The π-Base Model

for Small Semantic Databases

- → Steven Clontz, University of South Alabama

- → LMFDB, Computation, and Number Theory (LuCaNT) 2025
- → Wed, 2025 July 9, 11:30am

Topology is a dense forest of counterexamples. A usable map of the forest is a fine thing.

— Paraphrased from Mary Ellen Rudin's review of *Counterexamples in Topology*

A recent result (C, Giacopello 2025) in my subdiscipline of topology

Theorem 4.1. *The following are equivalent for any space.*

1. Bob $\uparrow_{\text{mark}} G_1(\mathcal{N}, \mathcal{N})$
2. Bob $\uparrow G_1(\mathcal{N}, \mathcal{N})$
3. Alice $\not\uparrow G_1(\mathcal{N}, \mathcal{N})$
4. Alice $\not\uparrow_{\text{pre}} G_1(\mathcal{N}, \mathcal{N})$ *(i.e.* $S_1(\mathcal{N}, \mathcal{N})$*)*
5. *Every network of the space contains a countable subcollection which is a network.*
6. *The space is countable and second-countable.*

In particular, $G_1(\mathcal{N}, \mathcal{N})$ *is determined.*

Question: Can the game be modified so that all networks contain only finite sets?

One direction is immediate:

The space is countable and second-countable. $\Longrightarrow$ $\textsc{Bob} \uparrow_{\text{mark}} G_1(\mathscr{N}, \mathscr{N})$

What would a counterexample to the converse look like?

$\textsc{Bob} \uparrow_{\text{mark}} G_1(\mathscr{N}, \mathscr{N})$ definitely implies countable…

Filter by Text

e.g. plank

Filter by Formula

countable+~Second countable+T_6

5 spaces satisfying Countable ∧ ¬Second countable ∧ T_6

Id	Name	Countable	Second countable	T_6
S23	Arens-Fort Space	✓	✗	✓
S96	Appert space Appert topology	✓	✗	✓
S111	Single ultrafilter subspace of $\beta\omega$ Single ultrafilter topology	✓	✗	✓
S131	Sequential fan with ω-many spines S_ω	✓	✗	✓
S156	Arens space S_2	✓	✗	✓

Cite as: The pi-Base Community. *π-Base, Search for `countable+~Second countable+$T_6$`*. Available at: https://topology.pi-base.org/spaces?q=countable%2B%7ESecond+countable%2B%24T_6%24 (Accessed: 2025-07-07).

Copy: Markdown Link

Space S000023

Arens-Fort Space

Let $X = \{(n, m) : n, m \in \omega\}$ with all singletons except $\{(0, 0)\}$ open. Define a set containing $(0, 0)$ to be open if and only if it contains all but a finite number of points in all but a finite number of columns.

Defined as counterexample #26 ("Arens-Fort Space") in DOI 10.1007/978-1-4612-6290-9.

This space is homeomorphic to a subspace of Arens space.

Show markdown

Properties Theorems References

count | Show All | ✓ ✕ | ✓ | ✕

Value	Id	Name	Source
✓	P57	Countable	
✕	P19	Countably compact	
✓	P32	Countably paracompact	
✓	P33	Countably metacompact	
✓	P81	Countably tight	

Space S23 | Property P27

Arens-Fort Space is not Second countable

Automatically deduced from the following:

Properties

Property	Value
Anticompact	✓
Countably infinite	✓
Extremally disconnected	✗
T_2	✓

Theorems

Id	If	Then
T281	T_2	R_1
T286	R_1	R_0
T44	Partition topology	Extremally disconnected
T466	Alexandrov ∧ R_0	Partition topology
T565	Locally finite	Alexandrov
T454	Countably infinite	Countable
T292	Anticompact ∧ k_1-space	Locally finite
T291	Anticompact ∧ Countable	Hemicompact

Property P000049

Extremally disconnected

The closure of every open set in X is open or, equivalently, clopen.

Equivalently, any two disjoint open sets have disjoint closures.

Defined in problem 15G of zbMATH 1052.54001 and problem 1H of DOI 10.1007/978-1-4615-7819-2.

DOI 1007/978-1-4612-6290-9 defines it on page 32 with the additional assumption of T_2, which we do not assume here.

Meta-properties

- This property is hereditary with respect to open sets (see Problem 15G.2 in zbMATH 1052.54001).
- This property is hereditary with respect to dense sets (see Math StackExchange 3769214).
- This property is hereditary with respect to locally dense sets (equivalent to previous two meta-properties; see also Proposition 1 of Math StackExchange 5025114). A set $A \subseteq X$ is called *locally dense* (or *preopen*) if every $x \in A$ has neighbourhood U with $U \cap A$ dense in U (equivalently, $A \subseteq \mathrm{int}(\overline{A})$).

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Theorems Spaces References

Id	If	Then
T10	Extremally disconnected ∧ Locally Hausdorff	Sequentially discrete
T44	Partition topology	Extremally disconnected

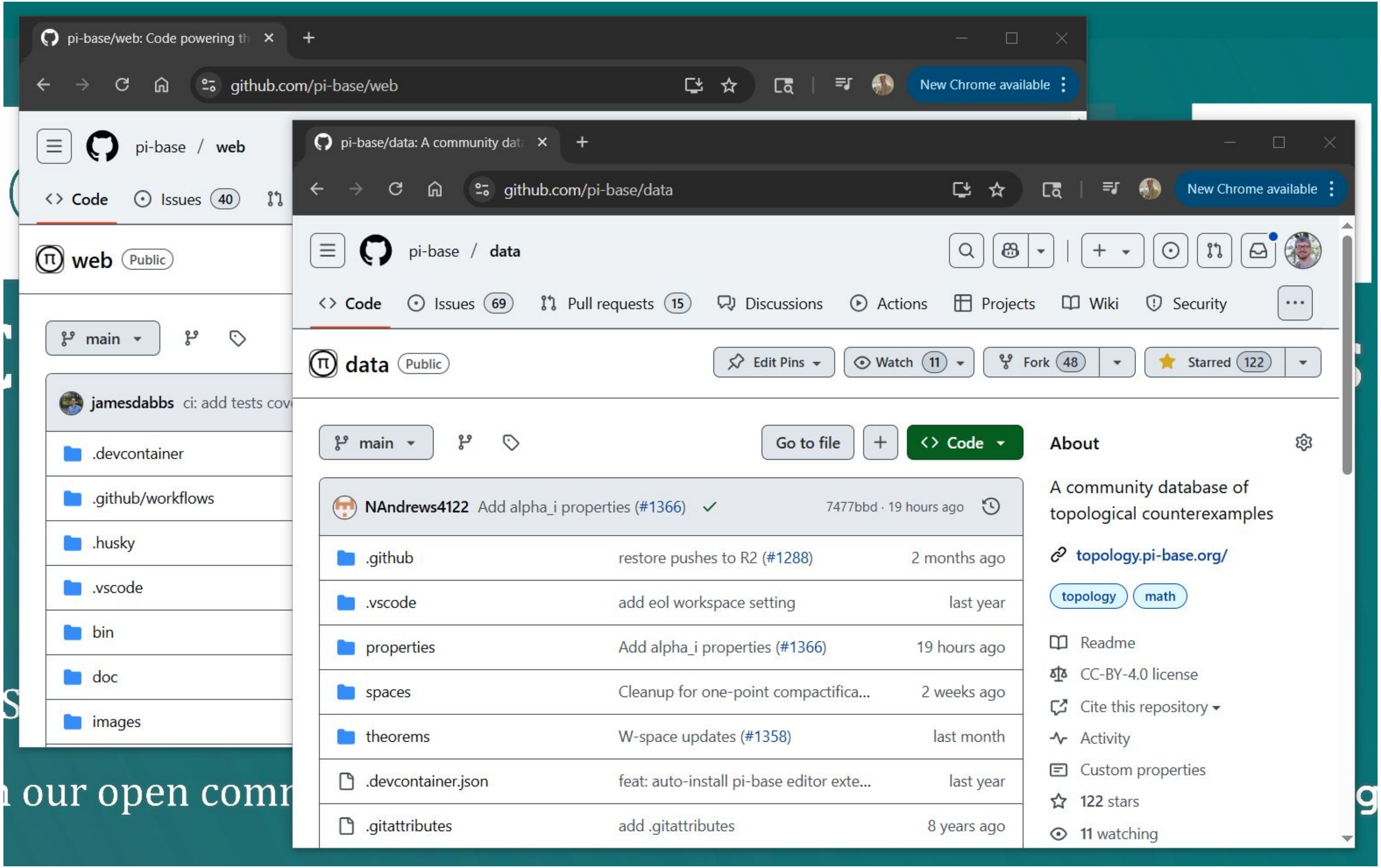
Aside: Davide Giacopello proved last week that

The space is countable and second-countable. $\Longleftrightarrow$ $\textsc{Bob} \uparrow_{\text{mark}} G_1(\mathcal{N}, \mathcal{N})$

It remains open whether $\textsc{Bob} \uparrow_{\text{mark}} G_1(\mathcal{N}, \mathcal{N})$

can be replaced by $\textsc{Alice} \not\uparrow G_1(\mathcal{N}, \mathcal{N})$

(But we're here to talk about databases, not topology!) 😅



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New Chrome available
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Edit Pins
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main
Go to file
Code
NAndrews4122 Add alpha_i properties (#1366)
7477bbd · 19 hours ago
.github
restore pushes to R2 (#1288)
2 months ago
.vscode
add eol workspace setting
last year
properties
Add alpha_i properties (#1366)
19 hours ago
spaces
Cleanup for one-point compactifica...
2 weeks ago
theorems
W-space updates (#1358)
last month
.devcontainer.json
feat: auto-install pi-base editor exte...
last year
.gitattributes
add .gitattributes
8 years ago
About
A community database of topological counterexamples
topology.pi-base.org/
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pi-base / data

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Files

main

Go to file

- S000020
- S000021
- S000022
- S000023
  - properties
    - P000003.md
    - P000049.md
    - P000136.md
    - P000181.md
    - P000203.md
  - README.md
- S000024

data / spaces / S000023 / properties / P000049.md

prabau Scrub uid's from traits (issue #58), part 1 (#126) — 154c4b8 · 4 years ago — History

Preview | Code | Blame — 10 lines (9 loc) · 198 Bytes — Raw

space	property	value	refs
S000023	P000049	false	doi: 10.1007/978-1-4612-6290-9_6 — name: Counterexamples in Topology

See item #9 for space #26 in {{doi:10.1007/978-1-4612-6290-9_6}}.

pi-base / data

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This pull request is waiting on your review. **Add your review**

Warsaw circle is not homogeneous #1367

Open **prabau** wants to merge 1 commit into `main` from `warsaw-homog`

Conversation 1 | Commits 1 | Checks 1 | Files changed 1 +8 −0

prabau commented last week (Collaborator)

The Warsaw circle (S205) is not homogeneous (P86).
https://topology.pi-base.org/spaces/S000205/properties/P000086

Based on #1365.

`Warsaw circle is not homogeneous` ✓ `d0c00e5`

yhx-12243 linked an issue last week that may be closed by this pull request

Trait Suggestion: the Warsaw circle S000205 does not satisfy Homogeneous P000086 #1365 Open

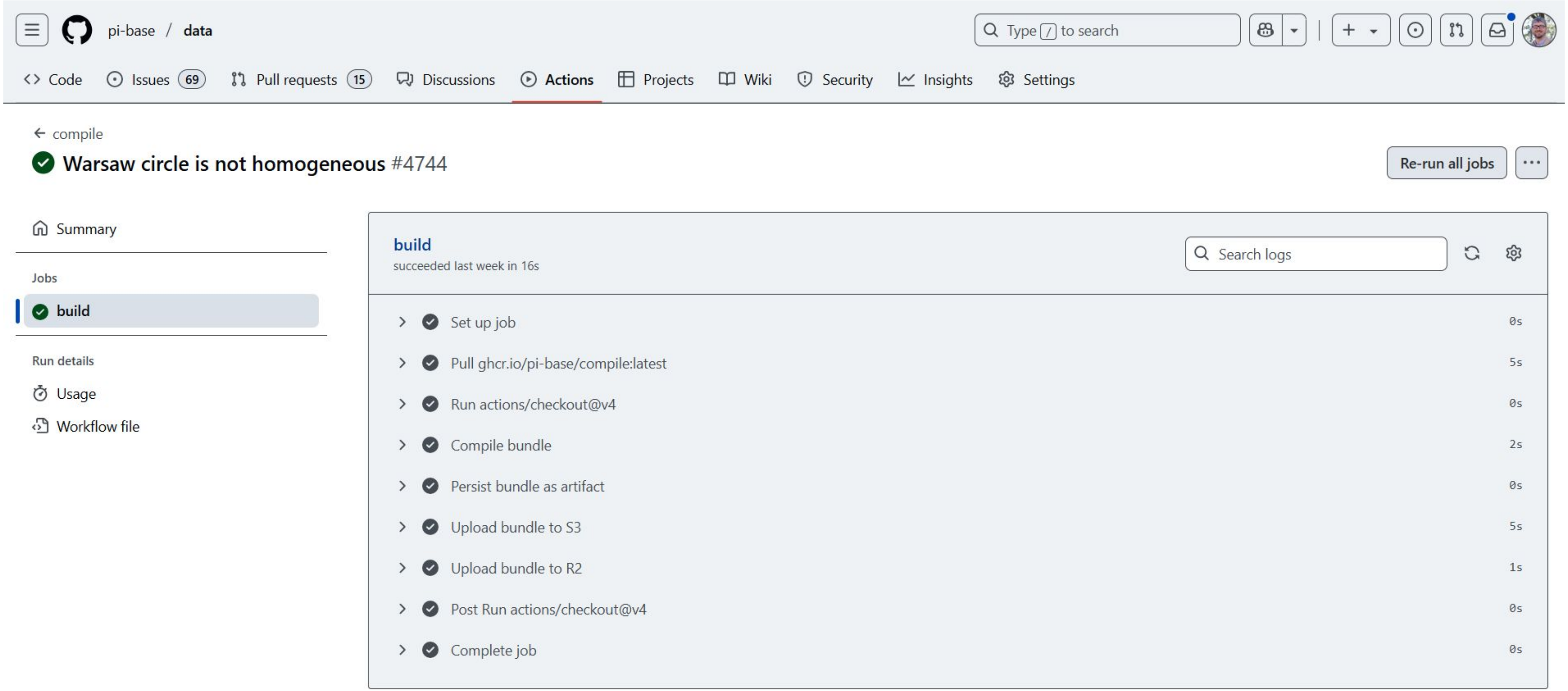
StevenClontz added trait under-review labels yesterday

Reviewers
StevenClontz
At least 1 approving review is required to merge this pull request.
Still in progress? Convert to draft

Assignees
No one—assign yourself

Labels
trait under-review

Projects
None yet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Warsaw circle is not homogeneous #4744
Re-run all jobs
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Jobs
build
Run details
Usage
Workflow file
build
succeeded last week in 16s
Search logs
Set up job 0s
Pull ghcr.io/pi-base/compile:latest 5s
Run actions/checkout@v4 0s
Compile bundle 2s
Persist bundle as artifact 0s
Upload bundle to S3 5s
Upload bundle to R2 1s
Post Run actions/checkout@v4 0s
Complete job 0s

π-Base Explore Spaces Properties Theorems Questions warsaw-homog Contribute Help

Viewer

Branch	main
SHA	72355584f23608b21f3c062c2f54425b33f7c4ce

Data

Using data hosted at

https://pub-65041ca69d744da88ade13abd31ad834.r2.dev/refs/heads/warsaw-homog.json

Host	https://pub-65041ca69d744da88ade13abd31ad834.r2.c
Branch	warsaw-homog
SHA	d0c00e56099d4eaf8faf3f11b29882d74053c550
Sync	just now

Entities

Spaces	210
Properties	206
Theorems	732
Traits	40553

Log to Console Editor Preview Reset

@danflapjax Pi-Base Tools

observablehq.com/@dinod123/pi-base-tools

Data last synchronized: just now

Space S205 | Property P86

Warsaw circle is not Homogeneous

At any point not in $\{0\} \times [-1, 1]$ the space is locally Euclidean, hence locally connected. But this does not hold at all points since Warsaw circle is not Locally connected.

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References

- No references available. If appropriate, please consider contributing a reference.

Limitations

π-Base Explore Spaces Properties Theorems Questions Advanced Contribute Help

Properties

cardinality

Id	Name	Description
P58	Cardinality $< \mathfrak{c}$ Smaller than the continuum, Cardinality $< \beth_1$	The cardinality of the space is less than the cardinality of $\mathbb{R}$.
P65	Cardinality $= \mathfrak{c}$ Continuum-sized, Cardinality $= \beth_1$	The cardinality of the space is equal to the cardinality of $\mathbb{R}$.
P78	Finite Cardinality $< \aleph_0$, Cardinality $< \beth_0$	The cardinality of the space is finite.
P181	Countably infinite Cardinality $= \aleph_0$, Cardinality $= \beth_0$	The cardinality of the space is equal to the cardinality of $\mathbb{N}$.
P59	Cardinality $\leq 2^{\mathfrak{c}}$ Smaller or same as the power set of the continuum, Cardinality $\leq \beth_2$	The cardinality of the space is at most the cardinality of $\mathcal{P}(\mathbb{R})$, the set of subsets of $\mathbb{R}$.
P57	Countable Cardinality $\leq \aleph_0$, Cardinality $\leq \beth_0$	The cardinality of the space is less than or equal to the cardinality of $\mathbb{N}$.
P114	Cardinality $= \aleph_1$	The cardinality of the space is equal to $\aleph_1$, the cardinality of the first uncountable ordinal ω_1