Harmonic Analysis and Convexity
Poster Session Abstracts
Tuesday, December 10, 2024

Optimal Hölder embeddings of groups of polynomial growth
Seung-Yeun Ryoo, California Institute of Technology

Assouad's embedding theorem constructs a Hölder embedding of a doubling metric space into a Euclidean space. Obtaining Hölder embeddings that are sharp both in the distortion and target dimension is of interest in metric embedding theory. We construct such Hölder embeddings in the category of finitely generated groups of polynomial growth and of nilpotent Lie groups. Previously, in the case of the Heisenberg group, such Hölder embeddings were constructed by T. Tao (2018) via a Nash--Moser iteration scheme and a quantitative topological argument. Our proof is free of this machinery and is simpler, as we use only the basic Lie group structure and the doubling property. We expect this proof method to extend to larger classes of manifolds.

Measure comparison problems for dilations of convex bodies
Malak Lafi, Kent State University

We study a version of the Busemann-Petty problem for log-concave measures, incorporating additional information about dilates. Our primary tool for addressing this question is a new
analog of the classical large deviation principle for log-concave measures, which depend on the norm of a convex body, which we hope will be of independent interest.

New criteria for solvability of the Dirichlet boundary value problem for elliptic PDE

Martin Ulmer, Brown University

We study an elliptic operator $L:=\mathrm{div}(A\nabla \cdot)$ on the upper half space $\mathbb{R}^n_+$. We give an overview over several conditions on the behavior of the matrix A that yield $\omega\in A_\infty(\sigma)$. These include the t-independence condition, the Carleson or DKP condition, and the Dini-type condition.
We also introduce the new mixed L^1-L^∞ condition on $\partial_t A$ and an L^1 Carleson condition on $\partial_t A(x,t)$ for $\mathbb{R}^2_+$ that extends the class of elliptic operators for which we have $\omega\in A_\infty(\sigma)$, i.e. solvability of the L^p Dirichlet problem for some $1<p<\infty$.

Dimension of the Intersection of Certain Cantor Sets in the Complex Plane

Vincent Shaw, Kent State University

We consider a retained digits Cantor set T based on digit expansions with Gaussian integer base. Let F be the set all z such that the intersection of T with its translate by z is non-empty and let Fβ be the subset of F consisting of all z such that the dimension of the intersection of T with its translate by z is β times the dimension of T. We find conditions on the retained digits sets under which the set Fβ is dense in F for all $0 \leq \beta \leq 1$.

Weighted Estimate for One-Sided Singular Integrals
Ljupcho Petrov, Washington University in St. Louis

We present a proof of the one-sided A_2 theorem in dimension one, with a logarithmic loss. This theorem involves one-sided Calderón-Zygmund operators (CZOs), which generalize standard CZOs by requiring the kernel $K(x,y)$ to vanish when $x < y$. Our proof extends the Pérez-Treil-Volberg result, reducing the norm estimate to testing on indicator functions. By combining this with the weak-(1,1) estimate of Riveros and Vidal and an extrapolation theorem, we obtain the one-sided A_2 estimate with a logarithmic loss.

This is joint work with A. Walton Green and Brett D. Wick.

Volume Inequalities for L_p-Sums of Anti-blocking Bodies
Auttawich Manui, Kent State University

We study the volume inequalities of Firey L_p-sums of anti-blocking bodies. We establish Plunnecke-Ruzsa type inequality, V. Milman inequality and Roger-Shephard inequality for L_p-sums of anti-blocking bodies. The sharp constants are provided in all those inequalities.