Webs and their intersections

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Webs in Algebra, Geometry, Topology and Combinatorics
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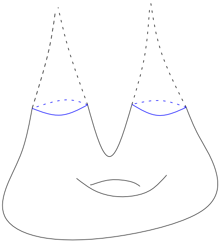
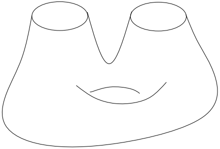
Fock–Goncharov Duality Conjecture

Conjecture (Fock–Goncharov 03')

The ***tropical integral points*** *of X parameterize the* ***canonical basis*** *of the regular function ring of the mirror $X^\vee$.*





$\mathcal{X}_{\mathrm{PGL}_n,\hat{S}}$

$\mathcal{A}_{\mathrm{SL}_n,\hat{S}}$ $\mathcal{L}_{\mathrm{PGL}_n,\hat{S}}$ $\mathcal{P}_{\mathrm{PGL}_n,\hat{S}}$

Fock–Goncharov Duality Conjecture

- The *marked surface* $\hat{S}$ is the connected oriented topological surface S with punctures m_p and finitely many marked points $m_b \subset \partial S$ considered up to isotopy.

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- Framing(decoration): The flat section of $\rho \times_G G/B$ $(\rho \times_G G/U)$ around m.
- $\mathcal{A}_{G,\hat{S}}$ consists of pairs (ρ, ξ) where unipotent bordered twisted representation $\rho \in \mathrm{Hom}(\pi_1(T^1S), G)/G$ with $\rho(\epsilon) = s_G$ and ξ is the decorations at $m_p \cup m_b$.
- $(\rho, \xi) \sim (g\rho g^{-1}, g\xi)$, $\forall g$.

Fock–Goncharov Duality Conjecture

- $\mathcal{X}_{G^L,\hat{S}}$ consists of pairs (ρ, ξ) where $\rho \in \mathrm{Hom}(\pi_1(S), G^L)/G^L$ and ξ is the framings at $m_p \cup m_b$.
- To match up with the dimension of $\mathcal{A}_{G,\hat{S}}$, Goncharov–Shen changed $\mathcal{X}_{G^L,\hat{S}}$ into $\mathcal{P}_{G^L,\hat{S}}$ where ξ is the decorations at m_b and framings at m_p.

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- The character variety $\mathcal{L}_{G^L,\hat{S}}$ consists of pairs (ρ,ξ) where $\rho \in \mathrm{Hom}(\pi_1(S), G^L)/G^L$ and ξ is the decorations at m_b.

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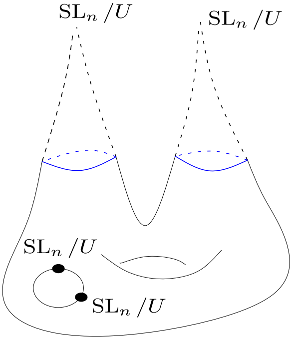
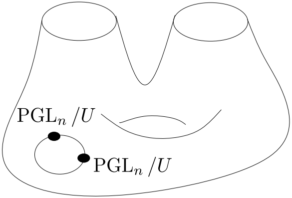
- $X = \mathcal{A}_{G,\hat{S}}$ then $X^V = \mathcal{P}_{G^L,\hat{S}}$ or $\mathcal{L}_{G^L,\hat{S}}$, and vice versa.





$\mathcal{A}_{\mathrm{SL}_n,\hat{S}}$ P $\mathcal{L}_{\mathrm{PGL}_n,\hat{S}}$

Goncharov–Shen adjusted the original Fock–Goncharov duality conjecture by adding the potential P.

Conjecture (Goncharov–Shen 13')

Let $\mathcal{L}_{G^L,\hat{S}}$ be the character variety. There is a holomogical mirror symmetry between $(\mathcal{A}_{G,\hat{S}}, P)$ and $\mathcal{L}_{G^L,\hat{S}}$.
The tropical integral points $\mathcal{A}^+_{G,\hat{S}}(\mathbb{Z}^t)$ parameterize the canonical basis of $\mathcal{O}(\mathcal{L}_{G^L,\hat{S}})$.

It has a beautiful **explicit** solution for PGL_2 after Fock and Goncharov using Thurston's transversely measured laminations.

Lamination

A **positive integer lamination** l on $\hat{S}$ is a formal sum

$$l = \sum n_i[\alpha_i] + \sum m_j[\beta_j], \quad n_i, m_j \in \mathbb{Z}_{>0}.$$

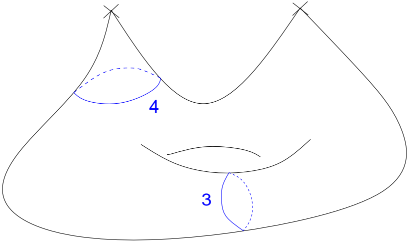


Figure: $3[\alpha_1] + 4[\beta_1]$

Theorem (Hoste–Przytycki 93')

All $M_l = \prod_i \mathrm{Tr}\rho(\alpha_i)^{n_i} \prod_j \mathrm{Tr}\rho(\beta_j)^{m_j}$ *form a linear basis of* $\mathcal{O}(\mathcal{L}_{\mathrm{SL}_2,\hat{S}})$.

Ideal triangulation

Given an **ideal triangulation**,

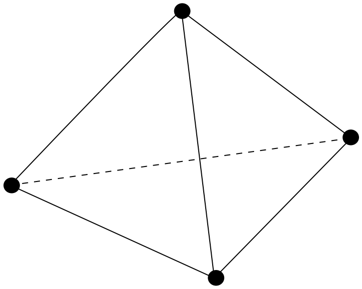


Figure: 4th punctured sphere case.

Laminations and tropical $\mathcal{A}$ coordinates

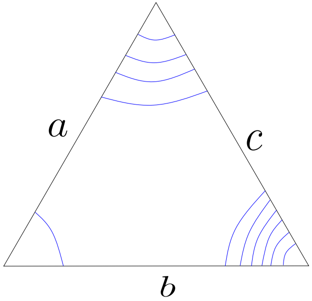


Figure: $a = \frac{1}{2} * 5 = \frac{5}{2}$, $b = \frac{1}{2} * 7 = \frac{7}{2}$, $c = \frac{1}{2} * 10 = 5$.

Lamination l intersects the ideal triangulation of $\hat{S}$ **minimally**. We assign 1/2 of the intersection number to each edge to obtain an element in $\mathcal{A}^{+}_{\mathrm{SL}_2,\hat{S}}(\mathbb{R}^t)_{\mathcal{T}}$.

Mutation for $\mathcal{A}$ coordinates

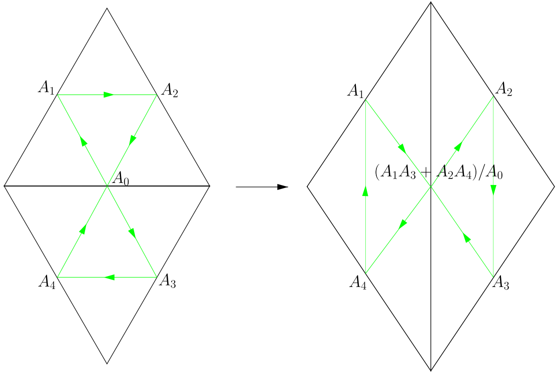


Figure: Mutation.

Mutation for tropical $\mathcal{A}$ coordinates

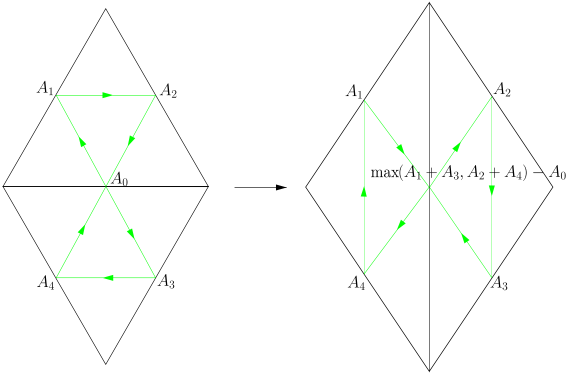


Figure: Tropical mutation.

Laminations and tropical $\mathcal{A}$ coordinates

- $\mathcal{A}^+_{\mathrm{PGL}_2,\hat{S}}(\mathbb{Z}^t)$ is the subset of $\mathcal{A}^+_{\mathrm{SL}_2,\hat{S}}(\mathbb{R}^t)$ such that for a given chart and any angle $a-b-c \in \mathbb{Z}_{\leq 0}$.

Theorem (Fock–Goncharov 03', Goncharov–Shen 13')

There is a $\mathrm{Mod}(\hat{S})$-equivariant bijection Φ from the space of positive integer laminations $\mathcal{W}_{\hat{S}}$ to $\mathcal{A}^+_{\mathrm{PGL}_2,\hat{S}}(\mathbb{Z}^t)$.

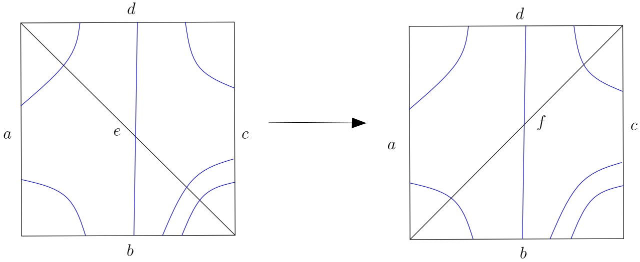


Figure: Proof: $a=1$, $b=2$, $c=3/2$, $d=3/2$, $e=2$, $f=3/2=\max\{a+c,b+d\}-e$.

Counting multicurves

$\mathcal{ML}_{g,n}$ space of geodesic measure laminations on $\hat{S} = S_{g,n}$, which is the $\mathbb{R}_{\geq 0}$ completion of the space of multicurves.
$\mathcal{T}_{g,n}$ and $\mathcal{ML}_{g,n}$ embedded into space of currents $\mathcal{C}(S) = \{\pi_1(S) - \text{invariant measure on } \mathbb{S}^1 \times \mathbb{S}^1 \backslash \Delta\}$.
μ the unique $\mathrm{Mod(S_{g,n})}$-invariant Thurston measure on $\mathcal{ML}_{g,n}$.

$$m(a) = \mu(\{x \in \mathcal{ML}_{g,n} \mid i(a,x) \leq 1\}).$$

$$B_{g,n} = \int_{\mathcal{M}_{g,n}} m(X) dX.$$

Theorem (Mirzakhani 08')

γ_0 multicurve on $S_{g,n}$. ρ hyperbolic metric on $S_{g,n}$.
Let $N_L^\rho = \#\{\gamma \in \mathrm{Mod(S_{g,n})} \cdot \gamma_0 \mid \ell_\rho(\gamma) \leq \mathrm{L}\}$. Then

$$\lim_{L \to +\infty} \frac{N_L^\rho}{L^{6g-6+2n}} = \frac{m(\rho) m(\gamma_0)}{B_{g,n}}.$$

Webs as integer higher laminations

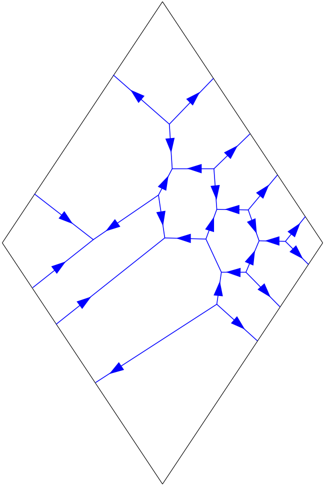
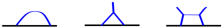


Figure: (Kuperberg, CMP 96') A SL_3-**web** is an oriented graph on the surface with 3-valent interior vertices such that the three oriented edges at any interior vertex point inward or outward.

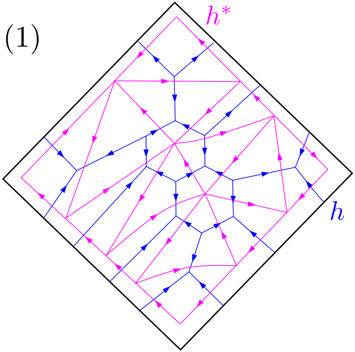
Webs as integer higher laminations

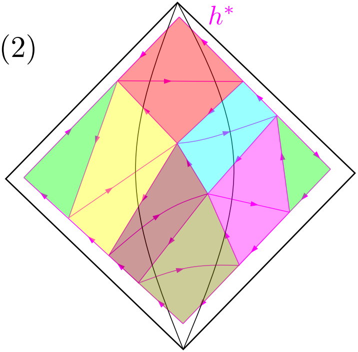
- A SL_3-web is **non-elliptic** if it does not contain 0-, 2-, 4-faces (cannot be decomposed by the skein relation any more).
- On $\hat{S}$, a non-elliptic SL_3-web is **reduced** if the edge on the boundary is 1-valence. And we do not have any path of ≤ 3 edges parallel to a boundary interval:



- Dual graph of a non-elliptic SL_3-web is triangulation of the surface.
- Non-elliptic = the dual graph is combinatorial $CAT(0)$—each interior vertex has valence ≥ 6.

Dual graph of a web





SL_3-skein algebra

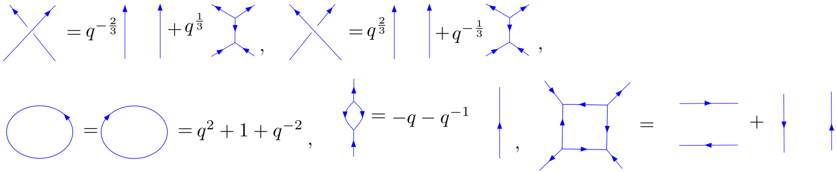


Figure: SL_3 skein relation

Theorem (Kuperberg 96' for polygon, Sikora–Westbury 07')

Reduced SL_3*-webs form a linear basis of the reduced* SL_3 *skein algebra.*

The classical limit of the web is the trace function of the web.

Theorem (Turaev 91')

The SL_n*-skein algebra is (almost) the quantization of* $\mathcal{O}(\mathcal{L}_{\mathrm{SL}_n,\hat{S}})$ *with respect to the Goldman Poisson structure.*

Given a split ideal triangulation $\mathcal{T}$ of the surface S,

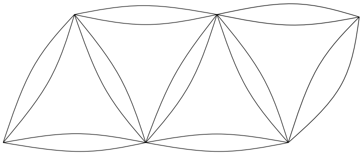
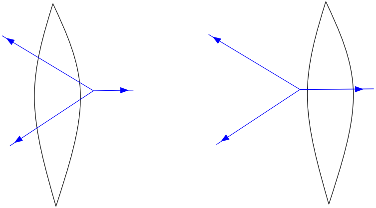


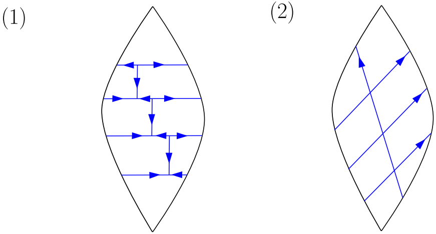
Figure: Split ideal triangulaton.

we could put reduced 3-web W in **good position** such that

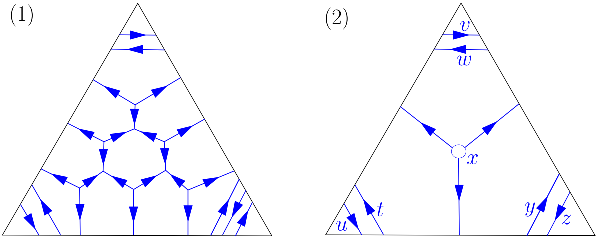
- W intersects $\mathcal{T}$ minimally,



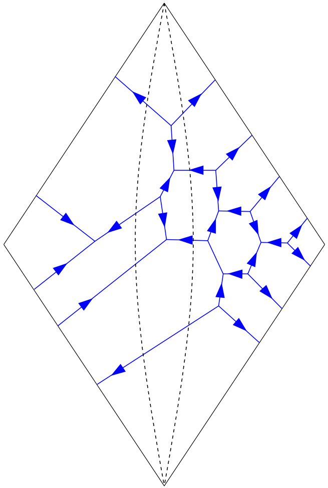
- the restriction of W to each bigon is a **minimal ladder**,



- the restriction of W to each ideal triangle is an **oriented honeycomb** with oriented arcs.



Web in good position



Basis assignments

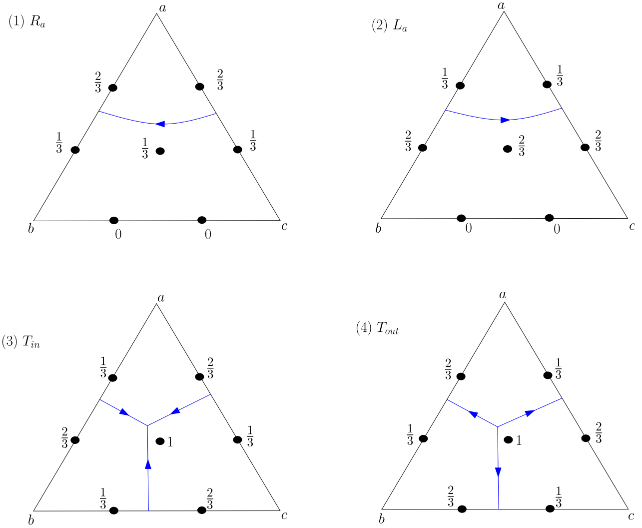


Figure: Tropical coordinate in triangle is linear combination of above basis elements with coefficients $(x, y, z, t, u, v, w) \in \mathbb{Z}^7_{\geq 0}$.

Gluing along the ideal edges

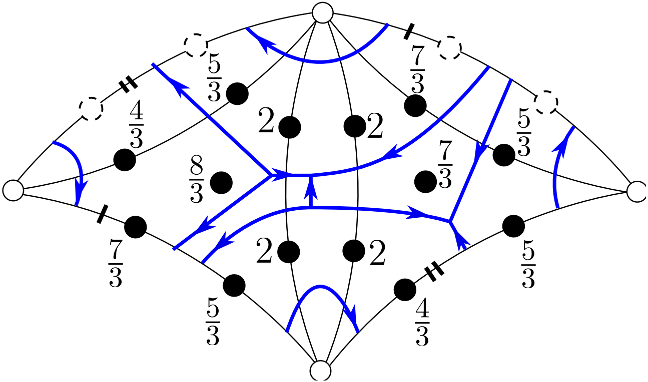


Figure: Example on $S_{1,1}$.

Webs and tropical $\mathcal{A}$ coordinates

Let $\mathcal{W}_{\hat{S}}$ be the space of reduced 3-webs up to homotopy.

Theorem (Douglas–S. 20')

$$\Phi : \mathcal{W}_{\hat{S}} \xrightarrow{\cong} \mathcal{A}^{+}_{\mathrm{PGL}_3,\hat{S}}(\mathbb{Z}^t).$$

More explicitly, we define $\Phi_{\mathcal{T}} : \mathcal{W}_{\hat{S}} \xrightarrow{\cong} \mathcal{A}^{+}_{\mathrm{PGL}_3,\hat{S}}(\mathbb{Z}^t)_{\mathcal{T}}$ *for any ideal triangulation* $\mathcal{T}$ *and* $\Phi_{\mathcal{T}'} = \mu_{\mathcal{T},\mathcal{T}'} \circ \Phi_{\mathcal{T}}$.

Theorem (Kim 20')

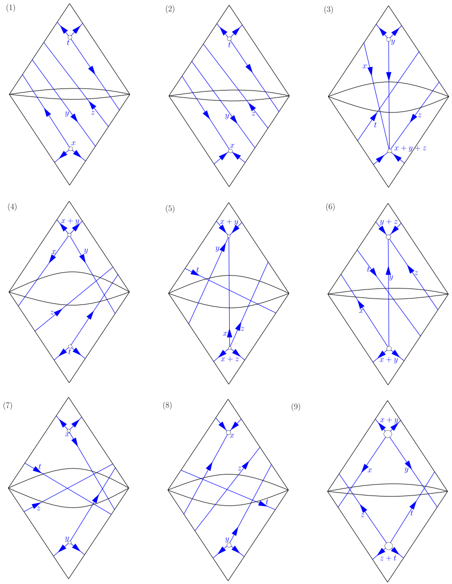
For $\hat{S} = S_{g,m}$, *there is a quantum trace map*

$$\mathrm{Tr}_q : \mathcal{RS}_q(\hat{S}) \to \mathcal{O}_q(\mathcal{P}_{\mathrm{PGL}_3,\hat{S}})$$

where the highest term degree of $\mathrm{Tr}_q(W)$ *is the same as* $\Phi(W)$.

Generalizing Bonahon–Wong 11's quantum trace map for SL_2. Lê and Yu 23' constructed the quantum trace map for SL_n.

Proof of flip equivariance 20'



Meaning of $\mathcal{A}^+_{\mathrm{PGL}_3,S}(\mathbb{Z}^t)$: Goncharov–Shen potential

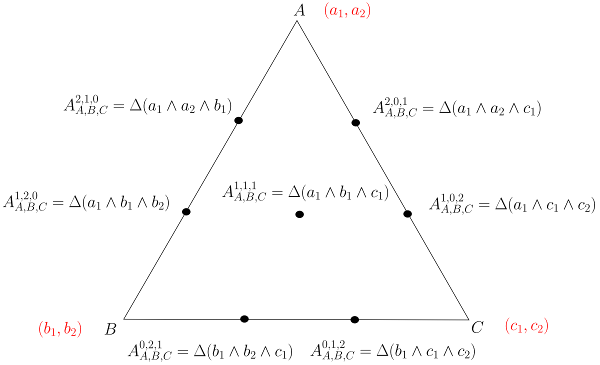


Figure: $\mathcal{A}$ **coordinates** for $n = 3$.

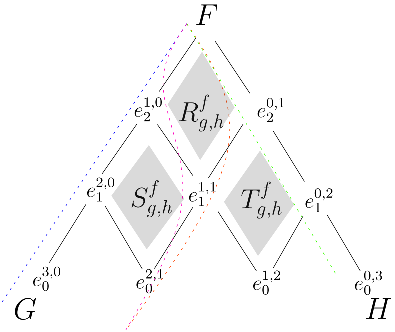


Figure: $\pi : G/U \to G/B$, $(F, \pi(H)) = u(F, \pi(G))$.

$$u = E_1(S_f^{g,h}) \cdot E_2(R_f^{g,h}) \cdot E_1(T_f^{g,h}) = \begin{pmatrix} 1 & S_f^{g,h} + T_f^{g,h} & S_f^{g,h} R_f^{g,h} \\ 0 & 1 & R_f^{g,h} \\ 0 & 0 & 1 \end{pmatrix},$$

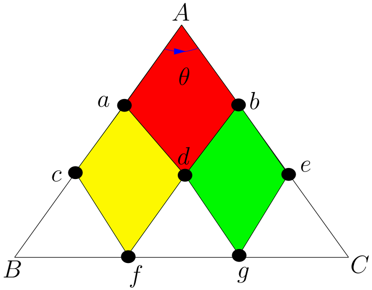


Figure: Rhombus terms.

The **Goncharov–Shen potential** for the marked ideal triangle $\theta = (A, B, C)$ is

$$P(\theta) = \frac{d}{ab} + \frac{af}{cd} + \frac{bg}{de}.$$

Meaning of $\mathcal{A}^+_{\mathrm{PGL}_3,S}(\mathbb{Z}^t)$: Goncharov–Shen potential

- The **Goncharov–Shen potential** is the $\mathrm{Mod}(S)$ **equivariant** regular function
$$P = \sum_\theta P(\theta)$$
summing over all the anticlockwise oriented marked ideal triangles on the surface.
- $P^t(\theta) = \max\{d - a - b, a + f - c - d, b + g - d - e\}$ where $a, b, c, d, e, f, g \in \mathbb{R}$.

Theorem (Goncharov–Shen, 13')

Let $\mathcal{A}^+_{G,\hat{S}}(\mathbb{R}^t)$ be the subset of $\mathcal{A}_{G,\hat{S}}(\mathbb{R}^t)$ with $P^t \leq 0$. Then $\mathcal{A}^+_{\mathrm{GL}_n,\Delta}(\mathbb{R}^t)$ coincides with the Knutson–Tao's hives.

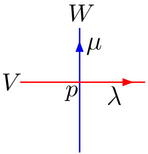
$\mathcal{A}^+_{\mathrm{PGL}_3,\hat{S}}(\mathbb{Z}^t)$ is the subset of $\mathcal{A}^+_{\mathrm{SL}_3,\hat{S}}(\mathbb{R}^t)$ such that for a given chart and any rhombus $\alpha^t \in \mathbb{Z}_{\leq 0}$. Here any rhombus $\alpha^t \in \mathbb{Z}_{\leq 0}$ holds for any triangulation.

Goncharov–Shen potential and other topics

1. (Goncharov–Shen 19') The Goncharov–Shen partial potentials play important roles in the quantization of the moduli spaces of G-local systems.
2. (Huang-S., 19') For each simple root and each puncture, the Goncharov–Shen partial potential is understood as generalized horocycle length which provides a family of McShane-type identities.
3. (Goncharov–S., 24') The Goncharov–Shen partial potentials is essential in the exponential volume of the moduli space of ideal hyperbolic surfaces with marked points on the boundary.

Intersection between two dual webs (Shen–S.–Weng)

- G-web: oriented graph with only 3-valent pointing inward (outward resp.) interior vertices where three edges colored by λ, μ, ν such that $\dim \left(V_\lambda \otimes V_\mu \otimes V_\nu\right)^G = 1$ ($\dim \left(V_\lambda^* \otimes V_\mu^* \otimes V_\nu^*\right)^G = 1$ resp.).
- $(G, \mathcal{A})$-web W: ends at boundary intervals, $(G^L, \mathcal{X})$-web V: ends at punctures or marked points,
- intersecting transversely, the **intersection number** of the **ordered** pair (W, V) at a point p



$$\epsilon_p(W, V) = < \lambda, \mu >$$

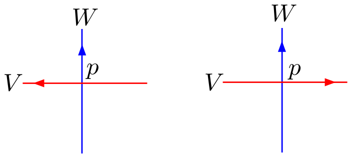
Then the **intersection number** of (W, V)

$$i(W, V) := \sum_{p \in W \cap V} \epsilon_p(W, V).$$

The **intersection number** of $([W], [V])$

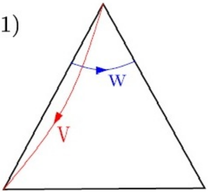
$$i([W], [V]) := \inf_{w \in [W], v \in [V]} \{i(w, v)\}.$$

SL_3 case



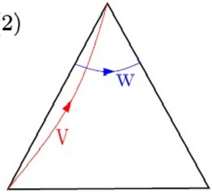
$\epsilon_p(W, V) = 1/3$ $\epsilon_p(W, V) = 2/3$

(1)



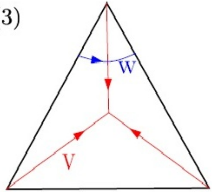
$i(W, V) = \frac{2}{3}$

(2)



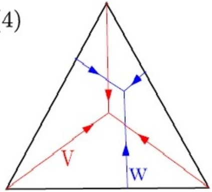
$i(W, V) = \frac{1}{3}$

(3)



$i(W, V) = \frac{2}{3}$

(4)



$i(W, V) = \frac{1}{3} + \frac{2}{3} = 1$

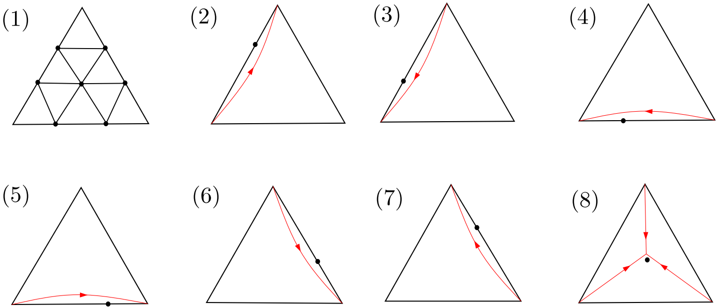


Figure: The $(SL_3, \mathcal{X})$-webs V_a corresponding to a in seed **s**.

For any seed **s** (collection of these $\mathcal{A}$ coordinates),

$$i_{\mathbf{s}} : \mathcal{W}_{\hat{S}} \to \left(\frac{1}{3}\mathbb{Z}_{\geq 0}\right)^N, \quad i_{\mathbf{s}}(W) := (i([W],[V_a]))_{a \in \mathbf{s}}.$$

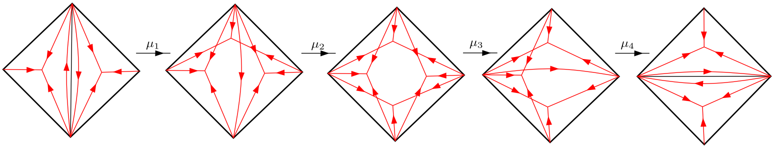
SL_3 case

Theorem (Shen–S.–Weng 23')

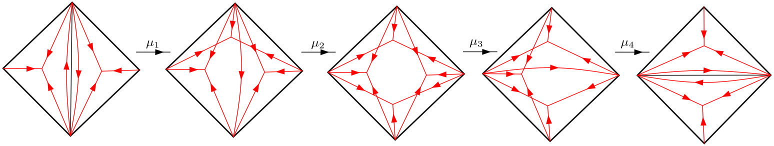
$i_{\mathfrak{s}} = \Phi_{\mathfrak{s}}$ in (Douglas–S. 20'). For any two seeds $\mathfrak{s}$ and $\mathfrak{s}'$ related by the mutation μ in the sequence of flip of diagonal of ideal triangulation $\mathcal{T}$, we have

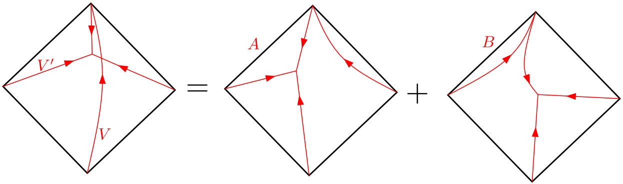
$$\mu \circ i_{\mathfrak{s}} = i_{\mathfrak{s}'}.$$

Particularly, applying to the seeds in the flip sequence of the square $\square$ implies mapping class group equivariance.



SL_3 case





SL_3 case

Simple examples:

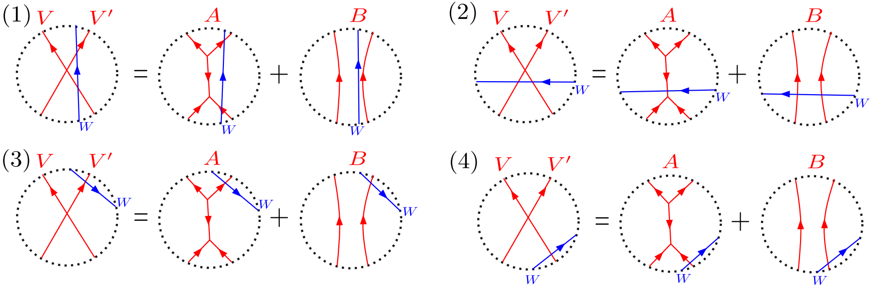
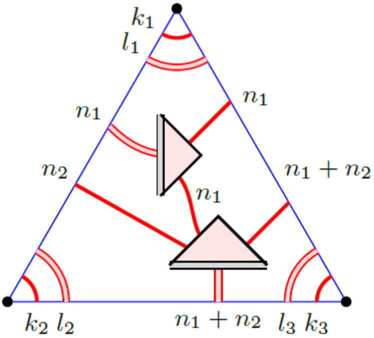


Figure: $i([W],[V]) + i([W],[V']) = \max\{i([W],[A]), i([W],[B])\}$.



Reduced Sp_4-webs in a triangle.

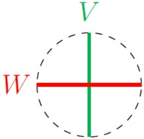
Flatting the reduced Sp_4-webs by

Obtaining reduced Sp_4-crossroad webs.

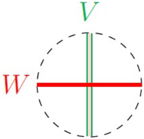
Theorem (Ishibashi–Yuasa 22')

Reduced Sp_4*-crossroad webs form a linear basis of clasped* Sp_4*-skein algebra.*

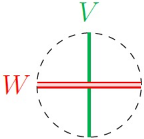
Sp_4 intersection number coordinates



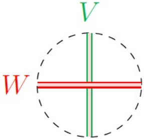
$\varepsilon_p(W,V)=1$



$\varepsilon_p(W,V)=1/2$

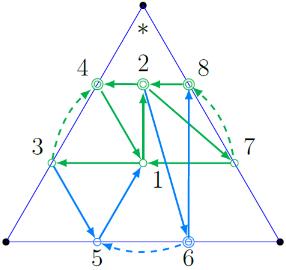


$\varepsilon_p(W,V)=1$

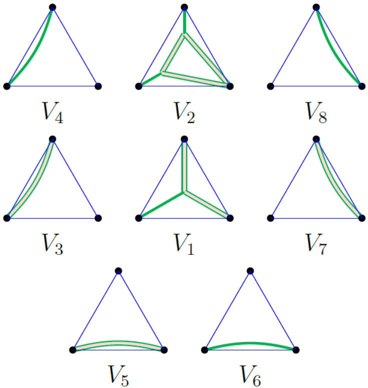


$\varepsilon_p(W,V)=1$

Sp_4 intersection number coordinates



$(\triangle, m, +)$



For any seed **s** (collection of these $\mathcal{A}$ coordinates),

$$i_{\mathbf{s}} : \mathcal{W}_{\hat{S}} \to \left(\frac{1}{2}\mathbb{Z}_{\geq 0}\right)^N, \quad i_{\mathbf{s}}(W) := (i([W],[V_a]))_{a \in \mathbf{s}}.$$

Sp_4 case

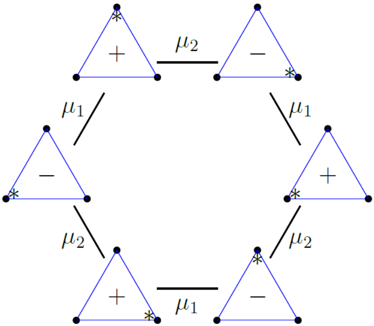
Let $\mathcal{L}^{\mathcal{A}}_{\mathrm{Sp}_4}(\hat{S}, \mathbb{Q})$ be the collection of disjoint unions of $\mathbb{Q}$-weighted peripheral elements and $\mathbb{Q}_{>0}$-weighted reduced Sp_4-crossroad webs.

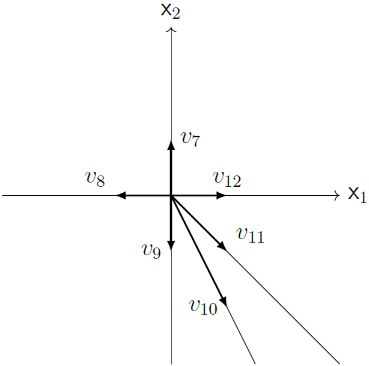
Theorem (Ishibashi–S.–Yuasa arXiv:2509.25014)

$i_{\mathbf{s}} : \mathcal{L}^{\mathcal{A}}_{\mathrm{Sp}_4}(\hat{S}, \mathbb{Q}) \to \mathbb{Q}^N$ *is a bijection.*

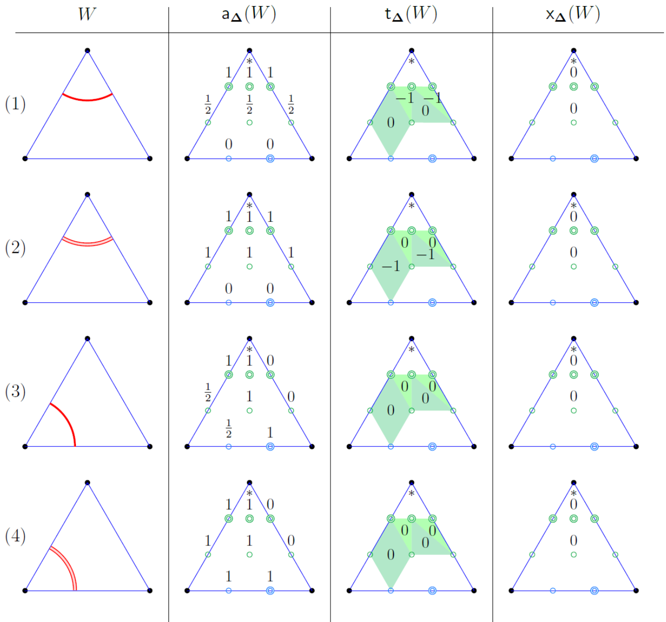
In our sequel paper, we will show that $i_{\mathbf{s}} : \mathcal{W}_{\hat{S}} \to \mathcal{A}^{+}_{\mathrm{SO}_5, \hat{S}}(\mathbb{Z}^t)$ is a bijection.

Sp_4 triangle case

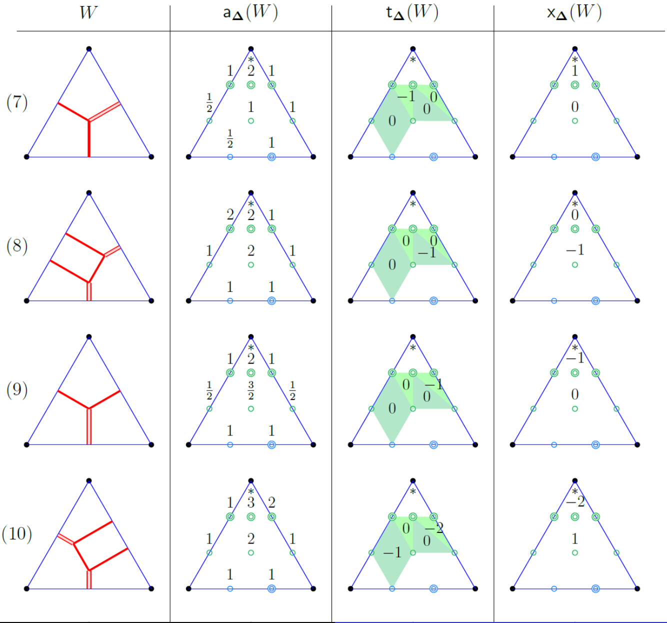




Sp_4 elementary webs



Sp_4 elementary webs



Webs and cluster algebra

Theorem (Gross–Hacking–Keel–Kontsevich, 14')

The Fock–Goncharov duality conjecture holds for certain cluster algebras under certain convexity condition.

Theorem (Goncharov–Shen 16'+ Fraser–Pylyavskyy 22')

Fock–Goncharov duality conjecture holds for $(\mathcal{A}_{\mathrm{SL}_n,\hat{S}}, \mathcal{P}_{\mathrm{PGL}_n,\hat{S}})$ *for* $(\hat{S}, n) \neq (S_{g,1}, 2)$ *after the existence of the Donaldson-Thomas transformation.*

Theorem (Mandel–Qin 23')

GHKK's theta basis is the same as bracelet basis for PGL_2.

Conjecture

GHKK's theta basis is the same as reduced web bracelet basis for PGL_3*?*

Theorem (Ishibashi–Kano, 24')

There is a natural bijection: $\mathcal{P}^{+}_{\mathrm{PGL}_3,\hat{S}}(\mathbb{Z}^t) \cong \mathcal{W}^{\mathcal{X}}_{\hat{S}}$.

1. The intersection pairing between webs induces a pairing:
$$I : \mathcal{A}^{+}_{\mathrm{PGL}_3,\hat{S}}(\mathbb{Z}^t) \times \mathcal{P}^{+}_{\mathrm{PGL}_3,\hat{S}}(\mathbb{Z}^t) \to \frac{1}{3}\mathbb{Z}.$$
2. By Fock–Goncharov duality, there is a pairing:
$$I_{FG} : \mathcal{A}(\mathbb{Z}^t) \times \mathcal{P}^{V}(\mathbb{Z}^t) \to \mathbb{Z}.$$
3. Given seed $\mathbf{s}$ and $(l,m) \in \mathcal{A}(\mathbb{Z}^t) \times \mathcal{P}^{V}(\mathbb{Z}^t)$, coordinates $(a_i)_i$ and $(x_i)_i$ for $\mathbf{s}$ has pairing: $< l, m >_{\mathbf{s}} := \sum_i a_i x_i$.

Conjecture (Shen–S.–Weng 23')

$$I(l,m) = I_{FG}(l,m) = \max_{\mathbf{s}} < l, m >_{\mathbf{s}} .$$
$$\forall l \in \mathcal{A}^{+}_{\mathrm{SL}_3,\hat{S}}(\mathbb{Z}^t), \forall m \in \mathcal{P}^{+}_{\mathrm{PGL}_3,\hat{S}}(\mathbb{Z}^t).$$

Further direction

1. **Positivity** for the Laurent polynomials under this Fock–Goncharov duality and the structure constants.
2. **Log concave** conjecture for coefficients of trace functions and structure constants. Zhichao Chen and Guanhua Huang have some progress on some examples arXiv:2408.03792.
3. Relation between webs and n-**graph weavings** (related to bipartite graphs, Legendrian links, spetral networks).

Further direction

1. Relation between k-**differentials** and webs.
2. Generalize Thurston's transversely measured laminations into **higher laminations** containing webs as integral points.
3. Generalize Mirzakhani's **counting problem** on multi-curves to webs. Distribution of webs. Random webs.

Thank you for your attention!

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