

Left regular bands with symmetry

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on joint work with Ben Steinberg (CUNY/CCNY)

Slides:



ICERM Workshop on Category Theory, Combinatorics, and Machine Learning

Today's focus

Joint work with Ben Steinberg



(CCNY/CUNY grad center)

But also... draws inspiration from + recovers results from...

- "Invariant theory for the face algebra of the braid arrangement"
↳ P.C., arXiv: 2404.00536
- "Invariant theory for the free left regular band and a q -analogue"
↳ arXiv: 2206.11406
↳ P.C. — Sarah Brauer (Brown U.) & Vic Reiner (UMN).



Outline

0. Motivating story

I. Left regular bands + their semigroup algebras

II. Left regular bands with symmetry + Guiding Questions

III. Our Findings

0. Motivating story

Solomon's descent algebra

- For $\sigma \in S_n$, the **descent set** $\text{Des}(\sigma) := \{ i : \sigma(i) > \sigma(i+1) \}$.

↳ Ex: $\text{Des}(\underline{24315}) = \{2, 3\}$.

← one-line notation

- Define **Solomon's descent algebra** by

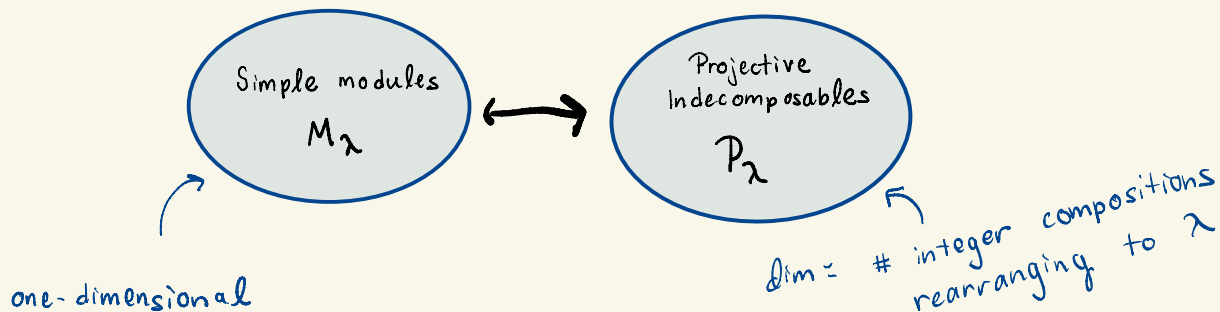
$$\Sigma_n := \text{span}_{\mathbb{C}} \left\{ \sum_{\text{Des}(\sigma) = J} \sigma : J \subseteq \{1, \dots, n-1\} \right\}.$$

↑
Thm (Solomon '76): Closed under multiplication!

- Remarkable** subalgebra of $\mathbb{C}S_n$: connections to S_n -characters, free Lie algebra, random walks on S_n

Structure of the descent algebra Σ_n

- non commutative and non semisimple
- Representation theoretic data indexed by partitions $\lambda \vdash n$



- Nice formulas for Cartan Invariants $\{ \overbrace{[P_\mu : M_\lambda]}^{\text{composition multiplicity}} : \lambda, \mu \vdash n \}$

Cartan Invariants of $\sum_n$

- A word on $\{1, 2, \dots, n\}$ is a **Lyndon word** if it's strictly lexicographically smaller than all its cyclic reorderings.

29299 ✓

2929 ✗

- Each word has a unique **factorization** into lexicographically (weakly) decreasing Lyndons.

Word w	Lyndon Factorization	Lyndon Type (w)
5614311236	$(\underbrace{56}_{11})(\underbrace{143}_8)(\underbrace{11236}_{13})$	$(13, 11, 8)$
121123	$(12)(1123)$	$(7, 3)$

Thm (Garsia-Zeutenauer, '89)

$$[P_\mu : M_\lambda] = \# \left\{ \begin{array}{l} \text{compositions } \alpha \\ \text{rearranging to } \mu \end{array} \mid \text{LyndonType}(\alpha) = \lambda \right\}$$

A strategy to understand Σ_n

Bidigare ('97), Schocker ('05), Salio la ('06-'08), ...

We can learn more about Σ_n by viewing it as an **invariant subalgebra** of an even **better understood finite-dimensional algebra**.

↳ Thm (Bidigare '97)

$$\Sigma_n^{\text{opp}} \cong (\mathbb{C} B)^{S_n}$$

a specific "left regular band" algebra

↳ New perspective simplified many known proofs + added new info

Idea of talk

"Left regular band" algebras

← Margolis - Saliola -
Steinberg
('15 - '21)

$(-)^G$

Invariant subalgebras of
"Left regular band" algebras

← ?



Tool



"poset cohomology"



Tool



group equivariant
"poset cohomology"

I. Left regular bands
+
their semigroup algebras

Left regular bands

A finite semigroup B is a **left regular band** (or an **LRB**) if for all $x, y \in B$:

- $x^2 = x$ (band)
- $xyx = xy$ (left regular)

Each LRB B has two associated posets:

Coarser: Support Semilattice

$$(\Lambda(B), \leq)$$

elements: principal left ideals
 $\{Bb : b \in B\}$

order: inclusion

More refined: Semigroup poset

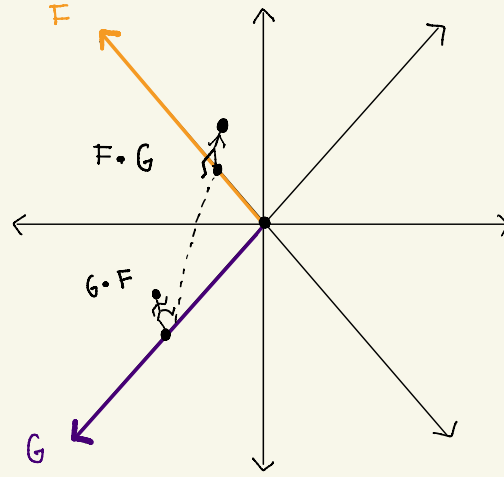
$$(B, \leq)$$

elements: B

order: $x \leq y$ iff $yx = y$

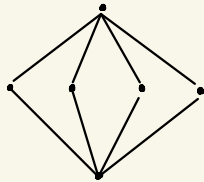
"Green's $\mathcal{R}$ -ordering on idempotents"

Example #1: Face monoid of a hyperplane arrangement



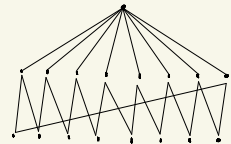
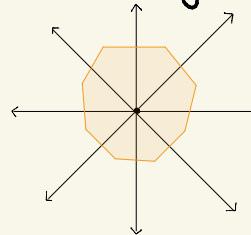
Support Semilattice

Lattice of **intersections** of hyperplanes under reverse inclusion.



Semigroup poset

The face poset of the **zonotope** of the arrangement.



Example #1: Face monoid of a hyperplane arrangement

Let $\mathcal{F}_n$ denote the face monoid of the **braid arrangement** associated to n .

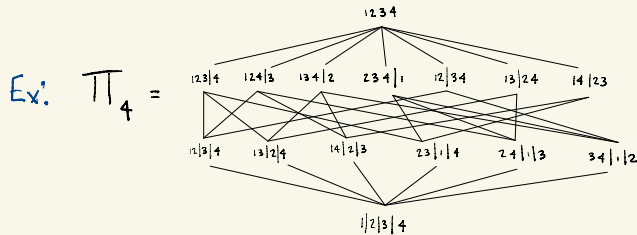
$$\{x_i = x_j : \overset{ii}{1 \leq i < j \leq n}\} \subseteq \mathbb{R}^n$$

Thm (Bidigare '97):

The S_n -invariant subalgebra $(\mathbb{K} \mathcal{F}_n)^{S_n}$ is anti-isomorphic to Solomon's descent algebra Σ_n .

Support Semilattice

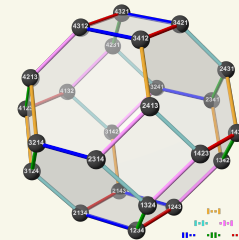
$\Lambda(\mathcal{F}_n) \cong$ **lattice of set partitions** Π_n .



Semigroup poset

$\mathcal{F}_n \cong$ face poset of n -permutahedron

Ex $\mathcal{F}_4$



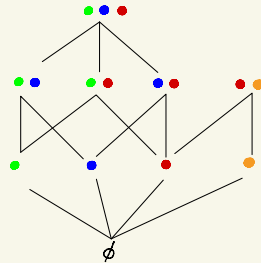
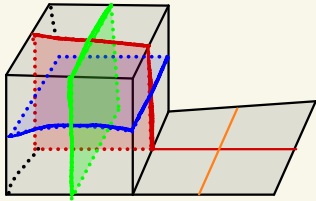
Example #2: Face semigroup of a CAT(0)-cube complex

Thm (Margolis-Saliola Steinberg '21, Bandelt-Chepoi-Knauer '15)

There is an LRB on the faces of any CAT(0)-cube complex.

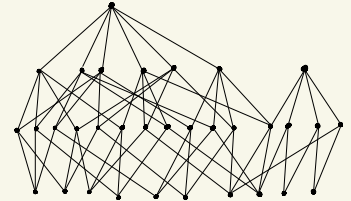
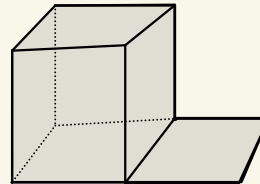
Support Semilattice

Intersection semilattice of **hyperplanes**
under reverse inclusion



Semigroup poset

Face poset of the CAT(0)-cube complex



Example #3: LRBs associated to geometric lattices

For each **geometric lattice** $\mathcal{L}$, Brown ('00) constructed an LRB $S(\mathcal{L})$.

$$S(\mathcal{L}) := \{ \text{prefixes } (x_1 < x_2 < \dots < x_k) \text{ of maximal chains in } \mathcal{L}^{\hat{0}} \}$$

with

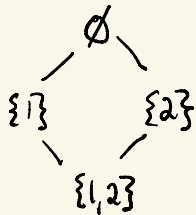
$$(x_1 < x_2 < \dots < x_k) \cdot (y_1 < y_2 < \dots < y_\ell) =$$

$$(x_1 < x_2 < \dots < x_k \leq x_k \vee y_1 \leq x_k \vee y_2 \leq \dots \leq x_k \vee y_\ell)^\wedge \quad \text{delete repeats!}$$

Support Semilattice

$\mathcal{L}^{\text{opp}}$

For $S(B_2)$:

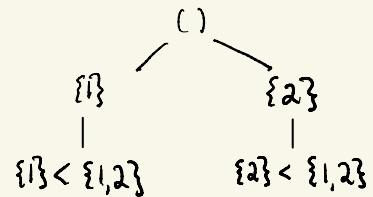


Semigroup poset

prefix ordering

For $S(B_2)$

Boolean lattice
on 2 elts



Why study LRBs?

- Probability:

- Motivated by understanding **mixing times** for  shuffling processes 
- ↳ Bidigare-Hanlon-Rockmore, Brown-Diaconis, Brown, + many others!

- Representation Theory of their semigroup algebras:

- Rich but tractable
- Aguiar, Bastiolas, Bidigare, Brauer, Brown, Mahajan, Margolis, Rhodes, Saliola, Schilling, Schocker, Schützenberger, Steinberg, and more ...

LRB Semigroup algebras

- The **LRB semigroup algebra** $\mathbb{C}B$ consists of elements of the form

$$\sum_{b \in B} c_b b \text{ for } c_b \in \mathbb{C}, \text{ with multiplication extended } \mathbb{C}\text{-linearly.}$$

Q: When does $\mathbb{C}B$ have a multiplicative identity?

Thm (Margolis - Saliola - Steinberg '21)

$\mathbb{C}B$ has a multiplicative identity $\iff$ semigroup poset (and certain subposets) are connected as graphs

We will assume B satisfies this property.

Representation theory of LRB algebras

Data is indexed by supports $X \in \Lambda(B)$.

Simple modules
 $\{M_X\}$

always one-dimensional!

Projective
Indecomposable
modules
 $\{P_X\}$

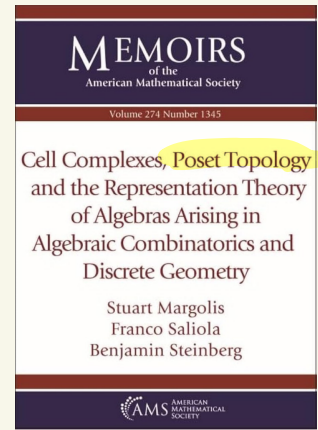
Idempotents
in a complete family
of primitive orthogonal
idempotents of $\mathbb{C}B$
 $\{E_X\}$

well-understood...
(Saliola '06, Aguiar-Mahajan '18)

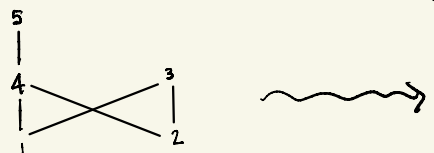
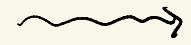
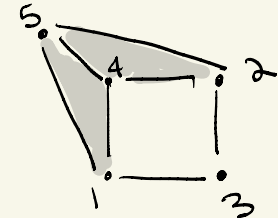
Representation theory of LRB algebras

Much is known about structure of $\mathbb{C}B$

- Cartan invariants
- Ext-quiver
- projective resolutions of simples
- Ext^i -groups between simples for all $i \geq 0$ / global dimension

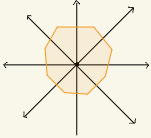
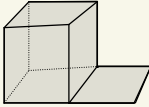


All of above: Expressed in terms of poset cohomology

- Poset $\mathcal{P}$   Simplicial complex $\Delta(\mathcal{P})$ 
- $\tilde{H}^i(\mathcal{P}) := \tilde{H}^i(\Delta(\mathcal{P}); \mathbb{C})$

CW LRBs

An LRB B is a **CW LRB** if its semigroup poset (and certain subsets) are face posets of **regular CW complexes**.

e.g. hyperplane LRBs  + CAT(0)-cube LRBs 

Even more is known for CW LRBs!

- **minimal** projective resolutions of simples
- **Koszul**, with Koszul dual incidence algebra $(\Lambda(B))^{\text{opp}}$

II.

LRBs with symmetry

+

Guiding Questions

LRBs with symmetry

Many LRBs in the literature are naturally equipped with symmetry groups.

Let G be a finite group $\curvearrowright B$ by semigroup automorphisms.

The G -invariant subalgebra is

$$(\mathbb{C}B)^G := \{x \in \mathbb{C}B : g(x) = x \quad \forall g \in G\}.$$

claim: $(\mathbb{C}B)^G$ is a combinatorially interesting algebra!

Q:

Can we use group-equivariant poset cohomology to understand it?

"Almost automatic" info on invariant subalgebras

Straightforward to use representation theory of $\mathbb{C}B$ to understand ...

- Data for $(\mathbb{C}B)^G$ is indexed by G -orbits of supports $[x] \in \Lambda(B)/G$.

Simple modules

$$\{M_{[x]}\}$$



still one-dimensional!

Projective
Indecomposables

$$\{P_{[x]}\}$$

Idempotents

in a complete family
of primitive orthogonal
idempotents of $(\mathbb{C}B)^G$.

$$\{E_{[x]}\}$$

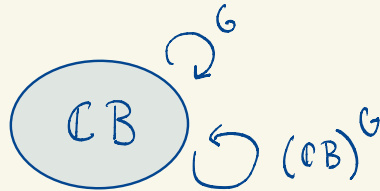
More mysterious aspects / Specific Questions

① When is $(\mathbb{C}B)^G$ commutative? When is it semisimple?

② If not semisimple: what are its Cartan invariants

$$[P_{[X]} : M_{[Y]}] \quad \text{for } [X], [Y] \text{ in } \Lambda(B)/G ?$$

③ (External representations, motivated by invariant theory):



Approach for ②+③: Invariant Peirce Decompositions

②

As $[X], [Y]$ vary in $\Lambda(B)/G$, what is the G -representation structure of

$$\underbrace{E_{[Y]} \mathbb{C}B E_{[X]}}_{\text{Invariant Peirce components}} ?$$

② Cartan invariants: $[P_{[X]} : M_{[Y]}] = \langle 1, E_{[Y]} \mathbb{C}B E_{[X]} \rangle_G$

③ $\mathbb{C}B \xrightarrow{G} (\mathbb{C}B)^G$: For an irreducible character χ of G ,
the χ -isotypic component $(\mathbb{C}B)^\chi$ is a $(\mathbb{C}B)^G$ -module,
with $[(\mathbb{C}B)^\chi : M_\chi] = \sum_{[X]} \dim(\chi) \langle \chi, E_{[Y]} \mathbb{C}B E_{[X]} \rangle_G$

III. Our Findings

Roadmap

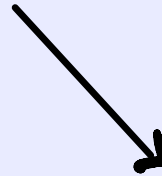
① When is $(\mathbb{C} B)^G$ **semisimple**? When is it **commutative**?

② + ③ Invariant Peirce Decompositions



④

$B = \text{CW LRB}$



⑤

$B = \text{Geometric lattice LRB}$

When are invariant subalgebras semisimple / commutative?

Thm (C. - Steinberg, 2025+)

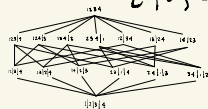
The following are equivalent:

- ① $(\mathbb{C}B)^G$ is semisimple
- ② $|B/G| = |\Lambda(B)/G|$
- ③ $(\mathbb{C}B)^G$ is commutative

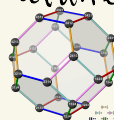
Example:

$$\Sigma_n \cong (\mathbb{C}\tilde{\mathcal{F}}_n^{\text{opp}})^{S_n}$$

$\Lambda(\tilde{\mathcal{F}}_n)$ = lattice of (unordered) set partitions of $\{1, 2, \dots, n\}$



$\tilde{\mathcal{F}}_n$ = face poset of permutahedron

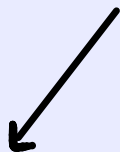


Σ_n is noncommutative and non semisimple.

Roadmap

① When is $(\mathbb{C} B)^G$ semisimple? When is it commutative?

② + ③ Invariant Peirce Decompositions



④

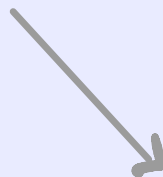
$B = \text{CW LRB}$



Braid Arrangement
LRB



CAT(0)-cube
LRBs



⑤

$B = \text{Geometric lattice LRB}$

Invariant Peirce decompositions for the braid arrangement face algebra

Thm (C., 2024)

$$\sum_{\substack{n \geq 0, \\ \lambda, \mu \vdash n}} \text{ch} \left(E_\lambda \subset \mathfrak{F}_n E_\mu \right) \underline{y}_\lambda \underline{z}_\mu = \prod_{\substack{\text{Lyndon} \\ \text{words } \underline{w}}} \sum_{\substack{\text{partitions} \\ \tau}} \mathcal{L}_\tau \left[h_{\underline{w}} \right] \underline{y}_{\tau \cdot |\underline{w}|} \underline{z}_{|\underline{w}| \tau}.$$

Frobenius characteristic map

higher Lie characters

plethysms

complete homogeneous symmetric fns

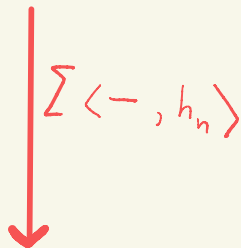
Other Notation

- $\underline{y}_\alpha := y_{\alpha_1} y_{\alpha_2} \dots y_{\alpha_k}$ (similarly for $\underline{z}_\alpha, h_\alpha$)
- $|\underline{v}|, |\underline{w}| :=$ sum of the parts of $\underline{v}$ and the letters of $\underline{w}$.
- $\underline{v} \cdot k :=$ scaling of the parts of $\underline{v}$ by k

Invariant Peirce decompositions for the braid arrangement face algebra

As a special case, recovers the Cartan invariants of Σ_n :

$$\sum_{\substack{n \geq 0, \\ \lambda, \mu \vdash n}} \text{ch} (E_\lambda (\mathbb{C} \tilde{\mathfrak{S}}_n) E_\mu) \bar{y}_\lambda \bar{z}_\mu = \prod_{\substack{\text{Lyndon} \\ \text{words } w}} \sum_{\tau \text{ partitions}} \mathcal{L}_\tau [h_w] \bar{y}_{z \cdot |w|} \bar{z}_{w|w|}$$



$$\sum_{\substack{n \geq 0, \\ \lambda, \mu \vdash n}} \text{ch} (E_\lambda (\mathbb{C} \tilde{\mathfrak{S}}_n)^{S_n} E_\mu) \bar{y}_\lambda \bar{z}_\mu = \dots = \sum_{\substack{n \geq 0, \\ \lambda, \mu \vdash n}} \# \{ \alpha \models n : \alpha \sim \mu \text{ and } \text{LyndonType}(\alpha) = \lambda \} \bar{y}_\lambda \bar{z}_\mu$$

Invariant Peirce decompositions for the braid arrangement face algebra

To come up with conjectures:



- Hyperplane Arrangements
- Descent algebras
- Symmetric Functions

S_n -isotypic subspaces (λ)

$\chi^{(111)}$					$M_{\square}$
$\chi^{(21)}$			$3M_{\square}$	$3M_{\square}$	$3M_{\square}$
$\chi^{(22)}$			$2M_{\square}$	$2M_{\square}$	$2M_{\square}$
$\chi^{(31)}$	$3M_{\square}$	$3M_{\square}$	$3M_{\square}$	$3M_{\square}$	$3M_{\square}$
$\chi^{(4)}$	$M_{\square}$	$M_{\square}$	$M_{\square}$	$M_{\square}$	$M_{\square}$
$\chi^{(4,1)}$	$6M_{\square}$	$6M_{\square}$	$6M_{\square}$	$6M_{\square}$	$6M_{\square}$
	$e_{2,1} e_1$	$e_{2,1} e_{2,1}$	$e_{2,1} e_{2,1}$	$e_{2,1} e_{2,1}$	$e_{2,1} e_{2,1}$
			μ		

Key tool for proof

Theorem: (implied by work of Saliola ('06) + Aguiar - Mahajan ('17))

Let $k := \ell(\mu) - \ell(\lambda)$. Then, as S_n -representations,

$$E_{\lambda} \subset \mathcal{F}_{\lambda} E_{\mu} \cong \bigoplus_{[X \leq Y]_{S_n}} \text{Block Size}(X) = \mu \quad \text{Block Size}(Y) = \lambda \quad \text{Det}(X) \otimes \tilde{H}^{k-2}(\Pi_n(X, Y)) \otimes \text{Det}(Y)$$

$\uparrow S_n$
 $\text{Stab}_{S_n}(X) \cap \text{Stab}_{S_n}(Y)$

Invariant Peirce decompositions for the braid arrangement face algebra

$$\text{Det}(X) \otimes \tilde{H}^{k-2}(\Pi_n(X, Y)) \otimes \text{Det}(Y)$$

Partition lattice

ADVANCES IN MATHEMATICS 104, 225-296 (1994)

The Homology Representations of the
Symmetric Group on Cohen-Macaulay Subposets
of the Partition Lattice

SHEILA SUNDARAM*

Department of Mathematics and Computer Science,
University of Miami, Coral Gables, Florida 33124

Determinant "twisting" Characters

- Set partition $X \longrightarrow$ subspace $V(X) \subseteq \mathbb{R}^n$

$$12|34|5|6 \longmapsto \text{span}_{\mathbb{R}} \{e_1 + e_2, e_3 + e_4, e_5, e_6\}$$

- $\text{Det}(X): \text{Stab}_{S_n}(X) \longrightarrow \{+1, -1\}$
 $g \longmapsto \text{Det}(g \curvearrowright V(X))$

Generalizing to CW LRBs:

Assume that B is a CW LRB.

Thm (C. - Steinberg, 2025+)

Let $k = \text{rk}(Y) - \text{rk}(X)$. As G -rep's,

$$E_{[Y]} \otimes E_{[X]} \cong \bigoplus_{\substack{[X' \leq Y']_G: \\ Y' \in [Y], X' \in [X]}} \text{Deg}(X') \otimes \tilde{H}_{\Lambda(B)}^{k-2}(X', Y') \otimes \text{Deg}(Y')$$

$\begin{array}{c} \uparrow G \\ \cdot \\ \downarrow G_{X'} \cap G_{Y'} \end{array}$

Degree Characters:

- $\text{Deg}(X)(g) \in \{+1, -1\}$ = degree of an unusual action g^Q (spherical poset associated to X).

Application: CAT(0)-cube LRBs

Assume $G \curvearrowright$ CAT(0)-cube complex $\mathcal{C}$ by isometries so that

- G permutes the faces of $\mathcal{C}$.
- If all $g \in G$ fix a face c of $\mathcal{C}$ setwise, then all $g \in G$ fix c pointwise.

Claim

Then,

$$\text{Deg}(X) \otimes H^{k-2}(X, Y) \otimes \text{Deg}(Y) \cong_{G_X \cap G_Y} \mathbb{1}.$$

Cor (C. - Steinberg, '25+)

In this case,

- Invariant Peirce decompositions are **permutation representations**.
- Cartan invariants of $(\mathbb{C}B)^G$ are easy to compute.

$$\hookrightarrow \left[p_{[X]} : M_{[Y]} \right] = \# \{ [X' \leq Y'] \mid X' \in [X] \text{ and } Y' \in [Y] \}.$$

Roadmap

① When is $(\mathbb{C} B)^G$ semisimple? When is it commutative?

② + ③ Invariant Peirce Decompositions



①

$B = CW LRB$

②

$B = \text{Geometric lattice LRB}$

"Derangement" representations

secretly

$\mathbb{C} B$ is hereditary

Derangement Numbers

A permutation is a **derangement** if it has no fixed points.

Let d_n denote the number of derangements in S_n .

↳ Ex $d_3 = 2 = \#\{231, 312\}.$

How to count d_n ?

• Typical: $n! = \sum_{j=0}^n \underbrace{\binom{n}{j} d_{n-j}}_{\text{perms w. } j \text{ fixed pts}} \longrightarrow d_n = \sum_{j=0}^n (-1)^j \binom{n}{j} (n-j)!$

• Seemingly little-known, but a **manifestly positive** formula:

$$d_n = \sum_{\alpha \models n} \binom{n}{\alpha} \prod_{i=1}^{\alpha} (\alpha_i - 1).$$

Derangement Representation of S_n

Desarmenién-Wachs ('88) introduced a representation of S_n with dimension d_n :

One possible
def:

$$\text{Der}(n) \cong \bigoplus_{k=0}^n (-1)^k \left(\text{sgn}_{S_k} \otimes \text{Reg}_{S_{n-k}} \right) \uparrow_{S_k \times S_{n-k}}^{S_n}$$

$\downarrow \dim_{\mathbb{C}} \text{ ---}$

$$d_n = \sum_{k=0}^n (-1)^k \binom{n}{k} (n-k)!$$

Thm (Brauer-C-Reiner '23): As S_n -rep's,

$$E_0 \subset S(B_n) \uparrow \cong \text{Der}(n)$$

+ other invariant Peirce decompositions can be expressed in terms of smaller $\text{Der}(k)!$

Brown's generalized derangement numbers

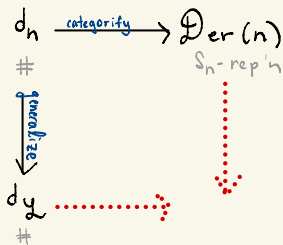
Let:

- $\mathcal{L}$ be a geometric lattice,
- $\tilde{\mathcal{F}}(\mathcal{L})$ be the set of maximal chains of $\mathcal{L}$.

Def (Brown '00):

$$d_{\mathcal{L}} := \sum_{x \in \mathcal{L}} \mu(\hat{0}, x) \cdot \# \tilde{\mathcal{F}}(\mathcal{L}[x, \hat{1}]) \quad \xrightarrow{\mathcal{L} = \mathcal{B}_n} \quad d_n = \sum_{k=0}^n (-1)^k \binom{n}{k} (n-k)!$$

Thm (implicit in Brown '00): $d_{\mathcal{L}} = \dim_{\mathbb{C}} E_{\hat{1}}(\mathbb{C}S(\mathcal{L})) E_{\hat{0}}$



Generalized Derangement Representations

For a geometric lattice $\mathcal{L}$,

Def

$$\text{Der}(\mathcal{L}) := G\text{-rep'n } E_{\hat{0}_{\mathcal{L}}} \oplus S(\mathcal{L}) \oplus E_{\hat{1}_{\mathcal{L}}}.$$

Thm (C. - Steinberg, '25+)

$$\text{Der}(\mathcal{L}) \cong \bigoplus_{m \geq 1} \left(\bigotimes_{i=0}^{m-1} \left(\mathbb{C}\{z: x_i \leq z < x_{i+1}\} - 1 \right) \right) \Big|_{\bigcap G_{x_i}}^G$$

G -orbits of chains in $\mathcal{L}$
 $[1 = x_0 < x_1 < \dots < x_m = \hat{0}]$

$$\begin{array}{ccc} \mathcal{L} = \mathcal{B}_n & \downarrow & \\ \text{Der}(n) \cong \bigoplus_{\alpha \models n} \left(\bigotimes_{i=1}^{\ell(\alpha)} S^{(\alpha_i-1, 1)} \right) & \begin{array}{c} \uparrow S_n \\ \downarrow S_\alpha \end{array} & \xrightarrow{\dim(-)} d_n = \sum_{\alpha \models n} \binom{n}{\alpha} \prod_{i=1}^{\ell(\alpha)} (\alpha_i - 1) \end{array}$$

Generalized Derangement Representations

Generalizing work of Brauer - C. - Reiner on $\mathcal{L} = \mathcal{B}_n$, $\mathcal{L} = \mathcal{B}_n(q)$:

Cor (C. - Steinberg, 2025+):

As G -rep's:

$$\textcircled{1} \quad E_{[Y]} \otimes S(\mathcal{L}) E_{[X]} \cong \bigoplus_{[X] \in \mathcal{L}/G} \text{Der}(\mathcal{L}[X, \uparrow]) \quad \begin{matrix} \uparrow G \\ G_X \end{matrix}$$

"All Invariant Peirce decompositions reduce to understanding derangement rep's"

$$\textcircled{2} \quad \text{Der}(\mathcal{L}) \cong \bigoplus_{[Y] \in \mathcal{L}/G} (-1)^{\text{rk}(Y)} \left(\widetilde{H}^{\text{rk}(Y)-2}(\emptyset, Y) \otimes \mathbb{C} \mathcal{F}(\mathcal{L}[Y, \uparrow]) \right) \quad \begin{matrix} \uparrow G \\ G_Y \end{matrix}$$

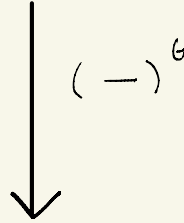
"Equivariant version" of $d_{\mathcal{L}} = \sum_{Y \in \mathcal{L}} \mu(\emptyset, Y) \cdot \# \mathcal{F}(\mathcal{L}[Y, \uparrow])$

Summary

"Left regular band" algebras

← Margolis-Saliola -
Steinberg
('15 - '21)

 Tool 
"poset cohomology"



Invariant subalgebras of
"Left regular band" algebras



 Tool 
group equivariant
"poset cohomology"

Thank you for listening !