Recursive Computations for Khovanov-Rozansky Homology

Misha Mazin

**Based on previous work with Nicolle González, Matt Hogancamp, and Carmen Caprau, and on recent conversations with Nicolle González and Eugene Gorsky.*

Categorified Young symmetrizers (Elias-Hogancamp)

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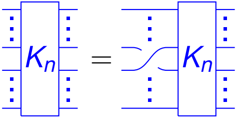
1. 

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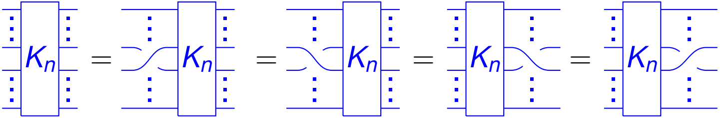
1. 

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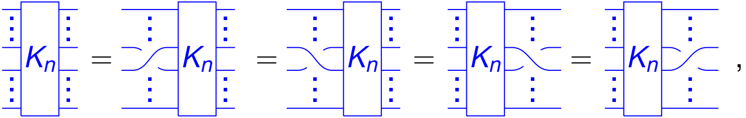
1. 

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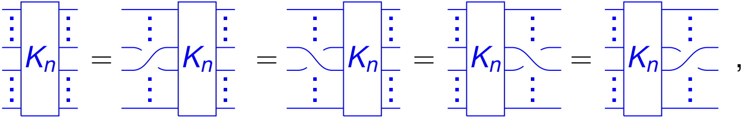
1. 

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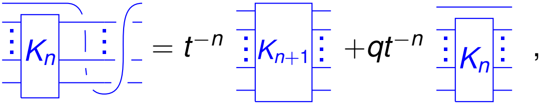

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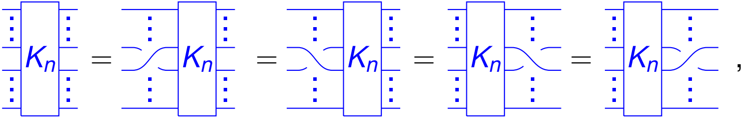
1. 

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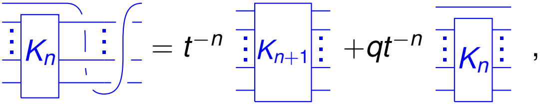
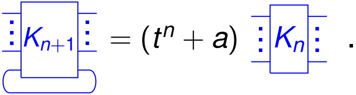


3. 

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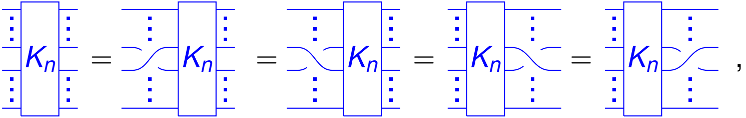
1. 

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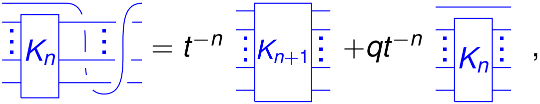
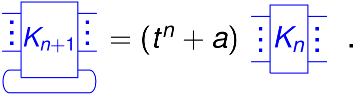
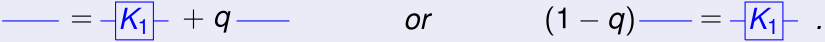


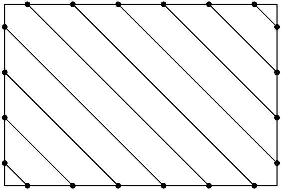
3. 

Remark

In particular, from (2) with $n = 0$ one gets



Hogancamp-Mellit recursions



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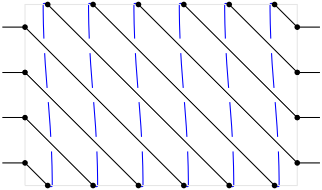

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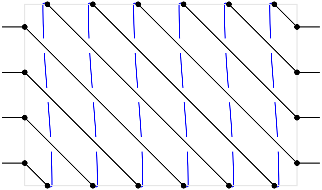

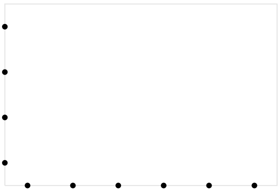

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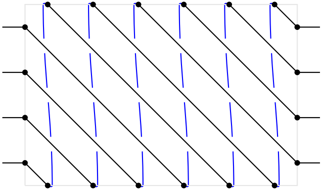

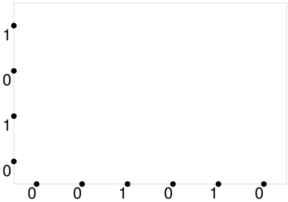

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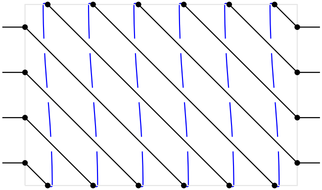

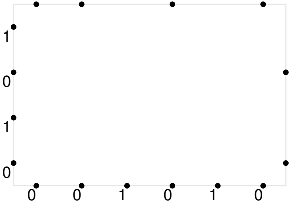

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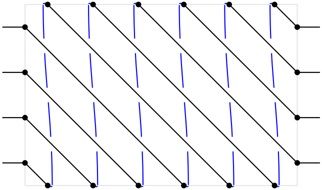

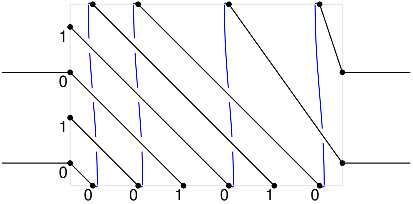

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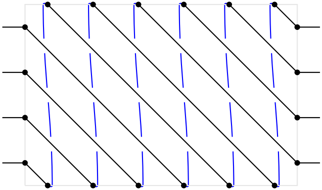

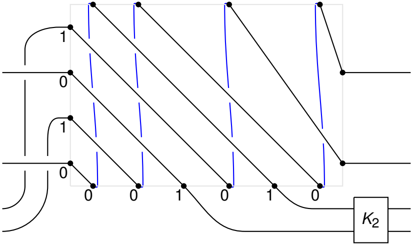

$R_{0101,001010}(q,t,a)$

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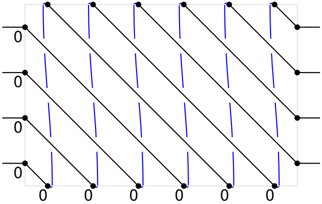

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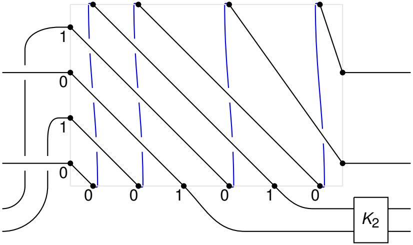

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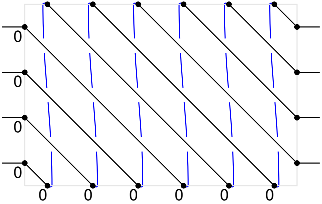

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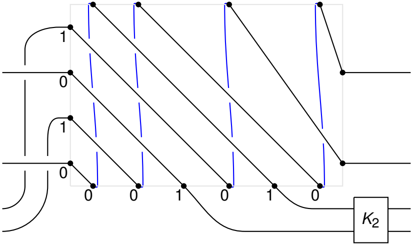


$R_{0101,001010}(q,t,a)$

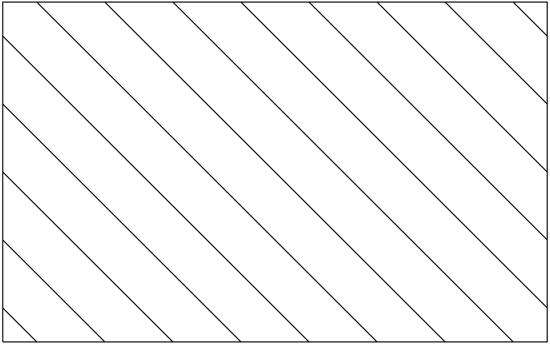
Theorem (Hogancamp-Mellit)

$$R_{0\mathbf{u},0\mathbf{v}} = t^{-|\mathbf{u}|}R_{\mathbf{u}1,\mathbf{v}1} + qt^{-|\mathbf{u}|}R_{\mathbf{u}0,\mathbf{v}0}, \quad R_{1\mathbf{u},0\mathbf{v}} = R_{\mathbf{u}1,\mathbf{v}}, \quad R_{\emptyset,0^n} = \left(\frac{1+a}{1-q}\right)^n,$$

$$R_{1\mathbf{u},1\mathbf{v}} = (t^{|\mathbf{u}|}+a)R_{\mathbf{u},\mathbf{v}}, \qquad R_{0\mathbf{u},1\mathbf{v}} = R_{\mathbf{u},\mathbf{v}1}, \quad R_{0^m,\emptyset} = \left(\frac{1+a}{1-q}\right)^m,$$

where $R_{\emptyset,\emptyset} := 1$.

Shortcut torus knots



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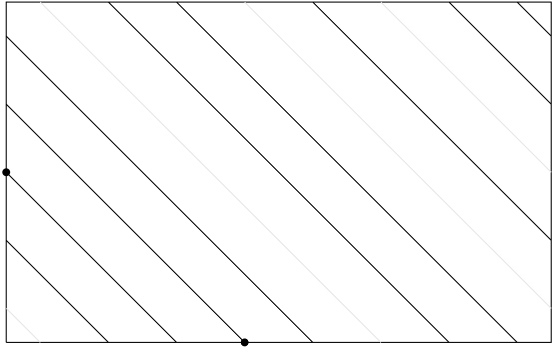

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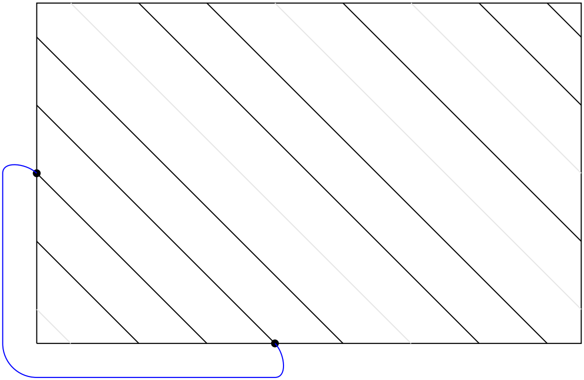

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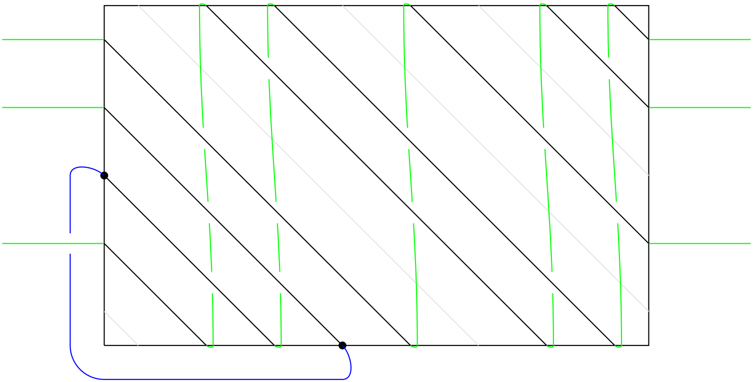

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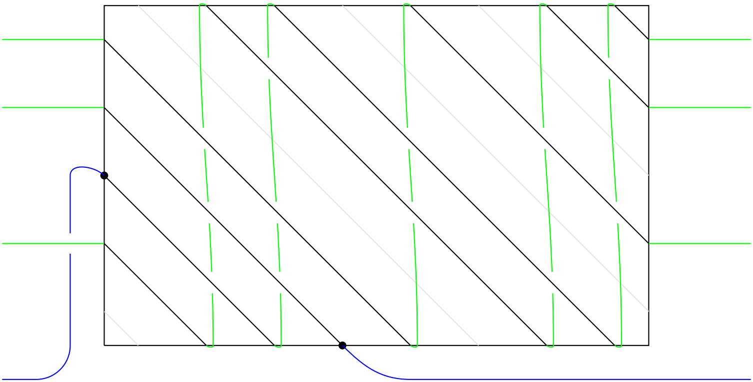

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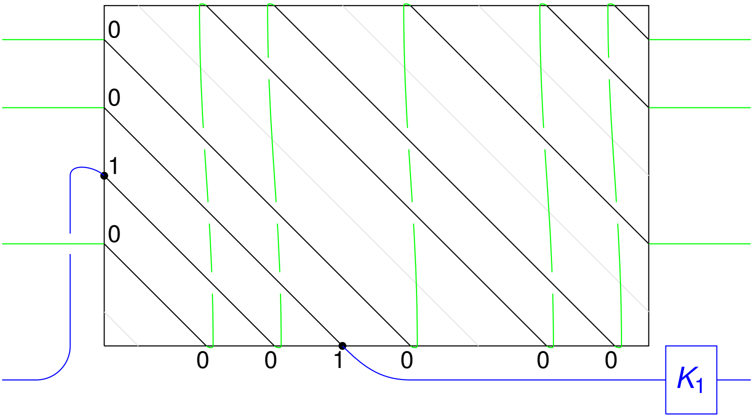

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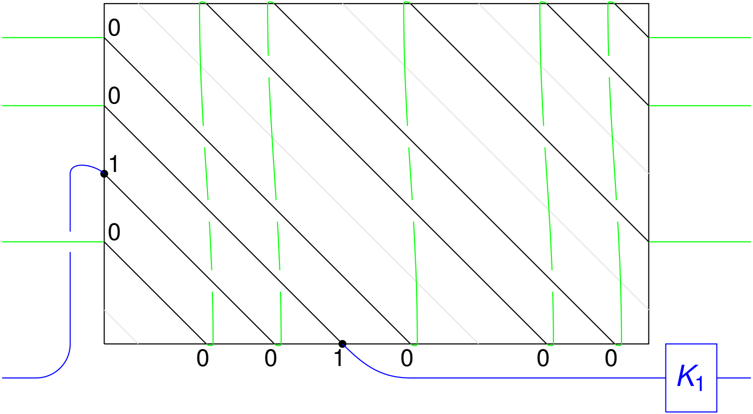
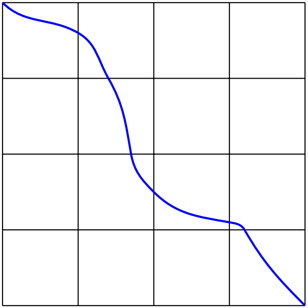


Theorem

All knots that appear in the Hogancamp-Mellit recursion are the shortcut torus knots (as above).

Monotone knots of Galashin-Lam



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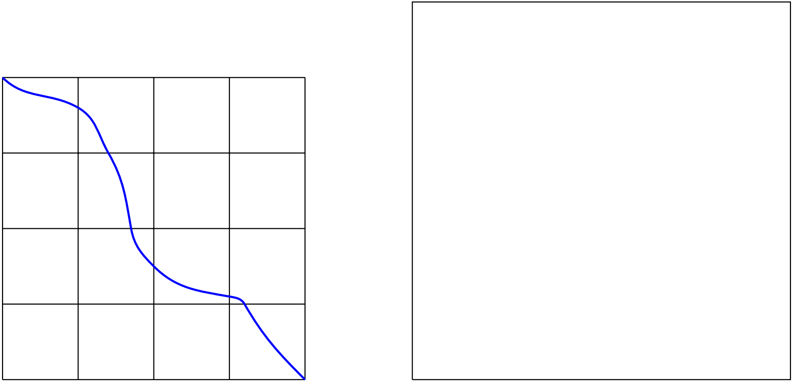

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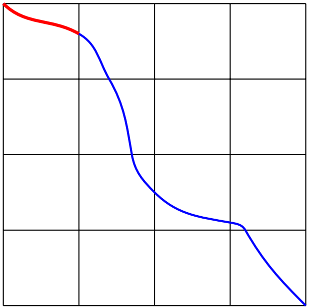

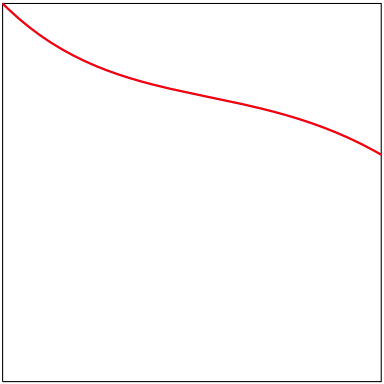

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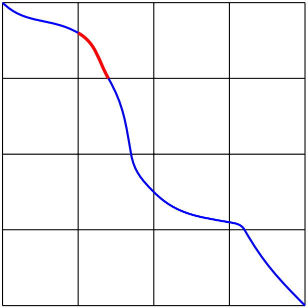

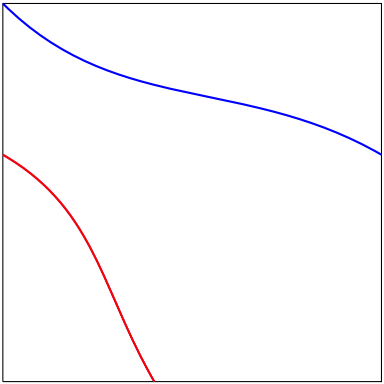

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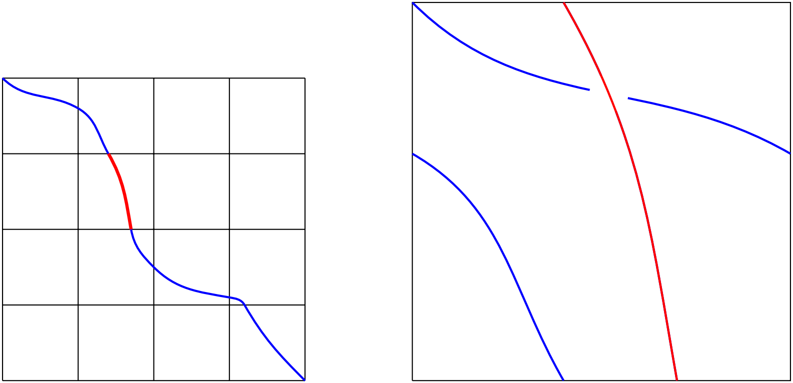

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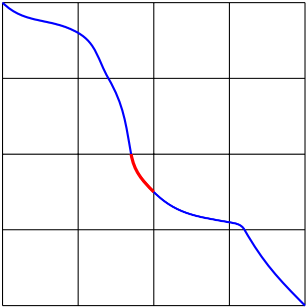

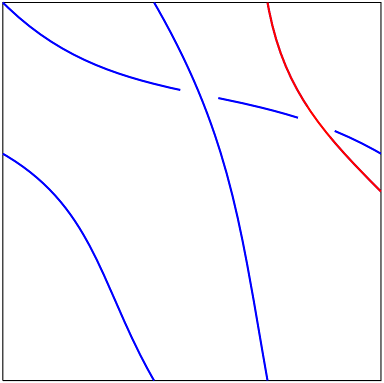

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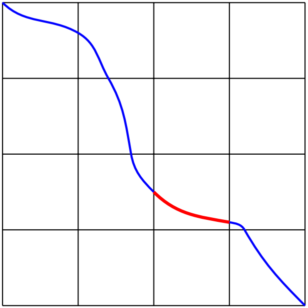

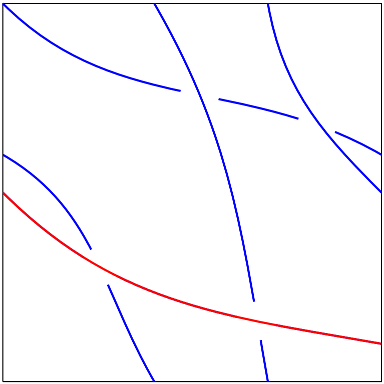

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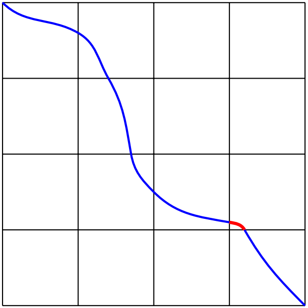
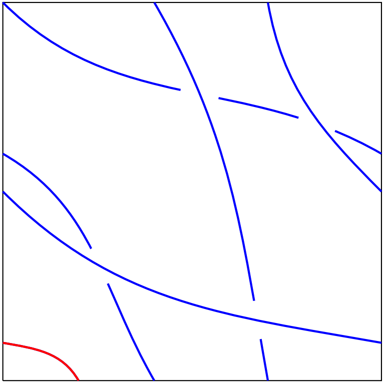
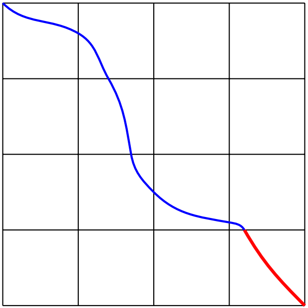
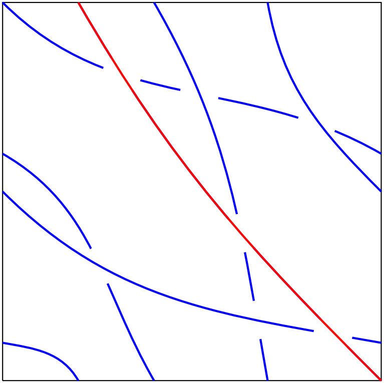




Monotone knots of Galashin-Lam





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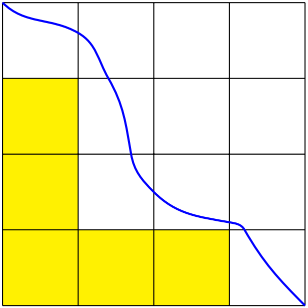
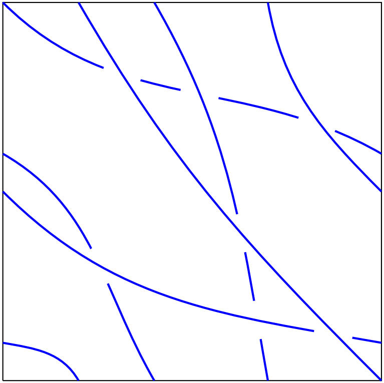
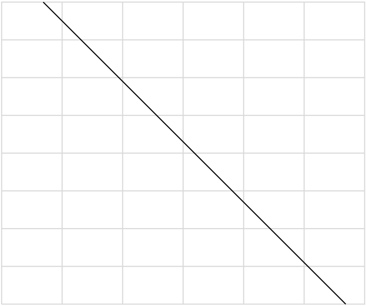


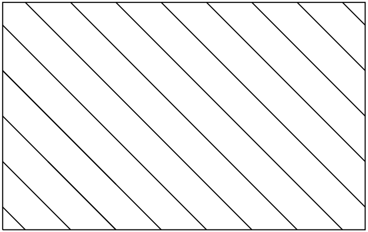


Theorem (Galashin-Lam)

Up to an isotopy, the knot only depends on the partition under the curve.

Monotone Knots of Triangular Partitions





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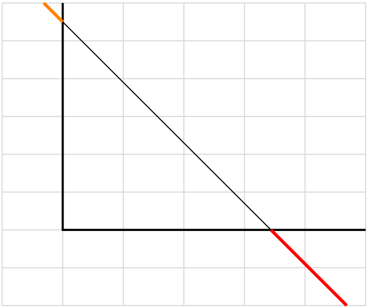

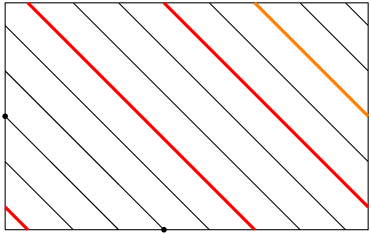

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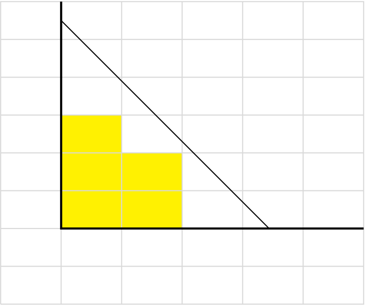

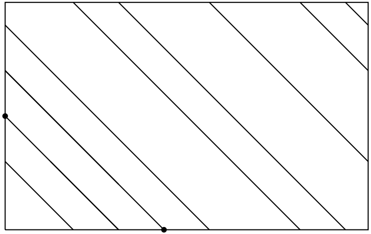

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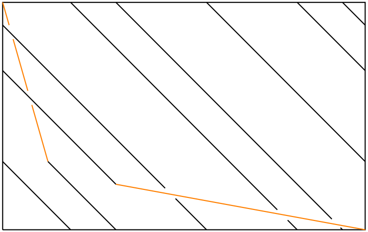

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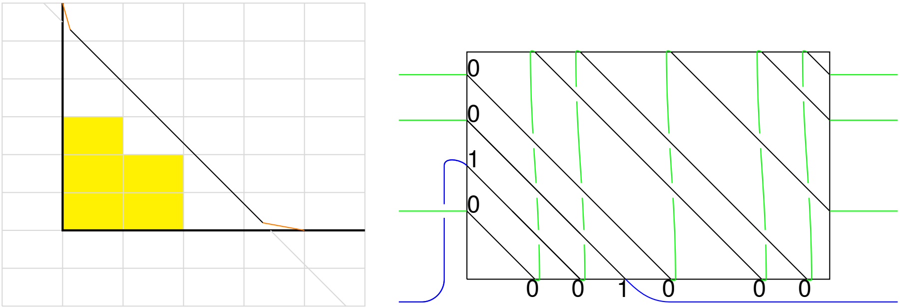
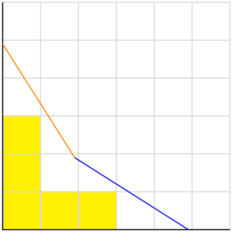
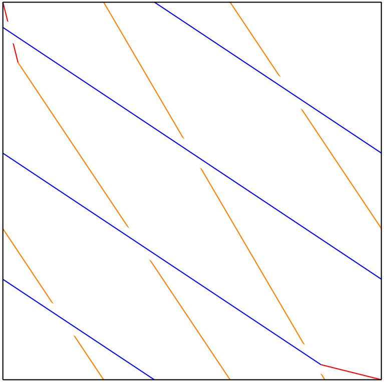


Theorem

Monotone knots of the triangular partitions are the shortcut torus knots.

Example: $(3, 1, 1)$





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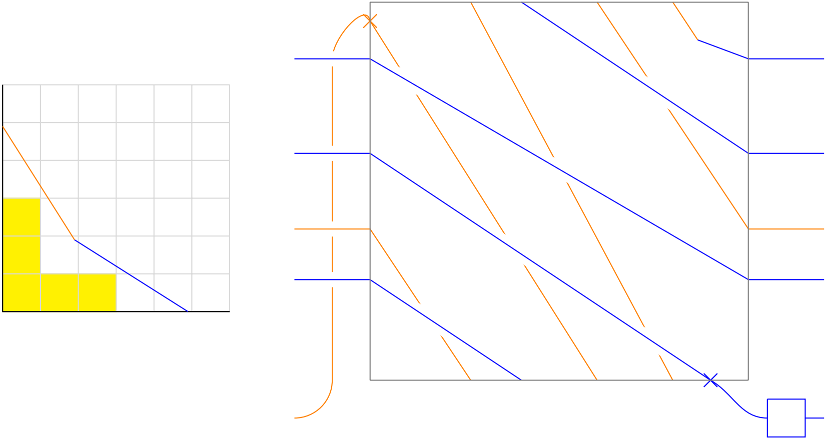

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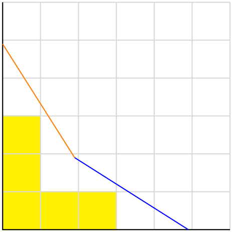

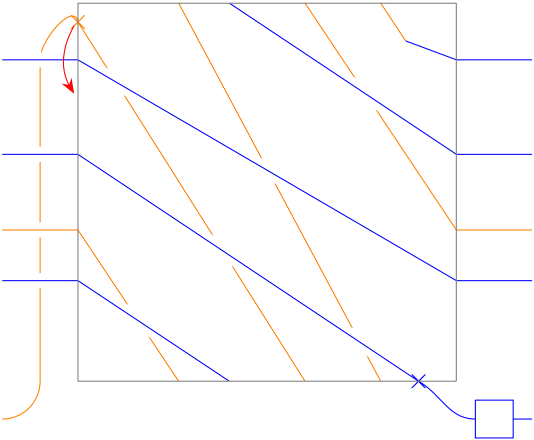

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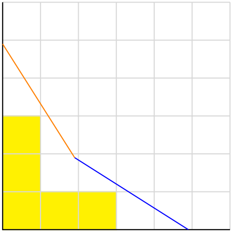

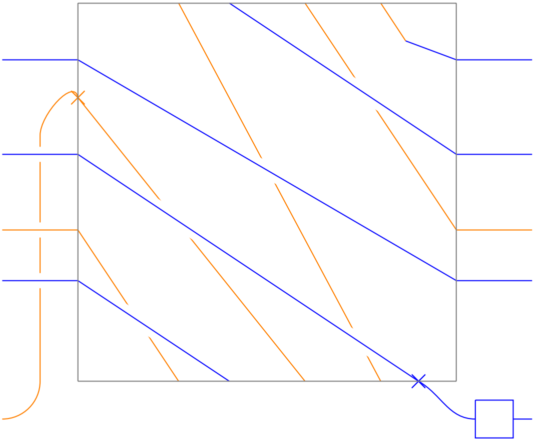

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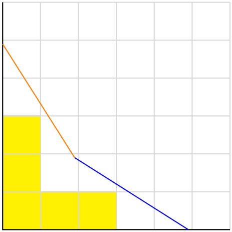

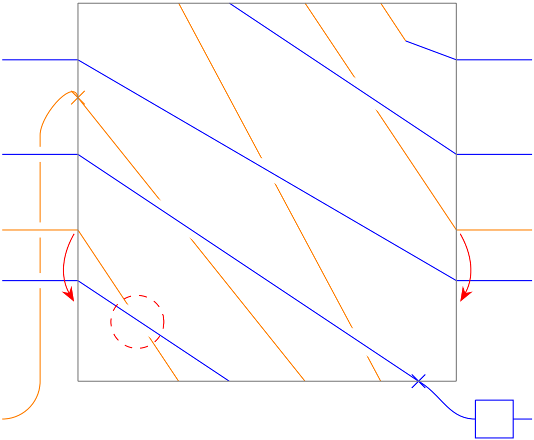

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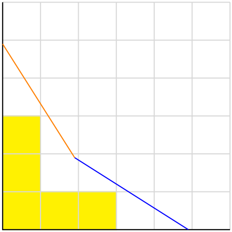

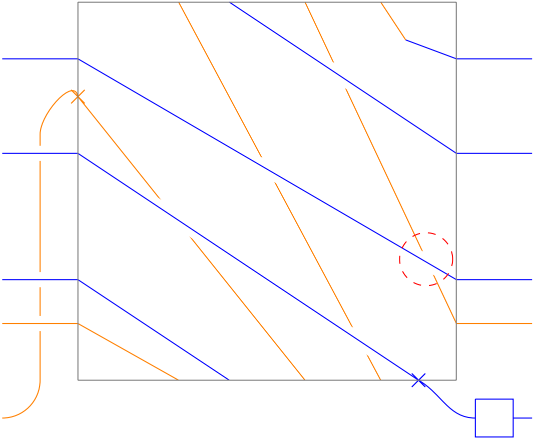

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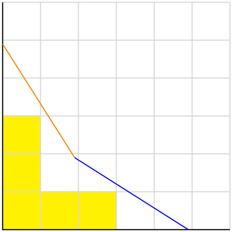

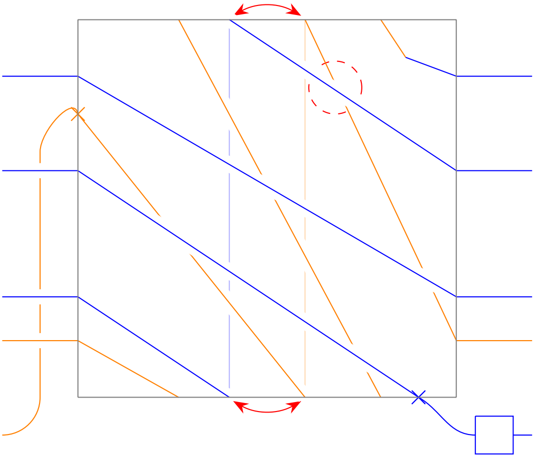

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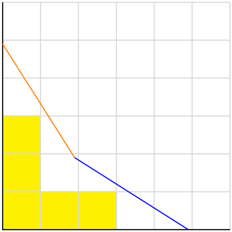

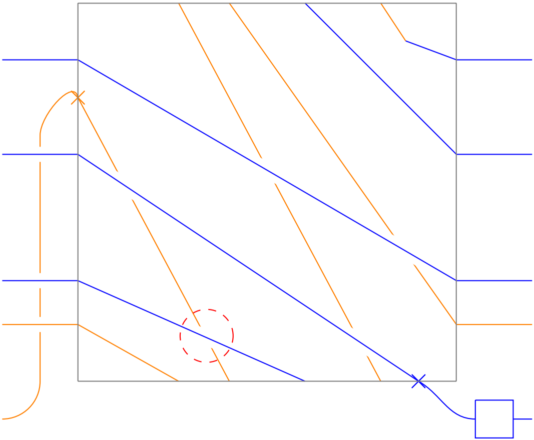

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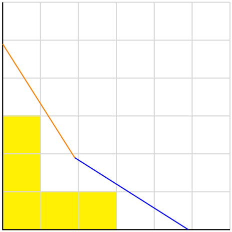

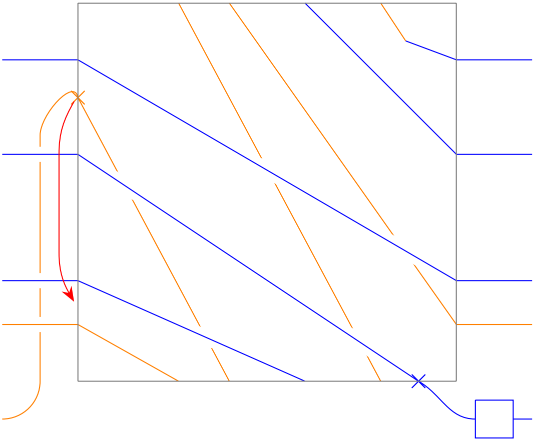

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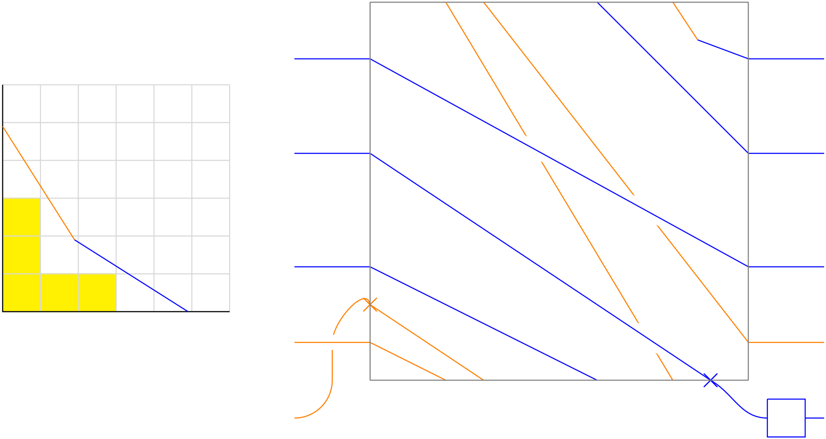

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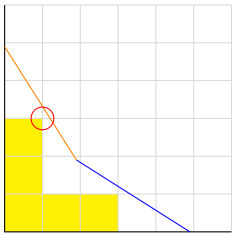

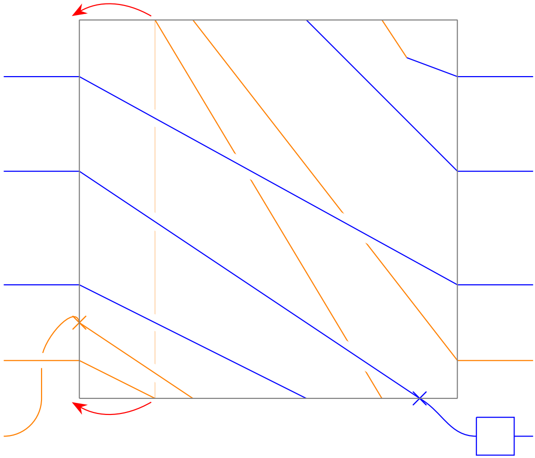

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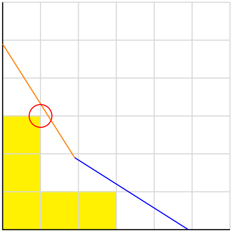

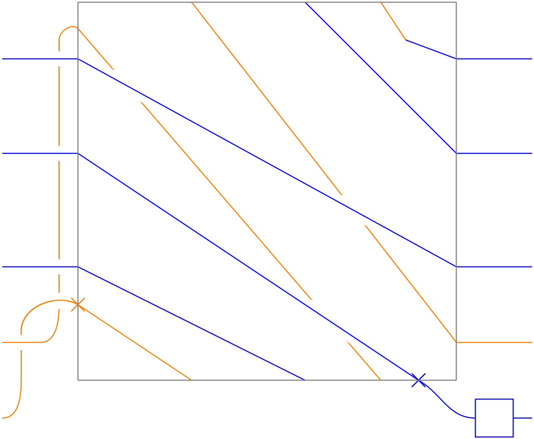

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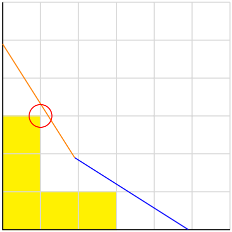
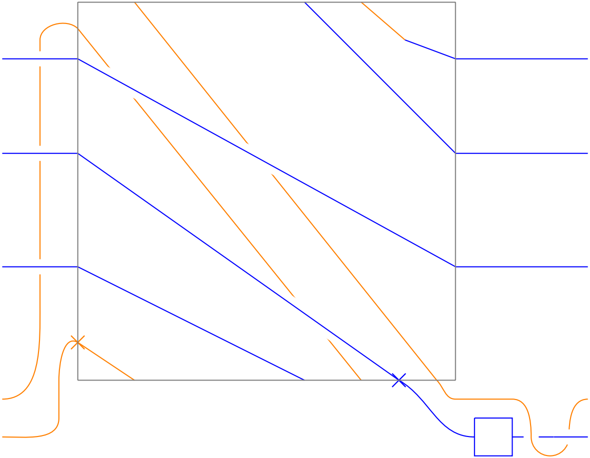
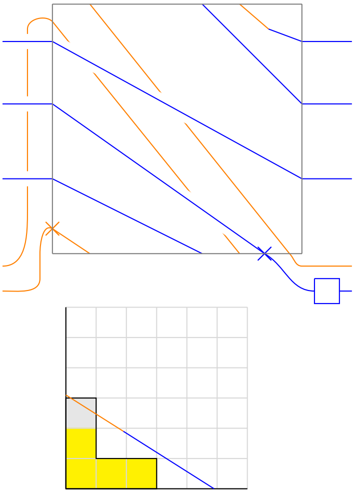
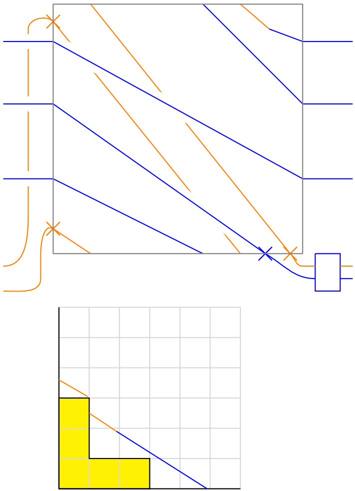




qt^{-1}  $+\ t^{-1}$ 

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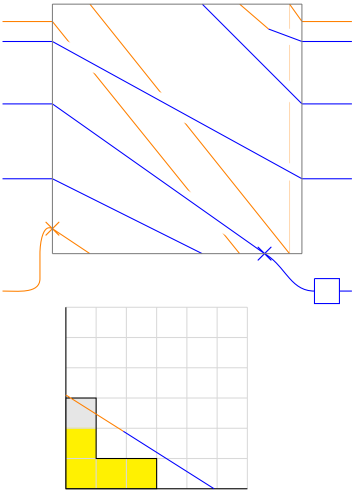

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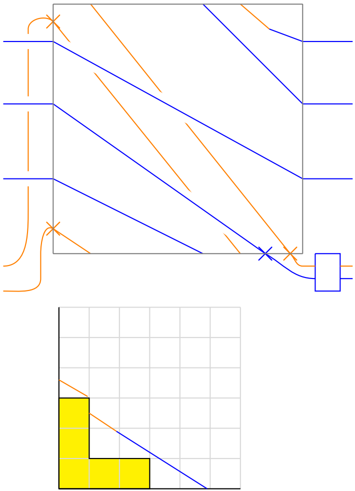
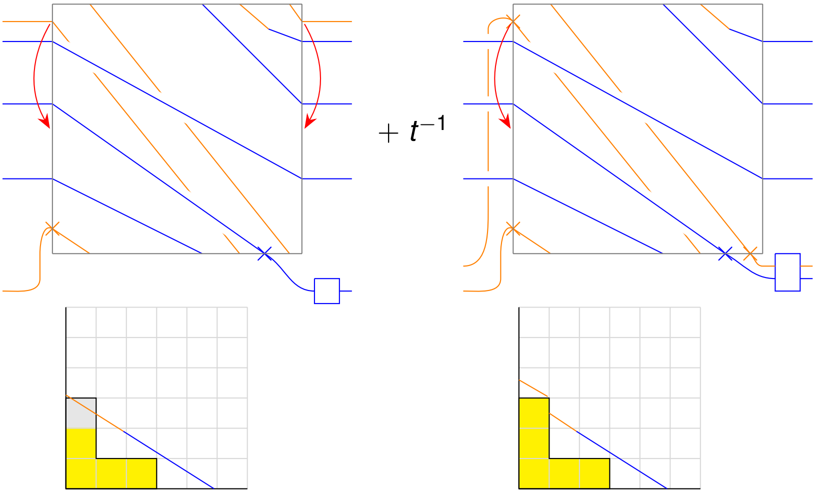


qt^{-1} $+\ t^{-1}$



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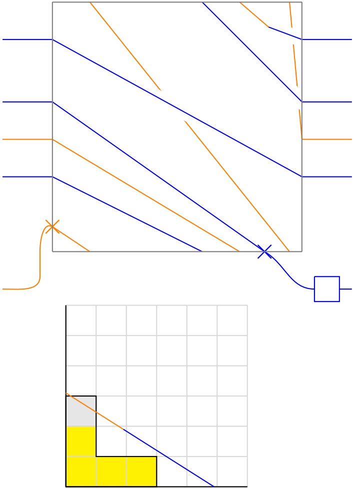

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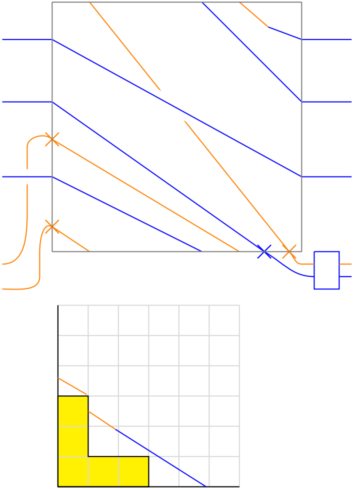

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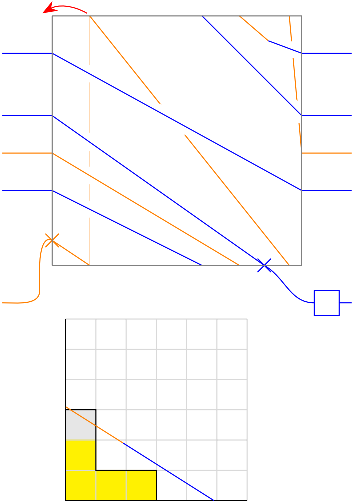

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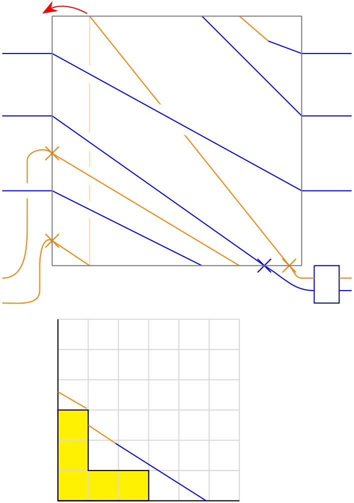

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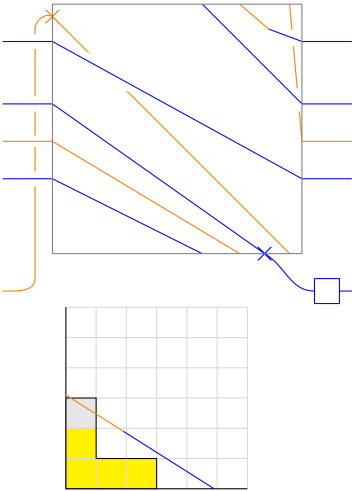

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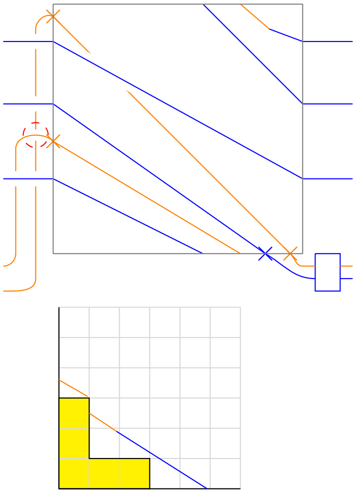

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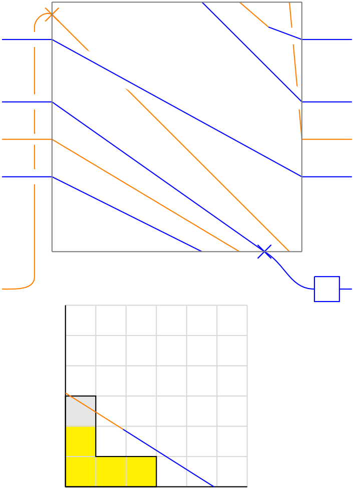

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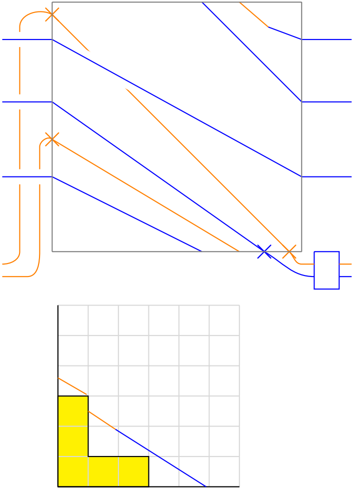

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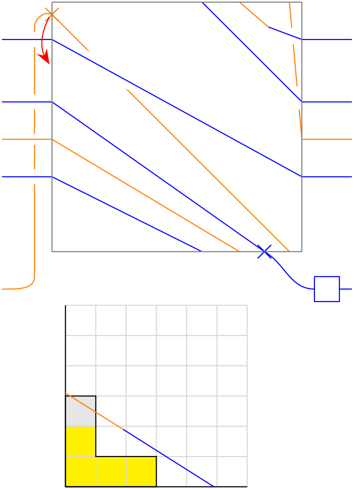

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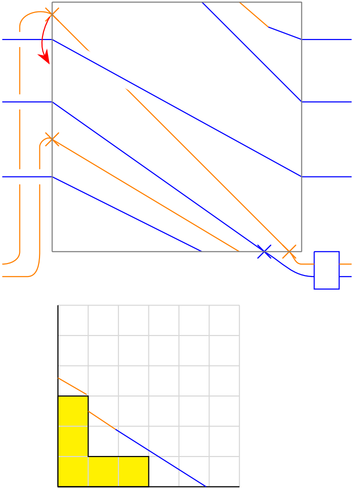

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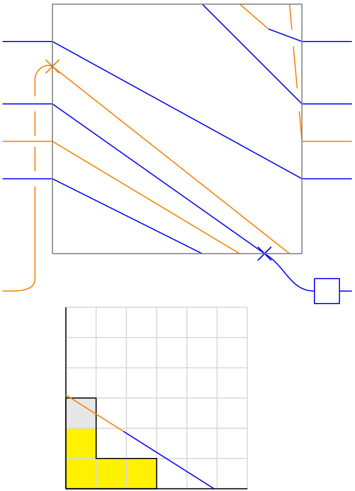

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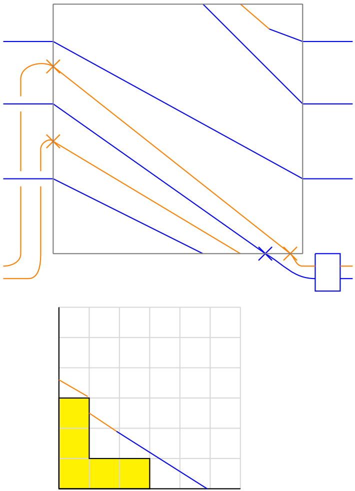

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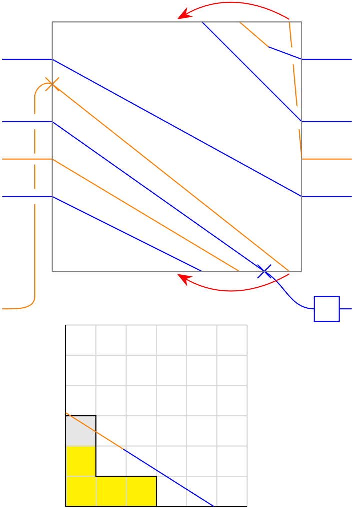

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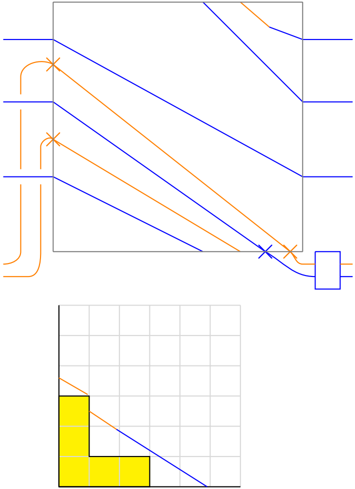

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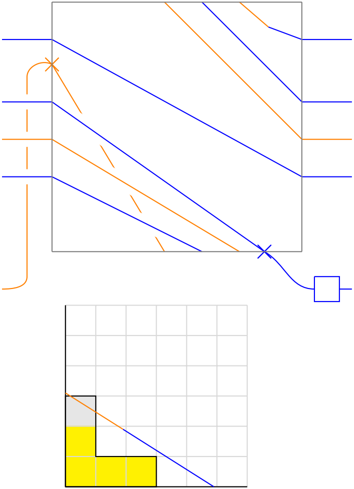

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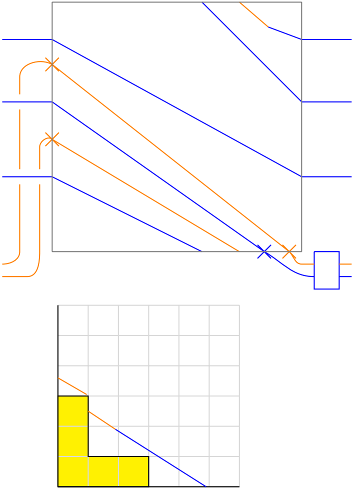

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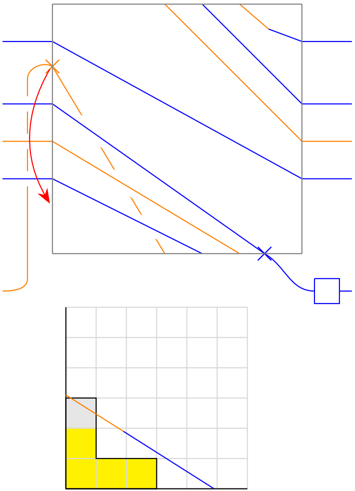

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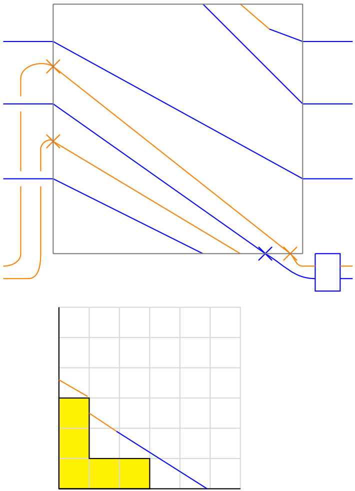

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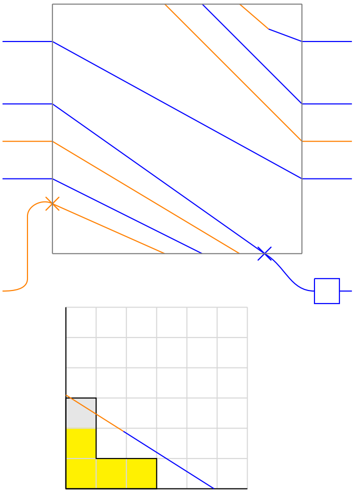

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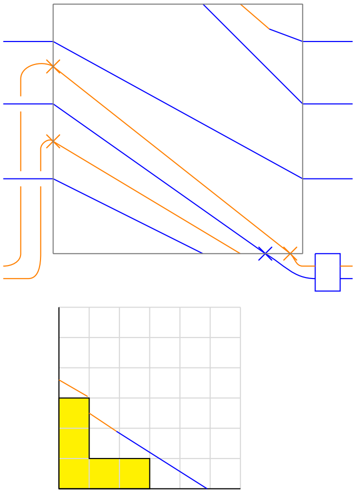

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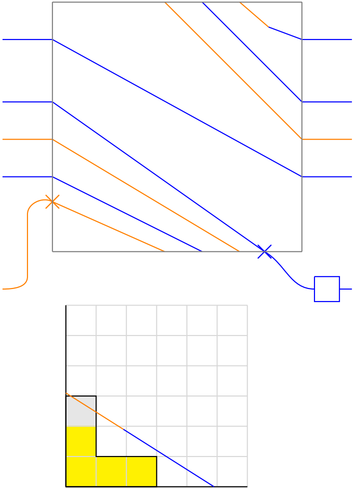

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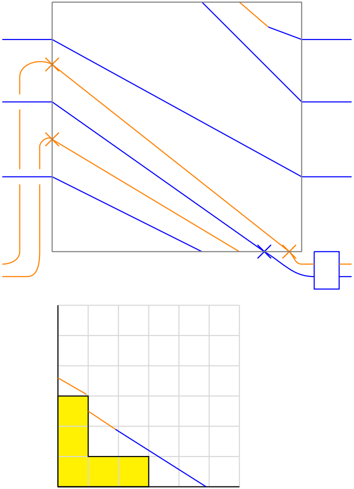
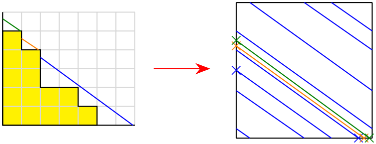
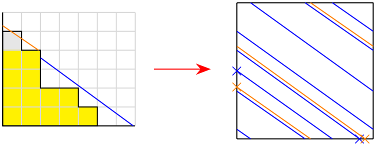


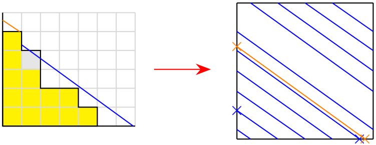
$$qt^{-1}R_{10000,0001} \quad + \quad t^{-1}R_{01010,0011}$$



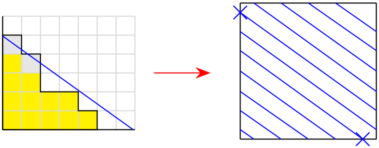
$t^{-3}\,T_{0010110,000111}$



$qt^{-3}\,T_{0011000,000011}$

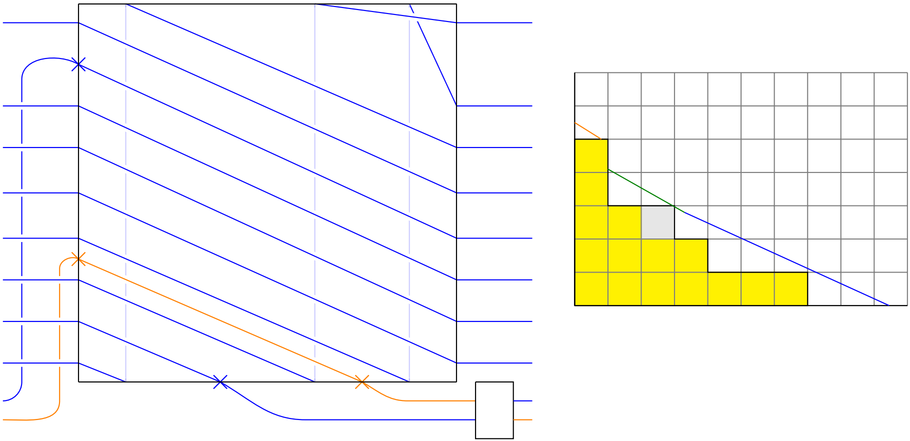


$qt^{-2}\,T_{0100010,000011}$



$q^2t^{-2}\,T_{0000001,00001}$

Troubles



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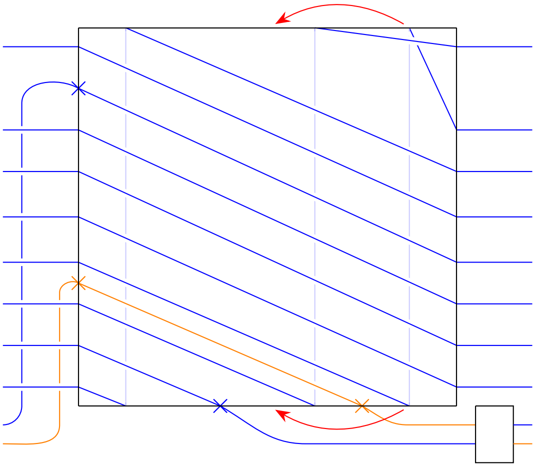

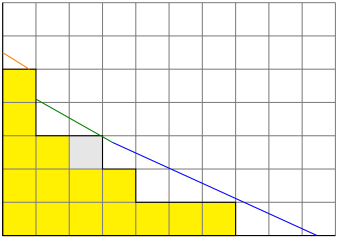

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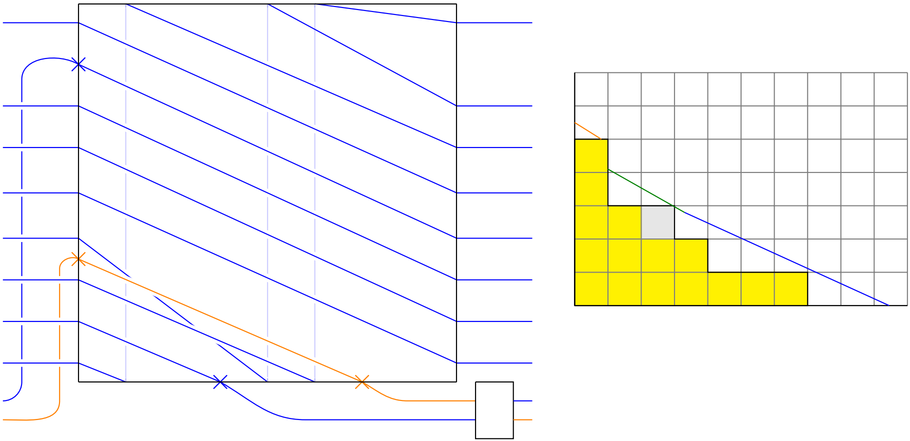

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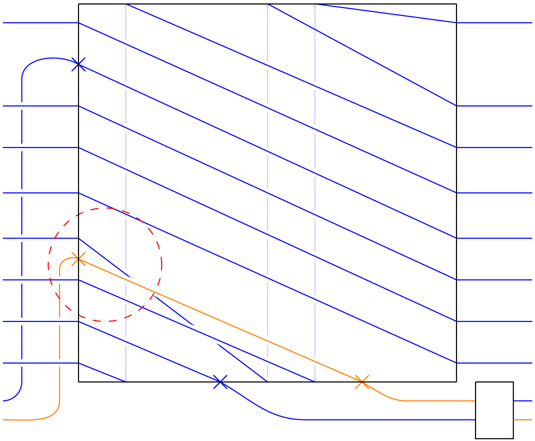

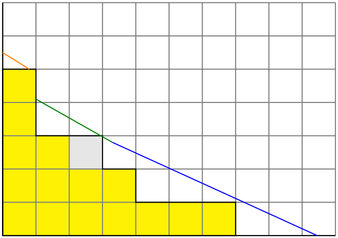

Results

Theorem (Almost)

Let λ be a concave partition such that there is a triangular subpartition $\tau \subset \lambda$ such that the boxes of $\lambda \setminus \tau$ belong to one column (or one row). Then the KR homology of the corresponding monotone knot is parity and can be recursively computed using symmetrizers.

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Same for two consecutive columns/rows.

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Same for two consecutive columns/rows.

Also, it should be relatively easy to obtain explicit area/dinv-type formulas in these cases.

Thank you!