# A Celebration of Paul Dupuis' Work

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## Big Thanks to...

- ICERM
- Co-organizers: Rami Atar and Kavita Ramanan
- All participants and speakers

# A Brief Biography

- B.S. Brown University(1981), M.S. Northwestern University (1982), Ph.D. Brown University (1985).
- Postdoctoral Fellow, IMA, University of Minnesota (1985-86), Visiting Assistant Professor, LCDS, Brown University (1986-1988), Assistant Professor, Department of Mathematics and Statistics, University of Massachusetts (1988-91).
- Division of Applied Math (DAM), Brown University (1991-current).
- Chair of DAM (2005-2011).
- IBM Professor of Applied Mathematics (2012-present).

# Biography(ctd.)

- Director of Lefschetz Center for Dynamical Systems (2013,2016-present).
- Fellow of the IMS, AMS, and SIAM.
- Author of 3 books and over 100 papers.
- Editor-in-Chief, *Applied Mathematics and Optimization*, 2003-2014.
- AE for *J. Theor. Prob.*, *ESAIM: Math. Model. and Num. Anal.*, *Bernoulli*, *Ann. App. Prob.*, *Stoch. Anal. App.*, *Ann. Prob.*, *Stoch. Proc. and App.*, *SIAM J. Cont. and Opt.*

# Ph.D. Students (18)

- Yixiang Mao (Harvard Dept. of Math.), completed 2020.
- Guo-Jhen Wu, completed 2019.
- Michael Snarski, completed December 2018.
- Wei Wu (co-advised with Kavita Ramanan), completed 2014.
- Dane Johnson, completed 2014.
- Yufei Liu, completed 2013.
- Yi Cai, completed 2012.
- Kenny Chowdhary (co-advised with Jan Hesthaven), 2012.
- Kevin Leder (co-advised with Hui Wang), completed 2008.
- Tom Dean, completed, 2007.
- Devin Sezer, completed 2006.
- Jim Zhang, completed 2005.
- John Curran, completed 2004.
- Hui-Ming Pai, completed 2003.
- Tao Pang (main supervisor Wendell Fleming), completed 2001.
- Adam Szpiro, completed 1999.
- Michelle Boue, completed 1997.
- Kavita Ramanan, completed 1997.

# Postdoctoral Fellows

- Rami Atar
- Amarjit Budhiraja (co-mentor, Harold Kushner)
- Arnab Ganguly (co-mentor, Kavita Ramanan)
- Markus Fischer
- Vaios Laschos (co-mentor, Kavita Ramanan)
- David Lipshutz (co-mentor, Kavita Ramanan)
- Pierre Nyquist
- Konstantinos Spiliopoulos
- Ruoyu Wu (co-mentor, Kavita Ramanan)
- Xiang Zhou

## Research Highlights

- Numerical Methods for Hamilton-Jacobi-Bellman Equations.
- Skorohod Problem, Constrained Diffusions, Queuing Networks.
- Accelerated Simulation Methods for Rare Events.
- Risk Sensitive and Robust Control and Optimization.
- Asymptotic Theory for Stochastic Approximation Schemes.
- Theory of Large Deviations.
- Other influential works.

# Numerical Methods for Hamilton-Jacobi-Bellman Equations.

- Developed in collaboration with Harold Kushner.

- Led to a book:
  - *Numerical Methods for Stochastic Control Problems in Continuous Time* (with H. J. Kushner), Second Revised Edition, Springer-Verlag, New York, 2001.

  - treats stochastic control problems for jump-diffusions in many settings: discounted, exit from a given set, optimal stopping, finite time, and ergodic cost. Allows for impulse and singular control.

  - computational schemes and convergence results for the Markov chain approximation method.

# Numerical Methods for Hamilton-Jacobi-Bellman Equations.

- Later works with Boué, Szpiro, Oliensis: Numerical methods for deterministic control.
- Boué and Dupuis (1999) consider a setting with affine control and quadratic cost.
  - Arise in calculus of variations problems, computer vision, large deviations, robust control and filtering.
  - Provide a Markov chain approximation method that is convergent, easily implementable, avoids self-transitions, and minimizes numerical diffusion.
  - Now known as the *Fast Sweeping Method* and has been very influential in Numerical Analysis.

# Skorohod Problem, Constrained Diffusions, and Queuing Systems.

- A one dimensional reflected Brownian motion (RBM) in $\mathbb{R}_+$ is a basic object that arisies in many settings.
- Such a process can be constructed by evaluating a 'path map' $\Gamma$ to a one dimensional Brownian motion.
- The map $\Gamma$ takes a càdlàg trajectory in $\mathbb{R}$ to a càdlàg trajectory in $\mathbb{R}_+$ and is known as the Skorohod map (Skorohod, 1961).
- Higher dimensional RBM also arise in many applications.

## Skorohod Problem, Constrained Diffusions, and Queuing Systems.

- E.g. in queuing theory one is ineterested in RBM in $\mathbb{R}^n_+$ with a given set of reflection directions at the $n$-faces: $\{d_i\}_{i=1}^n$.
- A basic result by Harrison and Reiman (1981) gives sufficient conditions on $\{d_i\}_{i=1}^n$ for wellposedness of the Skorohod problem.
- In a sequence of papers, Dupuis and Ishii (1991), Dupuis and Ramanan (1999a, 1999b), the sufficient condition is relaxed in a significant manner.
- Leads to a rich class of new queuing models that can be analyzed (e.g. the processor sharing model).

# Skorohod Problem, Constrained Diffusions, and Queuing Systems.

- Building on Dupuis and Ishii (1991), Dupuis and Nagurney (1993) developed a general framework of Projected Dynamical Systems.
    - Provides a dynamical interpretation to a class of variational inequalities.
    - Arise in economics, financial modeling, optimization, and operations research problems.
- Dupuis and Williams (1994), constructed Lyapunov functions to study ergodicity of semimartingale reflected Brownian motion.
    - Analogous Lyapunov constructions were used in many later papers to study stability and geometric ergodicity of constrained jump-diffusions.

## Skorohod Problem, Constrained Diffusions, and Queuing Systems.

- Dupuis and Ishii (1993) give a general theory for SDE with oblique reflections on piecewise smooth domains.
  - These results have found use in heavy traffic limit theory for stochastic networks.
- Dupuis and Ishii (1990, 1991(b)): Existence and uniqueness of viscosity solutions of fully nonlinear (degenerate) elliptic PDE with derivative BC in nonsmooth domains (with corners).
- Large deviation problems for constrained processes: Dupuis and Ramanan (1998a, 1998b, 2002), Dupuis and Atar (1999), B. and Dupuis (2003).

# Risk Sensitive and Robust Control and Optimization.

- Dupuis, James, and Peterson(2000): Formulate a general approach for robust control using a risk-sensitive cost criteria.
  - Key idea is the representation of a risk-sensitive cost as a supremum of ordinary costs over a family of model uncertainities.
  - Underlying this is the basic convex duality between exponential integrals and relative entropy divergence.
  - Developed extensively in later works: Boué and Dupuis(2001), Atar and Dupuis (2002), Atar, Dupuis and Shwartz (2003, 2004), Dupuis, Katsoulakis, Pantazis and Pléchăc (2016).
  - The framework adopted by economists Hansen and Sargent (Nobel prize winners) in their book *Robustness*.

# Risk Sensitive and Robust Control and Optimization.

- Novel information theoretic bounds on rare events and risk-sensitive cost, via Rényi family of relative entropies, developed in:
  Atar, Chowdhary, and Dupuis (2015); Dupuis, Katsoulakis, Pantazis and Rey-Bellet (2020)

- This approach used for:
  - robust LD bounds in queueing contexts: Atar, B., Dupuis, and Wu (2021)
  - neural network estimators: Birrell, Dupuis, Katsoulakis, Rey-Bellet and Wang (2021).

# Risk Sensitive and Robust Control and Optimization.

- A variational formula for Rényi divergences á la the Donsker-Varadhan formula for the KL divergence:
  Birrell, Dupuis, Katsoulakis, Rey-Bellet and Wang (2020).

- Interpolation between Rényi divergences and integral probability metrics, referred to as $(f, \Gamma)$-divergences, that are able to compare measures that are not absolutely continuous (e.g. when empirical measures are involved):
  Birrell, Dupuis, Katsoulakis, Pantazis and Rey-Bellet (2022).

# Accelerated Monte-Carlo and Subsolutions of HJB Equations.

- Importance sampling and particle splitting schemes for rare event probability estimation.
- Dupuis and Wang (2004, 2007) study importance sampling schemes using ideas of small noise stochastic differential games (SDG).
- The Issacs equation for the SDG turns out to be equivalen to a HJB equation for a control problem.
- Asymptotic optimality of an importance sampling scheme corresponded to a subsolution of the HJB equation.
- Simple form subsolutions can be constructed in many settings. Many later papers developed this theme: Dupuis, Sezer and Wang (2007), Dupuis, Leder and Wang (2007a, 2007b, 2009), Dupuis and Wang(2009), Dupuis, Spiliopoulos, and Wang (2012), Dupuis, Spiliopoulos, and Zhou (2015), Dupuis and Johnson(2017).

# Accelerated Monte-Carlo and Subsolutions of HJB Equations.

- The use of subsolutions of HJB equations was further developed for constructing particle splitting schemes for estimating rare event probabilities.
- This idea was first introduced in Dean and Dupuis (2009) and then further developed in Dean and Dupuis (2011), Dupuis, Kaynar, Ridder, Rubinstein and Vaisman (2012), Dupuis, Buijsrogge, and Snarski (2020).

# Parallel Tempering and Infinite Swapping.

- With Chemistry professor Doll, studied performance of parallel tempering Monte Carlo for large scale physical science problems.
- Using analysis of empirical measure large deviation results for Markov processes made a striking observation:
  - it is optimal for particles to swap at rate $\infty$ !
- Led to the development of the infinite swapping algorithm: Dupuis, Liu, Plattner, and Doll (2012).
- Further developed in Dupuis, Plattner, Doll, Wang, Liu and Gubernatis (2011), Doll, Dupuis, Plattner, Freeman and Liu (2012), Dupuis and Doll (2015), Dupuis, Doll, and Nyquist (2018), Dupuis and Wu (2022, 2024).
- Now part of the CHARMM molecular dynamics and simulation package maintained by Harvard.

# Theory of Large Deviations.

- Developed the *Weak Convergence Approach to the Theory of Large Deviation*, together with Richard Ellis in a book by the same title (over 500 citations).
- Starting point: Equivalence between a LDP and a Laplace Principle.
- Key observation: Stochastic control representations for Laplace functionals.
- Main ingrediaent: Convergence of value functions of stochastic control problems to those of certain deterministic control problems.
- The book considered LD problems for a broad range of discrete time stochastic dynamical systems (SDS): Empirical measures of Markov chains, Markov recursive systems with small noise, Markov chains with discontinuous statistics.
- It also had the hints and outlines of a general theory for continuous paramater SDS.

# Theory of Large Deviations.

- The breakthrough for continuous parameter models came in Boué and Dupuis (1998).
- Provided a stochastic control representation for exponential functionals of a finite dimensional Brownian motion- now known as the Boué-Dupuis Formula.
- This later extended to infinite dimensional Brownian motions (B., Dupuis and Maroulas (2008)) and then to infinite dimensional Lévy processes (B., Dupuis and Maroulas (2011)).
- Led to significant research activity in Friedlin-Wentzell theory for small noise SPDE with Gaussian and general Lévy noise.

# Theory of Large Deviations.

- Also led to study of many other types of LD problems for continuous time SDS:
    - Weakly interacting diffusions (B., Dupuis, and Fischer (2012)), Moderate Deviations for jump SDE (B., Dupuis, and Ganguly (2016)), multiscale jump-diffusions (Dupuis and Spiliopoulos(2012), B., Dupuis and Ganguly(2018)), Empirical measures of Markov processes (Dupuis and Lipshutz (2018), Dupuis, Laschos and Ramanan (2020), Dupuis and Wu (2022)), Random graph models (Bhamidi, B., Dupuis, Wu (2022)), Uniform LDP for SPDE and exit time asymptotics (Salins, B. and Dupuis (2019)).

- The book B. and Dupuis (2019) collects many results on LD problems for discrete time and continuous time SDS in finite and infinite dimensions, and also results on accelerated Monte-Carlo algorithms.

# Other Topics.

- Asymptotic analysis of stochastic approximation schemes and stochastic recursive algorithms. (Dupuis and Kushner (1985, 1986, 1987(a,b,c), 1989(a,b))

- Explicit solutions of calculus of variations problems arising from LD analysis of discrete random structures. (Dupuis, Nuzman, Whiting, Zhang (2004, 2006, 2007, 2008(a,b)), Bhamidi, B. Dupuis, Wu(2022)).

- Variational approaches to image matching and MAP estimator interpretation. (Dupuis, Grenander, Miller(1998), B., Dupuis and Maroulas(2010)).

Thanks!