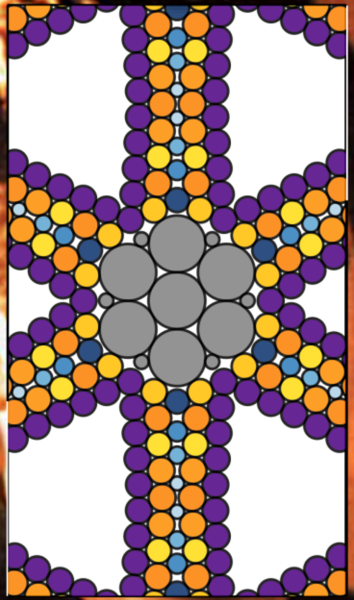
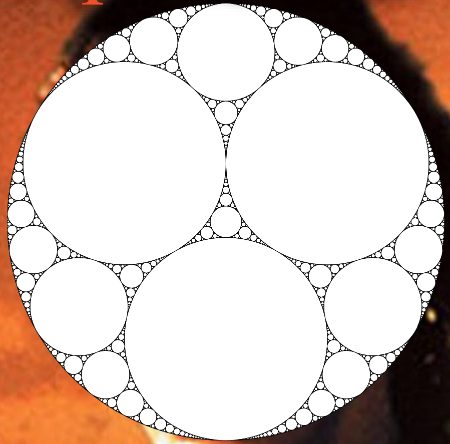
BEST OF THE BEST

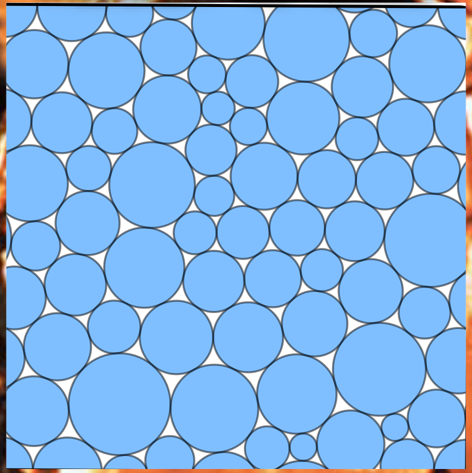
Dionysian



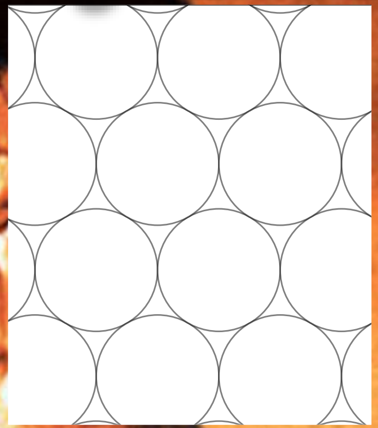
Apollonian



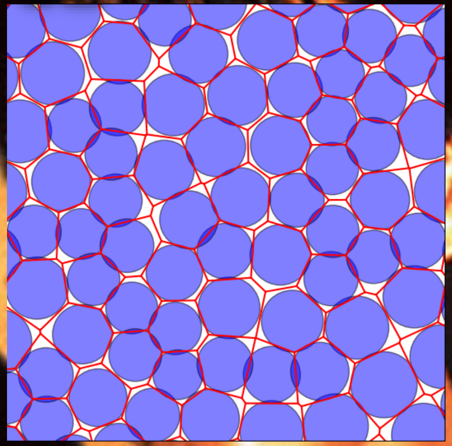
Ideal Glass



Crystalline



Hyperuniform



Eric Corwin

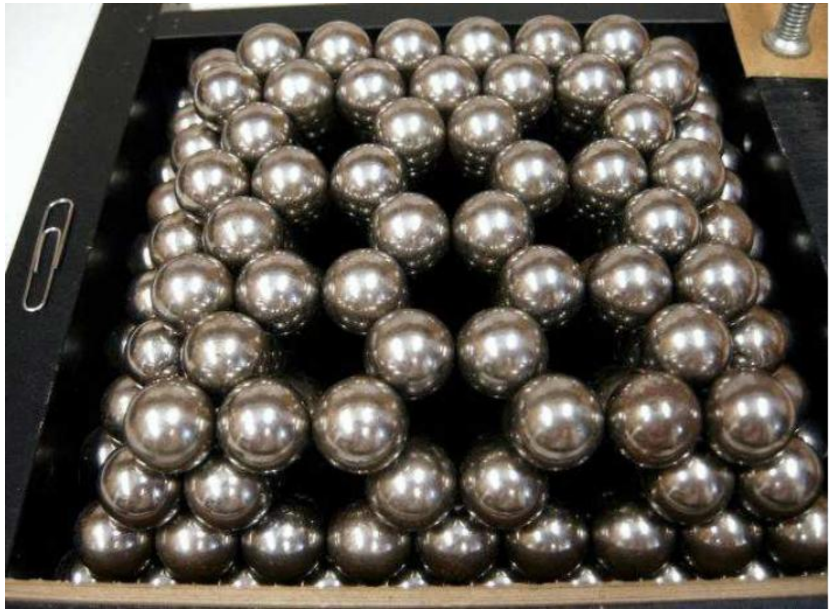
University of Oregon

Part 1, Best ≡ Lowest Density Jammed Packing

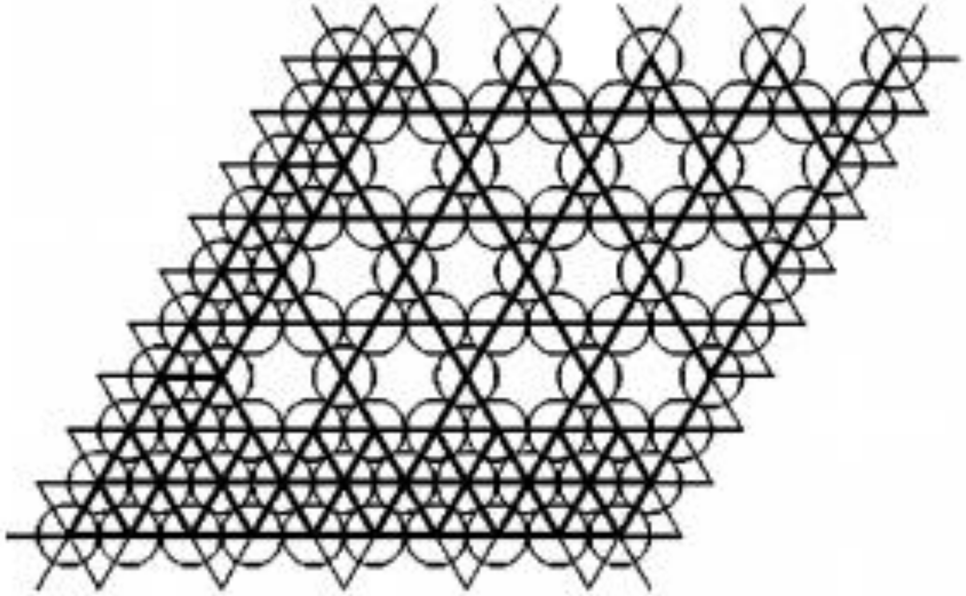
RC Dennis, EI Corwin, PRL (2021)

Tunneled Crystals: Sparsest (Then) Known

Density = 0.494

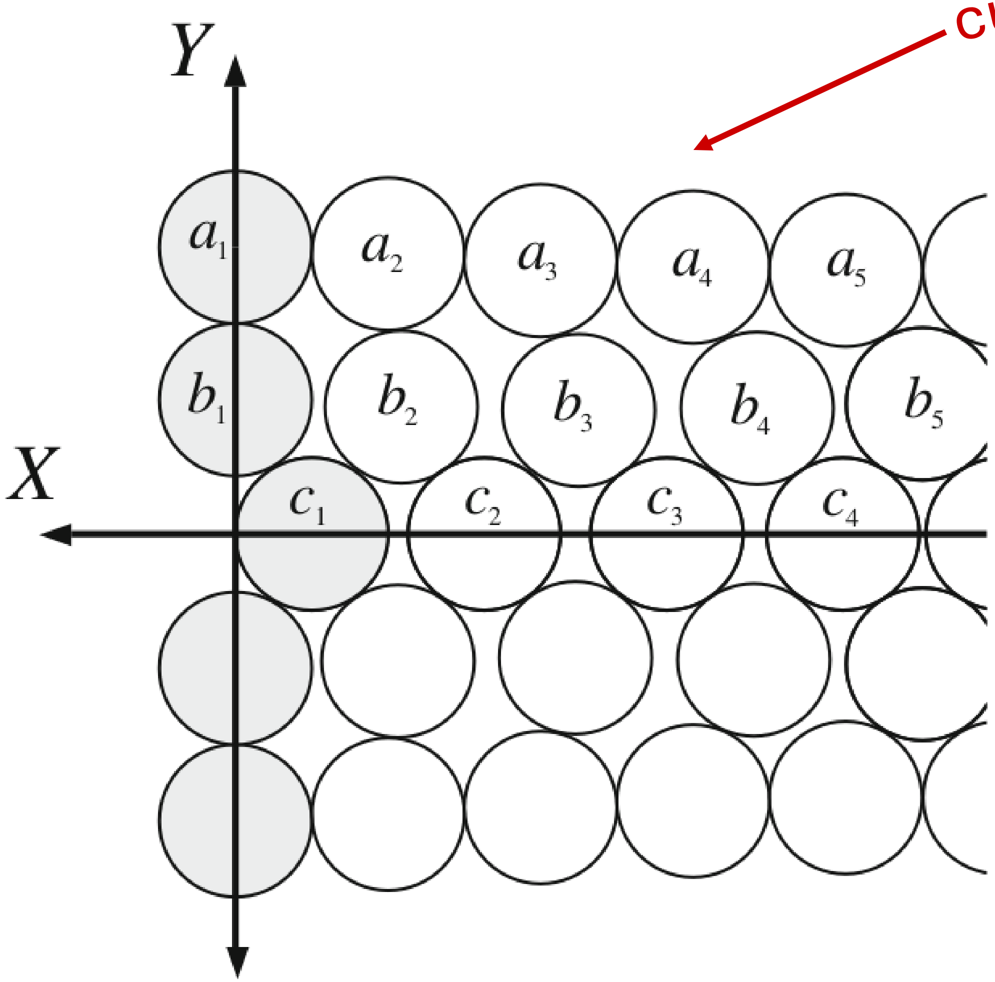


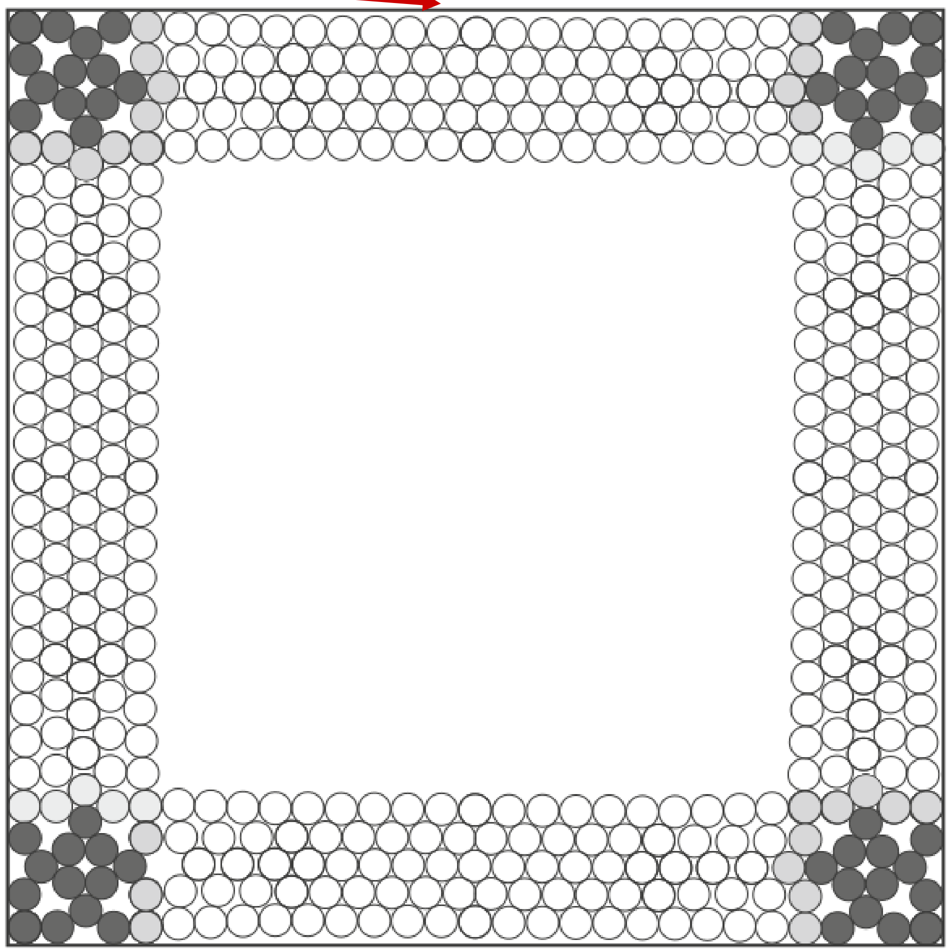
Density = 0.680



S Torquato, FH Stillinger, *Journal of Applied Physics* (2007)

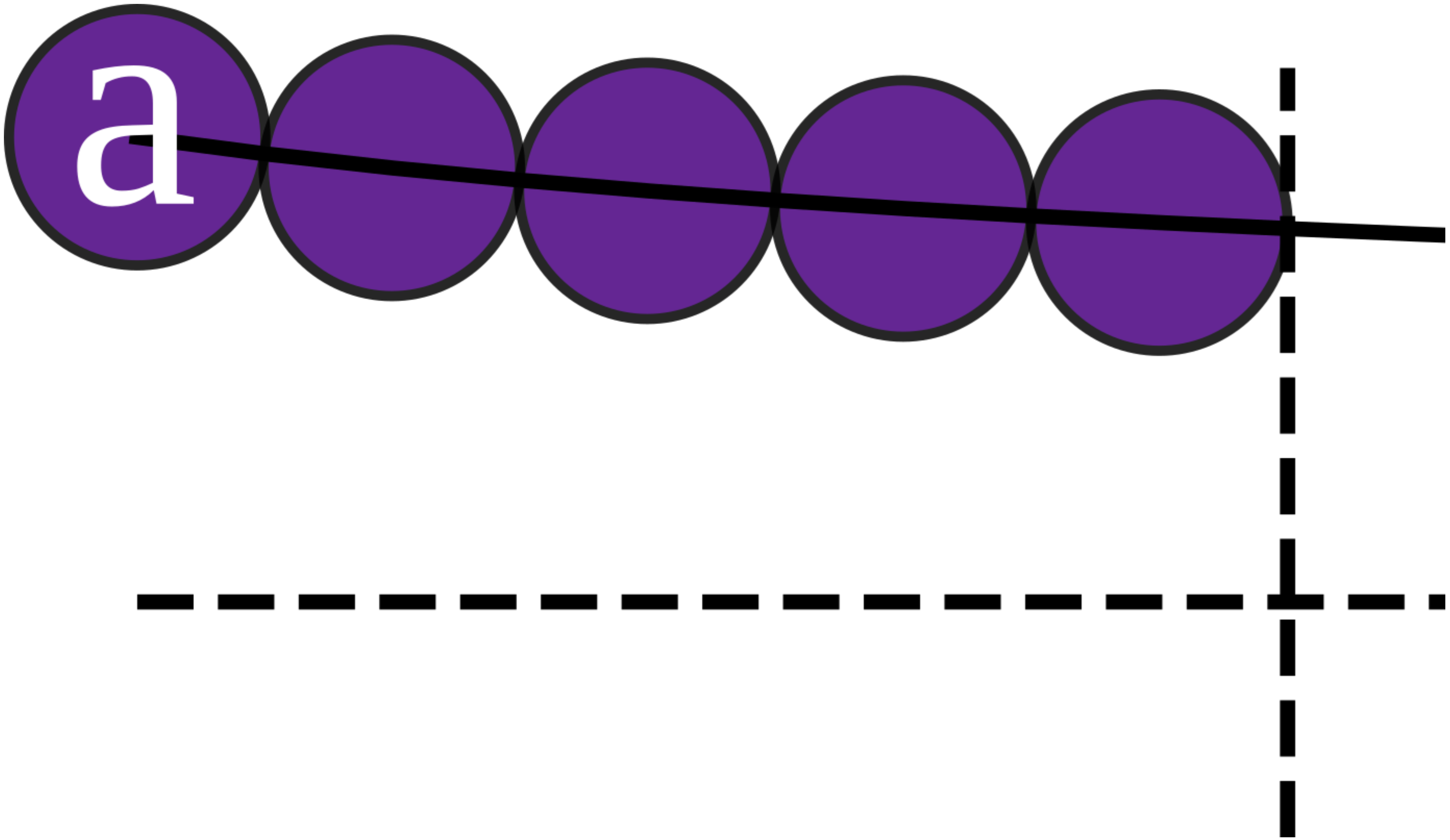
Just Locally Stable: Böröczky's Packing



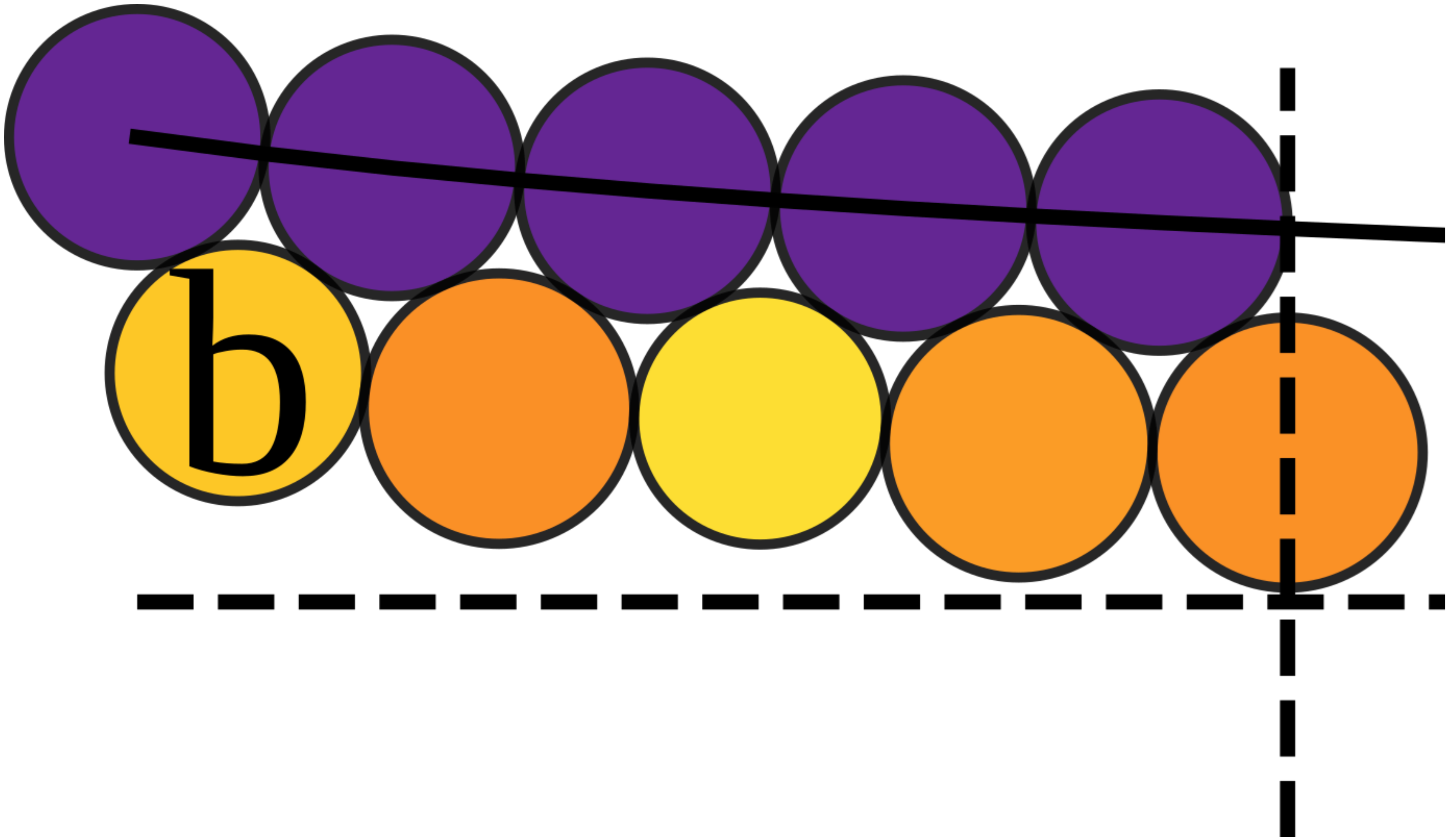


Kahle, Matthew. "Sparse Locally-Jammed Disk Packings." *Annals of Combinatorics* 16 (2012): 773-780.

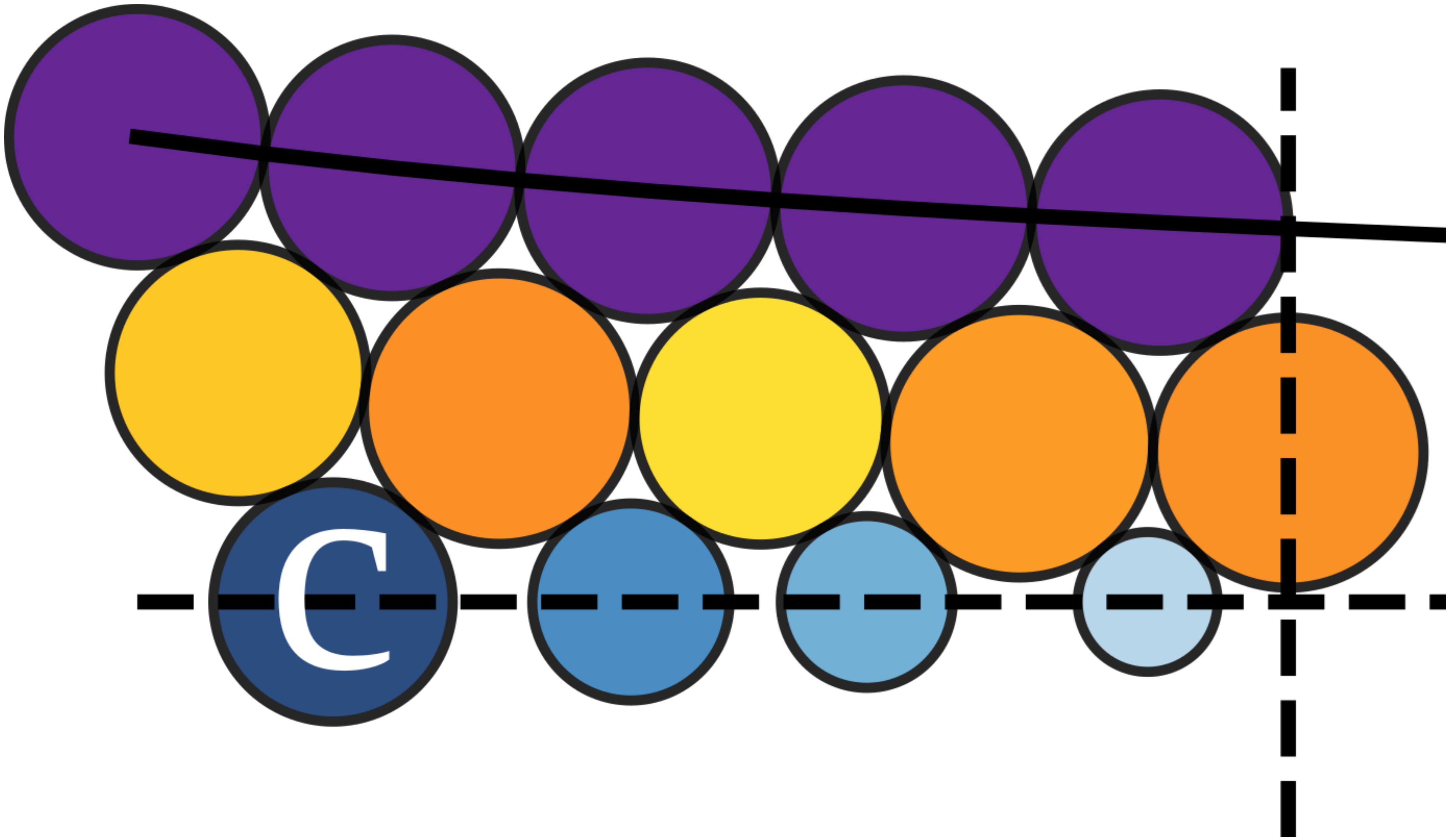
Proposing a Dionysian Packing



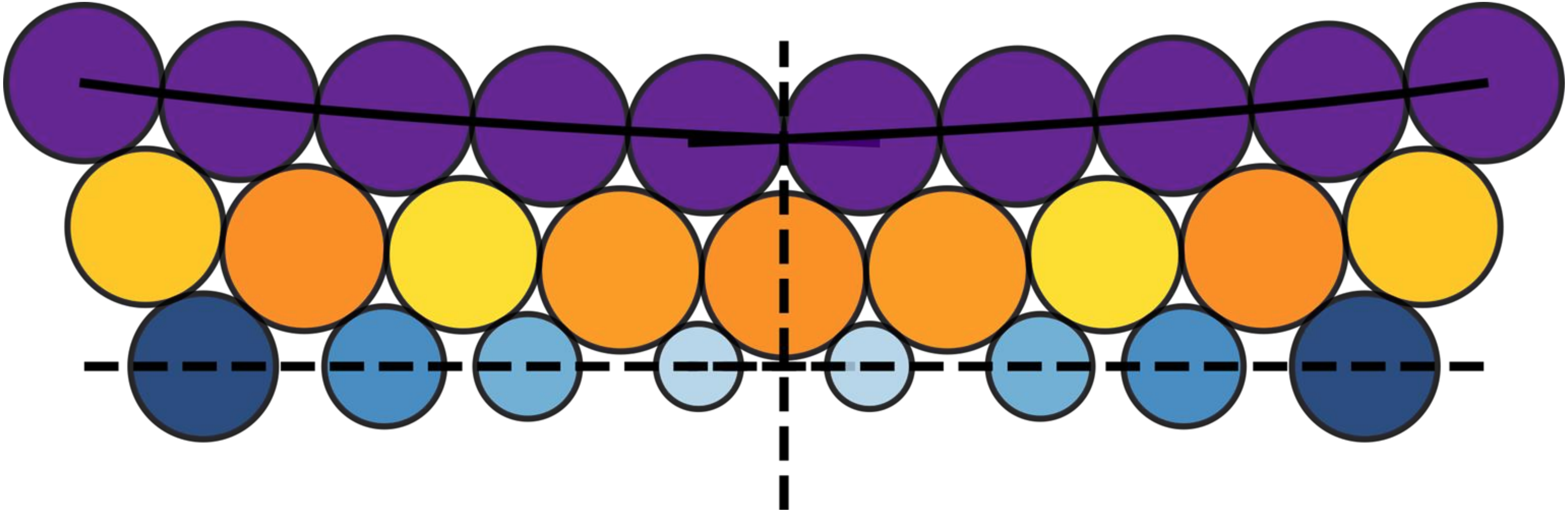
Proposing a Dionysian Packing



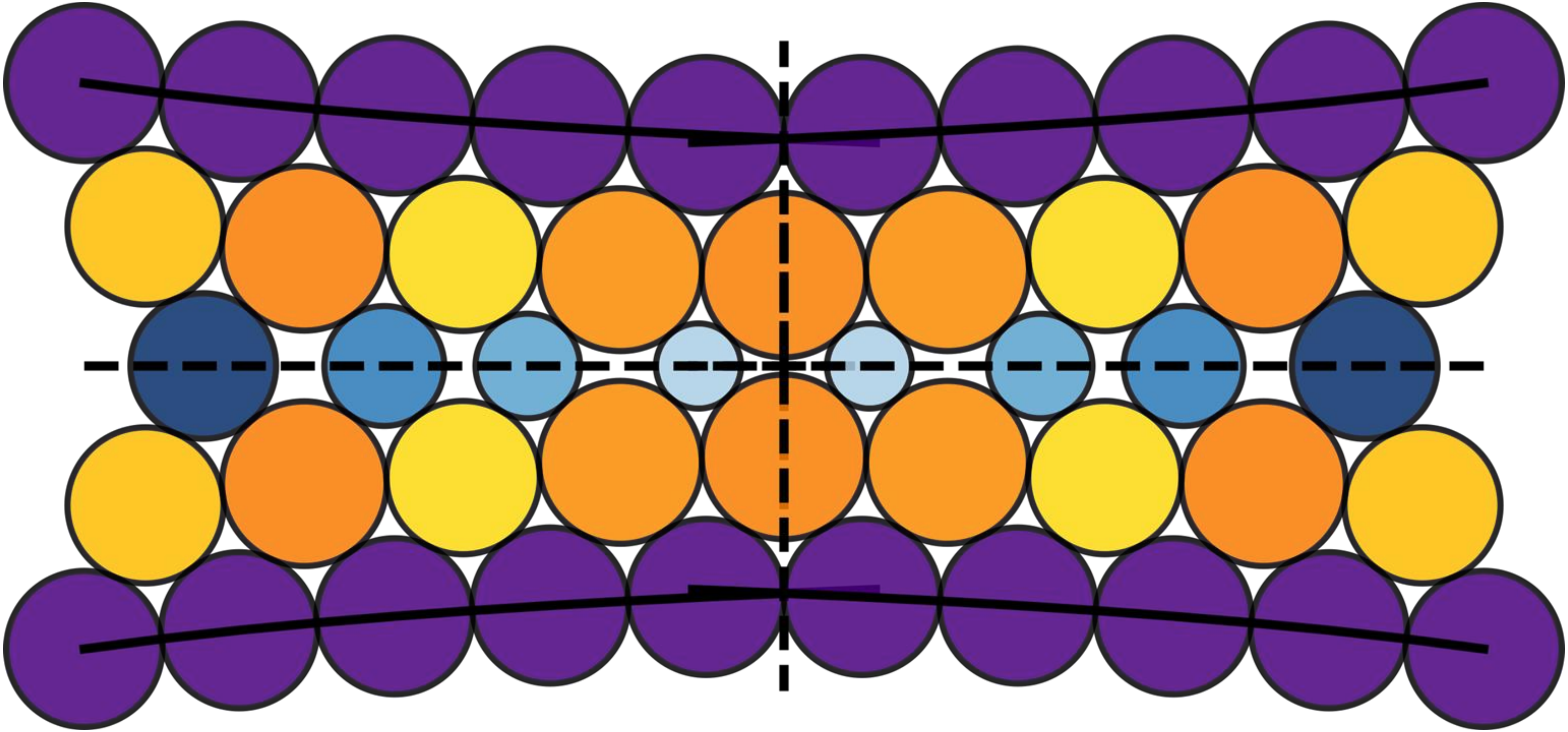
Proposing a Dionysian Packing



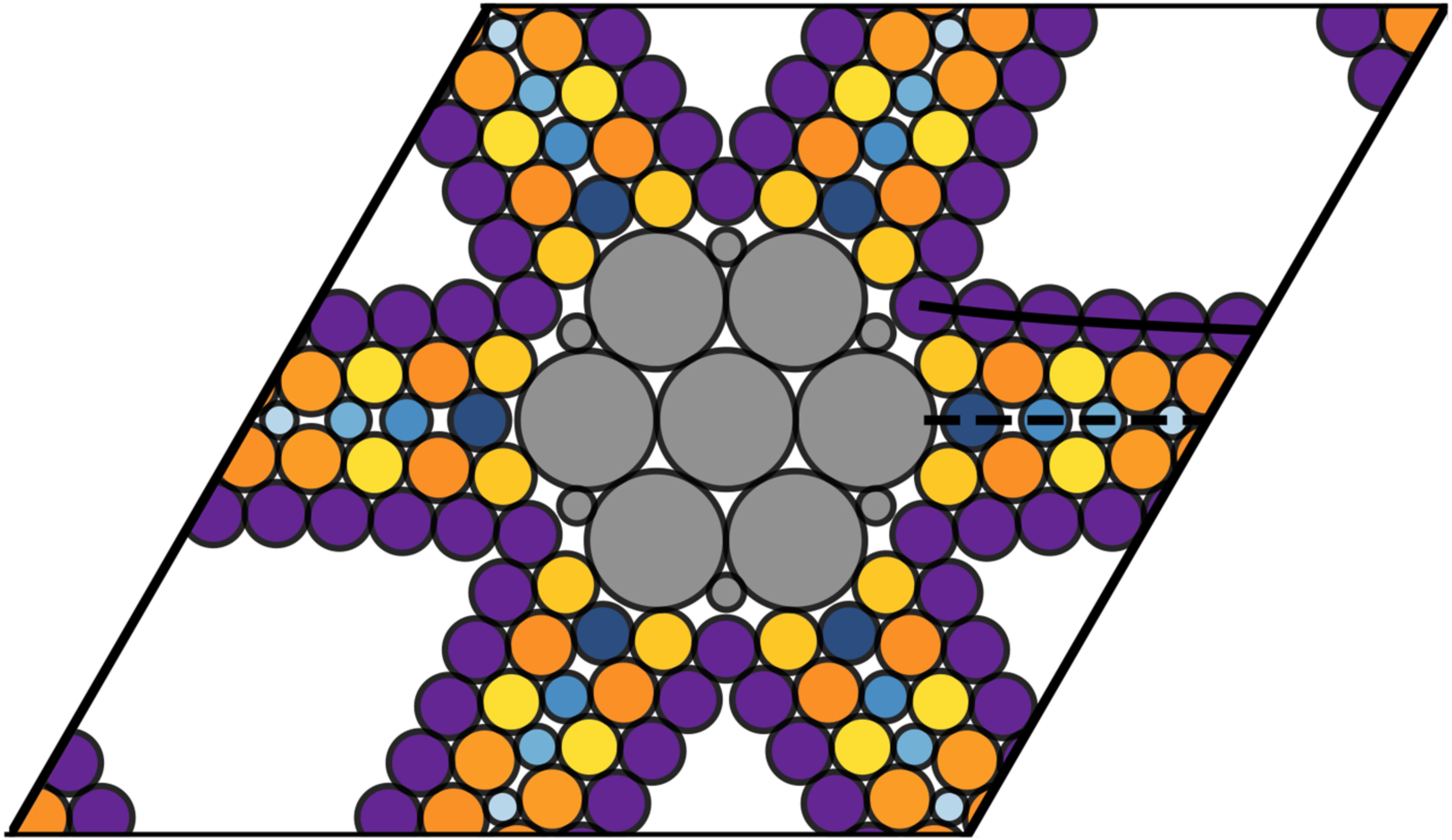
Proposing a Dionysian Packing



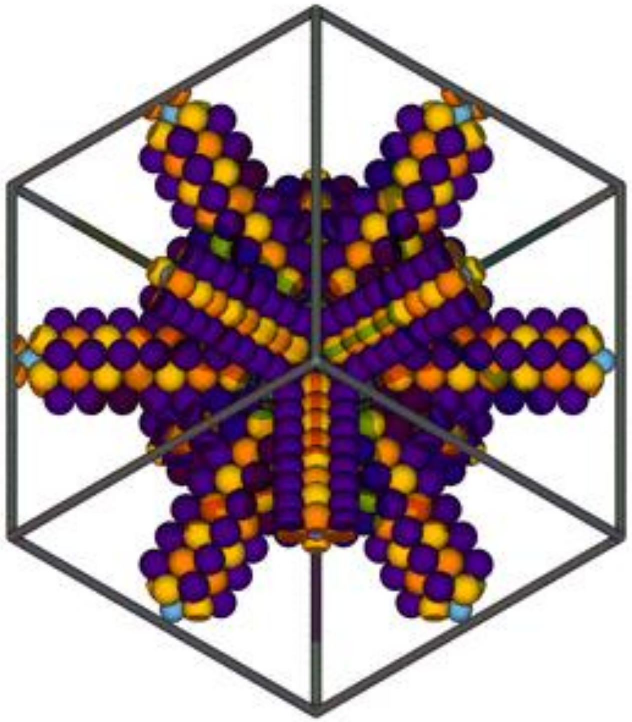
Proposing a Dionysian Packing



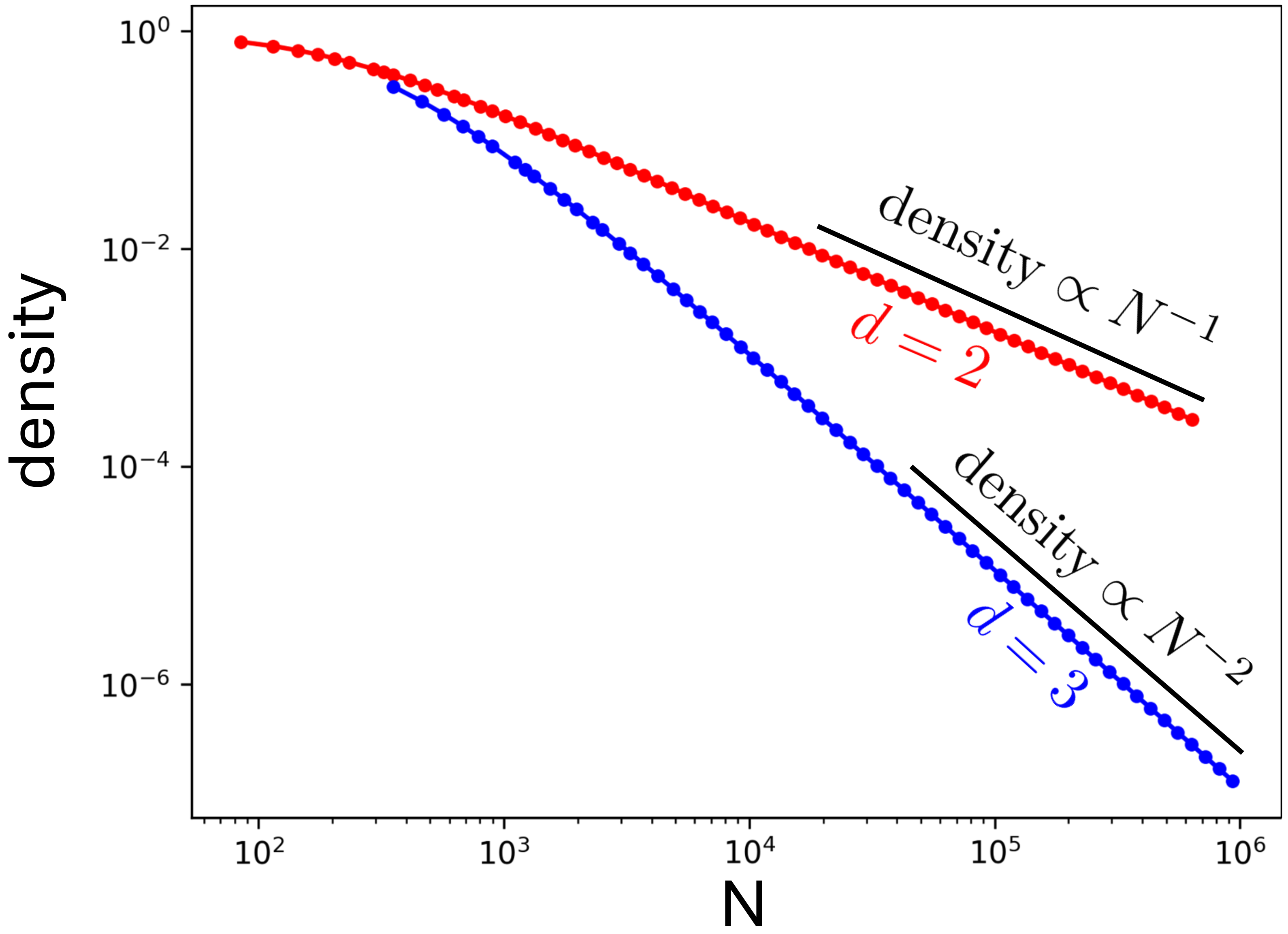
Proposing a Dionysian Packing



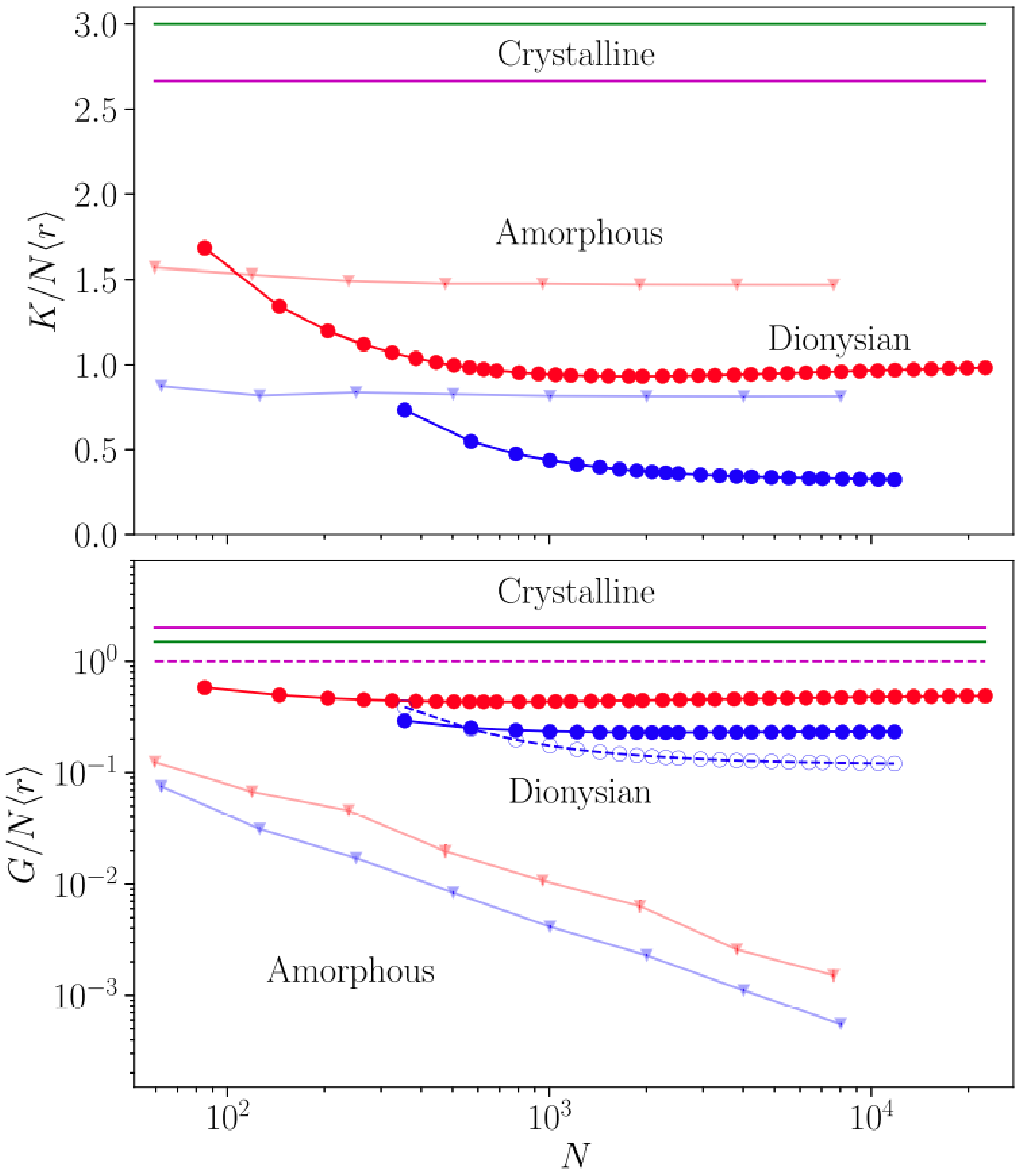
3D Dionysian Packing



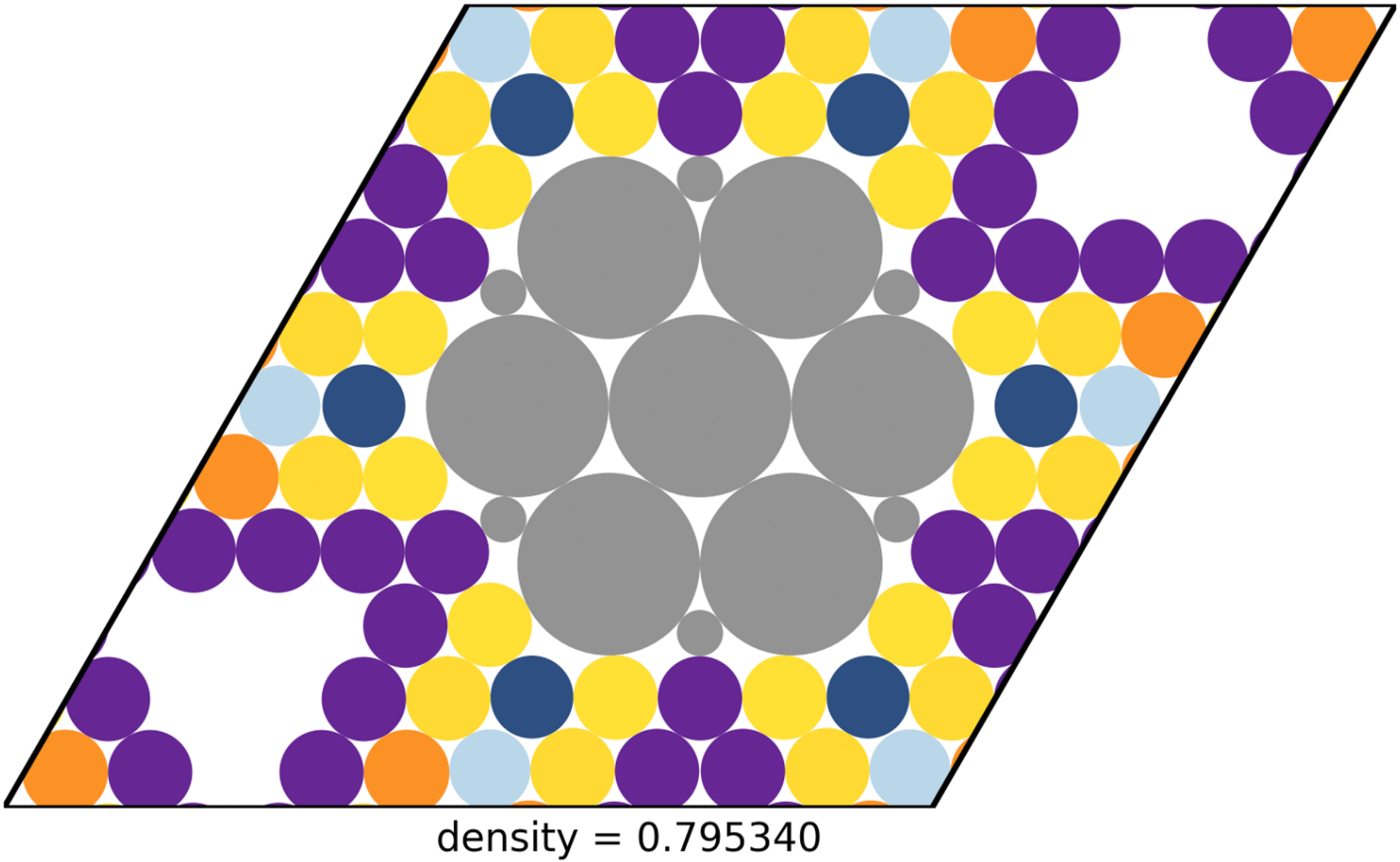
Density as a Function of Particle Number



Mechanically Stable at all N



Dionysian Packing is Best: Stable at Zero Density



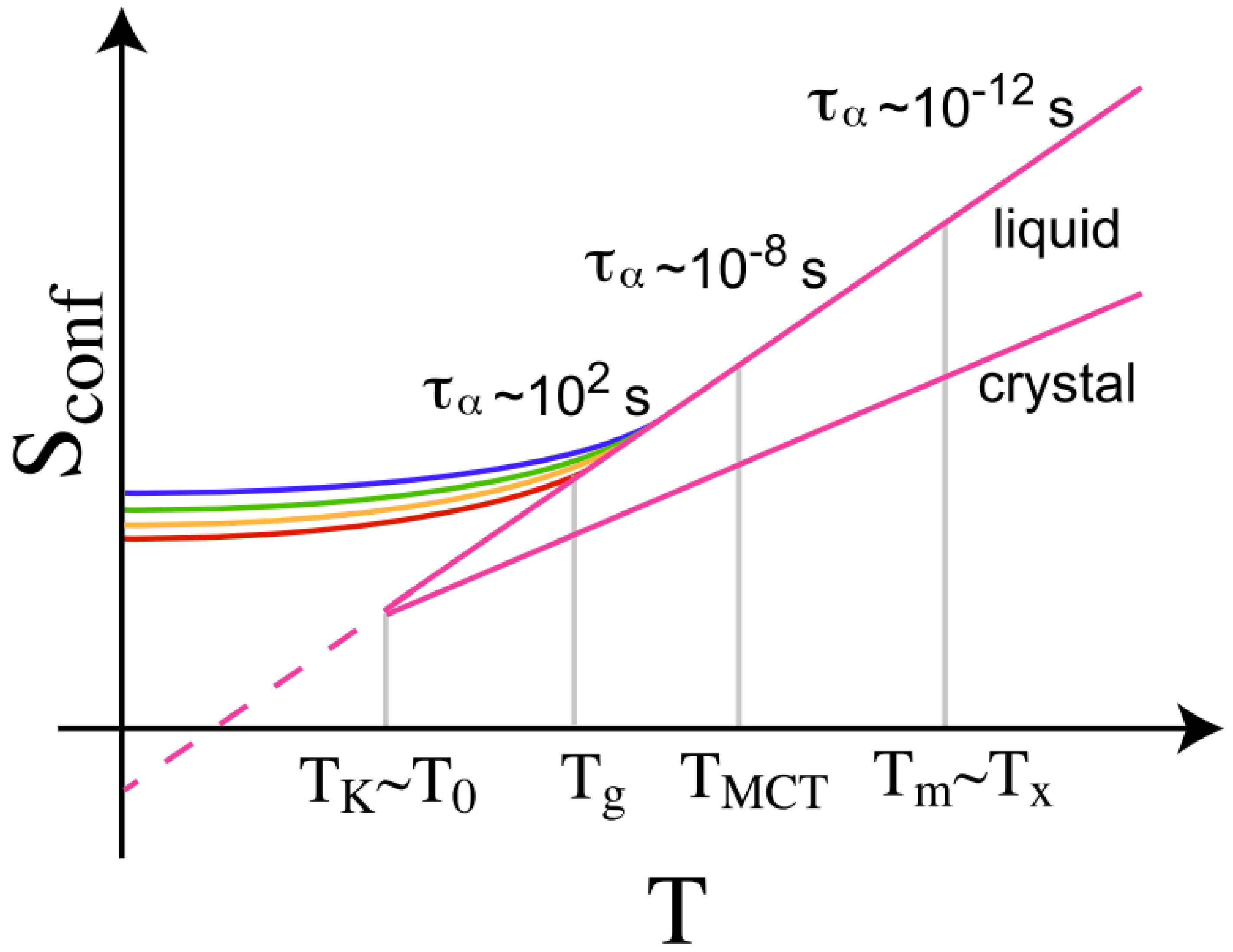
Part 3, Best ≡ Densest Amorphous Packing

V Bolton-Lum, RC Dennis, P Morse, EI Corwin, arXiv:2404.07492 (2024)

Kauzmann Paradox: An Amorphous Crystal

At low temperature

- Volume
- Heat Content
- Specific Heat
- Entropy of the forecasted supercooled liquid is lower than the crystal



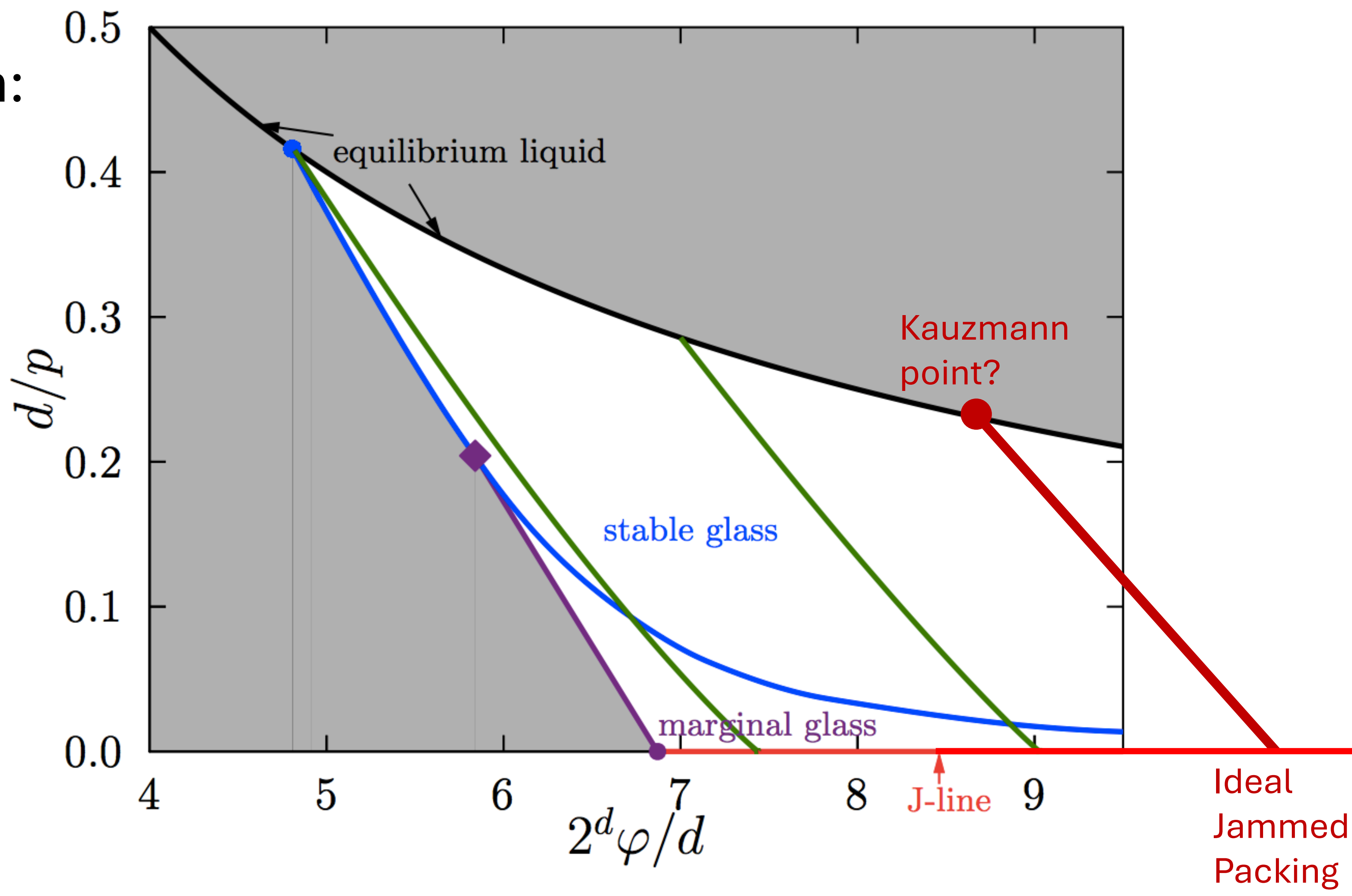
Kauzmann, Chem. Rev. 1948
Royall, et al J. Phys.: Condens. Matt. 2018

But, You Can Never **Ever** Get There

Past dynamical transition:

- Timescales diverge
- Fall out of equilibrium
- Glassy behavior

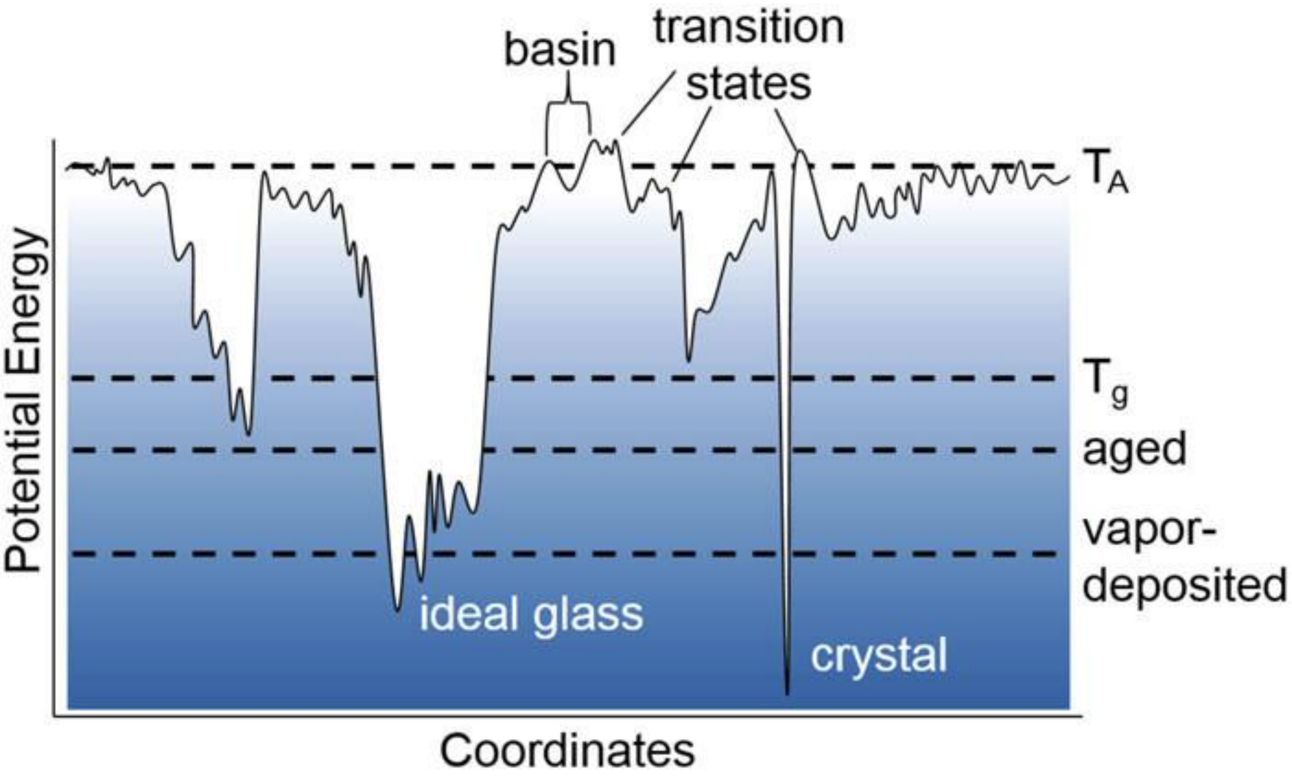
No thermal path to the ideal glass other than to wait forever

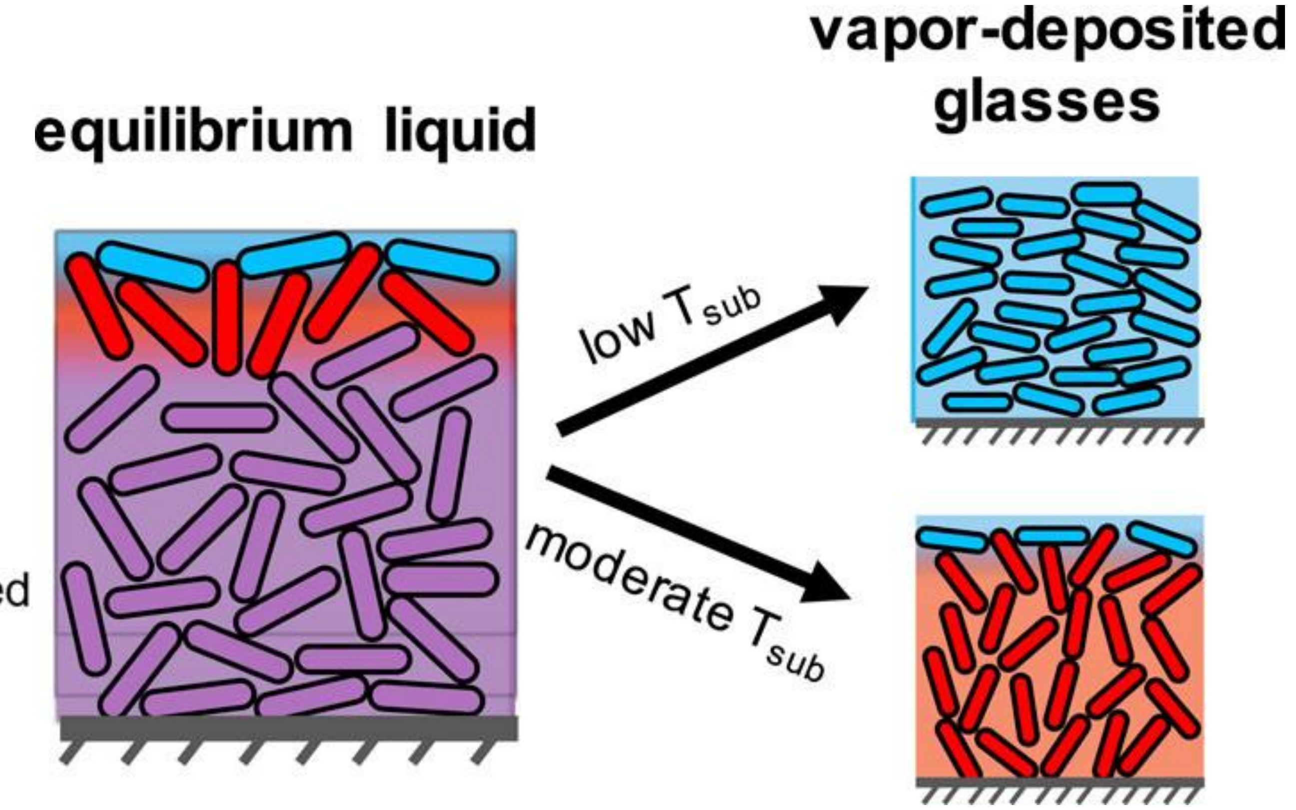


We can try to wait forever: 320 Million Year Old Amber

https://news.siu.edu/2009/10/100209tjc9084.php

Vapor deposition speeds up clock

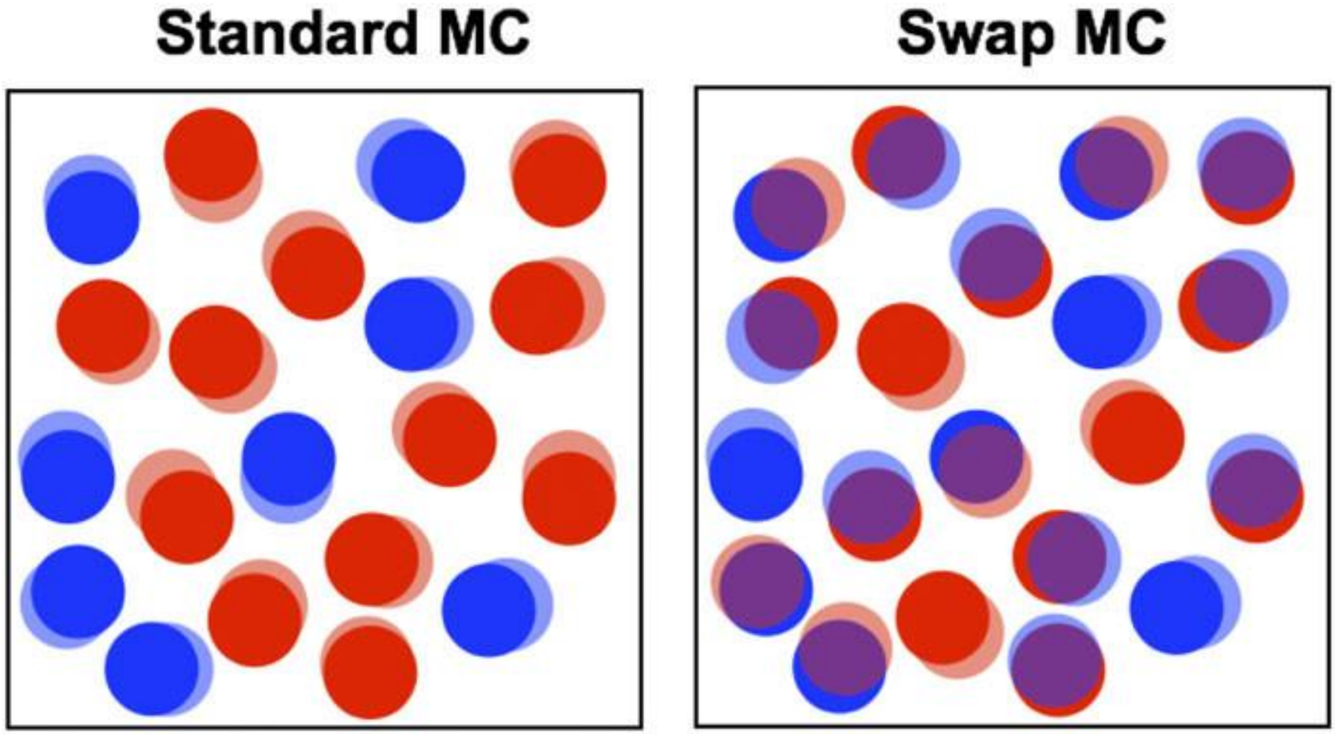




M. D. Ediger; *J. Chem. Phys.* **147,** 210901 (2017)

Numerical thermal systems speed up the clock

- Swap Monte Carlo
  - Particles exchange radii
- Breathing Modes
  - Radii fluctuate with an energy cost function proportional to $(r_i - r_{i0})^2$



Approaching infinity, faster, still takes infinite time

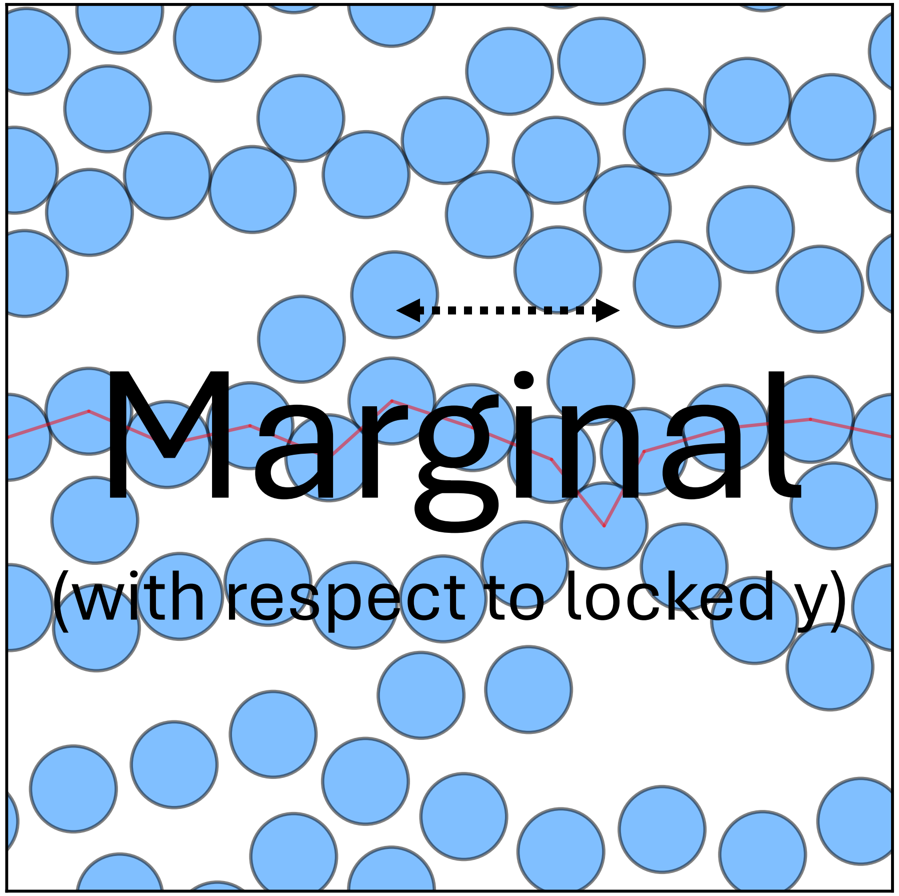
Andrea Ninarello, Ludovic Berthier, and Daniele Coslovich Phys. Rev. X **7**, 021039 (2017)
Harukuni Ikeda; Francesco Zamponi; Atsushi Ikeda; *J. Chem. Phys.* **147,** 234506 (2017)
Geert Kapteijns, Wencheng Ji, Carolina Brito, Matthieu Wyart, and Edan Lerner Phys. Rev. E **99**, 012106 (2019)

Trivial ultrastability: Freeze degrees of freedom

2D system with locked y-coordinate

1 DOF, 2 contacts per particle

Isostatic



2D system with locked y-coordinate

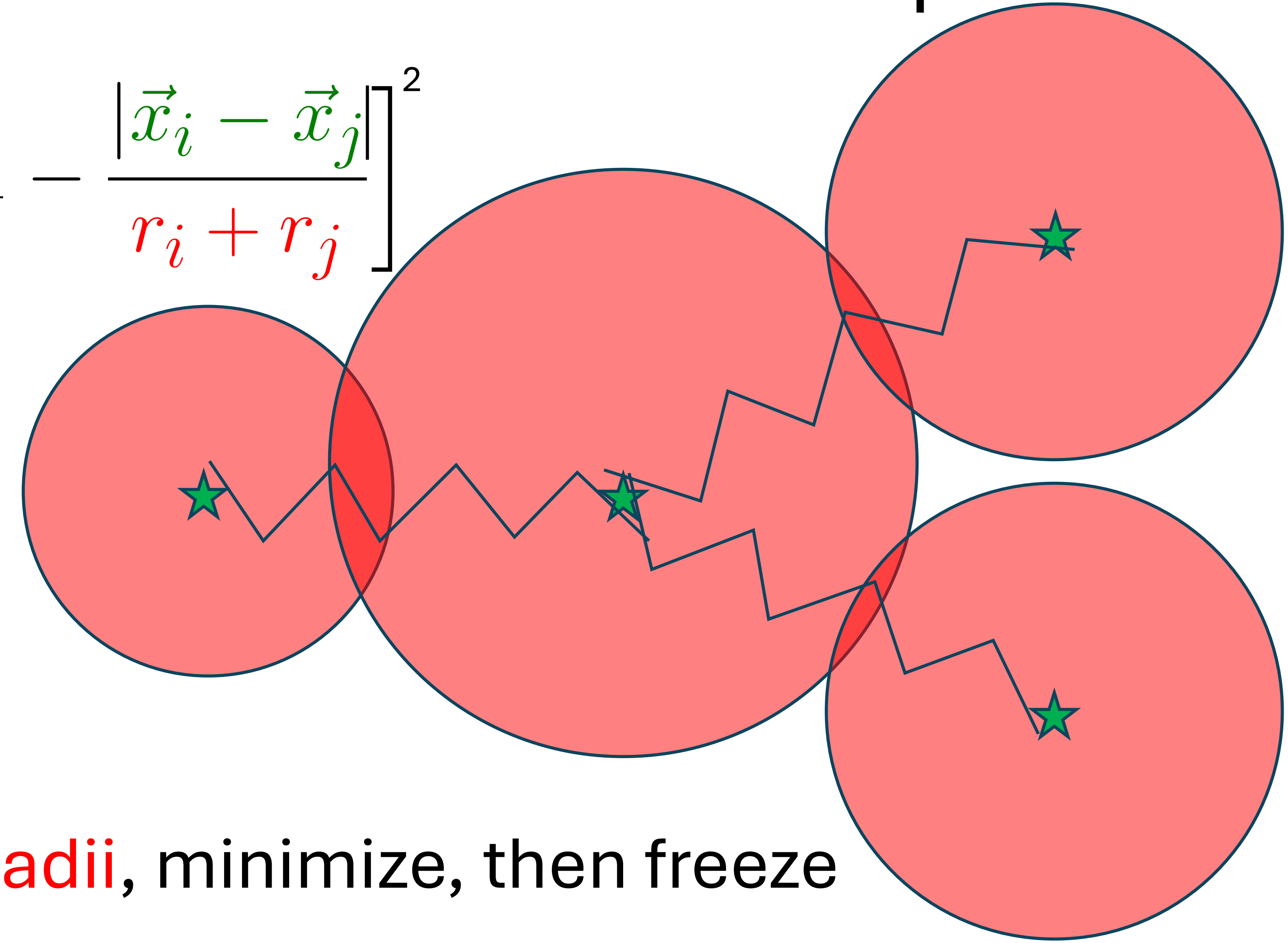
2 DOF, 4 contacts per particle

Very Hyper-static

What degrees of freedom can we exploit?

$$V_{ij} = \frac{k_i k_j}{k_i + k_j}\left[1 - \frac{|\vec{x}_i - \vec{x}_j|}{r_i + r_j}\right]^2$$

Positions
Radii
Stiffnesses



Free Radii, minimize, then freeze

VF Hagh, SR Nagel, AJ Liu, ML Manning, EI Corwin, PNAS 119

How to minimize with respect to radii DOFs

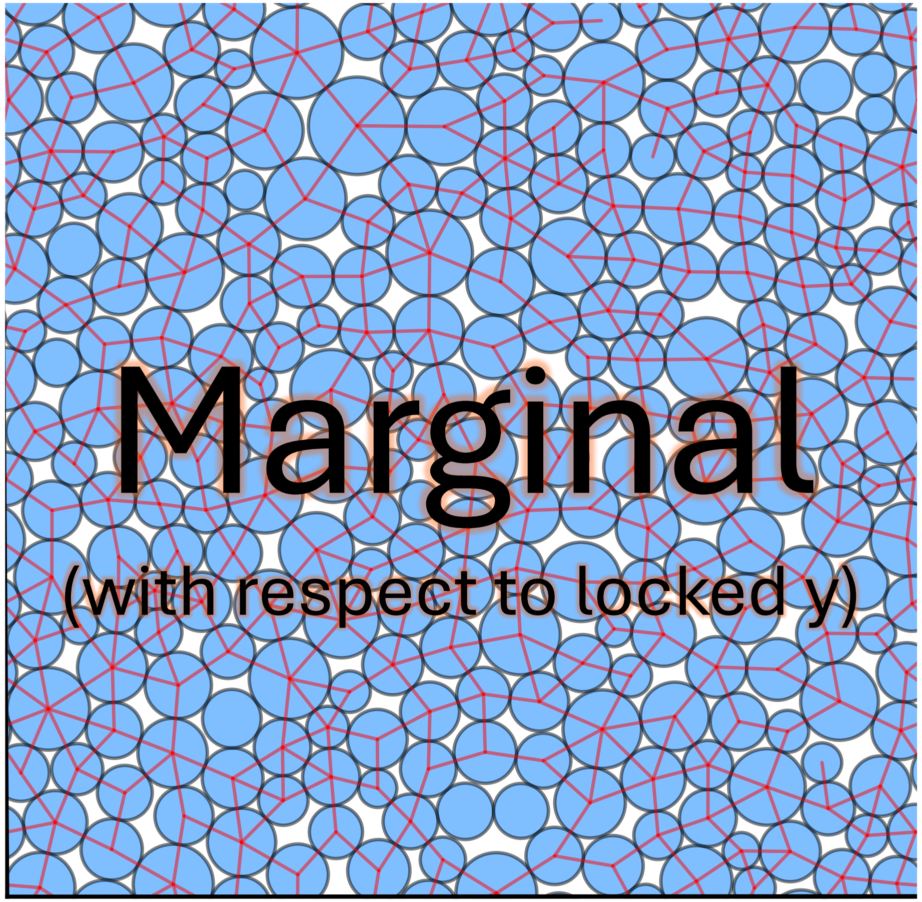
1. Fix φ by constraining $\sum r_i^d$
2. Require that no radii can become negative by constraining $\sum r_i^{-d}$
3. Require that no particle grows too big by constraining $\sum r_i^{2d}$
4. Orthonormalize these constraints to find constrained subspace
5. During minimization project motion into the subspace perpendicular to these constraints

   Three extra constraints to avoid "run away" scenarios

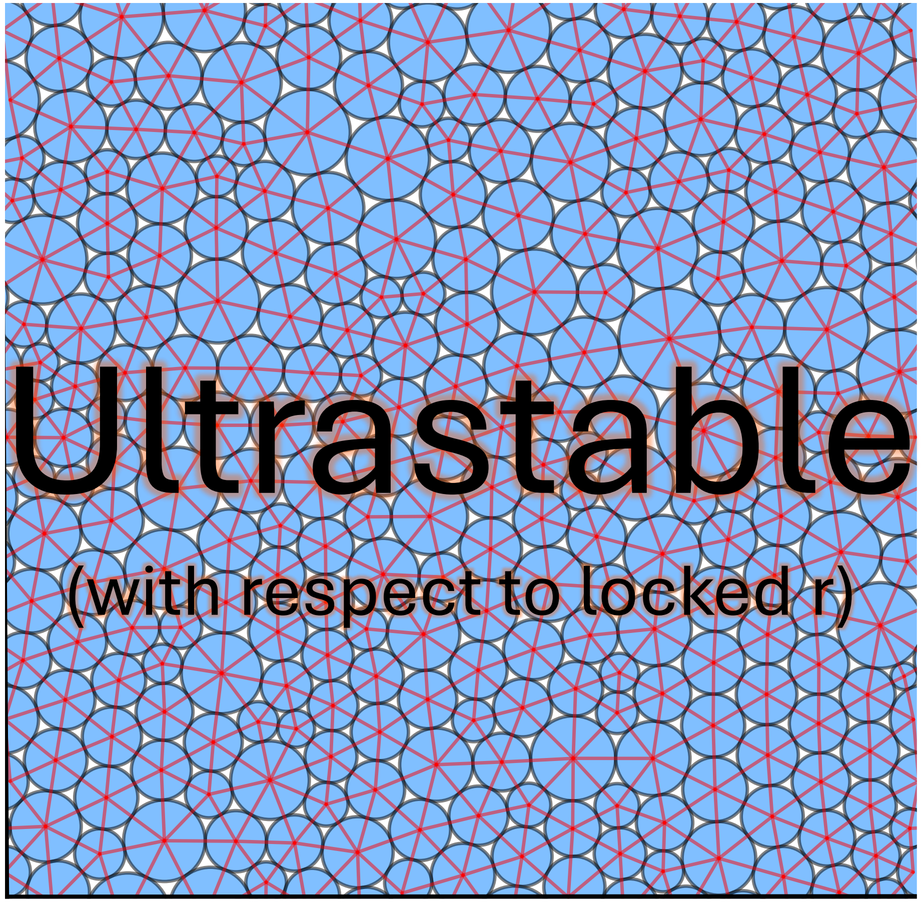
6. Refine away defects with "CirclePack" (Orick, Stephenson, Colins, Computational Geometry 64 (2017) or using Lagrange Multipliers

Ideal ultrastability: Unlock degrees of freedom

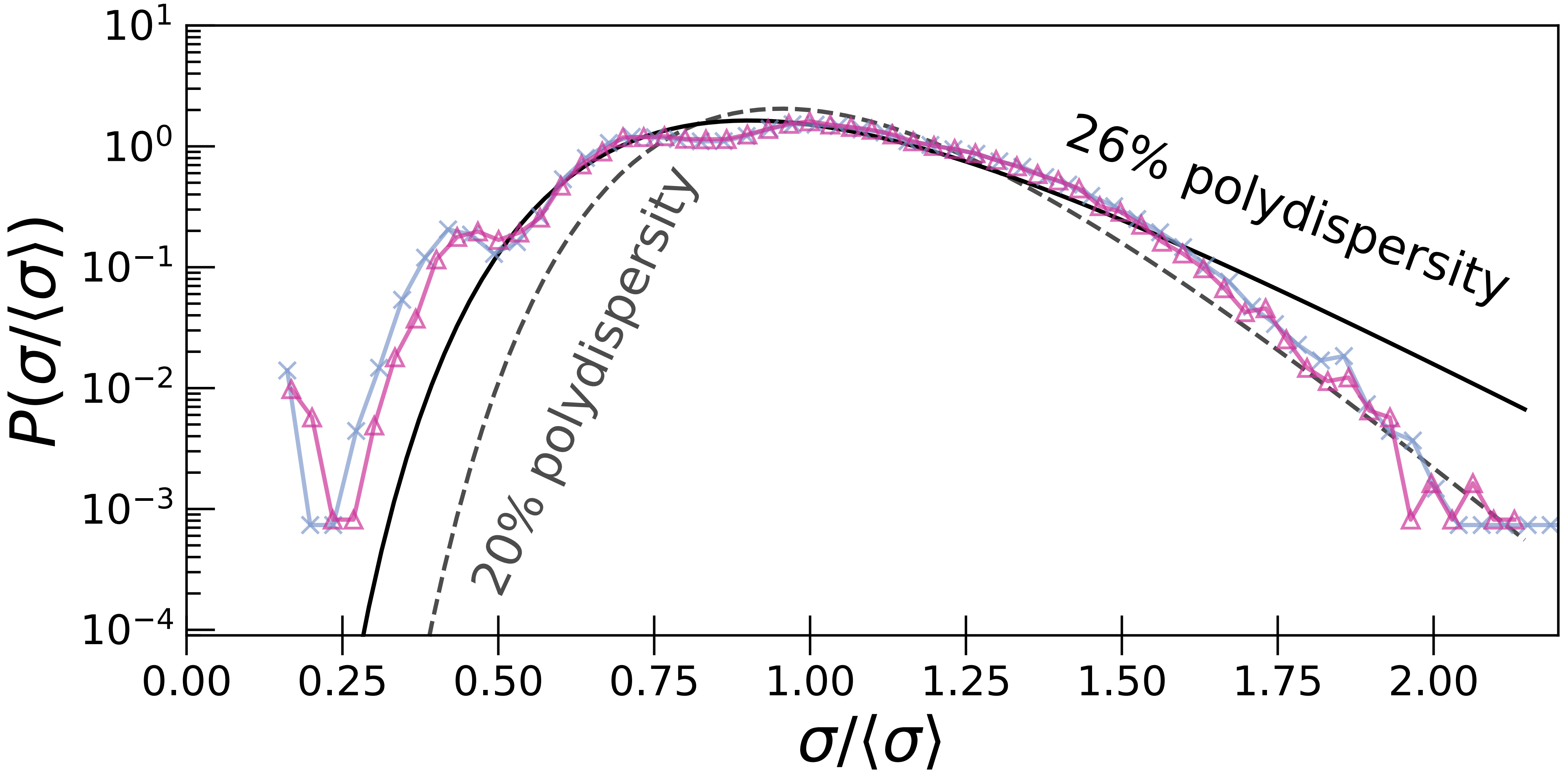
2D system with locked radii
2 DOF, 4 contacts per particle
Isostatic



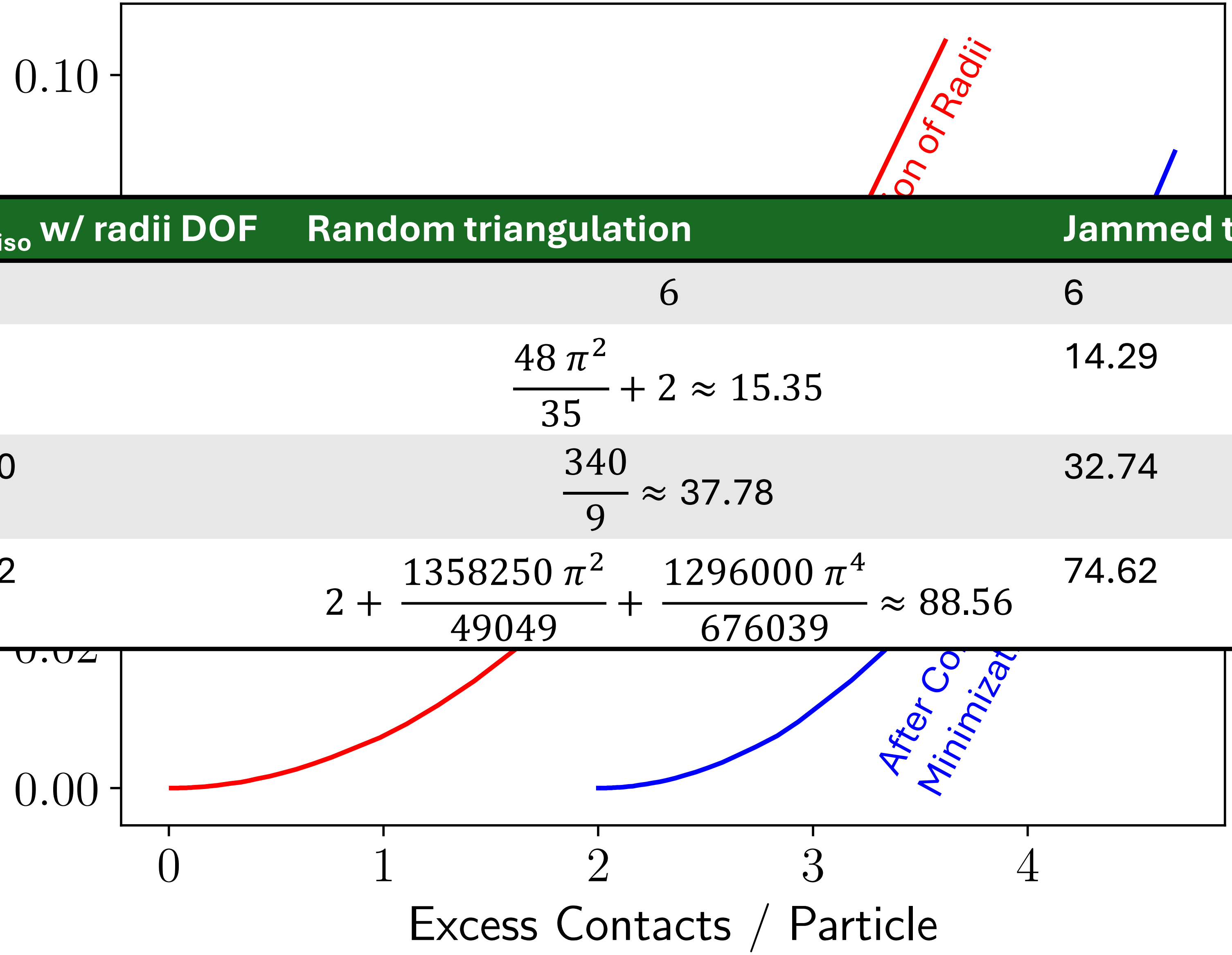
2D system with locked radii
3 DOF, 6 contacts per particle
Very Hyper-static



Distribution of radii is unchanged

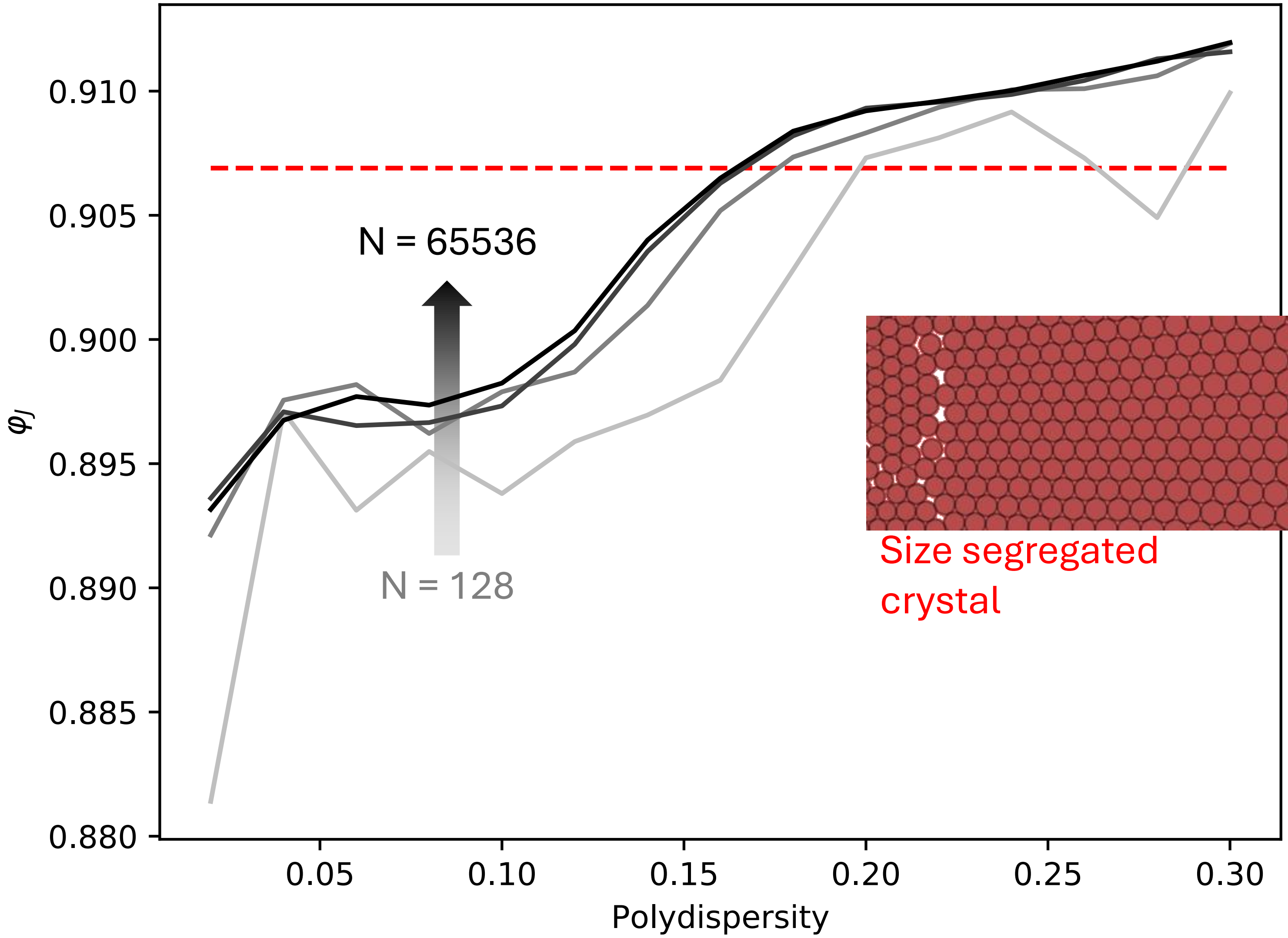


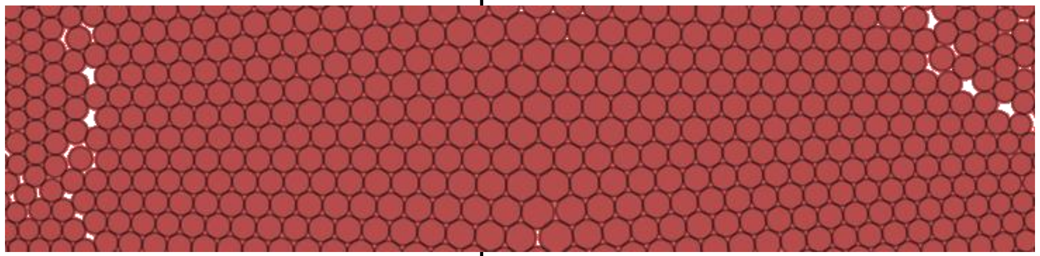
Radii DOFs shift isostatic point by 2



d	Z_{iso}	Z_{iso} w/ radii DOF	Random triangulation	Jammed triangulation
2	4	6	6	6
3	6	8	$\frac{48\,\pi^2}{35} + 2 \approx 15.35$	14.29
4	8	10	$\frac{340}{9} \approx 37.78$	32.74
5	10	12	$2 + \frac{1358250\,\pi^2}{49049} + \frac{1296000\,\pi^4}{676039} \approx 88.56$	74.62

Denser than the size segregated crystal



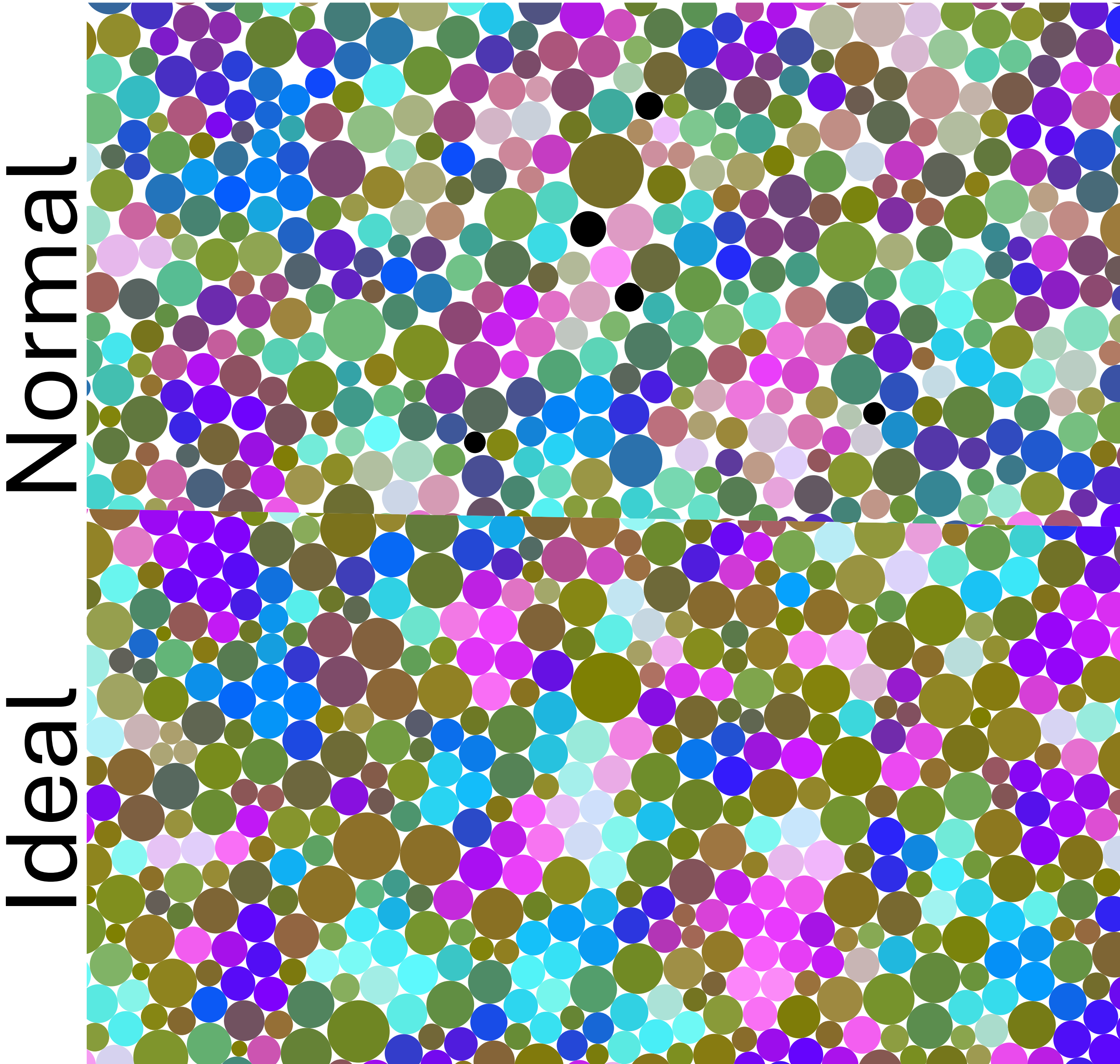


Size segregated crystal

Denser for polysdispersities > 0.15

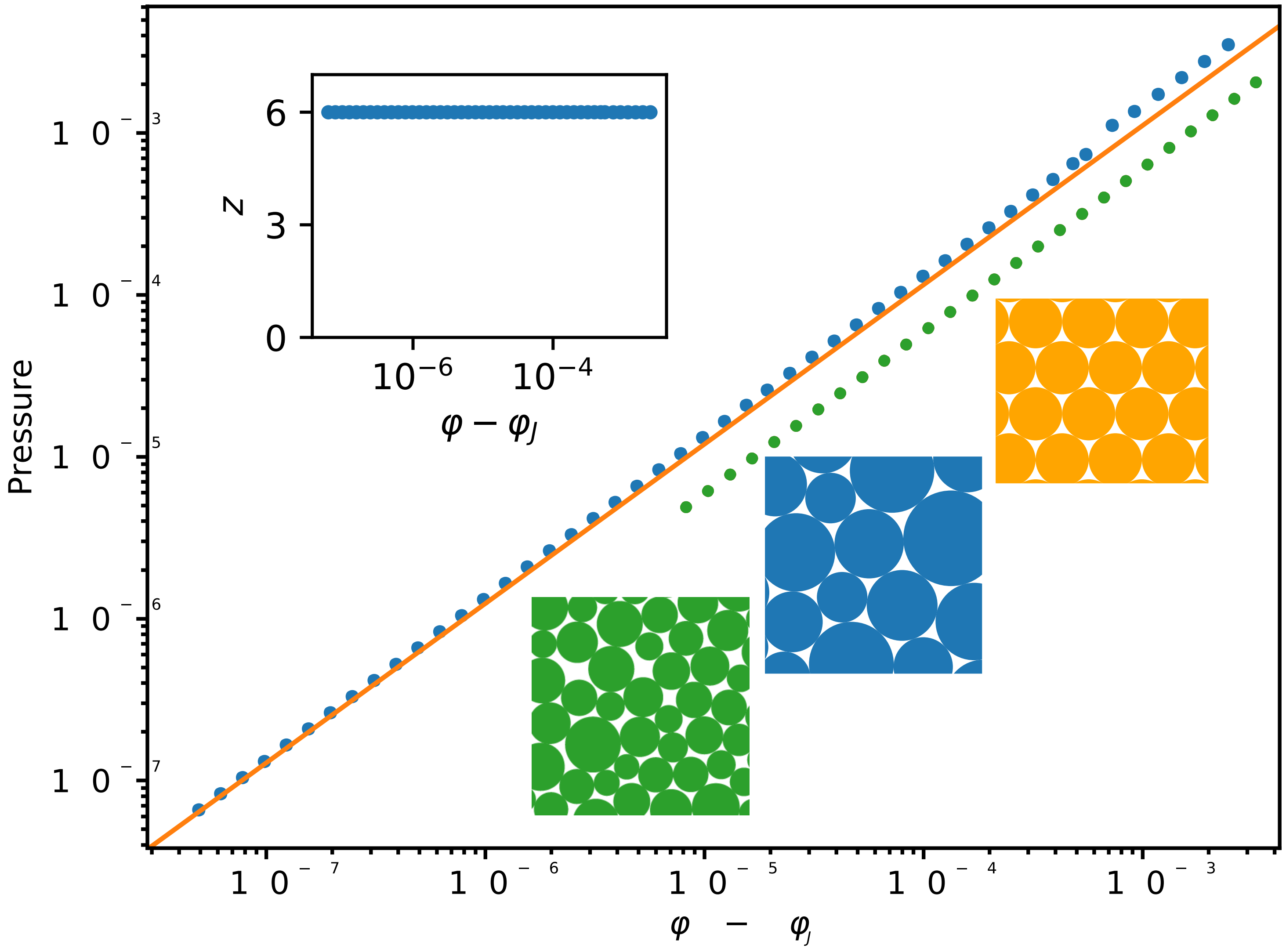
Claim: A zero T (polydisperse) ideal glass/packing must be…

1. Critically Jammed
2. Triangulated
3. Amorphous (no long range order)
4. Mechanically ultra-stable
5. Hyperuniform
6. Anomalously high melting $T=T_K$



1. Ideal glass is critically jammed

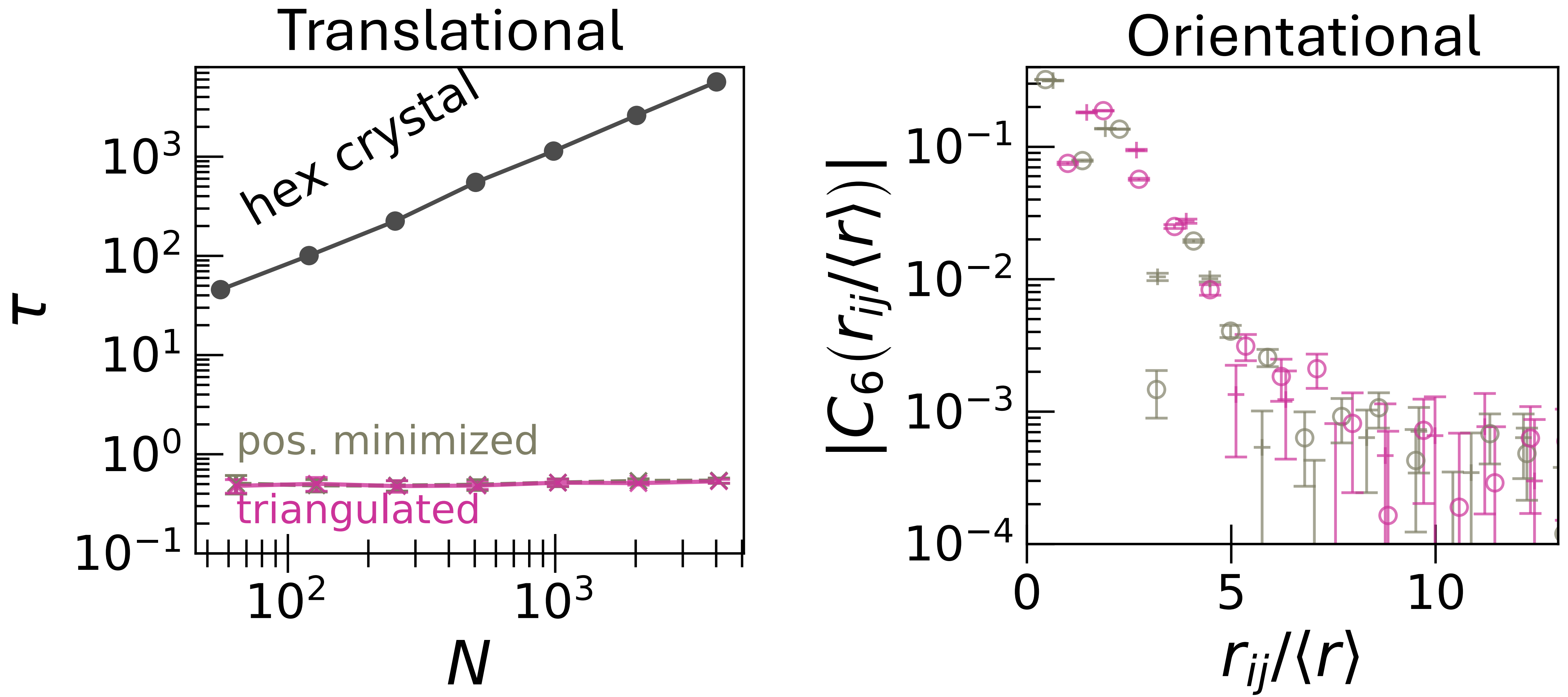
- Structure jammed at zero pressure
- Contacts **don't** decrease with density/pressure
- Same pressure scaling as crystal



2. Ideal glass is triangulated (by construction)

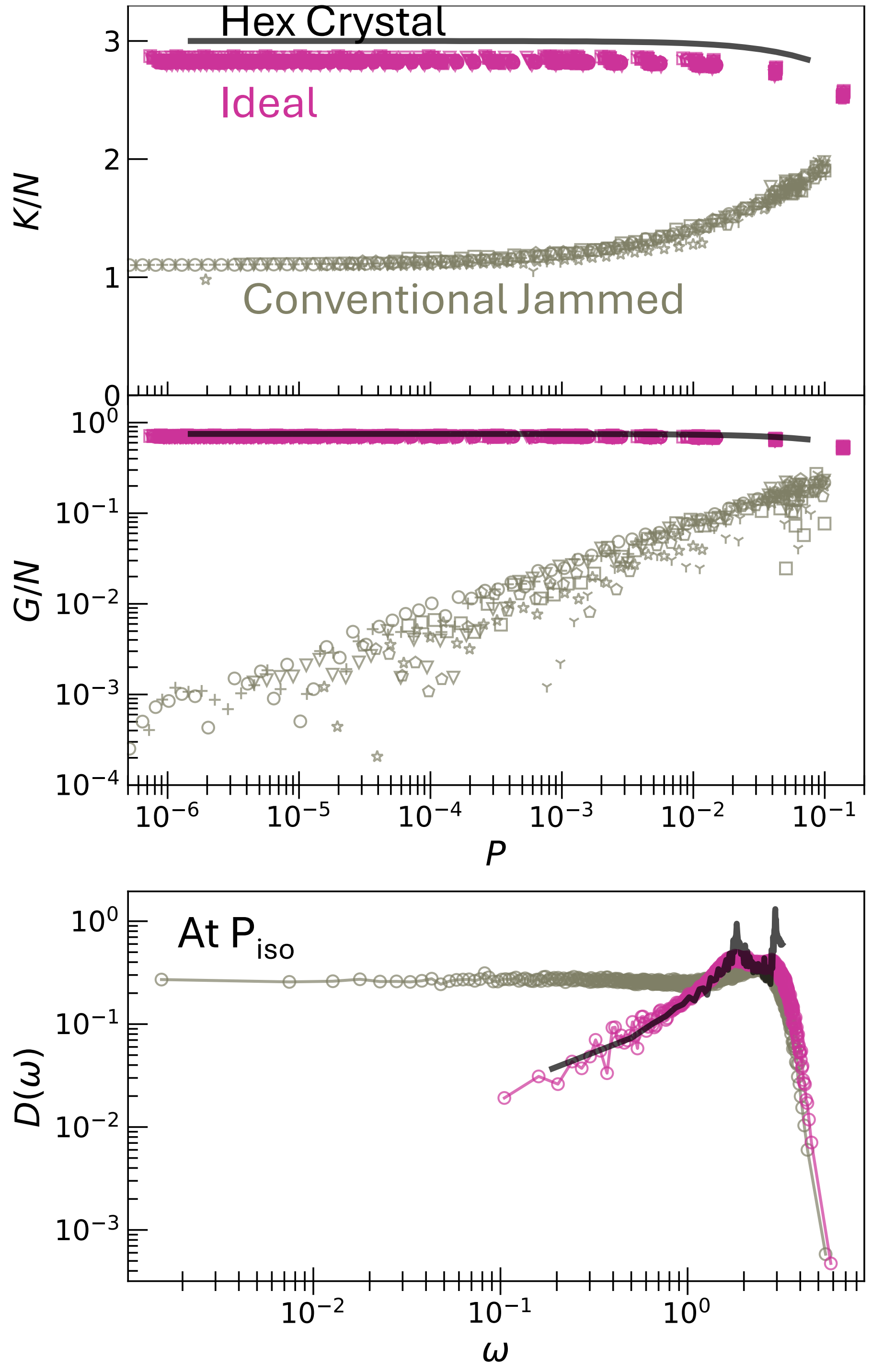
- Many possible disk packings for any connectivity graph
- $S_{graph} \leq S_{conf}$
- Theorem: triangulated graphs on spheres and torii have zero entropy
- Theorem: triangulated graphs are 1-1 with triangulated packings
- Therefore, triangulated packings have zero configurational entropy

3. Ideal glass is amorphous

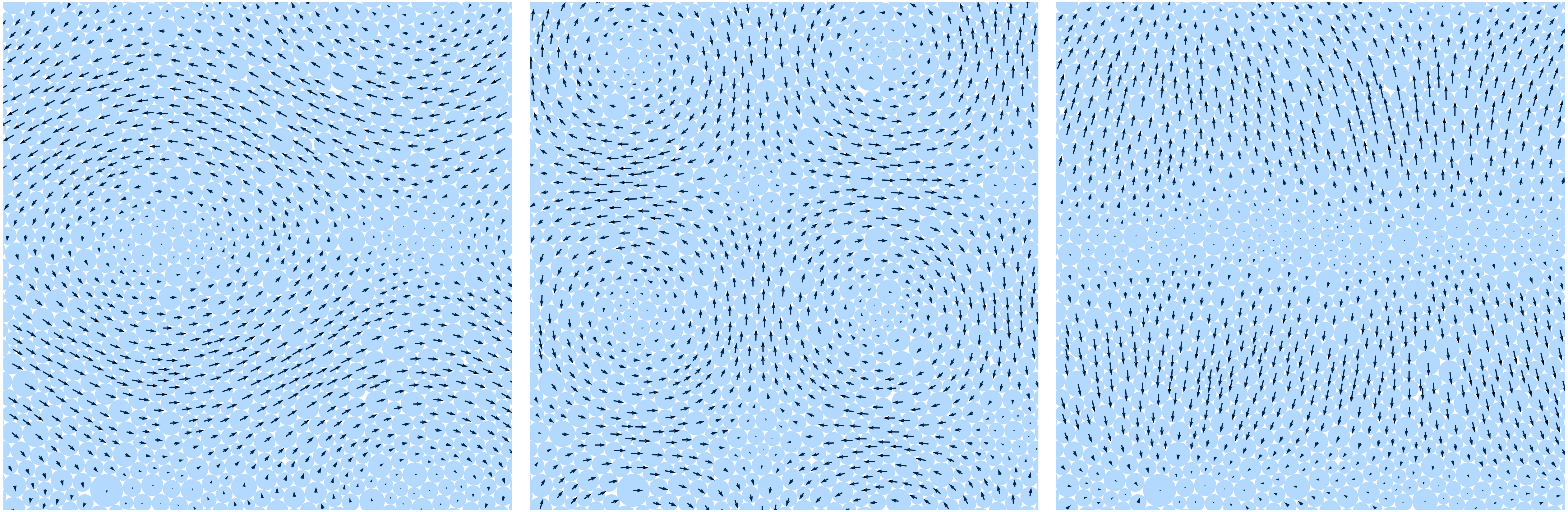


4. Ideal glass is ultrastable

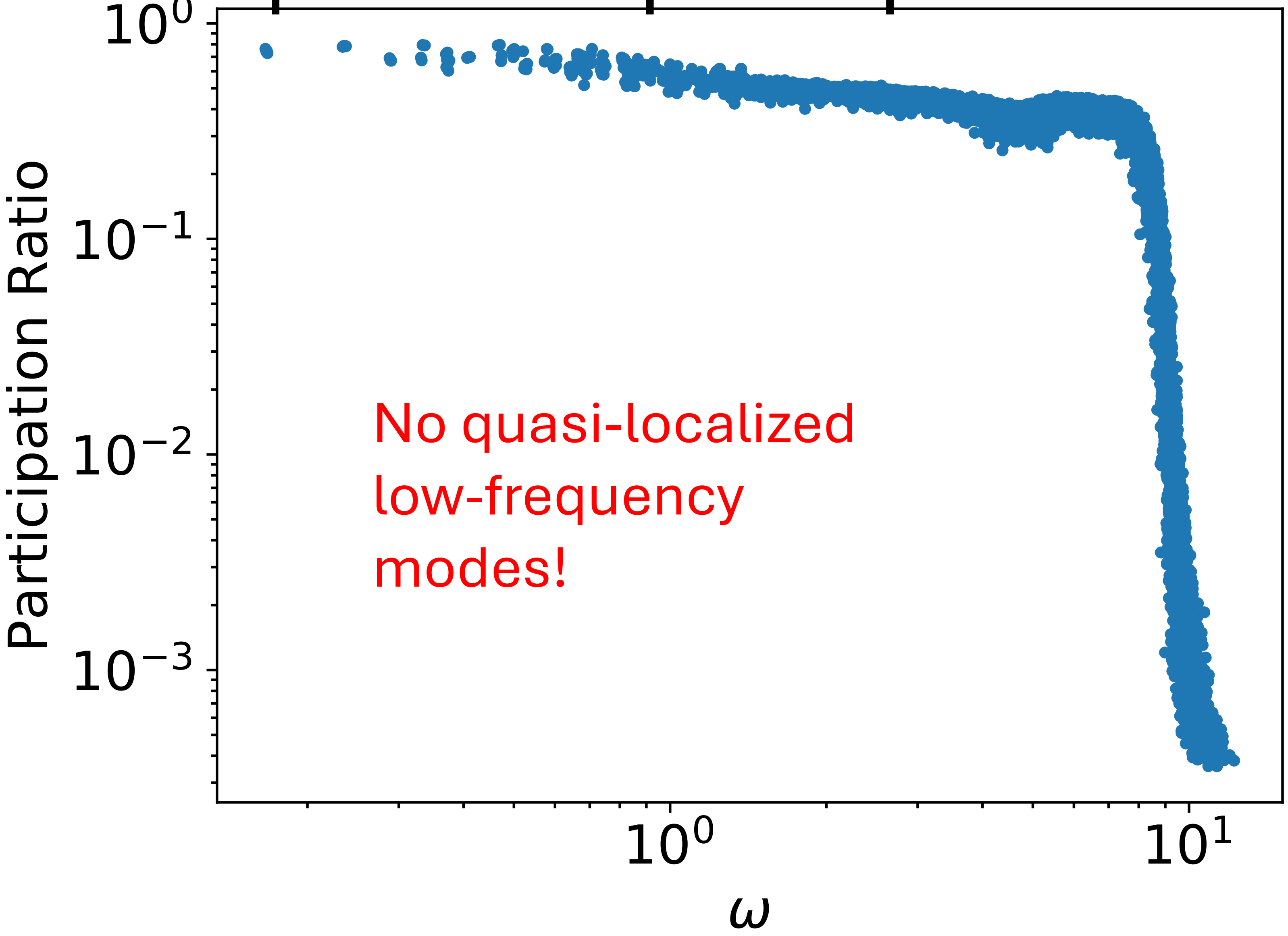
- Shear and bulk moduli of crystal
- No changes with system size
- Vibrational density of states of crystal
- Debye scaling at low frequency



Low frequency modes look like phonons…

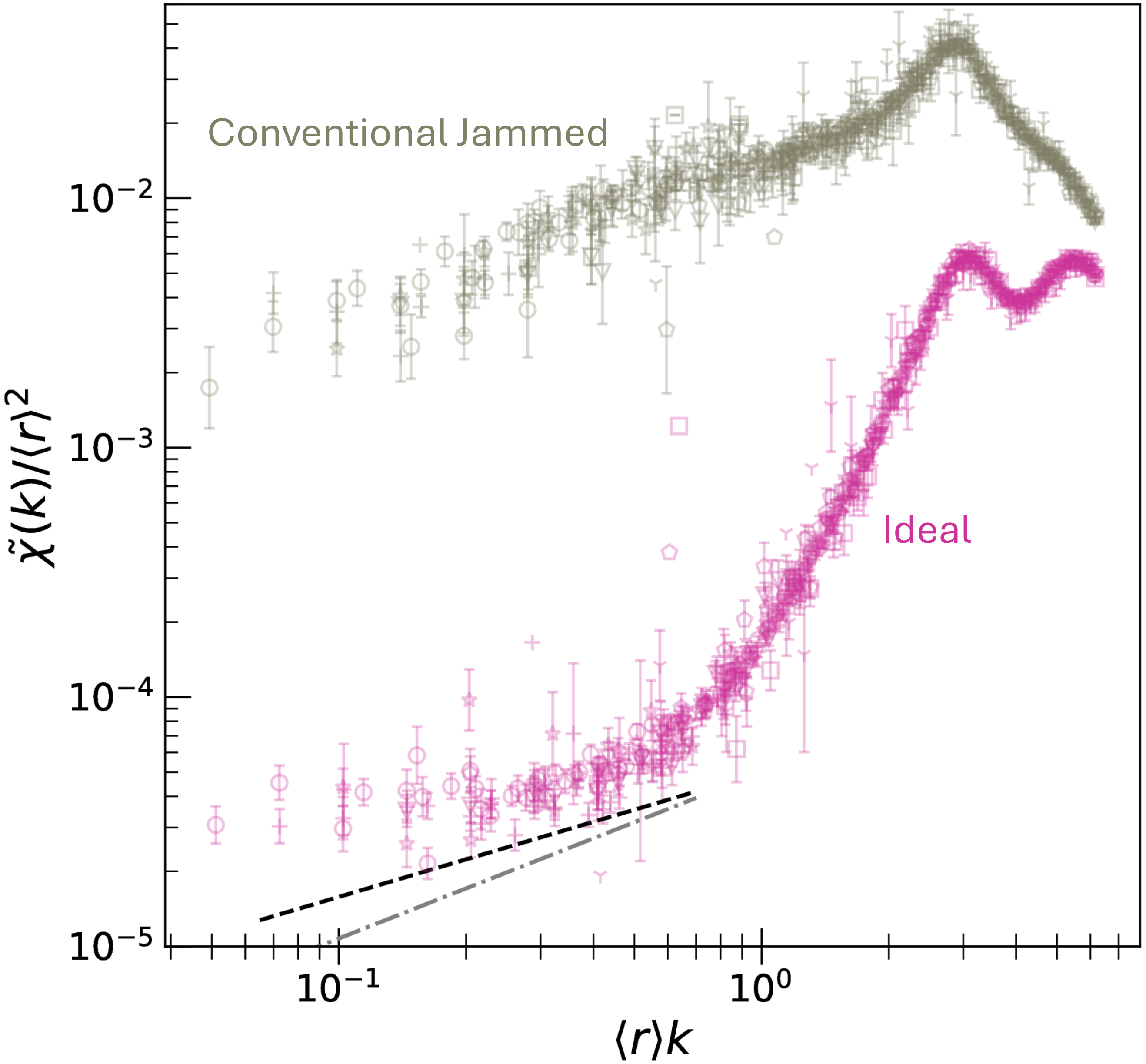


... and have phononic participation ratios



5. Ideal glass is hyperuniform

- Every sub-region should be ideal -> suppressed density fluctuations
- Fluctuations mean one could replace low density region with a copy of a high density region

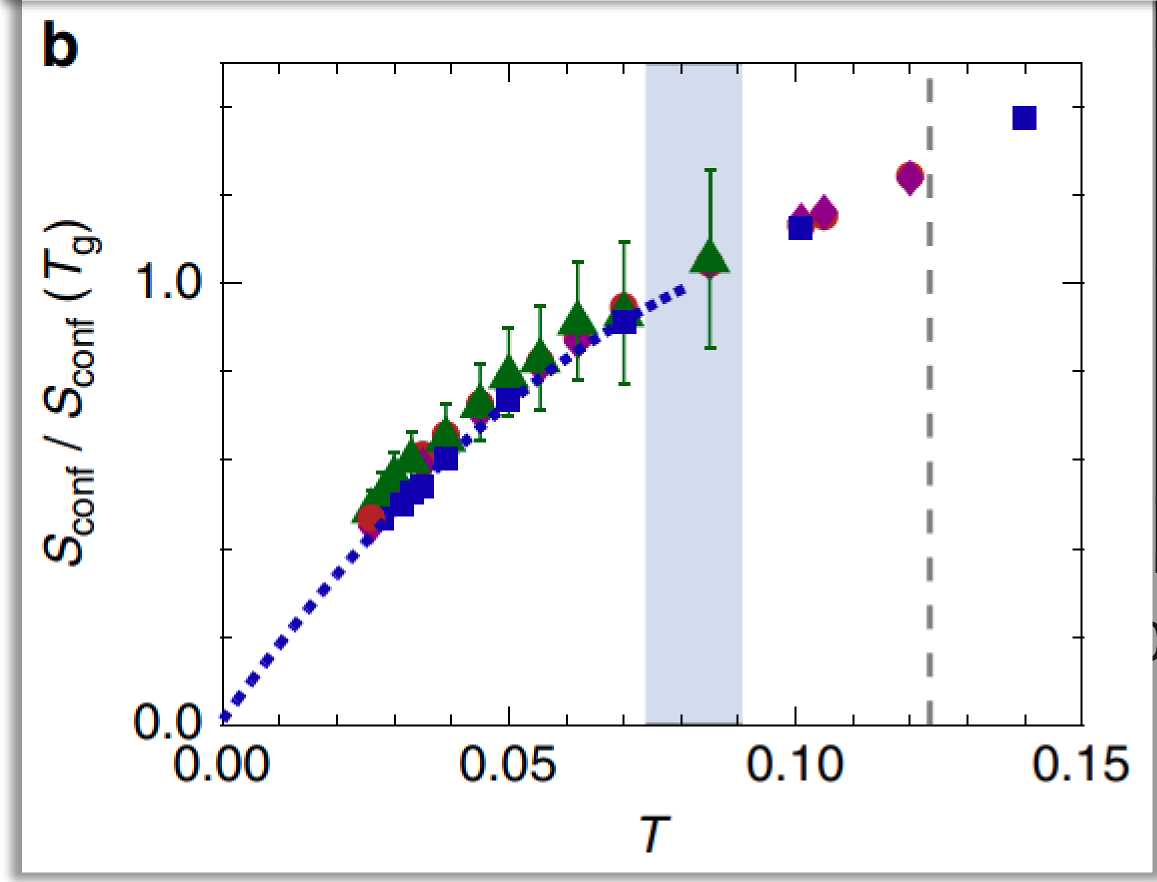


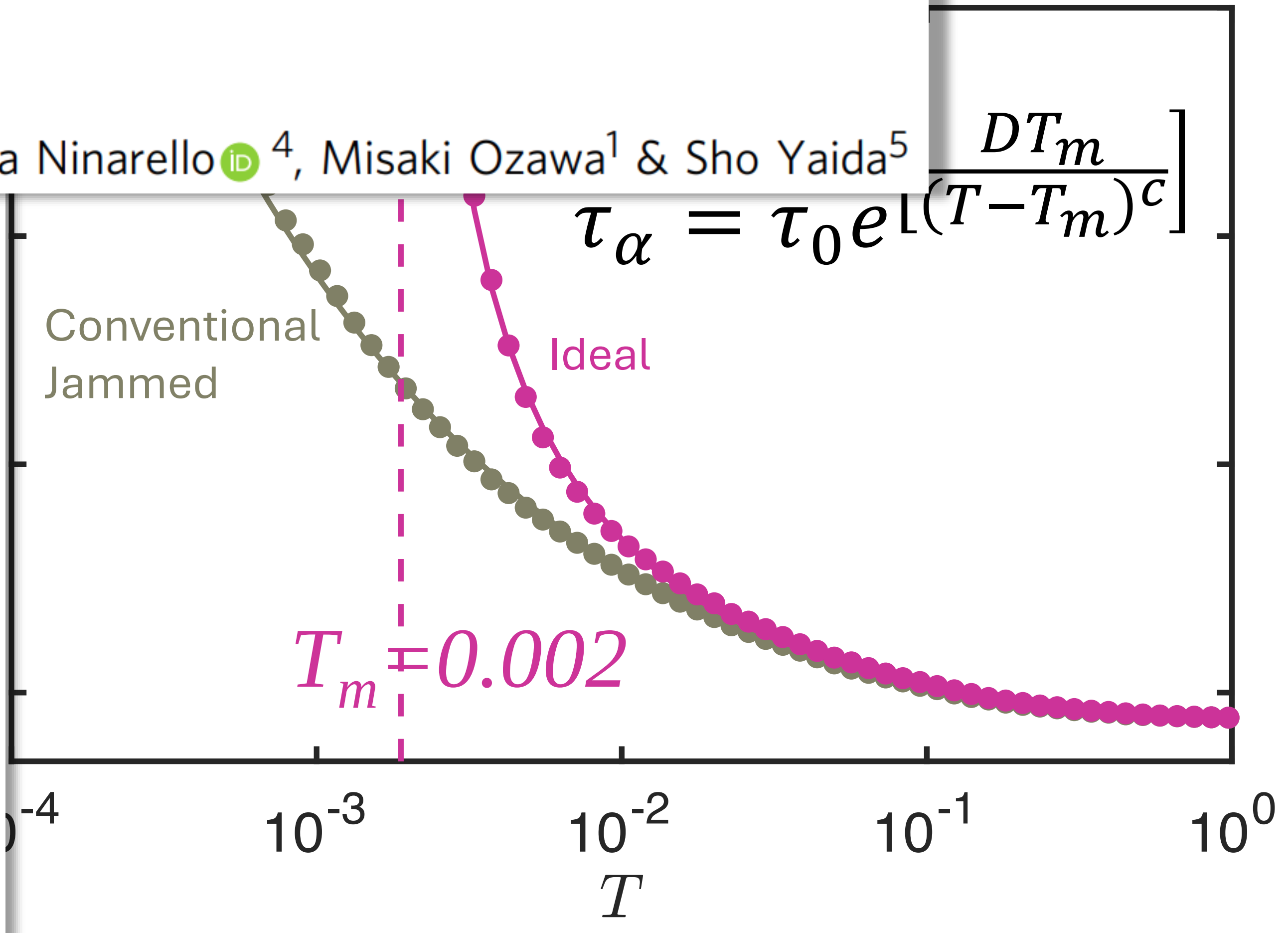
ARTICLE

https://doi.org/10.1038/s41467-019-09512-3 OPEN

Zero-temperature glass transition in two dimensions

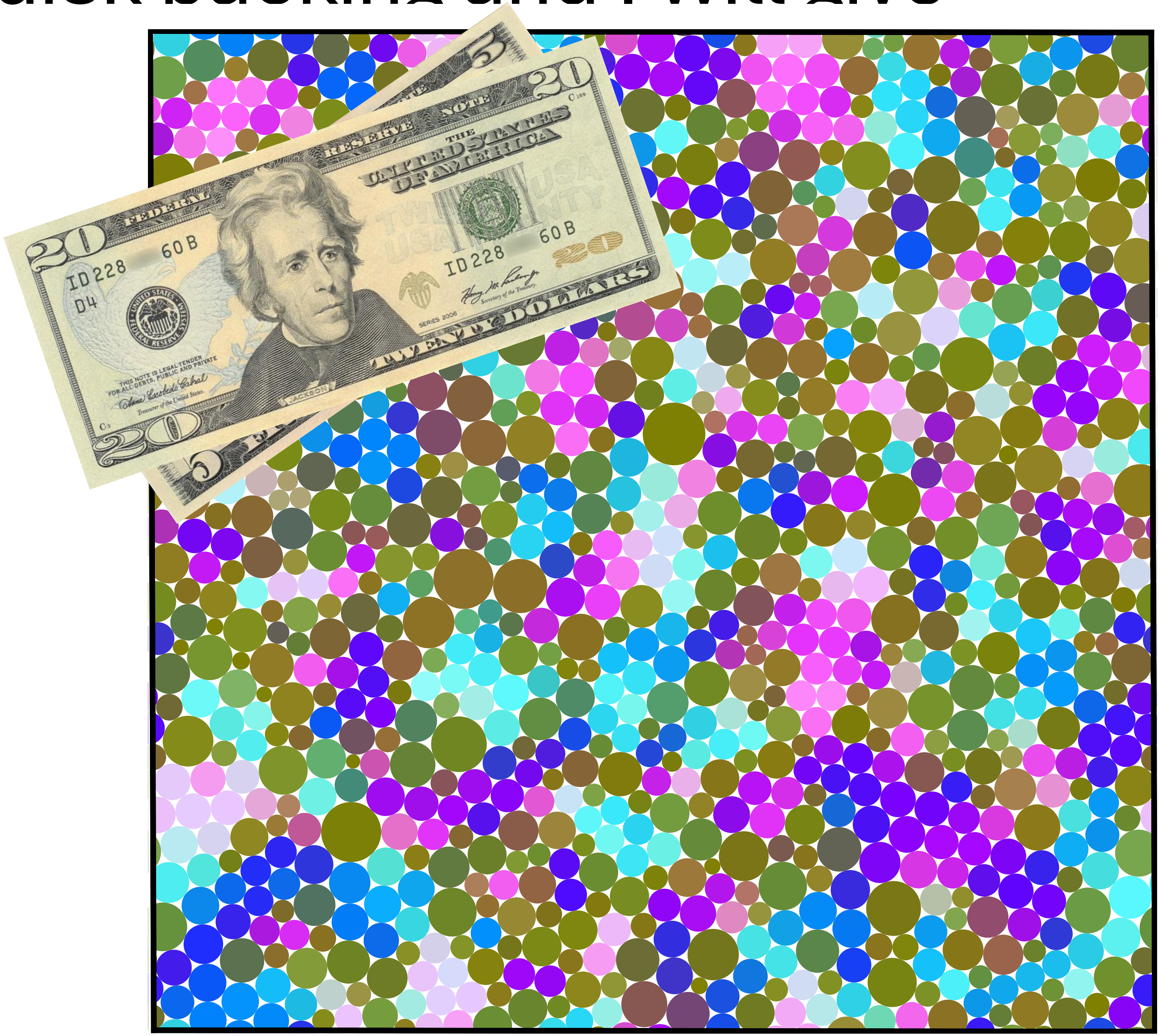
Ludovic Berthier[1], Patrick Charbonneau[2,3], Andrea Ninarello[4], Misaki Ozawa[1] & Sho Yaida[5]





Show me a better disk packing and I will give you ~~$5~~$20

- Ideal glass exists!
- "*bona fide* thermodynamic phase"
- Made using radii minimization and CirclePack
- Need more DOFs to play same game in higher d
- There's a whole world of ideal glass physics to explore




BEST OF THE BEST
Dionysian
Apollonian
Ideal Glass
Crystalline
Hyperuniform