

Theorem 1 (August 2025)

There exists an 8-vertex paper torus.

Theorem 2 (July 2025)

There does not exist a 7-vertex paper torus.

[arXiv:2507.14998](https://arxiv.org/abs/2507.14998)

8 vertex existence proof:

1. Use supervised hill-climbing to find an example that is (say) 10^{-30} away from flat and (say) 10^{-4} robustly embedded.
2. Apply an effective version of the Inverse Function Theorem to show that there is a nearby embedded example that is perfectly flat.

example coords

0	1	2
0	1	3
0	2	4
0	3	5
0	4	6
0	5	6
1	2	5
1	3	4
1	4	7
1	5	7
2	4	7
2	5	6
2	6	7
3	4	6
3	5	7
3	6	7

+0.755 +0.650 z_0

-0.455 +0.345 z_1

-0.170 +1.140 z_2

+0.455 -0.345 z_1

-0.755 -0.650 z_0

-0.090 +0.665 0

+0.170 -1.140 z_2

+0.090 -0.665 0

$z_0 = 0.9805\ 0571\ 5859\ 7793\ 5561\ 6538\ 2008\ 5693$

$z_1 = 0.9902\ 8162\ 4334\ 3054\ 2934\ 3176\ 1585\ 8328$

$z_2 = 0.9765\ 3883\ 4703\ 1231\ 7624\ 1842\ 4567\ 2434$

Effective IFT: details

The Jacobian

$$\begin{bmatrix} -0.91 \dots & +0.74 \dots & +0.39 \dots \\ +0.74 \dots & -1.92 \dots & +1.14 \dots \\ +0.39 \dots & +1.14 \dots & -0.06 \dots \end{bmatrix}$$

The inverse matrix

$$\begin{bmatrix} -0.56 \dots & +0.23 \dots & +0.76 \dots \\ +0.23 \dots & -0.04 \dots & +0.64 \dots \\ +0.76 \dots & +0.64 \dots & +0.57 \dots \end{bmatrix}$$

7 vertex nonexistence proof:

1. An embedded 7-vertex torus cannot have all vertices on its convex hull boundary. (Bokowsky and Eggert, 1991.)

2. At a vertex not on the convex hull boundary, the cone angle exceeds 2π . This is by Crofton's formula.