A database of finite groups in the LMFDB

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LuCaNT
July 9, 2025

Why another database of groups?

Existing databases: Small groups DB, Transitive groups DB, GroupNames, ...

- ▶ **Dynamically searchable**. We have precomputed invariants and properties of groups and stored them; users can search on these values.
- ▶ **Free**. Most computations done in Magma (closed source, $$$), but the data is now publicly available.
- ▶ **Easy**[1] **to use**. Intuitive search interface. No programming knowledge required!
- ▶ **Aggregates groups from many sources**. Small groups DB, Transitive groups DB, intransitive subgroups of S_n, groups of Lie type, sporadic simple groups, etc.

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[1]The views expressed in this talk are the speaker's own and do not reflect those of MIT, ICERM, or the United States government.

Why another database of groups?

- **Searches on subgroups and complex characters!** Computed and stored subgroups, along with their inclusions. Knowing all normal subgroups = knowing all surjective homomorphisms. Can use subgroup search to find all extensions, e.g., find all extensions of F_9 by D_{10}.
- **Connections to other parts the LMFDB.** Groups occur in many places in the LMFDB: as automorphism groups, monodromy groups, Galois groups, inertia groups, torsion subgroups, component groups, etc. The groups database helps connect objects related by a given group.

Connections
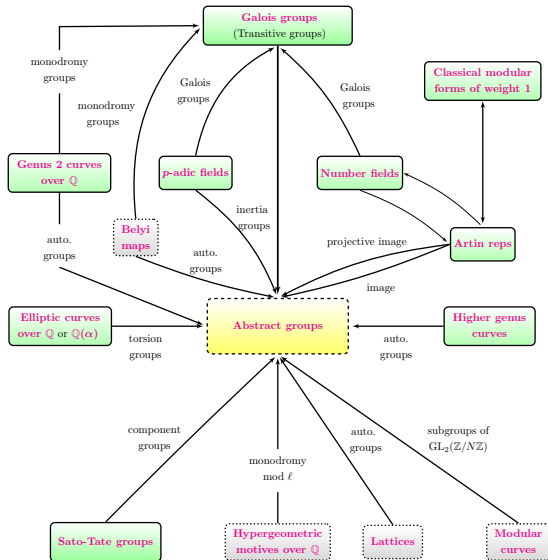
Scope and sources of data

- All small groups of order up to 2,000, except those of order > 500 divisible by 128.
- All transitive subgroups of S_n for $n \leq 47$, except those with $n = 32$ and order between 512 and $4 \cdot 10^{10}$.

▶ Small groups: 257,936

▶ Transitive groups: 235,919

▶ Intransitive groups: 5,444

▶ Lie Type: 2,201

▶ Crystallographic (CARAT): 189

▶ $GL_n(\mathbb{F}_q)$ subgroups: 3,018

▶ $GL_2(\mathbb{Z}/N\mathbb{Z})$ subgroups: 29,771

▶ Perfect: 123

▶ Chevalley: 13

▶ Sporadic: 9

▶ Small group auts: 283

▶ Transitive group auts: 498

▶ Aut. groups of curves: 530

Computational details

▶ **Subgroups**. We typically compute subgroups up to conjugacy. However, not always practical: C_2^{10} has 229,755,605 subgroups, but only 11 up to automorphism. In these cases, we compute subgroups up to automorphism using conjugacy in the holomorph $G \rtimes \mathrm{Aut}(G)$.

▶ **Isomorphisms of transitive groups**. The transitive groups DB contains transitive subgroups of S_n up to conjugacy in S_n. But these could still be abstractly isomorphic! We created a hash function to distinguish transitive groups that are abstractly isomorphic.
Running the hash on the 408,641,062 groups of order 1,536 produced 408,597,690 distinct values, with maximum cluster size 72.

Thank you!

- https://beta.lmfdb.org/Groups/Abstract/
- https://github.com/roed314/FiniteGroups