Functions are All You Need

Nathan Carter, May 20, 2025

with apologies to Vaswani et al. 2017

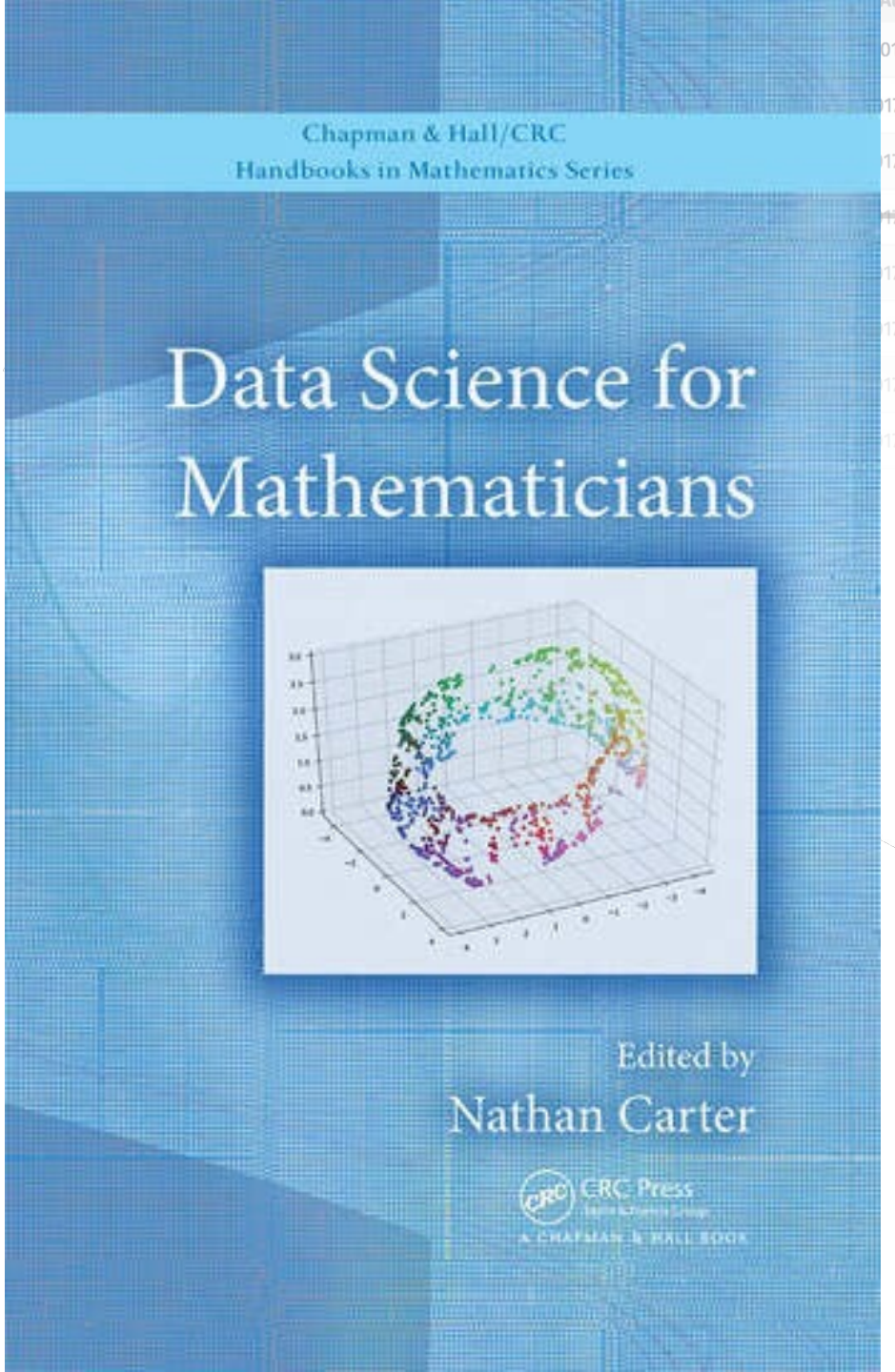
BENTLEY UNIVERSITY

Center for Analytics and Data Science

Data Science for Mathematicians

Chapman & Hall/CRC
2020

18 authors



Aspects of functions

- Plugging in values
- Drawing a graph
- Domain and range
- Vertical line test
- Table of (x, y) pairs
- Derivatives
- Antiderivatives
- Codomain
- Image, inverse image
- (Non)deterministic
- Multivariate and vector functions
- Injective and surjective
- Embeddings
- Continuity
- Homomorphisms
- Distance functions
- Metric spaces
- Function spaces
- Approximating a function
- Well-definedness
- Iteration
- Tangent planes
- Etc…

Outline: Three examples

of how familiarity with functions accelerates data science learning

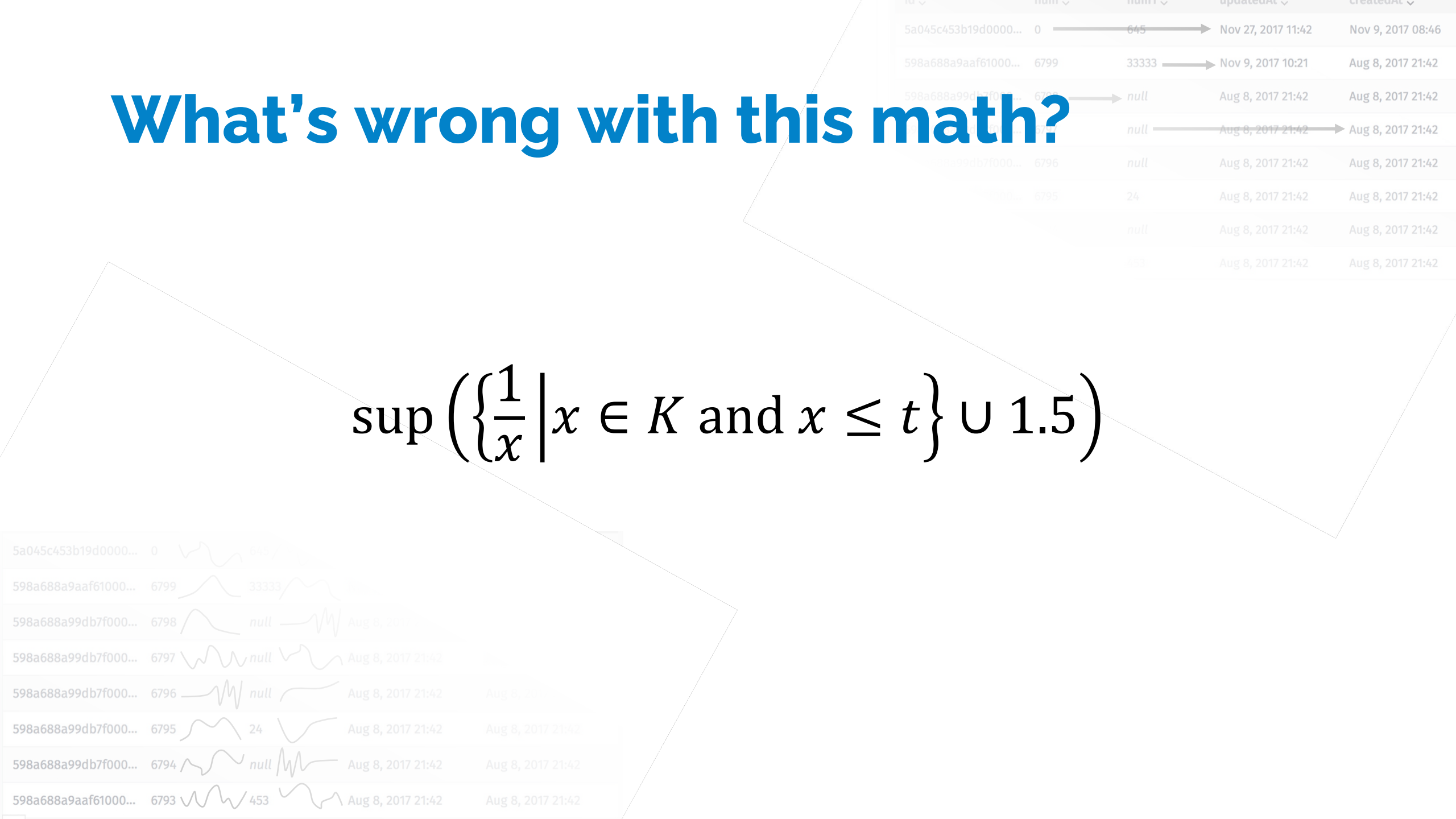
1. Composition of functions
   - Why this matters: Education

2. Tables as functions
   - Why this matters: Working with data

3. Artificial Intelligence (AI)
   - Why this matters: Asking important questions

What's wrong with this math?

$$\sup\left(\left\{\frac{1}{x}\middle|x \in K \text{ and } x \leq t\right\} \cup 1.5\right)$$



```
p = seq( 0.1, 0.9, 0.1 )
strength = 100
ematrix = matrix( 0, strength, length(p) )
for (i in 1:strength) {
  for (j in 1:length(p)) {
    simulated.data = rbinom( 100, 1, p[j] )
    disc.sim.data = discretize(
      simulated.data, numBins = 2)
    ematrix[i,j] = entropy(
      disc.sim.data,method = c("CS"),
      unit = c("log2"))
  }
}
ematrix
```

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p = seq( 0.1, 0.9, 0.1 )
strength = 100
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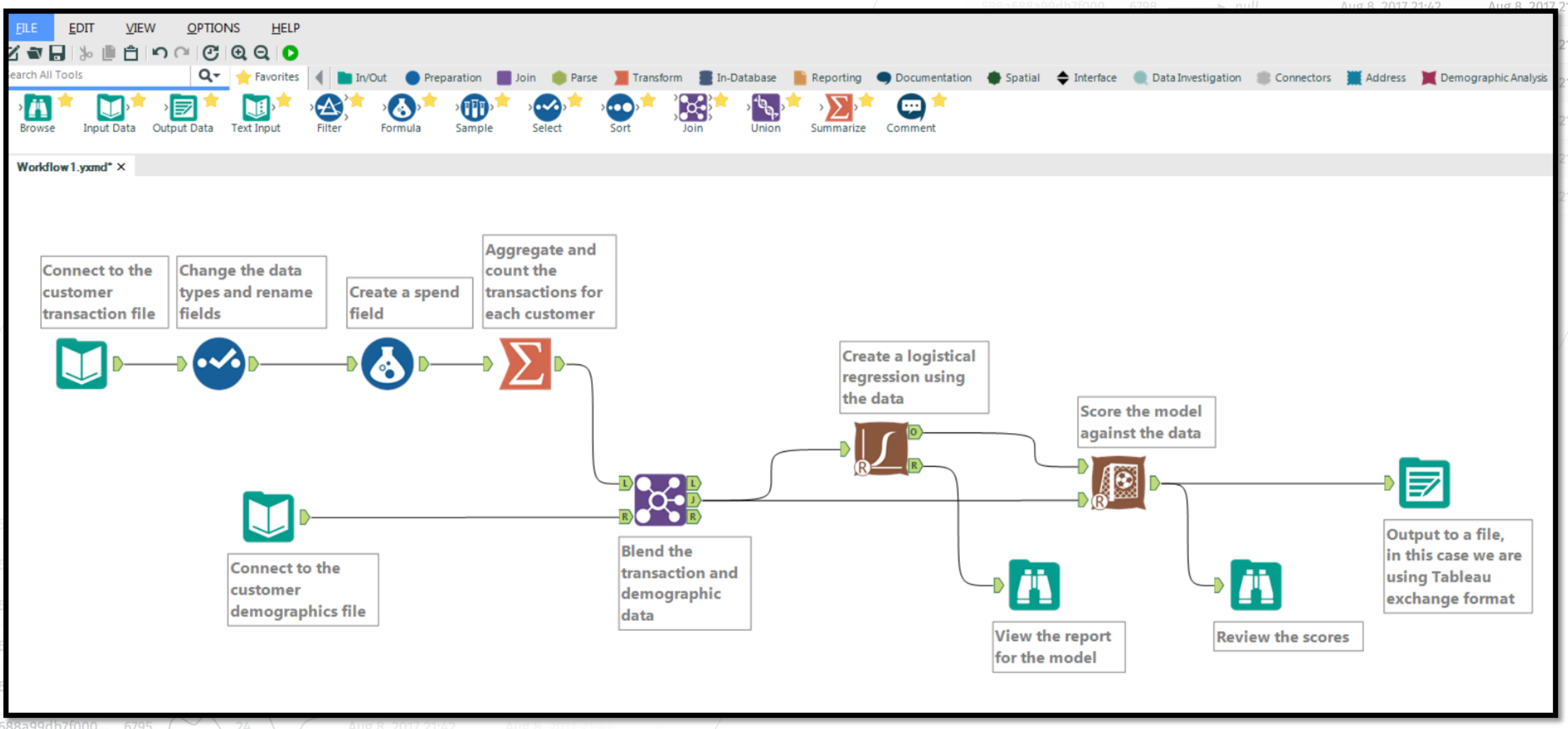

Alteryx Designer, image taken from Microsoft Revolution Analytics Blog

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Merging often uses tables as functions

Date	Tm	Opp	Quarter	Time	Down	ToGo	Location	Receiver	Yds	...
10/12/2009	Ravens	Packers	4	2:40	4	21	RAV 19	Tandon Doss	63	...
11/9/2009	Bengals	Ravens	4	0:02	4	15	CIN 49	A.J. Green	51	...
12/28/2009	Packers	Bears	4	0:46	4	8	CHI 48	Randall Cobb	48	...
10/5/2009	Dolphins	Ravens	4	1:30	4	10	MIA 20	Brandon Gibson	46	...
12/28/2009	Cowboys	Eagles	4	3:57	4	9	PHI 32	Dez Bryant	32	...
11/23/2009	Packers	Vikings	4	1:17	4	6	MIN 40	James Joones	28	...
12/7/2009	Eagles	Lions	4	2:00	4	12	DET 37	Brent Celek	27	...
10/5/2009	Titans	Chiefs	4	0:14	4	12	KAN 38	Jackie Battle	24	...
12/28/2009	Saints	Buccaneers	4	7:40	4	3	TAM 40		24	...
11/17/2009	Patriots	Panthers	4	0:47	4	10	NWE 20	Rob Gronkowski	23	...
9/14/2009	Jaguars	Raiders	4	1:27	4	12	JAX 18	Ace Sanders	23	...
9/28/2009	Buccaneers	Cardinals	4	9:22	4	1	TAM 25		22	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	

Merging often uses tables as functions

Player	Tm	Year	Age	Pos	G	GS	Att	Yds	TD	Lng	Y/A	Y/G	…
Laurence Maroney	NE	2009	24	rb	15	5	194	757	9	45	3.9	50.5	…
Sammy Morris	NE	2009	32	rb	12	5	73	319	2	55	4.4	26.6	…
Fred Taylor	NE	2009	33		6	1	63	269	4	19	4.3	44.8	…
Ben Watson	NE	2009	29	TE	16	7	0	0	0	0		0	…
Sam Aiken	NE	2009	29	wr	14	7	0	0	0	0		0	…
Chris Baker	NE	2009	30	te	16	7	0	0	0	0		0	…
Joey Galloway	NE	2009	38		3	2	0	0	0	0		0	…
Isaiah Stanback	NE	2009	25		6	2	0	0	0	0		0	…
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	

Merging often uses tables as functions

Play Record

Receiver

Date	Tm	Opp	Quarter	Time	Down	ToGo	Location	Receiver	Yds	...
10/12/09	Ravens	Packers	4	2:40	4	21	RAV 19	Tandon		
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9/28/09	Buccaneer	Cardinals	4	9:22	4	1	TAM 25		22	...
...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...

Player

Player Stats

Player	Tm	Year	Age	Pos	G	GS	Att	Yds	TD	Lng	Y/A	Y/G	...
Laurence Maroney	NE	2009	24	rb	15	5	194	757	9	45	3.9	50.5	...
Sammy Morris	NE	2009	32	rb	12	5	73	319	2	55	4.4	26.6	...
Fred Taylor	NE	2009	33		6	1	63	269	4	19	4.3	44.8	...
Kevin Faulk	NE	2009	33	RB	15	7	62	335	2	29	5.4	22.3	...
Tom Brady *	NE	2009	32	QB	16	16	29	44	1	9	1.5	2.8	...
BenJarvus Green-Ellis	NE	2009	24		12	0	26	114	0	29	4.4	9.5	...
Brian Hoyer	NE	2009	24		5	0	10	25	1	20	2.5	5	...
Wes Welker *+	NE	2009	28	WR	14	13	5	36	0	11	7.2	2.6	...
Julian Edelman	NE	2009	23	WR	11	7	2	5	0	5	2.5	0.5	...
Matthew Slater	NE	2009	24		14	0	1	6	0	6	6	0.4	...
Brandon Tate	NE	2009	22		2	1	1	11	0	11	11	5.5	...
Randy Moss	NE	2009	32	WR	16	16	0	0	0	0		0	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...

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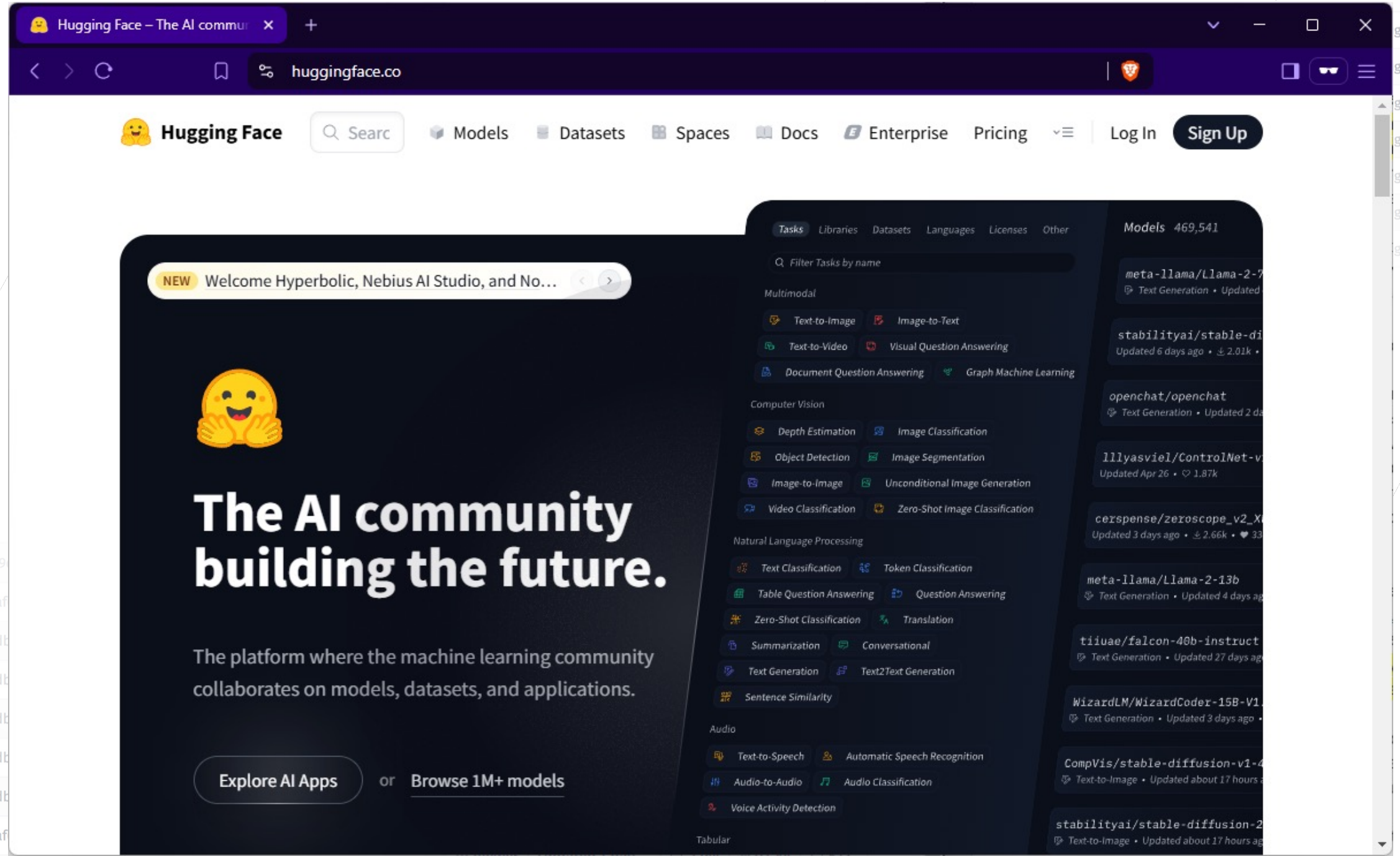
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Document Question Answering
Graph Machine Learning
Computer Vision
Depth Estimation
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Object Detection
Image Segmentation
Image-to-Image
Unconditional Image Generation
Video Classification
Zero-Shot Image Classification
Natural Language Processing
Text Classification
Token Classification
Table Question Answering
Question Answering
Zero-Shot Classification
Translation
Summarization
Conversational
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Voice Activity Detection
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Updated 3 days ago • ↓ 2.66k • ♥ 33
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Text Generation • Updated 4 days ag
tiiuae/falcon-40b-instruct
Text Generation • Updated 27 days ag
WizardLM/WizardCoder-15B-V1.
Text Generation • Updated 3 days ago •
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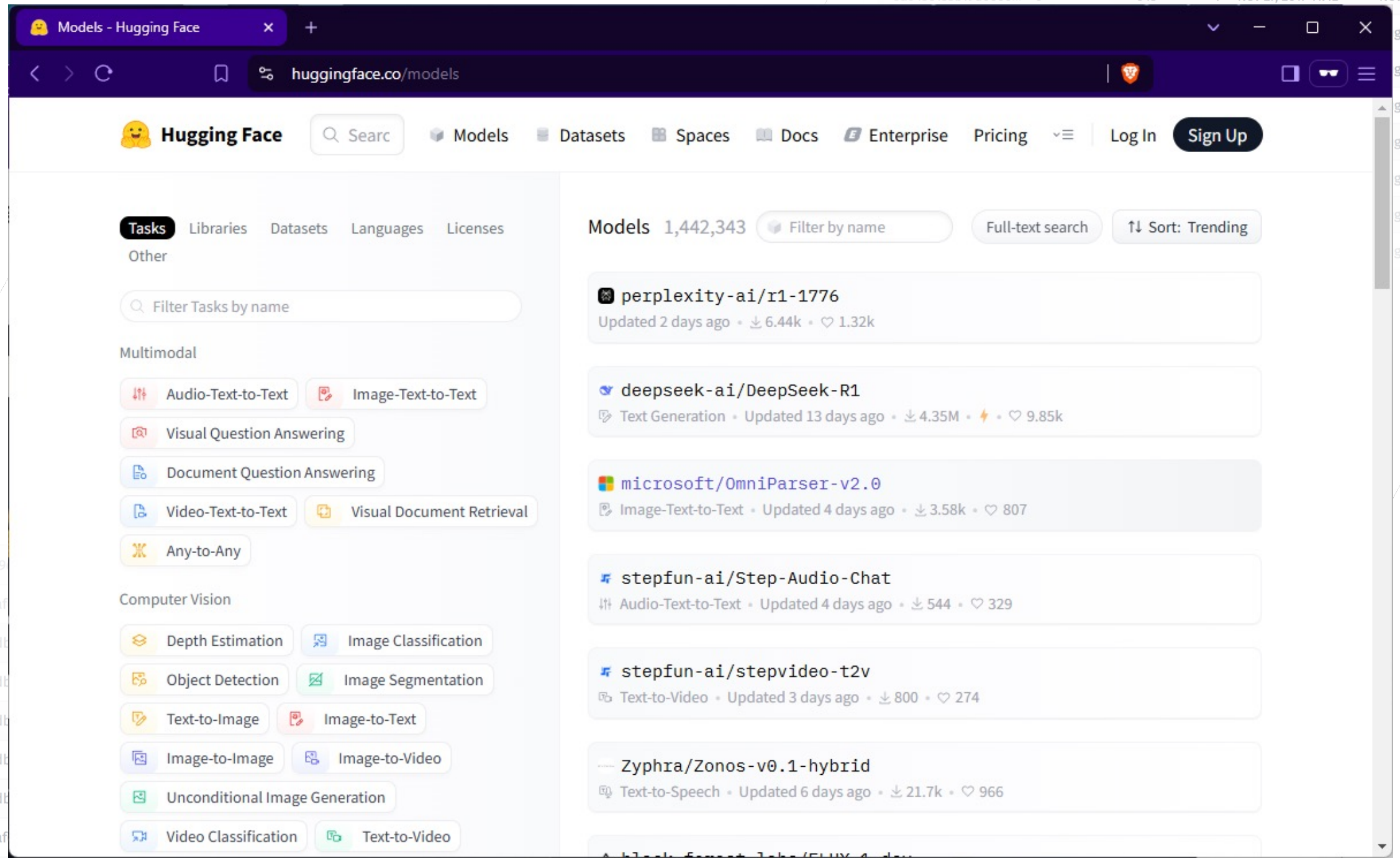
Text-to-Image • Updated about 17 hours
stabilityai/stable-diffusion-2
Text-to-Image • Updated about 17 hours ag


Models - Hugging Face
huggingface.co/models
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Enterprise
Pricing
Log In
Sign Up
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Libraries
Datasets
Languages
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Filter Tasks by name
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Image-Text-to-Text
Visual Question Answering
Document Question Answering
Video-Text-to-Text
Visual Document Retrieval
Any-to-Any
Computer Vision
Depth Estimation
Image Classification
Object Detection
Image Segmentation
Text-to-Image
Image-to-Text
Image-to-Image
Image-to-Video
Unconditional Image Generation
Video Classification
Text-to-Video
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Full-text search
Sort: Trending
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Updated 2 days ago • 6.44k • 1.32k
deepseek-ai/DeepSeek-R1
Text Generation • Updated 13 days ago • 4.35M • 9.85k
microsoft/OmniParser-v2.0
Image-Text-to-Text • Updated 4 days ago • 3.58k • 807
stepfun-ai/Step-Audio-Chat
Audio-Text-to-Text • Updated 4 days ago • 544 • 329
stepfun-ai/stepvideo-t2v
Text-to-Video • Updated 3 days ago • 800 • 274
Zyphra/Zonos-v0.1-hybrid
Text-to-Speech • Updated 6 days ago • 21.7k • 966

What is Image-Text-to-Text? - H

huggingface.co/tasks/image-text-to-text

Tasks

Image-Text-to-Text

Image-text-to-text models take in an image and text prompt and output text. These models are also called vision-language models, or VLMs. The difference from image-to-text models is that these models take an additional text input, not restricting the model to certain use cases like image captioning, and may also be trained to accept a conversation as input.

Inputs

Text Prompt

Describe the position of the bee in detail.

Image-Text-to-Text Model

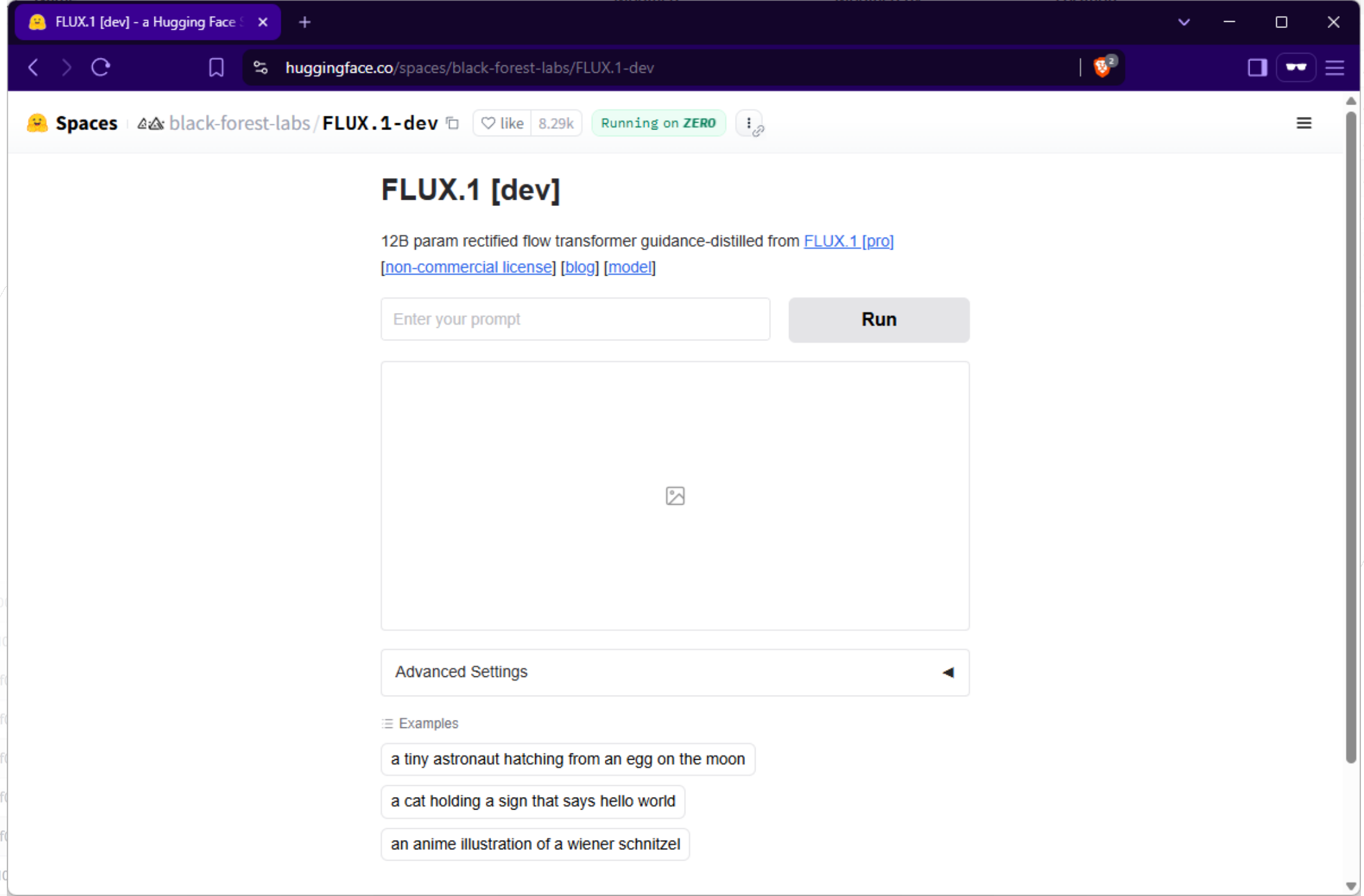
Output

Answer

The bee is sitting on a pink flower, surrounded by other flowers. The bee is positioned in the center of the flower, with its head and front legs sticking out.

Understanding AI through Functions

1. AI is just a collection of functions


FLUX.1 [dev] - a Hugging Face
huggingface.co/spaces/black-forest-labs/FLUX.1-dev
Spaces
black-forest-labs / FLUX.1-dev
like 8.29k
Running on ZERO
FLUX.1 [dev]
12B param rectified flow transformer guidance-distilled from FLUX.1 [pro]
[non-commercial license] [blog] [model]
Enter your prompt
Run
Advanced Settings
Examples
a tiny astronaut hatching from an egg on the moon
a cat holding a sign that says hello world
an anime illustration of a wiener schnitzel

FLUX.1 [dev] - a Hugging Face S

huggingface.co/spaces/black-forest-labs/FLUX.1-dev

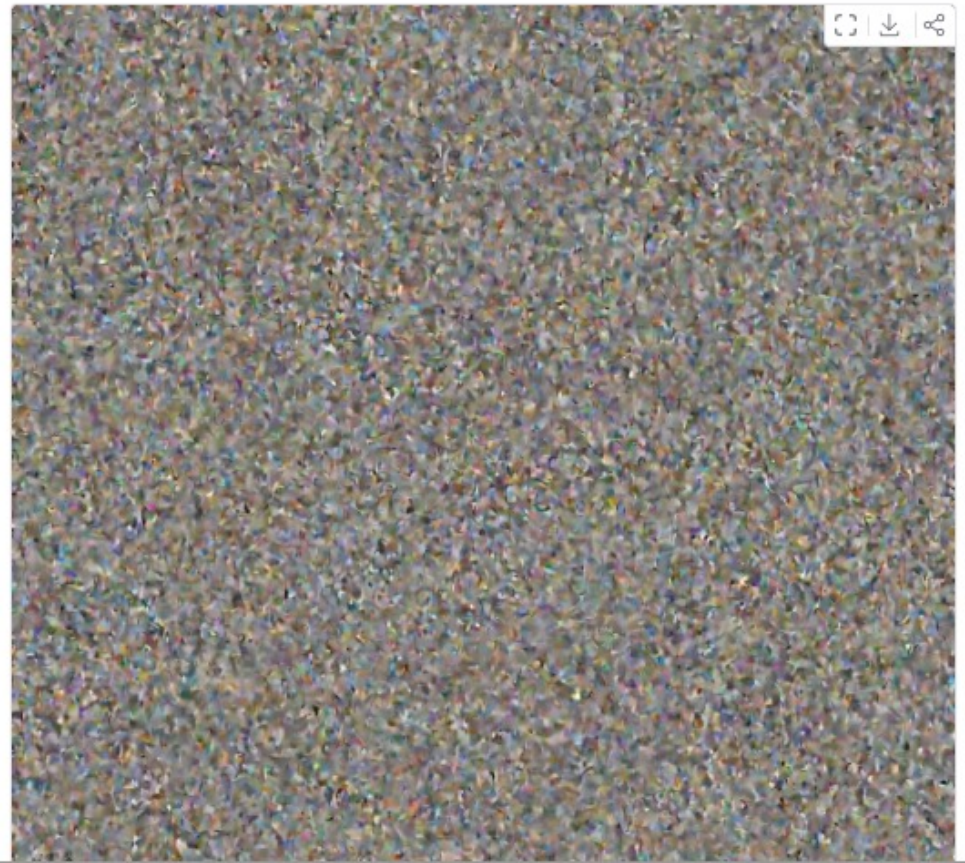
Spaces | black-forest-labs / FLUX.1-dev | like 8.29k | Running on ZERO | 2

FLUX.1 [dev]

12B param rectified flow transformer guidance-distilled from FLUX.1 [pro]
[non-commercial license] [blog] [model]

adorable capybaras playing in the sunshine

Run



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FLUX.1 [dev] - a Hugging Face

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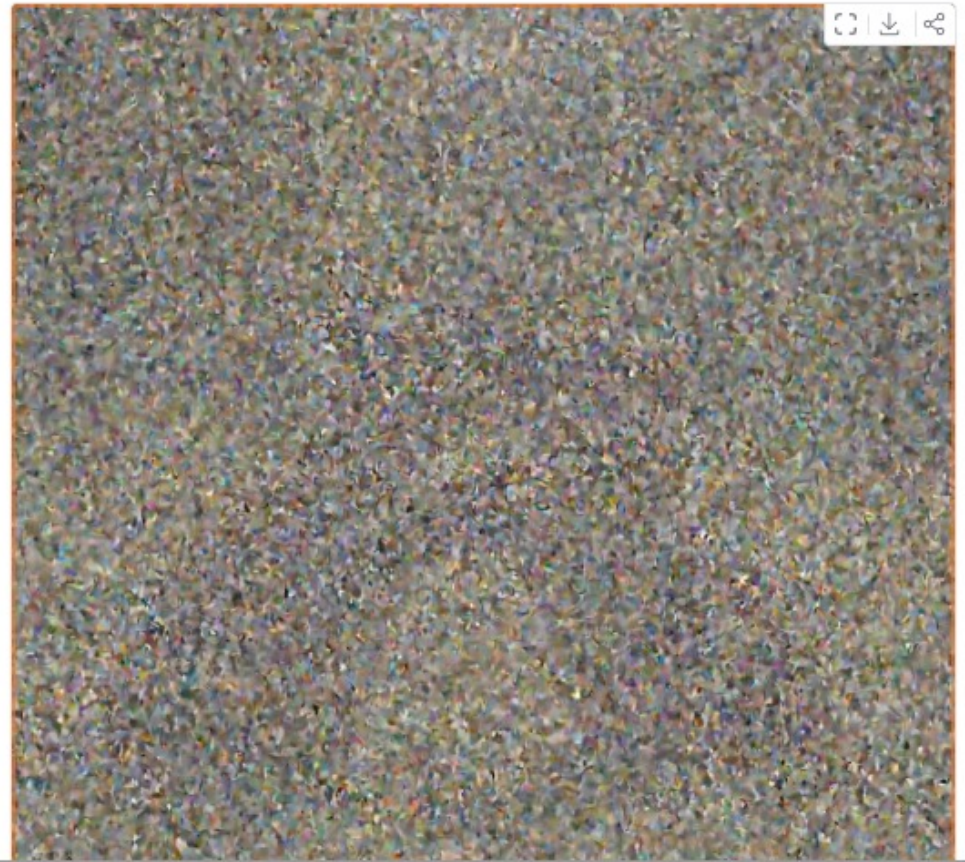

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FLUX.1 [dev] - a Hugging Face S

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FLUX.1 [dev]

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FLUX.1 [dev] - a Hugging Face

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FLUX.1 [dev]

12B param rectified flow transformer guidance-distilled from FLUX.1 [pro]
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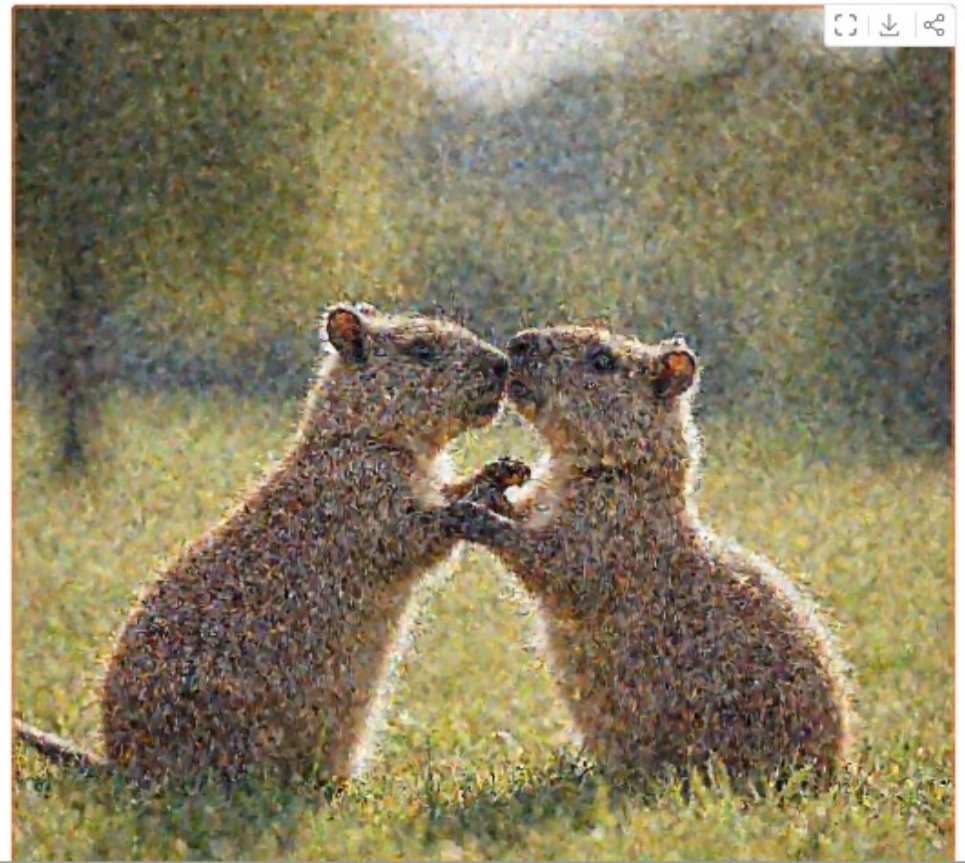

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FLUX.1 [dev] - a Hugging Face

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FLUX.1 [dev]

12B param rectified flow transformer guidance-distilled from FLUX.1 [pro]
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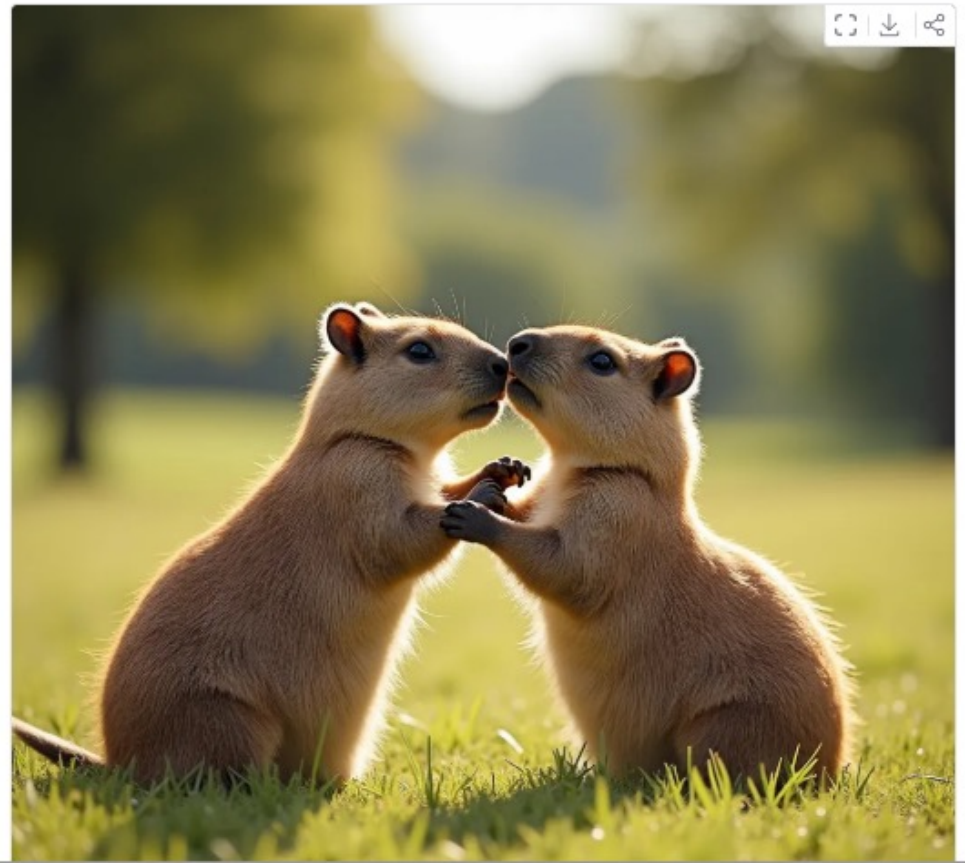

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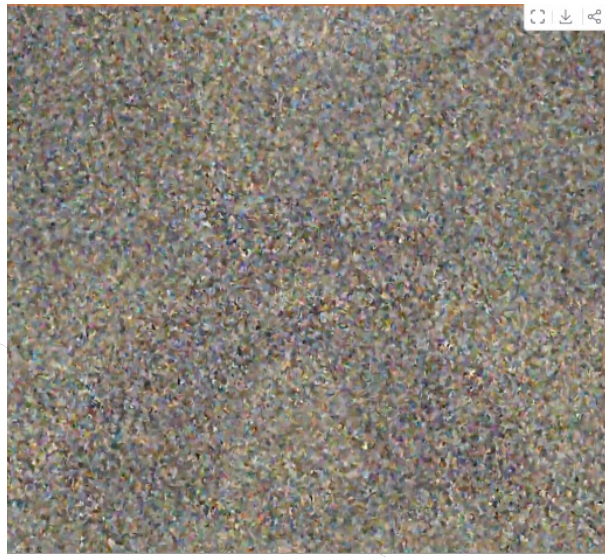
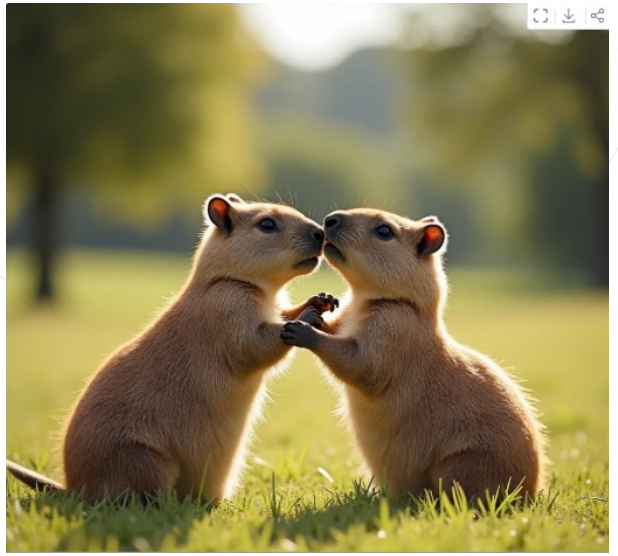
i = random noise

f



f $\cdots$ f



$f^*(i)$

Understanding AI through Functions

1. AI is just a collection of functions
2. A function alone is not always a product
   - Sometimes other functions surround it
   - Sometimes user interface innovations are needed

History lesson?

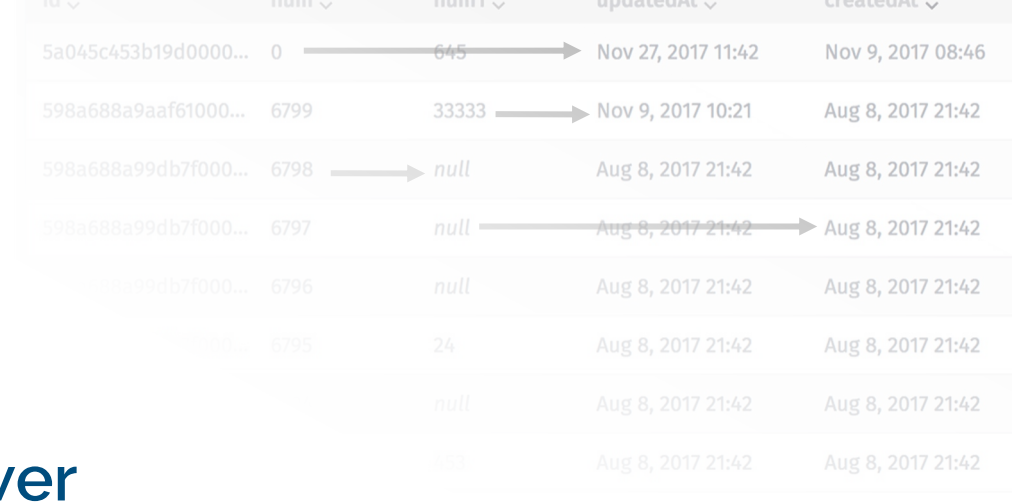
My prompt	TinyLlama 1.1B Chat v1.0's answer
What happened in 1776?	In 1776, the American Revolutionary War began with the Battles of Lexington and Concord, which marked the beginning of the American Revolution. The war was fought between the American colonies and the British Empire…

History lesson?

My prompt	TinyLlama 1.1B Chat v1.0's answer
What happened on July 1, 1962?	On July 1, 1962, the Soviet Union launched Sputnik 1, the first artificial satellite into orbit. This event marked the beginning of the Space Race between the United States and the Soviet Union, …

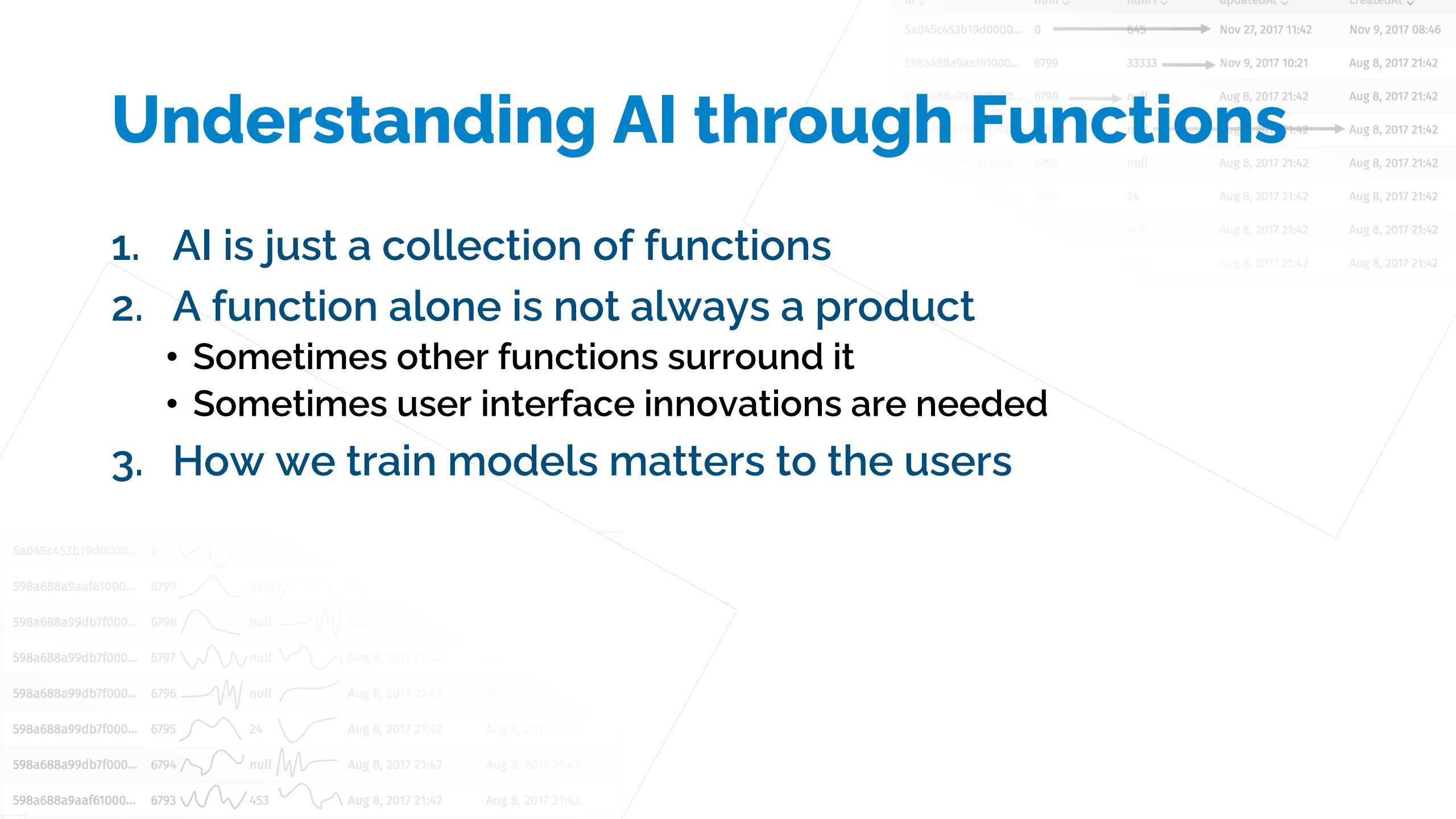
History lesson?

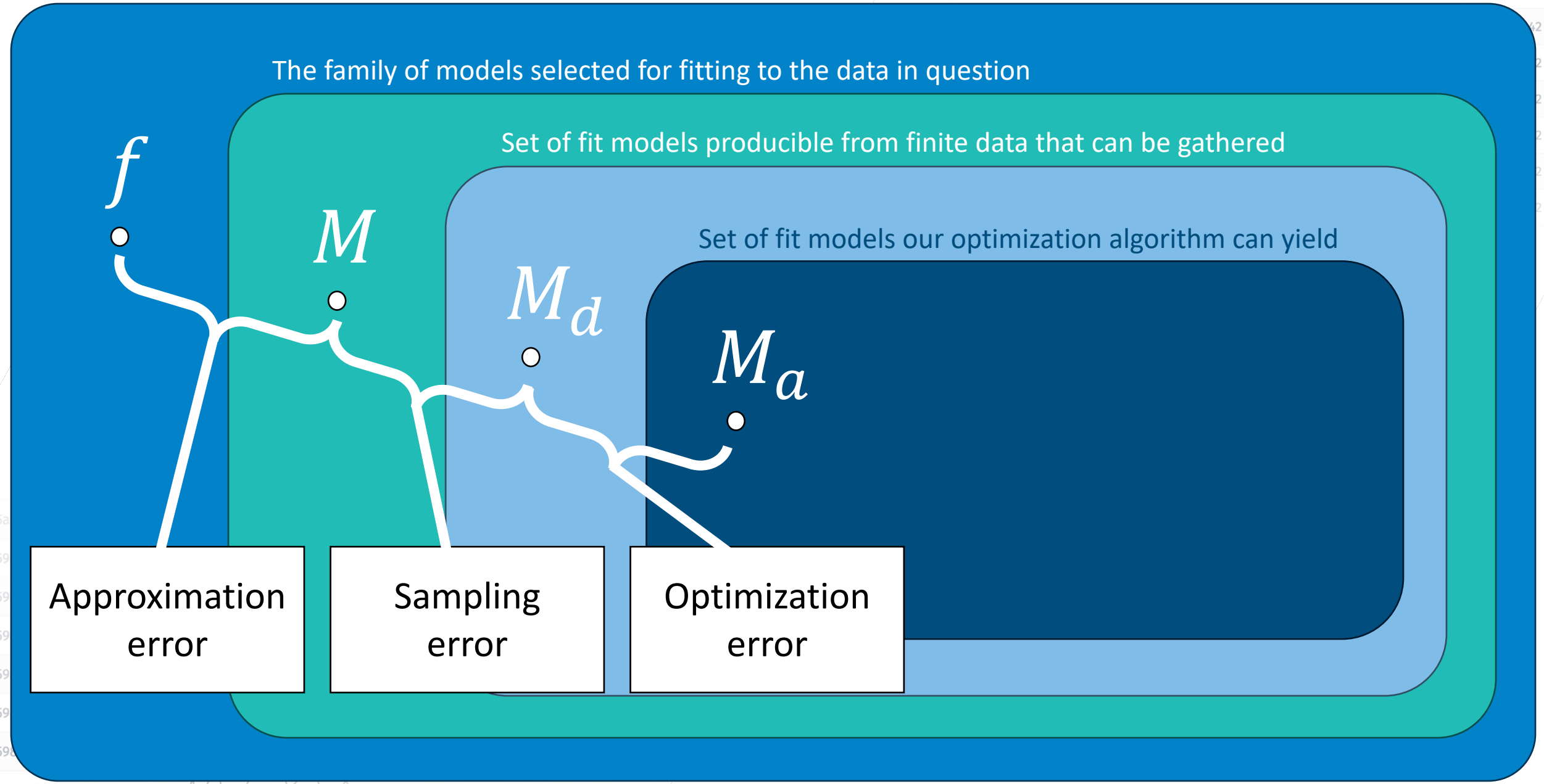
My prompt	A more correct answer
What happened on July 1, 1962?	The nations of Rwanda and Burundi gained independence from Belgium.



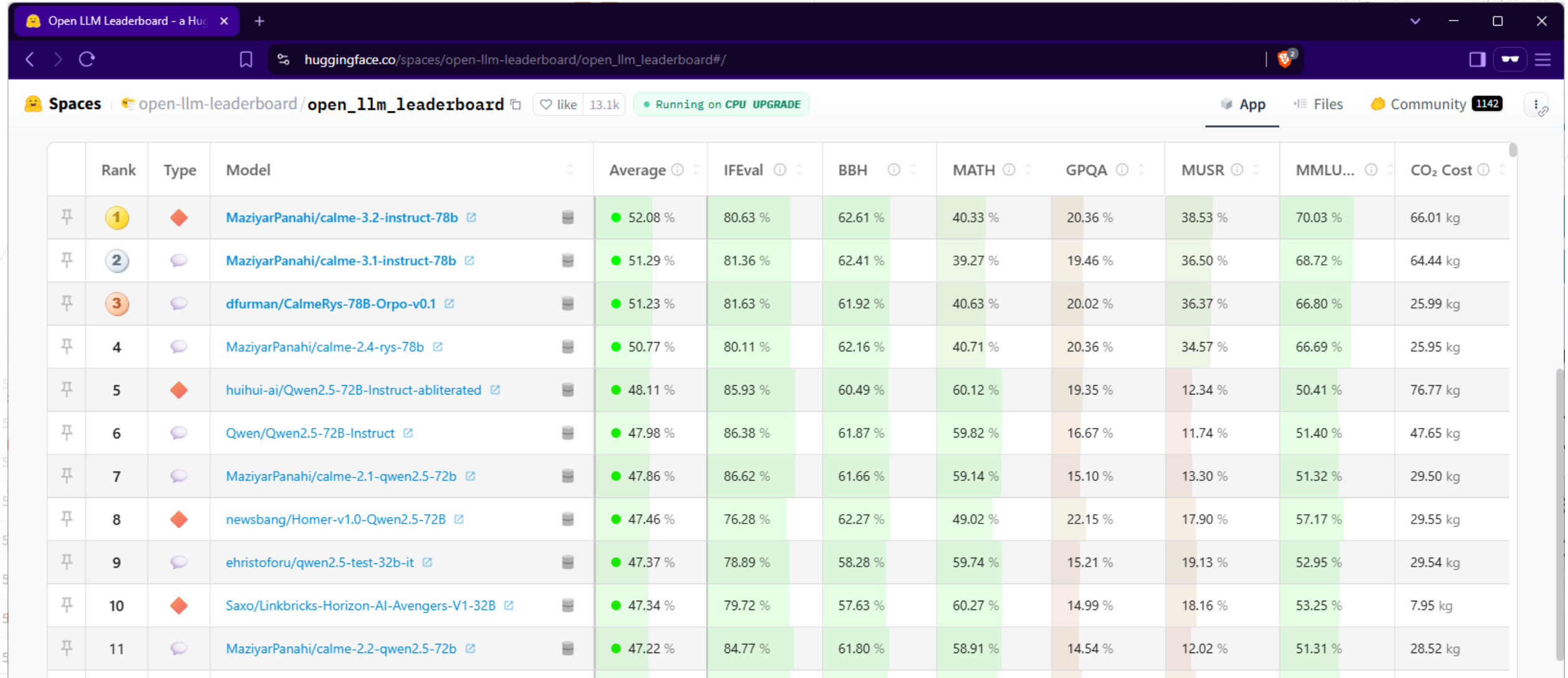
Understanding AI through Functions

1. AI is just a collection of functions
2. A function alone is not always a product
   - Sometimes other functions surround it
   - Sometimes user interface innovations are needed
3. How we train models matters to the users




Universe of all possible functions with appropriate domain and codomain
The family of models selected for fitting to the data in question
Set of fit models producible from finite data that can be gathered
Set of fit models our optimization algorithm can yield
f
M
M_d
M_a
Approximation error
Sampling error
Optimization error

Evaluating AI Models



	Rank	Type	Model	Average	IFEval	BBH	MATH	GPQA	MUSR	MMLU...	CO_2 Cost
	1	◆	MaziyarPanahi/calme-3.2-instruct-78b	52.08 %	80.63 %	62.61 %	40.33 %	20.36 %	38.53 %	70.03 %	66.01 kg
	2	💬	MaziyarPanahi/calme-3.1-instruct-78b	51.29 %	81.36 %	62.41 %	39.27 %	19.46 %	36.50 %	68.72 %	64.44 kg
	3	💬	dfurman/CalmeRys-78B-Orpo-v0.1	51.23 %	81.63 %	61.92 %	40.63 %	20.02 %	36.37 %	66.80 %	25.99 kg
	4	💬	MaziyarPanahi/calme-2.4-rys-78b	50.77 %	80.11 %	62.16 %	40.71 %	20.36 %	34.57 %	66.69 %	25.95 kg
	5	◆	huihui-ai/Qwen2.5-72B-Instruct-abliterated	48.11 %	85.93 %	60.49 %	60.12 %	19.35 %	12.34 %	50.41 %	76.77 kg
	6	💬	Qwen/Qwen2.5-72B-Instruct	47.98 %	86.38 %	61.87 %	59.82 %	16.67 %	11.74 %	51.40 %	47.65 kg
	7	💬	MaziyarPanahi/calme-2.1-qwen2.5-72b	47.86 %	86.62 %	61.66 %	59.14 %	15.10 %	13.30 %	51.32 %	29.50 kg
	8	◆	newsbang/Homer-v1.0-Qwen2.5-72B	47.46 %	76.28 %	62.27 %	49.02 %	22.15 %	17.90 %	57.17 %	29.55 kg
	9	💬	ehristoforu/qwen2.5-test-32b-it	47.37 %	78.89 %	58.28 %	59.74 %	15.21 %	19.13 %	52.95 %	29.54 kg
	10	◆	Saxo/Linkbricks-Horizon-AI-Avengers-V1-32B	47.34 %	79.72 %	57.63 %	60.27 %	14.99 %	18.16 %	53.25 %	7.95 kg
	11	💬	MaziyarPanahi/calme-2.2-qwen2.5-72b	47.22 %	84.77 %	61.80 %	58.91 %	14.54 %	12.02 %	51.31 %	28.52 kg

Understanding AI through Functions

1. AI is just a collection of functions
2. A function alone is not always a product
   - Sometimes other functions surround it
   - Sometimes user interface innovations are needed
3. How we train models matters to the users
4. There is no one best AI model
   - Some models are specialists and some are generalists
   - Benchmarks are functions that help us tell the difference

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What are the model's inputs and outputs?

How will the user interact with the model?

What was the model trained on?

How did you measure success?

Outline: Three examples

of how familiarity with functions accelerates data science learning

1. Composition of functions
   - Why this matters: Education

2. Tables as functions
   - Why this matters: Working with data

3. Artificial Intelligence (AI)
   - Why this matters: Asking important questions