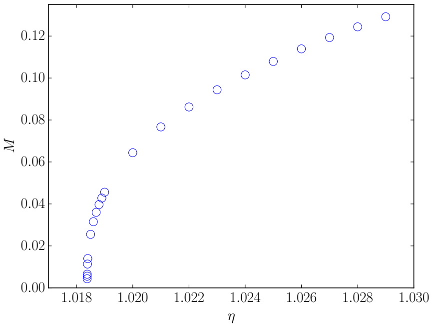


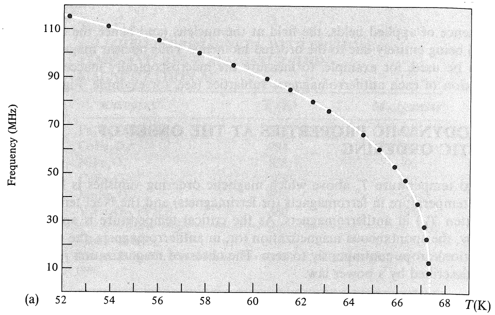
[Choptuik, 1998]

Looks familiar?



Looks familiar?





Critical Phenomena...

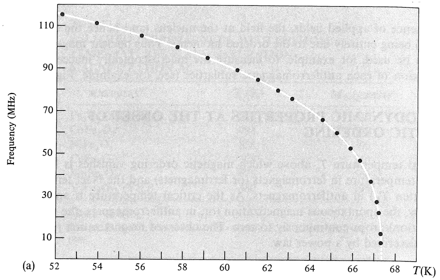
THERMODYNAMIC PROPERTIES AT THE ONSET OF MAGNETIC ORDERING

The critical temperature T_c above which magnetic ordering vanishes is known as the Curie temperature in ferromagnets (or ferrimagnets) and the Néel temperature (often written T_N) in antiferromagnets. As the critical temperature is approached from below, the spontaneous magnetization (or, in antiferromagnets, the sublattice magnetization) drops continuously to zero. The observed magnetization just below T_c is well described by a power law.

$$M(T) \sim (T_c - T)^{\beta}, \qquad \textbf{(33.1)}$$

where β is typically between 0.33 and 0.37 (see Figure 33.4).

The onset of ordering is also signaled as the temperature drops to T_c from above,



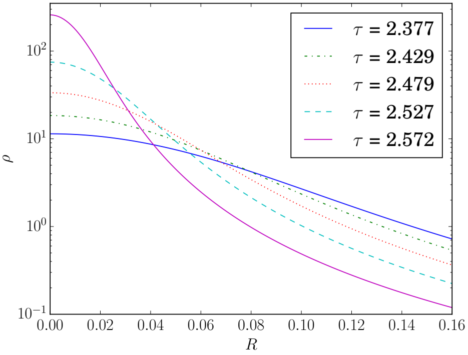
Magnetic field M in MnF_2 versus temperature T
[Ashcroft & Mermin, *Solid State Physics*, 1976]

Power-law scaling in vicinity of critical parameters with *universal* critical exponent

Critical Solution

Radiation fluid ($P = \rho/3$) fine-tuned to critical parameter [Evans & Coleman(1994)]

- plot ρ versus R at different times...

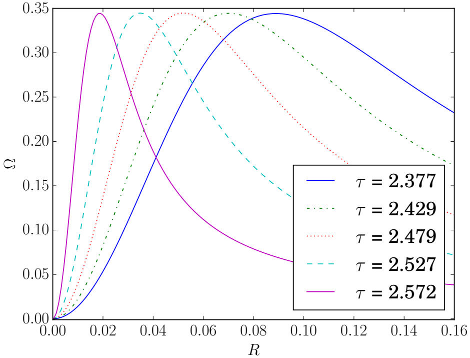


[TWB & Montero, 2016]

Critical Solution

Radiation fluid ($P = \rho/3$) fine-tuned to critical parameter [Evans & Coleman(1994)]

- plot ρ versus R at different times...
- plot $\Omega \equiv 4\pi\rho R^2$ versus R...



[TWB & Montero, 2016]

Critical Solution

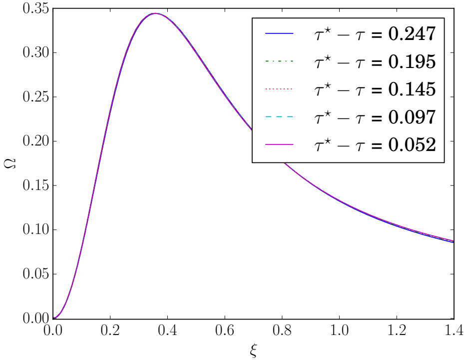
Radiation fluid ($P = \rho/3$) fine-tuned to critical parameter [Evans & Coleman(1994)]

- plot ρ versus R at different times...
- plot $\Omega \equiv 4\pi\rho R^2$ versus R...
- plot Ω versus

$$\xi \equiv \frac{R}{\tau_* - \tau}$$

with *accumulation time* $\tau_* = 2.624$

Critical solution is *self-similar* and *universal*

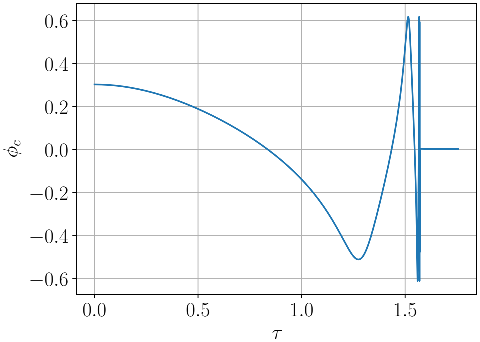


[TWB & Montero, 2016]

Critical Solution for scalar field

- Look at scalar field ϕ at center ($r = 0$)
- plot as function of proper time τ
- oscillations accumulate at accumulation time

$$\tau_* \simeq 1.5698$$



Critical Solution for scalar field

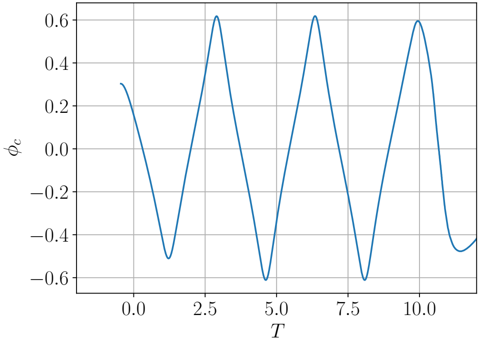
- Look at scalar field ϕ at center $(r = 0)$
- plot as function of proper time τ
- oscillations accumulate at accumulation time

$$\tau_* \simeq 1.5698$$

- plot as function of *self-similar time*

$$T \equiv -\ln(\tau_* - \tau)$$

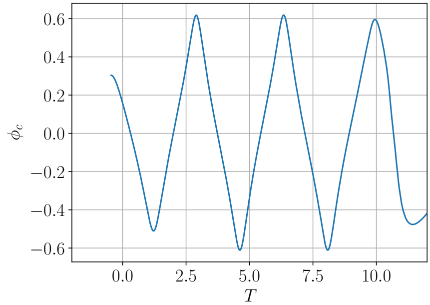
Critical solution performs oscillations with period Δ in T: *discrete self-similarity*

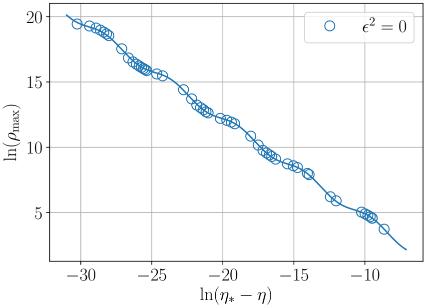


Continuous versus discrete self-similarity

Self-similarity can be...

- *continuous* (CSS) (e.g. fluid)
- *discrete* (DSS) (e.g. scalar fields):
  expect periodic "wiggles" super-imposed on scaling laws
  [Gundlach, 1997; Hod & Piran, 1997]





Critical Phenomena in Gravitational Collapse

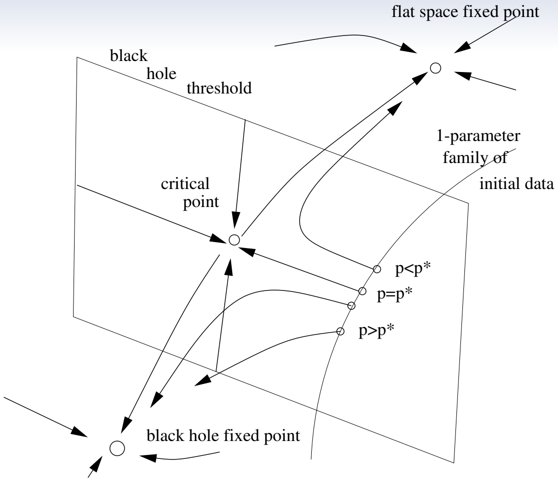
- Consider matter model (e.g. scalar field, fluid, vacuum...)
- Consider family of initial data parametrized by η and evolve...
- Critical parameter η_* separates:
  - *supercritical* data: form black hole
  - *subcritical* data: don't
- in vicinity of η_* observe *critical phenomena*:
  - dimensional quantities display *scaling*, e.g.

$$M \simeq (\eta - \eta_*)^\gamma$$

    with *critical exponent* γ: depends on matter model, but not on parametrization of initial data
  - spacetime approaches universal *self-similar solution*

Phase-space picture

- (infinite-dimensional) phase-space of initial data
- critical solution acts as intermediate attractor of co-dimension one
- possesses single unstable mode, Lyapunov exponent λ
- critical exponent given by $\gamma = 1/\lambda$ [Koike *et.al.*, 1995; Maison, 1996]

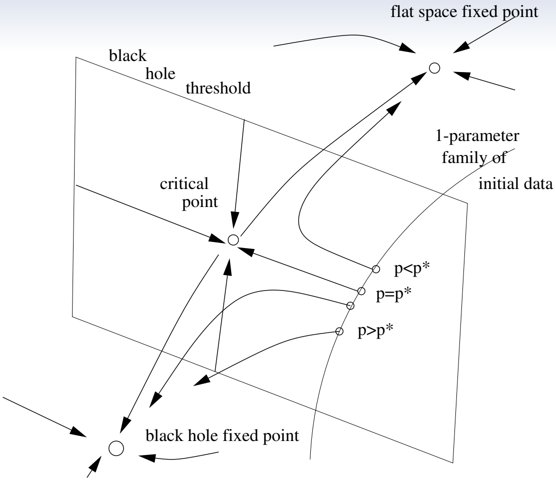


[Gundlach & Martín-García, 2007]

Phase-space picture

- (infinite-dimensional) phase-space of initial data
- critical solution acts as intermediate attractor of co-dimension one
- possesses single unstable mode, Lyapunov exponent λ
- critical exponent given by $\gamma = 1/\lambda$ [Koike *et.al.*, 1995; Maison, 1996]

Does this picture persist in the absence of spherical symmetry?



[Gundlach & Martín-García, 2007]

Things that we thought we knew in the absence of spherical symmetry...

VOLUME 70, NUMBER 20 PHYSICAL REVIEW LETTERS 17 MAY 1993

Critical Behavior and Scaling in Vacuum Axisymmetric Gravitational Collapse

Andrew M. Abrahams[a]
Center for Radiophysics and Space Research, Cornell University, Ithaca, New York 14853

Charles R. Evans
Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27599
(Received 22 December 1992)

We report a second example of critical behavior in gravitational collapse. Collapse of axisymmetric gravitational wave packets is computed numerically for a one-parameter family of initial data. A black hole first appears along the sequence at a critical parameter value $p^\star$. As with spherical scalar-field collapse, a power law is found to relate black-hole mass (the order parameter) and critical separation: $M_{\rm BH} \propto |p - p^\star|^\beta$. The critical exponent is $\beta \simeq 0.37$, remarkably close to that observed by Choptuik. Near-critical evolutions produce echoes from the strong-field region which appear to exhibit scaling.

But...

Despite many attempts...

[Alcubierre *et.al.*, 2000; Garfinkle & Duncan, 2001; Santamaria, 2006; Rinne, 2008; Sorkin, 2011; Hilditch *et.al.*, 2013]

... it has been difficult to reproduce this:

- many simulations crash
- others produce results in apparent conflict with A&E

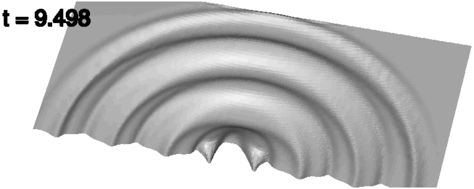
(more on this later...)

Critical collapse without spherical symmetry: scalar fields

- *All nonspherical perturbations of the Choptuik spacetime decay* (linear perturbations) [Martín-García & Gundlach, 1999]

Critical collapse without spherical symmetry: scalar fields

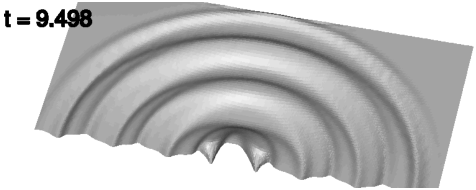
- *All nonspherical perturbations of the Choptuik spacetime decay* (linear perturbations) [Martín-García & Gundlach, 1999]
- But *nonlinear* aspherical perturbations grow and lead to bifurcation [Choptuik *et.al.*, 2003; TWB, 2018; Marouda *et.al.*, 2024]



[Choptuik *et.al.*, 2003]

Critical collapse without spherical symmetry: scalar fields

- *All nonspherical perturbations of the Choptuik spacetime decay* (linear perturbations) [Martín-García & Gundlach, 1999]
- But *nonlinear* aspherical perturbations grow and lead to bifurcation [Choptuik *et.al.*, 2003; TWB, 2018; Marouda *et.al.*, 2024]
- Inclusion of angular momentum leads to *different* critical solution [Choptuik *et.al.*, 2004; Marouda 2025]



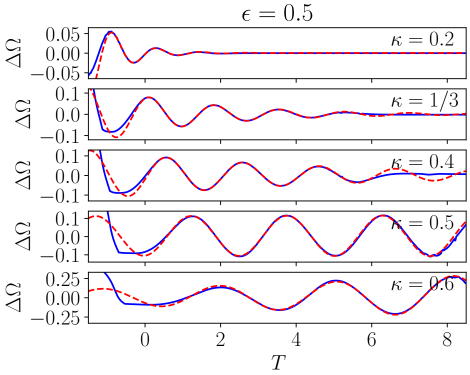
[Choptuik *et.al.*, 2003]

Critical collapse without spherical symmetry: ideal fluids

Fluid with equation of state

$$P = \kappa\rho$$

- (non-rotating) aspherical deformations ($\ell = 2$) unstable for $\kappa \gtrsim 0.49$ [Gundlach, 2002; TWB & Montero, 2015; Celestino & TWB, 2018]



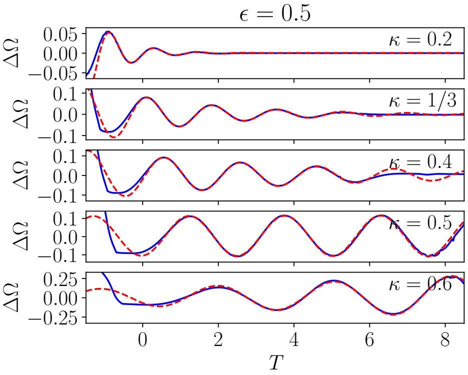
[Celestino & TWB, 2018]

Critical collapse without spherical symmetry: ideal fluids

Fluid with equation of state

$$P = \kappa\rho$$

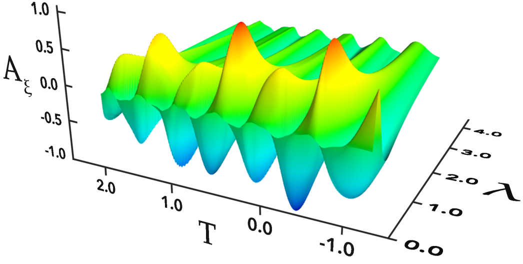
- (non-rotating) aspherical deformations ($\ell = 2$) unstable for $\kappa \gtrsim 0.49$ [Gundlach, 2002; TWB & Montero, 2015; Celestino & TWB, 2018]
- (rotational) perturbations ($\ell = 1$) unstable for $\kappa < 1/9$ [Gundlach, 2002; TWB & Gundlach, 2016; Gundlach & TWB, 2016, 2017]



[Celestino & TWB, 2018]

Critical collapse without spherical symmetry: electromagnetic waves

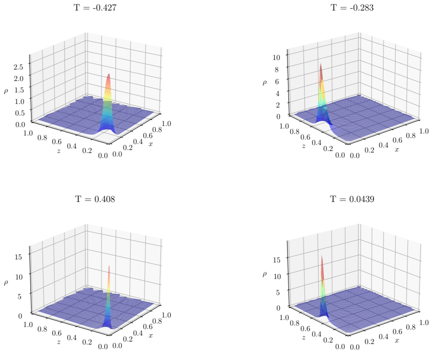
- Dipolar electromagnetic waves feature only approximately DSS critical solution [TWB *et.al.*, 2019]
- Competition between different degrees of freedom? [Gundlach *et.al.*., 2019]



[TWB *et.al.*, 2019]

Critical collapse without spherical symmetry: electromagnetic waves

- Dipolar electromagnetic waves feature only approximately DSS critical solution [TWB *et.al.*, 2019]
- Competition between different degrees of freedom?
  [Gundlach *et.al.*., 2019]
- Do quadrupolar electromagnetic waves have distinct critical solution?
  [Perez Mendoza & TWB, 2021; Gray & Choptuik, 2023]



[Perez Mendoza & TWB, 2021]

Things that we don't know in the absence of spherical symmetry

In absence of spherical symmetry, threshold of black hole formation significantly more complicated

- threshold solution not exactly self-similar?
- threshold solution not unique?
- in some cases evidence for bifurcation?
- single or multiple accumulation points?

Things that we don't know in the absence of spherical symmetry

In absence of spherical symmetry, threshold of black hole formation significantly more complicated

- threshold solution not exactly self-similar?
- threshold solution not unique?
- in some cases evidence for bifurcation?
- single or multiple accumulation points?

Revisit critical collapse of gravitational waves

GW initial data: Teukolsky waves

Wave-like solution to linearized Einstein equations [Teukolsky, 1982; Rinne, 2008], "dressed up" to satisfy constraints

- E.g.: choose moment of time symmetry: momentum constraint satisfied identically
- Construct spatial metric from seed functions $F = F(r \pm t)$ and its derivatives, e.g.

$$C = \frac{1}{4}\left(\frac{F^{(4)}}{r} + \frac{2F^{(3)}}{r^2} + \frac{9F^{(2)}}{r^3} + \frac{21F^{(1)}}{r^4} + \frac{21F}{r^5}\right)$$

- Adopt spatial metric as conformally related metric
- Solve Hamiltonian constraint to construct (non-linear) initial data: invert *non-flat* Laplace operator

Adopted by Abrahams & Evans (1993) (see also Rostworowski, 2025)

GW initial data: Brill waves

Construct time-symmetric vacuum initial data [Brill, 1959]

- choose axisymmetric seed function, e.g.

$$q(r,\theta) = Ar^2 \sin^2\theta e^{-r^2} = A\rho^2 e^{-(\rho^2+z^2)}$$

- deform conformally related spatial metric (in cylindrical coordinates)

$$dl^2 = \psi^4 \left(e^q (d\rho^2 + dz^2) + \rho^2 d\phi^2 \right)$$

- solve linear *flat* elliptic equation for conformal factor

$$\nabla^2\psi = -\frac{\psi}{8}\left(\frac{\partial^2 q}{\partial\rho^2} + \frac{\partial^2 q}{\partial z^2}\right)$$

Adopted by almost everybody since Abrahams & Evans (1993)

Evolution

- Many previous attempts used 1+log slicing,

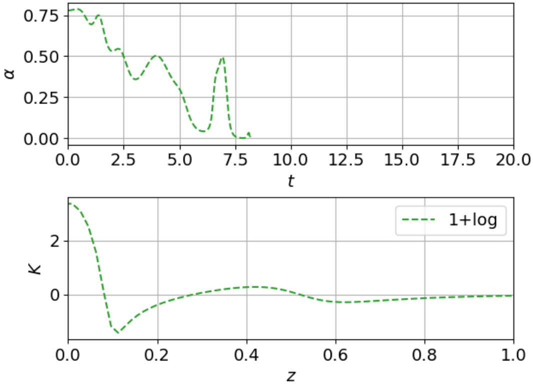
$$(\partial_t - \beta^i \partial_i)\alpha = -\alpha^2 f(\alpha) K$$

with

$$f(\alpha) = 2/\alpha$$

[Bona *et.al.*, 1995]

- Very successful in many cases, but can lead to coordinate shocks... [Alcubierre, 1997; 2003]



[TWB, Gundlach & Hilditch, 2023]

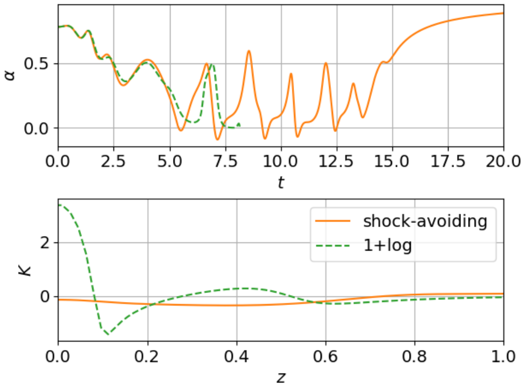
Evolution

Consider alternatives...

- `sphGR`: *shock-avoiding* slicing condition in BSSN: use

  $$f(\alpha) = 1 + 1/\alpha^2$$

  [Alcubierre, 1997; TWB & Hilditch, 2022]
- `prague`: approximate maximal slicing in BSSN
  [Ledvinka & Khirnov, 2018]
- `bamps`: gauge-source functions in generalized harmonic formalism
  [Hilditch *et.al.*, 2017]



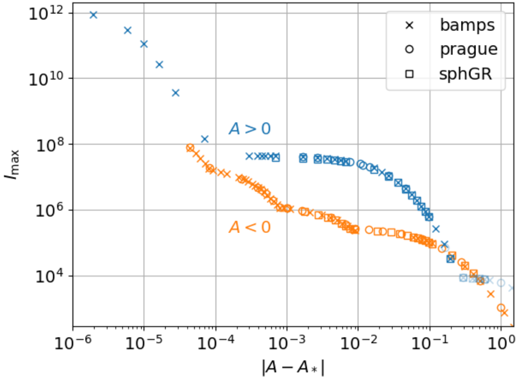
[TWB, Gundlach & Hilditch, 2023]

Code comparison

- Choose Brill initial data with both $A > 0$ and $A < 0$
- plot maximum attained curvature invariant I for subcritical data
- all three codes agree very well
- But: no evidence for universal power law

$$I \simeq |A - A_*|^{-4\gamma}$$

plus periodic wiggles with universal γ



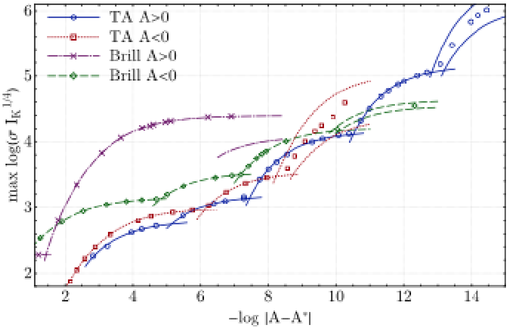
[TWB *et.al.*, 2023]

Universality...

Unlike in spherically symmetric case...

- γ depends on family
- threshold solution family-dependent
- No clear evidence for threshold solutions being DSS

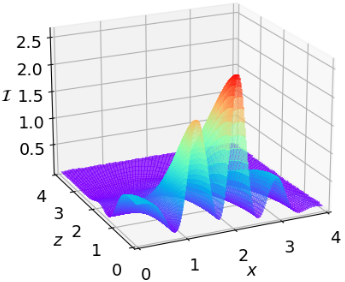
But: do gravitational-wave families with DSS threshold solutions exist??



[Ledvinka & Khirnov, 2021]

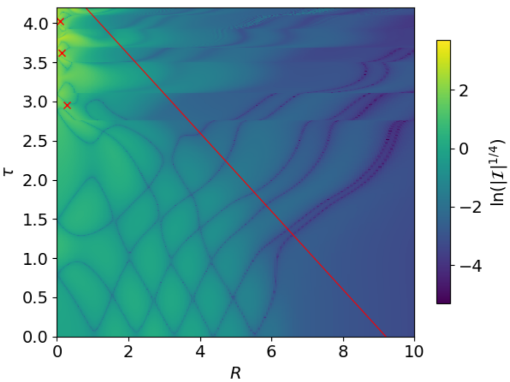
Evolution of Teukolsky waves

- Consider superposition of quadrupolar ($\ell = 2$) Teukolsky waves
- Use `sphGR` code (BSSN in spherical polar coordinates)
- evolve with shock-avoiding slicing condition
- analyze Weyl scalar $\mathcal{I}$ [TWB, Gundlach & Hilditch, 2023]



Quadrupolar wave

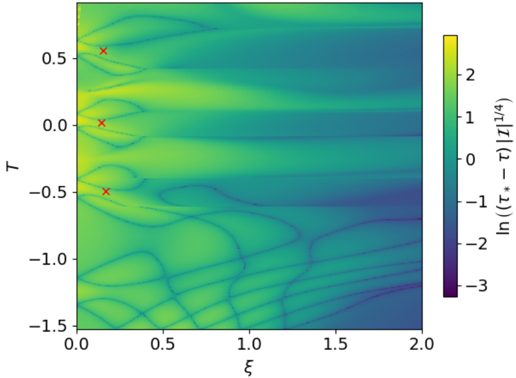
- Weyl scalar $\mathcal{I}$ for near-critical $\ell = 2$ solution in equatorial plane
- plot as function of R and τ...



[TWB, Gundlach & Hilditch, 2023]

Quadrupolar wave

- Weyl scalar $\mathcal{I}$ for near-critical $\ell = 2$ solution in equatorial plane
- plot as function of R and τ...
- plot as function of ξ and T...

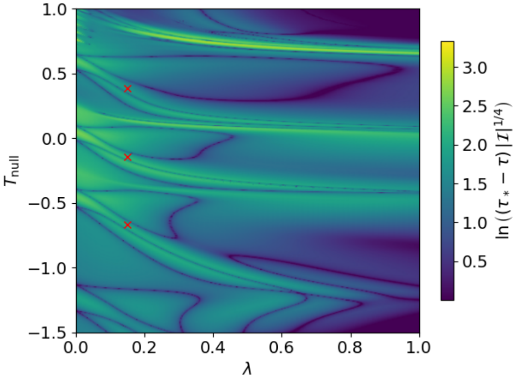


[TWB, Gundlach & Hilditch, 2023]

Quadrupolar wave

- Weyl scalar $\mathcal{I}$ for near-critical $\ell = 2$ solution in equatorial plane
- plot as function of R and τ...
- plot as function of ξ and T...
- plot as function of null coordinates λ and T_{null}

Approximate DSS with period $\Delta \simeq 0.52$, consistent with Abrahams & Evans (1993)

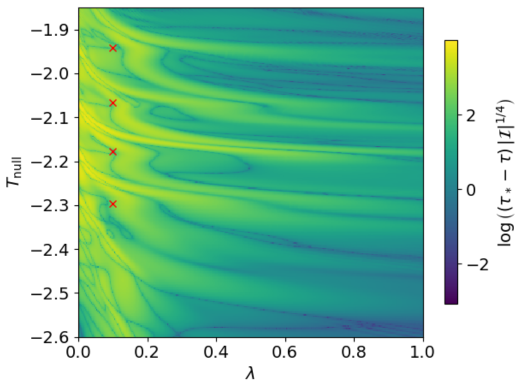


[TWB, Gundlach & Hilditch, 2023]

Hexadecapolar wave

- Repeat analysis for $\ell = 4$
- find approximate DSS with period $\Delta \simeq 0.1$

Quadrupolar and hexadecapolar threshold solutions are distinct



[TWB, Gundlach & Hilditch, 2023]

GW initial data: Nakamura waves

Analytical wave-like solution to linearized Einstein equations [Nakamura, 1984], “dressed up” to satisfy constraints

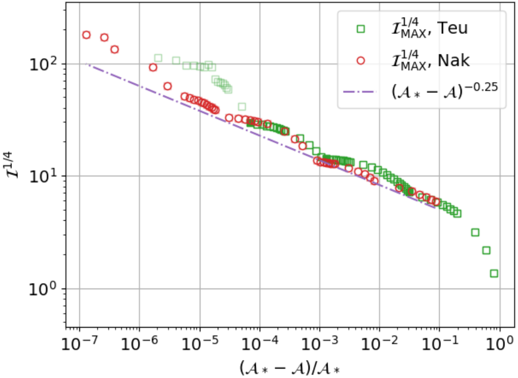
- E.g.: choose moment of time when $\gamma_{ij} = \eta_{ij}$
- Construct analytical solution to momentum constraint from seed functions $F = F(r \pm t)$, and its derivatives
- Solve Hamiltonian constraint to construct (non-linear) initial data: invert *flat* Laplace operator

Quadrupolar Teukolsky and Nakamura waves

- For both families find approximate scaling with

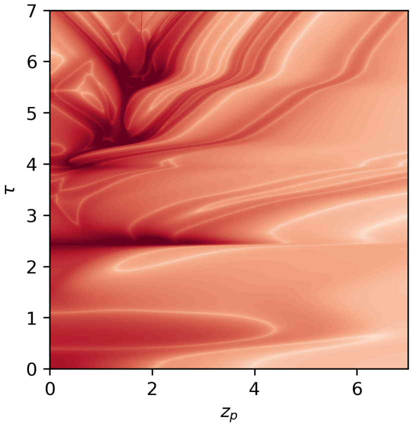
$$\gamma \simeq 0.25$$

- threshold solutions appear to be approximately, but not exactly DSS



Does every family have DSS threshold solution?

- Consider $\mathcal{I}$ for Brill waves with $A > 0$ (focus of most previous studies)
- No evidence for self-similarity with accumulation point at center



[TWB *et.al.*, 2023]

Summary

- Critical phenomena in gravitational collapse
- Universality, self-similarity, and scaling
- Well understood in spherical symmetry...
- ... but less so in absence of spherical symmetry
- Recent progress for vacuum gravitational waves:
  - replace 1+log slicing with other choice
  - good agreement between independent codes
  - no evidence for universal critical solution
  - for some families there exist at least approximate DSS threshold solutions...
  - ... but possibly not for others

Why has it been so difficult to reproduce Abrahams & Evans?

- Numerical issues:
  - popular slicing condition (1+log) fails in these particular applications
  - use different slicing condition
- Absence of uniqueness:
  - different families of initial data lead to different threshold solutions (at least at current level of fine-tuning)
  - does *not* present conflict with Abrahams & Evans (1993)