Category Theory, Combinatorics, and Machine Learning

Poster Session Abstracts

September 17, 2025

The Transition Matrix Combinatorics of Polysymmetric Functions

Aditya Khanna, Virginia Tech

Asvin G. and Andrew O'Desky introduced the algebra of polysymmetric functions (PSym) as a tensor product of copies of symmetric functions algebra (Sym). This object has interesting geometric and representation theoretic connections but in this poster we will concern ourselves with the combinatorics of the transition matrix coefficients. One way to construct a basis of PSym is by considering the tensor product of Sym basis. On the other hand, the authors define non pure-tensor bases namely H, E, E+, P as generalizations of the Sym bases. In this poster, we will show how we can compute the transition matrix coefficients between the non-pure tensor and pure-tensor bases using Pieri-like rules on the tensor diagrams of tableaux.

Plethysm, categorification, and Stanley's hook-content formula

Álvaro Gutiérrez, University of Bristol

A number of q-binomial identities suggest the existence of isomorphisms between certain SL(2) representations, of plethystic type. We construct explicit isomorphisms between the representations, which categorify the q-binomial identities. The method is novel, and brings symmetric functions into representation theory without taking characters. A particular nice source of examples is Stanley's hook-content formula.

Joint with Á. Martínez, M. Szwej, and M. Wildon.

A hidden symmetry of refined canonical stable Grothendieck polynomials

Joseph Pappe & Jianping Pan, ICERM

Refined canonical stable Grothendieck polynomials introduced by Hwang, Jang, Kim, Song, and Song have two combinatorial models: one using hook-valued tableaux and the other using pairs of a semistandard Young tableau and an exquisite tableau. We discover a novel connection between the two models via the uncrowding map of Pan, Pappe, Poh, and Schilling, the jeu de taquin algorithm of Goulden and Greene, and the tableau switching of Benkart, Sottile, and

Stroomer. This connection allows us to obtain another combinatorial model for the refined canonical stable Grothendieck polynomials in terms of biflagged tableaux, which naturally appear in the characterization of the image of the uncrowding map.

Kronecker coefficients, polytopes, and complexity

Aram Bingham, Universidad de Chile

The Kronecker coefficients problem is one of the last major open questions in the classical representation theory of symmetric groups. It asks for a combinatorial rule describing the decomposition of tensor products of irreducible symmetric group representations. Solutions are known only in limited special cases. Kronecker coefficients have also been the subject of much recent research motivated by the geometric complexity theory (GCT) program, which hypothesized efficient computation of these values as part of a strategy to separate the computational complexity classes P and NP. We obtain results on computing these coefficients in certain cases using discrete volumes of polytopes.

Double Boxes and Double Dimers

Tatyana Benko, University of Oregon

We give a combinatorial proof of a result in rank 2 Donaldson-Thomas theory, which states that the generating function for certain plane-partition-like objects, called double-box configurations, is equal to a product of MacMahon's generating function for (boxed) plane partitions. In our proof, we first give the correspondence between double-box configurations and double-dimer configurations on the hexagon lattice with a particular tripartite node pairing. Using this correspondence, we apply graphical condensation and double-dimer condensation to prove the result.

Lift of a Kostka matrix problem to NSym and a reduction to a solution in Sym

Kyle Celano, Wake Forest University

Egecioglu and Remmel provide a combinatorial proof (using special rim hook tableaux) that the product of the Kostka matrix K and its inverse K^{-1} equals the identity matrix I. They then pose the problem of proving the reverse identity K^{-1}K =I combinatorially. Sagan and Lee prove a special case of this identity using overlapping special rim hook tableaux. Loehr and Mendes provide a full proof using bijective matrix algebra that relies on the Egecioglu--Remmel map. In this poster, we solve the problem in full generality independent of the Egecioglu--Remmel bijection. To do this, we start by proving NSym versions of both Kostka matrix identities using sign-reversing involutions involving the tunnel hook coverings recently of Allen and Mason. Then we modify our sign-reversing involutions to reduce to Sym.

Conditions for mutation-acyclic quivers

Scott Neville, ICERM

Totally proper quivers have many useful properties including powerful mutation invariants. We show that every mutation-acyclic quiver (i.e., a quiver that is mutation equivalent to an acyclic one) is totally proper. This yields new necessary conditions for a quiver to be mutation-acyclic, many of which are easily computed. We present several of these conditions.

The Chromatic Symmetric Function For Unicyclic Graphs

Lisa Johnston, University of California

Motivated by the question of which structural properties of a graph can be recovered from the chromatic symmetric function (CSF), we study the CSF of connected unicyclic graphs. While it is known that there can be non-isomorphic unicyclic graphs with the same CSF, we find experimentally that such examples are rare for graphs with up to 17 vertices. In fact, in many cases we can recover data such as the number of leaves, number of internal edges, cycle size, and number of attached non-trivial trees, by extending known results for trees to unicyclic graphs. These results are obtained by analyzing the CSF of a connected unicyclic graph in the \emph{star-basis} using the deletion-near-contraction (DNC) relation developed by Aliste-Prieto, Orellana and Zamora, and computing the ``leading'' partition, its coefficient, as well as coefficients indexed by hook partitions. We also give explicit formulas for star-expansions of several classes of graphs, developing methods for extracting coefficients using structural properties of the graph.

Quiver presentations for KLR algebras and Hecke categories

Alice Dell'Arciprete, ICERM

The ultimate goal of representation theory is to obtain a complete understanding of the submodule structure of some algebraic objects. In this poster we will tackle this problem for KLR algebras.
In particular, we will discover the rich algebraic structure of KLR algebras through an exciting interplay with the diagrammatic Hecke categories of maximal parabolics of finite symmetric groups. We will reveal how combinatorics (in the shape of Dyck tableaux) plays a huge role in understanding the structure of these algebras. Even further we go beyond just looking at sets of

Dyck tableaux (which enumerate the q-decomposition numbers) — we uncover the relationships for passing between them. This "meta-Kazhdan-Lusztig combinatorics" is, in fact, rich enough to fully determine the Ext-quiver and the relations presentation of these algebras.

The quasisymmetric Macdonald polynomials are quasi-Schur positive at t = 0

Kartik Singh, University of Waterloo

The quasisymmetric Macdonald polynomials $G_{\gamma}(X;q,t)$ are a quasisymmetric refinement of the $P_{\lambda}(X;q,t)$'s that specialize to the quasisymmetric Schur functions $\QS_{\gamma}(X)$. We study the $t=0$ specialization $G_{\gamma}(X;q.0)$, which can be described as a sum over weighted multiline queues. We show that $G_{\gamma}(X;q,0)$ expands positively in the quasisymmetric Schur basis and give a charge formula for the quasisymmetric Kostka--Foulkes polynomials $K_{\gamma,\alpha}(q)$ in the expansion $G_{\gamma}(X;q,0)=\sum_{\alpha}K_{\gamma,\alpha}(q)\QS_{\alpha}(X)$.

Diagonal supersymmetry for coinvariant rings

John Lentfer, University of California

The classical coinvariant ring was generalized by Haiman (1994) to the diagonal coinvariant ring, which consists of a polynomial ring in two sets of variables quotiented by the ideal generated by polynomials invariant under the diagonal action of the symmetric group, without constant term. Recently there has been much interest in studying (k,j)-bosonic-fermionic coinvariant rings, which are defined analogously for k sets of commuting (bosonic) and j sets of anticommuting (fermionic) variables. We prove the "diagonal supersymmetry" conjecture of Bergeron (2020), which asserts that the multigraded Frobenius series of a (k,j)-bosonic-fermionic coinvariant ring can be expressed in terms of universal coefficients, super Schur functions, and Frobenius characters. We compute some of these universal series coefficients and discuss applications.