Patterns, Dynamics, and Data in Complex Systems
Poster Session Abstracts

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Fractal opinions among interacting agents

Fei Cao, University of Massachusetts Amherst

We investigate an opinion model consisting of a large group of interacting agents, whose opinions are represented as numbers in [-1,1]. At each update time, two random agents are selected, and the opinion of the first agent is updated based on the opinion of the second (the "persuader'"). We derive the mean-field kinetic equation describing the large population limit of the model, and we provide several quantitative results establishing convergence to the unique equilibrium distribution. Surprisingly, in some range of the model parameters, the support of the equilibrium distribution exhibits a fractal structure, linking the mean-field description of our opinion dynamics to the concept of Bernoulli convolutions studied extensively in the fractal geometry literatures. This provides a new mathematical description for the so-called opinion fragmentation phenomenon.

Consensus on Higher Order Networks

Joe Geisz Colorado State University

We discuss mathematical models of consensus on networks, at three levels of complexity. First, graphs as models for networks are explained and a model for consensus on a graph is derived using the graph Laplacian. Next, the idea of higher-order networks is introduced, and simplicial homology using real coefficients is used to define the Hodge Laplacian. These tools allow the consensus model to be applied on edges and higher order connections. Finally, simplicial sheaves are explored as a model for higher-dimensional state spaces. A toy model of birds coming to consensus while flocking is discussed as an example of consensus on a sheaf.

Global Dynamics of an Electroencephalographic Mean Field Model of the Neocortex

Farshad Shirani, Emory University

In this poster presentation, I show key analytical and computational results on a mean field model of EEG activity in the neocortex. The model is a degenerate system of coupled ODEs and PDEs, with subtle solution irregularities. I show that the model possesses bounded absorbing sets for all plausible values that its biophysical parameters can take. For some sets of parameter values, I show that the equilibrium set of the model is not compact, implying further that the global attracting set of the model is infinite-dimensional---a property that sounds crucial for the functionality of the neocortex. I also present computational results on generation and spatial propagation of transient gamma oscillations in the solutions of the model. The results specifically identify important challenges in interpreting and modelling temporal patterns of EEG recordings, originating mainly from the low spatial resolution of EEG electrodes.

Pattern Formation in Cell Membranes - An interplay of Eckhaus and Turing Instabilities

Sebastian Suckau, Hamburg University

Recently, bulk-surface models have been derived to study the dynamics of cell proliferation for the MIN protein system, in particular when exposed to advection in the cell interior. We study travelling wave bifurcations in a reduced bulk integrated model, which forms a spatially one-dimensional reaction-diffusion-advection system with six components. Due to the conservation laws innate to the MIN system, a complex and fascinating combination of conservative Turing-Hopf- and Eckhaus-type instabilities can be observed. The model background stems from E. coli bacteria, where the MIN protein system plays an important part in localizing the cell center during cell division. It consists of the MinC, MinD and MinE proteins, whose chemical interactions lead to oscillations of MinD and MinE along the cell. These oscillations induce a time-average gradient that initializes cell division. Similar mechanisms have been observed or suspected in other bacteria [Feddersen et al., Dynamics of the Bacillus subtilis Min System, mbio 2021]. The model we study was proposed in [Meindlhumer et al., Directing Min protein patterns with advective bulk flow, nature communications, 2023]. This is joint work with Jens Rademacher (University of Hamburg).

The emergence of stable spatiotemporal patterns in a mean-field model for neuronal activity

Ying Liu, University of Iowa

In this work we use bifurcation theory techniques and numerical simulations to study the emergence of stable spatiotemporal patterns in a mean-field model for neuronal activity. We identify sufficient conditions on the model's parameters for the generation of traveling waves (TWs), standing waves (SWs) and modulated waves (MWs). We show how the relative contribution of the intrinsic cell dynamics, the network structure, and certain features of a feedback connectivity loop (slow vs fast and weak vs strong, negative feedback component) lead to the selection of TW, SW, and MW spatiotemporal patterns.

Exponential Dichotomies in Spatial Evolutionary Equations for Elliptic PDEs

Alanna Haslam-Hyde, Boston University

Exponential dichotomies, when they exist, provide powerful information about the structure of bounded solutions even in the case of an ill-posed evolutionary equation. The method of spatial dynamics, in which one views a spatial variable as a time-like evolutionary variable, allows for the use of classical dynamical systems techniques, such as exponential dichotomies, in broader contexts. This has been utilized to study stationary solutions of PDEs on spatial domains with a distinguished unbounded direction (e.g. the real line or a channel of the form $R\times\Omega$). Recent work has shown how to extend the spatial dynamics framework to elliptic PDEs posed on general multi-dimensional spatial domains. In this poster we show that, in the same context, exponential dichotomies do exist, thus allowing for their use in future analyses of coherent structures, such as spatial patterns in reaction-diffusion equations on more general domains.

InfoMap for absorbing random walks

Esteban Vargas Bernal, Arizona State University

InfoMap is a community detection algorithm based on random walks with no absorbing states. We adapt this algorithm to account for the effect of distinct absorption rates in an absorbing random walk. We apply this adaptation to disease spread where we find communities informed by both disease contact network and treatment. Here, the flow of the disease on the contact network can be interpreted as a random walk and disease treatment can be interpreted as absorption.

Linearly Stable Spatially Periodic Traveling Vegetation Stripes

Daniel Shvartsman, University of California Irvine

The phenomenon of vegetation pattern formation has been observed in a variety of ecological contexts, particularly in semi-arid ecosystems. The formation of such patterns can be seen as an adaptation by the ecosystem to resource scarcity. On desert hillsides, vegetation stripes have been observed to move uphill at a constant rate, over large timescales (mm/yr), while maintaining their profile. We seek to show that these phenomena are well described by periodic traveling wave solutions to the Klausmeier reaction diffusion advection PDE, which describes interaction of water and vegetation. Our work is focused on the analysis of this model in the case of a sloped planar domain, where the advection of water dominates diffusion. We construct a family of far-from-onset traveling wave train solutions using geometric singular perturbation theory. Our aim is to show that such solutions are linearly stable to 2D perturbations using exponential trichotomies and Lin's method.

Bayesian parameter inference in agent-based models of zebrafish patterns using TDA

Yue Liu, Purdue University

Complex patterning resulting from the collective behaviour of individual agents is present across a wide range of biological systems. Agent-based models (ABM) offer a flexible and natural framework for capturing these pattern formation processes. Inferring parameter values in these models is crucial for understanding the underlying mechanisms and for validating models. However, inferring the parameters in such models poses significant challenges. First, direct comparison of patterns using simple metrics may fail to account for the qualitative features of the pattern important to biology. Second, the high computational cost of simulating ABMs limits the number of simulations that can feasibly be conducted for inference. We demonstrate that combining topological data analysis (TDA) with approximate approximate Bayesian computation (AABC) is a computationally feasible approach to addressing these challenges. We focus on an existing agent-based model of pattern formation in zebrafish skin, and we show how to estimate parameters in this complex, stochastic model.

A Polar Approach to Fully Localised Planar Patterns
Dan J Hill, Saarland University

Rigorously studying spatially localised patterns is particularly difficult in more than one spatial dimension, since we no longer have a single time-like variable in which localisation occurs. Even existence proofs of well known localised planar patterns (such as patches of rolls or hexagons) remain open problems, let alone the stability or invasion behaviour of such structures.

This poster presents a summary of recent work on studying spatially localised planar patterns using polar coordinates and an angular Fourier expansion, where localisation is restricted to the radial direction. This includes existence proofs for a Galerkin approximation of localised dihedral patterns (with David Lloyd and Jason Bramburger), a function space theory for the analysis of radial Fourier coefficients (with Mark Groves), and work in progress on a radial Lyapunov--Schmidt-style existence proof for fully localised planar patterns (with Mark Groves).

ESMDA Applications in Epidemiology and Atmospheric Science
Emmanuel Fleurantin, George Mason University

This work demonstrates the versatility of Ensemble Smoother with Multiple Data Assimilation (ESMDA) for parameter estimation and uncertainty quantification in complex dynamical systems. We apply ESMDA to two distinct applications: a multi-population epidemiological model examining COVID-19 transmission dynamics across demographic groups, and a reduced-order atmospheric model investigating stratospheric circulation patterns. Results show ESMDA effectively handles time-varying parameters and nonlinear dynamics while providing robust uncertainty quantification. The method successfully captures both rapid epidemiological changes during intervention periods and sudden stratospheric warming events, validating against real-world observations.

Multi-patch Disease Surveillance via Wastewater Data
Erik Bergland, One Health Trust

During the COVID-19 pandemic, wastewater and environmental surveillance (WES) data proved to be extremely useful for identifying variants and estimating infection levels. In this work, we analyze the problem of optimally allocating resources for disease surveillance via WES across a heterogeneous network of subpopulations, or "patches". We propose a simplified flow-kick model of an epidemic, characterized by a Poisson process to determine the arrival of the disease to a particular subpopulation, with the spread determined by linear dynamics. By appending a simplified condition for detecting the epidemic, we can explicitly compute the expected cost burden in each patch due to running a surveillance program, as well as the cost associated with treating cases that escape surveillance. In the 2-patch case, the results of both subpopulations cooperating to minimize the total costs are compared to the Nash equilibrium obtained from each subpopulation accounting only for its own costs. Further explorations of more realistic detection models are carried out via numerical simulations.

Nonlinear stability of two-dimensional periodic waves in parabolic systems with conservation laws
Aric Wheeler, Duke University

We show that assuming the background periodic wave is diffusively stable, a stronger form of spectral stability, then the wave is nonlinearly stable even in the presence of conservation laws. The key difference with the case without conservation law analyzed by Melinand-Rodrigues is that even for extremely nice perturbations the linearized semigroup decays at a slow rate and so phase modulations play a deeper role. This work is joint with L. Miguel Rodrigues.

Needle in a haystack: First passage time of T-cells searching for cognate antigen presentation cells

Tony Wong, UCLA

The adaptive immune system requires effective recognition of foreign antigens. While in the lymph node, immune T-cells search and bind to antigen-presenting cells (APCs) that express fragments of a specific pathogen, activating the immune response. Each T-cell, however, is specific only to a very small fraction of ``cognate" APCs, leading to many dead-ends when ``non-cognate" APCs are encountered. We present a three-dimensional model whereby the searcher T-cell engages, successfully with cognate, and unsuccessfully with non-cognate, APCs through a multi-stage process, is subject to degradation, diffuses in and can exit from the lymph node. Activation of the immune response is represented as the full binding of aT-cell with its cognate APC. We calculate the probability of binding and the corresponding first passage time to successful binding to a cognate APC. We model the multi-stage engagement process with non-cognate APCs as a linear Markov chain, and via a kinetic proof-reading scheme to study which configurations lead to enhanced binding between T-cells and cognate APCs. Our mathematical model can be adapted to different assumptions on APC distribution and abundance, boundary conditions, and interaction kinetics between T-cells and APCs.