Circle Packings, Minimal Surfaces, and Discrete Differential Geometry
Poster Session Abstracts
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Discrete Differential Geometric patterns from a Finite Element perspective
Kaibo Hu, University of Edinburgh

Compatible and structure-preserving discretization in computational electromagnetism and fluid dynamics inspired the development of Finite Element Differential Forms and Finite Element Exterior Calculus (FEEC). Discrete differential forms and the Whitney forms were interpreted and generalized as finite elements. Tensor-valued problems draw attention in recent developments of FEEC. Efforts were made to extend compatible finite element discretisation of differential forms to geometric objects and structures, such as curvature, connection, and torsion tensors. On the continuous, the Bernstein-Gelfand-Gelfand (BGG) construction derives a large class of differential complexes from the de Rham complexes; on the discrete level, canonical finite elements generalizes the Regge calculus from quantum and numerical relativity. In this presentation, we review the history and discuss some progress in this direction.

Exploring the Homogeneity of Disordered Minimal Surfaces
Matthias Himmelmann, ICERM

Bicontinuous geometries, both ordered and amorphous, are commonly found in many soft matter systems. Ordered bicontinuous phases can be modeled by periodic minimal surfaces, including Schoen's (G)yroid or Schwarz's (P)rimitive and (D)iamond surfaces. By contrast, a minimal surface model for amorphous phases has been lacking. Such disordered and, on average, negatively curved interfaces occur in various organic scenarios, such as photonic crystals, in sea urchin skeletons and in sponge-like carbon phases. Here, we study minimal surface models for amorphous bicontinuous phases. Using the Surface Evolver software with a novel topology-stabilizing minimization scheme, we numerically construct amorphous minimal surfaces from both a continuous random network model for the amorphous Diamond and from a randomly perforated parallel sheet model. As per Hilbert's embedding theorem, the Gaussian curvature of these surfaces cannot be constant. Our analysis of Gaussian curvature variances finds no substantial long-wavelength curvature variations in the amorphous Diamond minimal surfaces. However, their Gaussian curvature variance is substantially larger than that of the cubic G, P and D surfaces. Our work demonstrates the superior curvature homogeneity of the cubic G, P and D surfaces compared to their entropy-favored amorphous counterparts and to other periodic minimal surfaces. This general geometric result is relevant to bicontinuous structure formation in soft matter and biology across all length scales.

The Xarax Unlinks are Physically Gordon
Rob Kusner, University of Massachusetts Amherst

We prove a 2-component unlink configuration proposed by Constant Xarax is physically Gordian: there is no thickness-preserving isotopy to the standard unlink configuration without increasing the length of some component. We also show a related 3-component unlink configuration of Xarax is physically Gordian as well. Our main tools are a sharp version of Schur's Lemma, and a characterization of "4-surround loops" bounding "4-apertures" — non-positively curved embedded disks containing the centers of 4 disjoint unit balls in 3-space — in a neighborhood of the planar length-minimizing 4-surround loops. It remains an open question whether there is a nearby "ideally" Gordian unlink configuration, in the sense that length-trading among components is allowed: a configuration which is a local minimum for its total length subject to the thickness constraint, and thus balanced. (joint with Wöden Kusner, Lawrence University)

Minimal surfaces, maximal surfaces and Ramanujan's identities

Rahul Singh, Indian Institute of Technology Patna

In this poster, we present an interesting connection between examples of zero mean curvature surfaces in Euclidean and Lorentz-Minkowski spaces and some special Ramanujan's identities.

Minimal Surfaces via Checkerboard Patterns

Felix Dellinger, TU Vienna

Checkerboard patterns are quadrilateral nets with regular combinatorics in which every second face is a parallelogram. These structures provide a natural framework for discrete differential geometry: parallelogram faces define first-order properties, while the remaining faces encode second-order differential properties. This enables the consistent definition of conjugate, principal, Koenigs, and isothermic checkerboard patterns. A discrete shape operator, obtained by mapping parallelograms to corresponding parallelograms in the Gauss image, aligns with the notion of principal checkerboard patterns. Checkerboard patterns satisfy the transformation group principle. In particular, principal curvature and isothermic checkerboard patterns are Möbius-invariant, while Koenigs checkerboard patterns remain invariant under projective transformations and dualization. This allows for a geometric construction: a planar isothermic checkerboard pattern can be mapped to the unit sphere via a Möbius transformation, then dualized to yield a discrete minimal surface. The discrete shape operator defined on the resulting minimal checkerboard pattern has zero mean curvature. Checkerboard patterns naturally arise from midpoint subdivision of a general quadrilateral net with regular combinatorics. They are in one-to-one correspondence with the pair of diagonal nets of the original net, establishing a direct connection to binets.

Embeddings and Classification of Icosahedra from Tetrahedra

Vanishree Krishna Kirekod, RWTH Aachen University

The realization of Platonic solids (and thus polyhedra) in $\R^3$ has fascinated mathematicians since ancient Greece. While solving a system of distance equations is a well-established method for realizing polyhedra in $\R^3$, more efficient alternatives have been explored over the years. One such approach is inscribing one polyhedron in another. Here, we embed an octahedron in $\R^3$ by inscribing it in a tetrahedron. Further, we embed an icosahedron by placing the coordinates of its vertices on (the linear span of) the edges of the inscribed octahedron. By considering a tetrahedron with congruent triangles having arbitrary edge lengths, we obtain a wide range of embeddings of an icosahedron from this method. We study these icosahedra by classifying them with respect to the congruence types of triangles and investigate properties like convexity and automorphism groups.

Orthodiagonal Maps, Tilings of Rectangles, and their Convergence to Conformal Maps

David Pechersky, Beijing Institute of Mathematical Sciences and Applications (BIMSA)

A classic result of Brooks, Smith, Stone and Tutte associates to any weighted planar graph with distinguished source and sink vertices, a tiling of a rectangle by smaller subrectangles. This tiling can be viewed as the discrete analogue of the conformal map that maps a simply connected domain with four distinguished boundary points to a rectangle, so that the four boundary points are mapped to the four corners of the rectangle. We make this intuition precise by showing that for orthodiagonal maps- a wide class of weighted planar graphs that are a good approximation of continuous 2D space- the tiling maps associated to finer and finer orthodiagonal approximations of a simply connected domain with four marked boundary points, converge uniformly on compacts to the aforementioned conformal map.