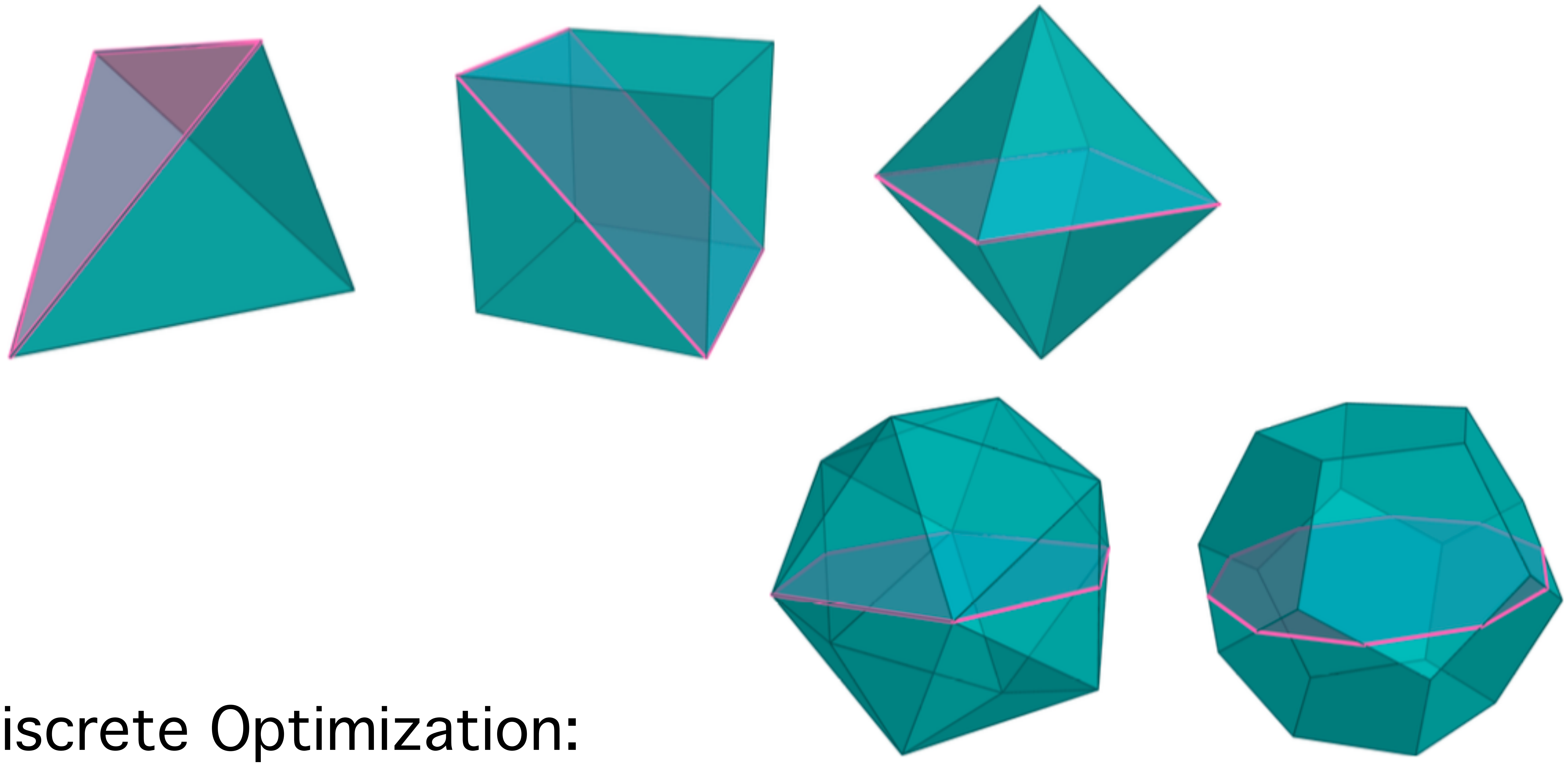
Slices of convex bodies



Chiara Meroni

ETH zürich
Institute for Theoretical Studies

Discrete Optimization:
Mathematics, Algorithms, and Computation

ICERM, August 2024

Busemann–Petty problem

1956 ⟶ 1999

ETH zürich

Let K and L be convex bodies in $\mathbb{R}^d$. Assume that for every hyperplane H through the origin,

$$\mathrm{vol}\,(K \cap H) \leq \mathrm{vol}\,(L \cap H).$$

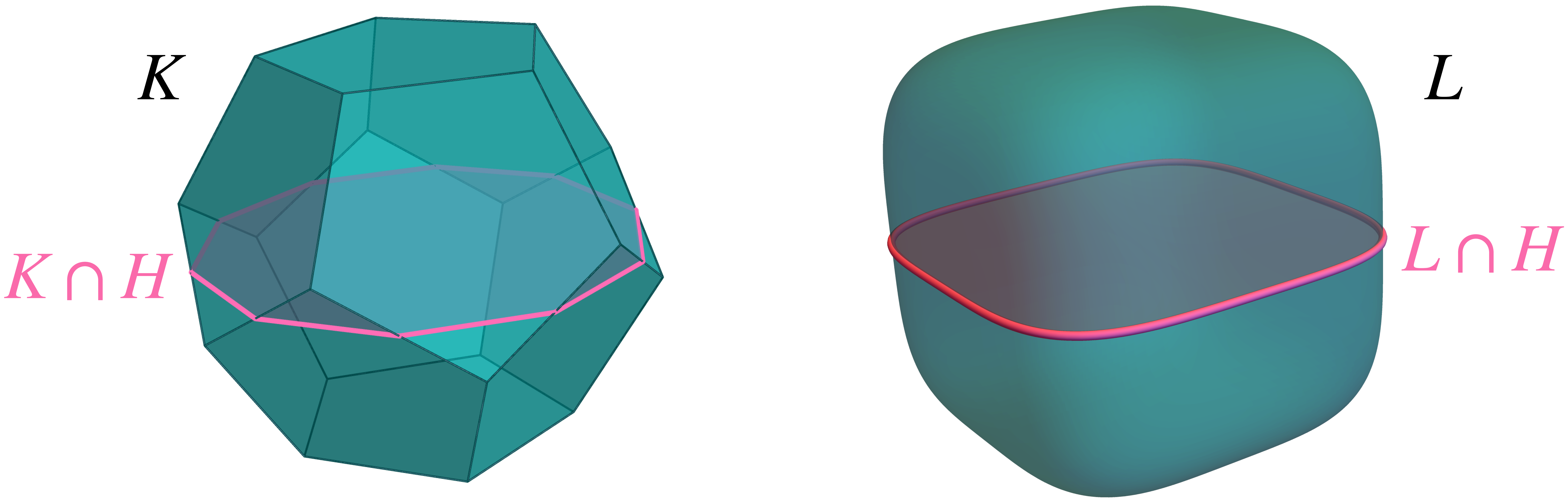
Busemann–Petty problem

1956 ⟶ 1999

ETH zürich

Let K and L be convex bodies in $\mathbb{R}^d$. Assume that for every hyperplane H through the origin,

$$\mathrm{vol}\,(K \cap H) \leq \mathrm{vol}\,(L \cap H).$$



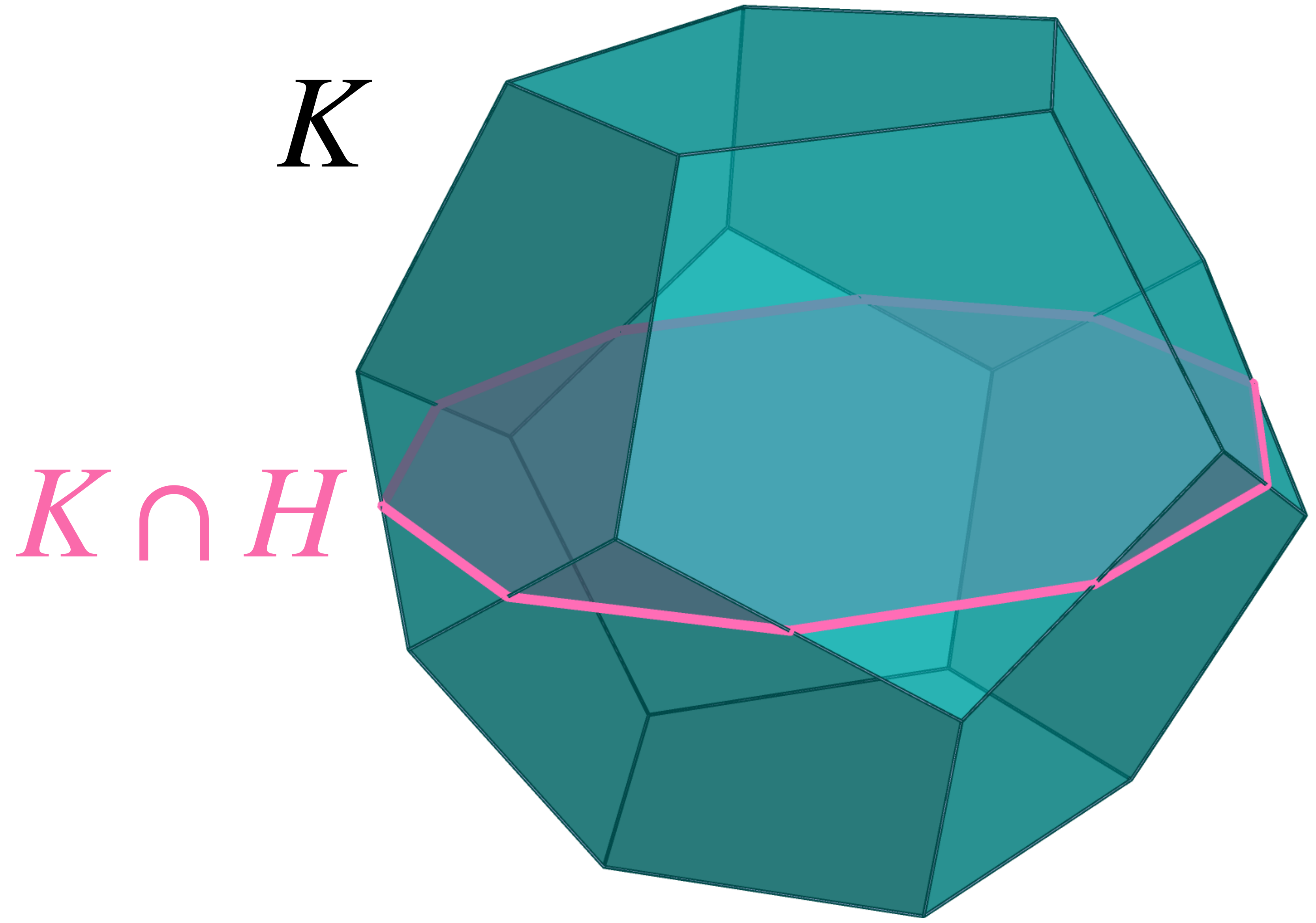
Busemann–Petty problem

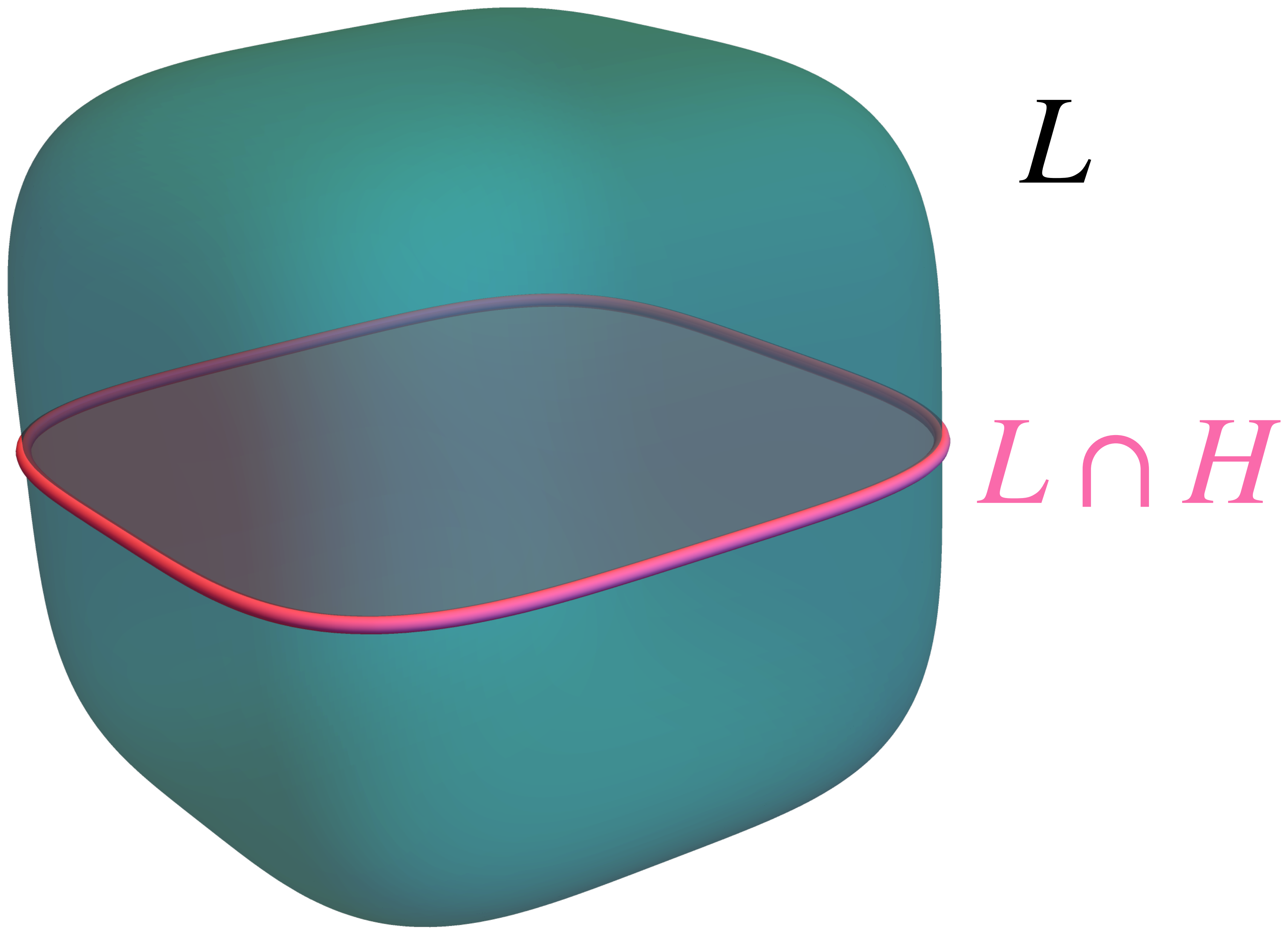
1956 ⟶ 1999

ETH zürich

Let K and L be convex bodies in $\mathbb{R}^d$. Assume that for every hyperplane H through the origin,

$$\mathrm{vol}\,(K \cap H) \leq \mathrm{vol}\,(L \cap H).$$





Does this imply that $\mathrm{vol}\,(K) \leq \mathrm{vol}\,(L)$?

Busemann–Petty problem

$\mathrm{vol}(K) \leq \mathrm{vol}(L)$?

ETH zürich

In general NOT true!

Busemann–Petty problem

$\mathrm{vol}(K) \leq \mathrm{vol}(L)$?

In general NOT true!

It holds for convex bodies in $\mathbb{R}^d$ when $d \leq 4$.

Busemann–Petty problem

$\mathrm{vol}(K) \leq \mathrm{vol}(L)$?

ETH zürich

In general NOT true!

It holds for convex bodies in $\mathbb{R}^d$ when $d \leq 4$.

Let's refine it:

Bourgain's slicing conjecture (1986):
$\mathrm{vol}(K) \leq C\ \mathrm{vol}(L)$,
where C does not depend on d.

Very active research topic in functional analysis, convex geometry, tomography,...

Busemann–Petty problem

$\text{vol}(K) \leq \text{vol}(L)$?

ETH zürich

In general NOT true!

It holds for convex bodies in $\mathbb{R}^d$ when $d \leq 4$.

Let's refine it:

Bourgain's slicing conjecture (1986):
$\text{vol}(K) \leq C\ \text{vol}(L)$,
where C does not depend on d.

Very active research topic in functional analysis, convex geometry, tomography,...

Rephrase:

Bourgain's slicing conjecture (1986):
There exists $C > 0$ such that for all d and all convex bodies $K \subset \mathbb{R}^d$ of volume 1 there exists a hyperplane H satisfying $\text{vol}(K \cap H) > \frac{1}{C}$.

Best slices

ETH zürich

This motivates the study of extremal slices of convex bodies

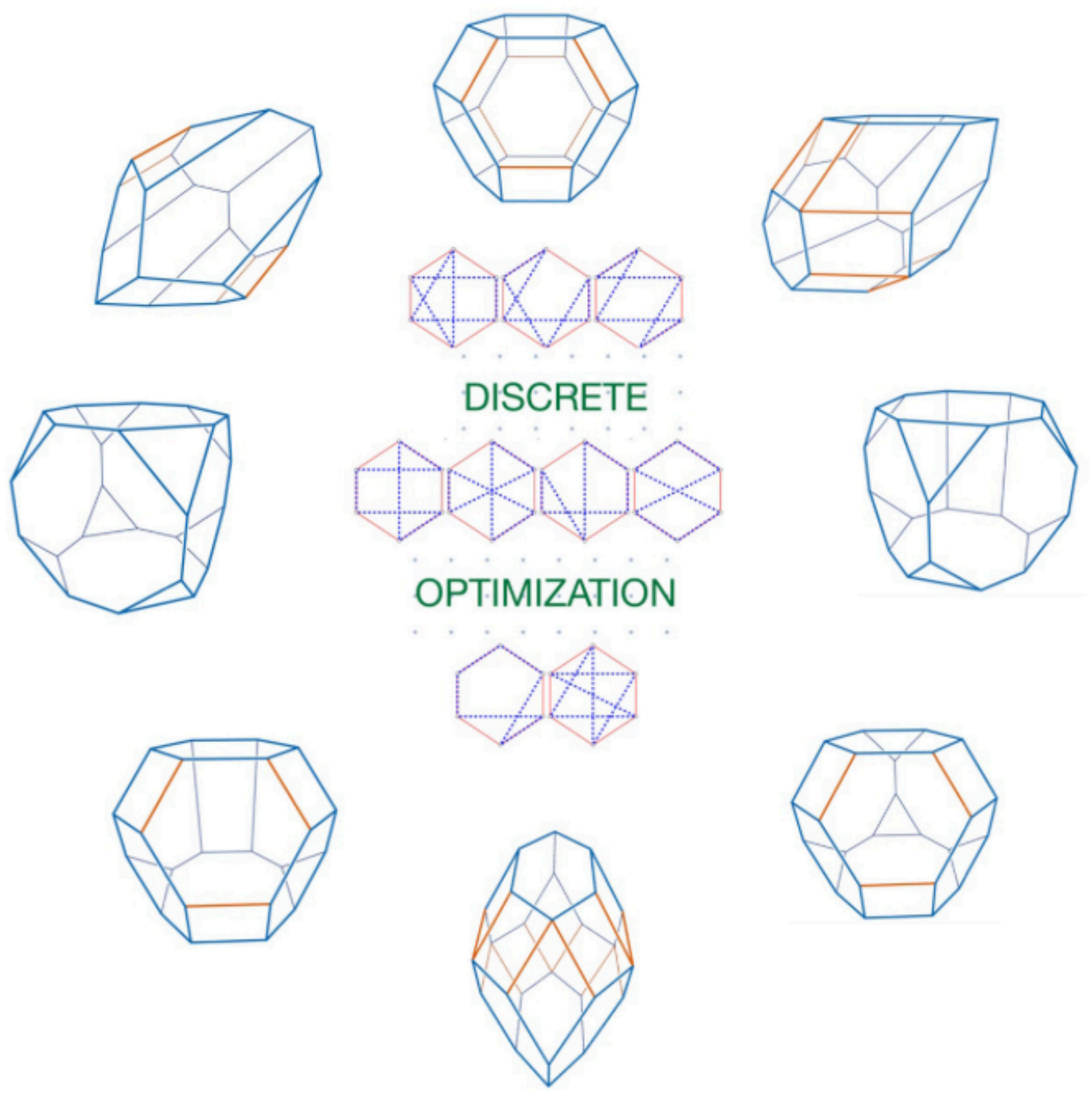
Best slices

ETHzürich

This motivates the study of extremal slices of convex bodies

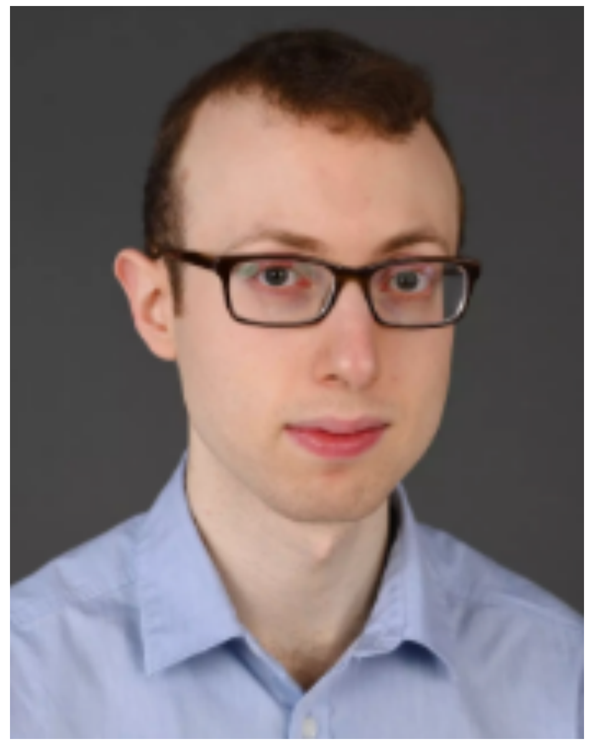
Many collaborators
and collaborations

that started at



Jesús A. De Loera

Mauricio Velasco



Jared Miller

Marie-Charlotte Brandenburg

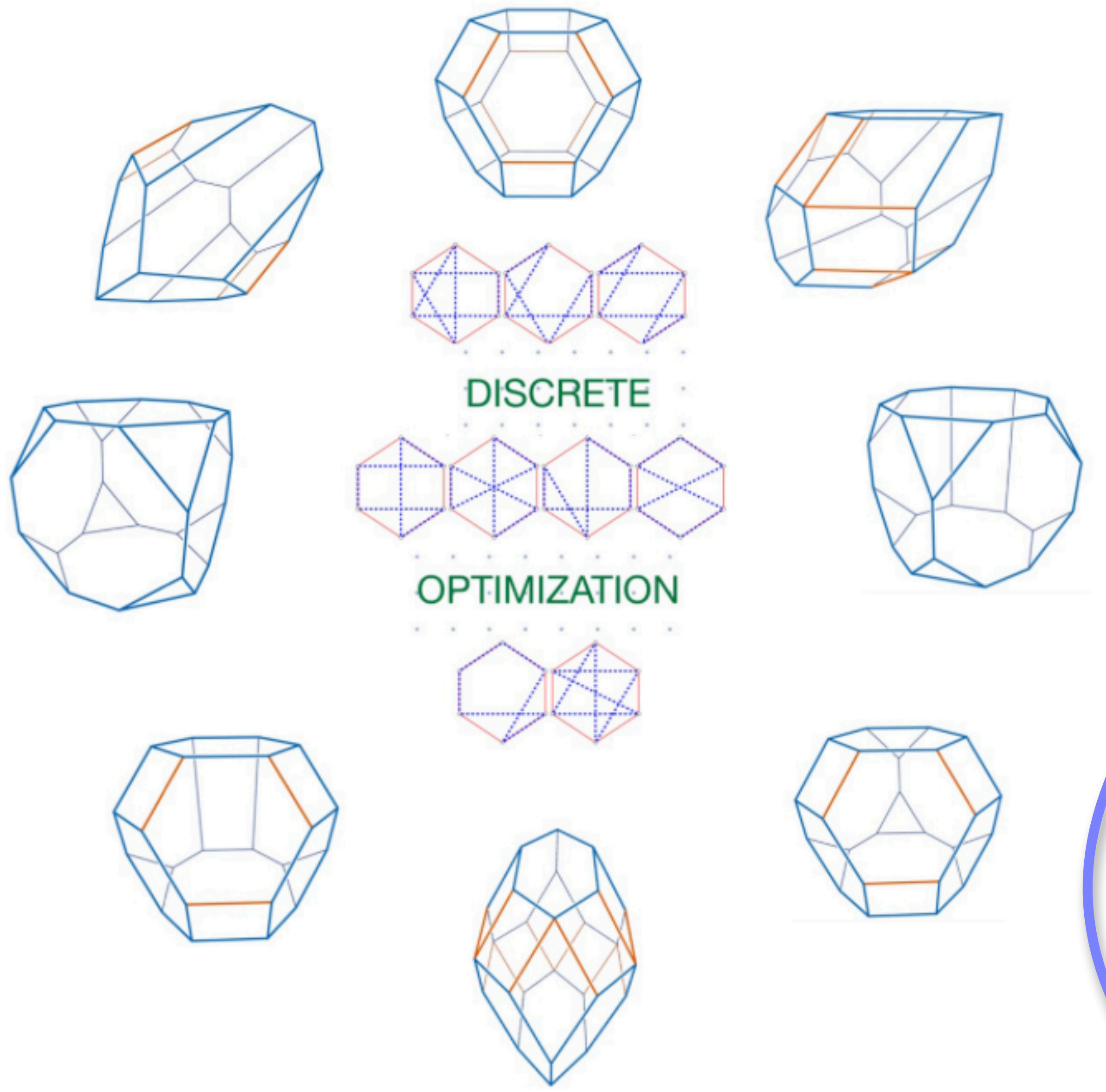
Matteo Tacchi

Best slices

This motivates the study of extremal slices of convex bodies

Many collaborators
and collaborations

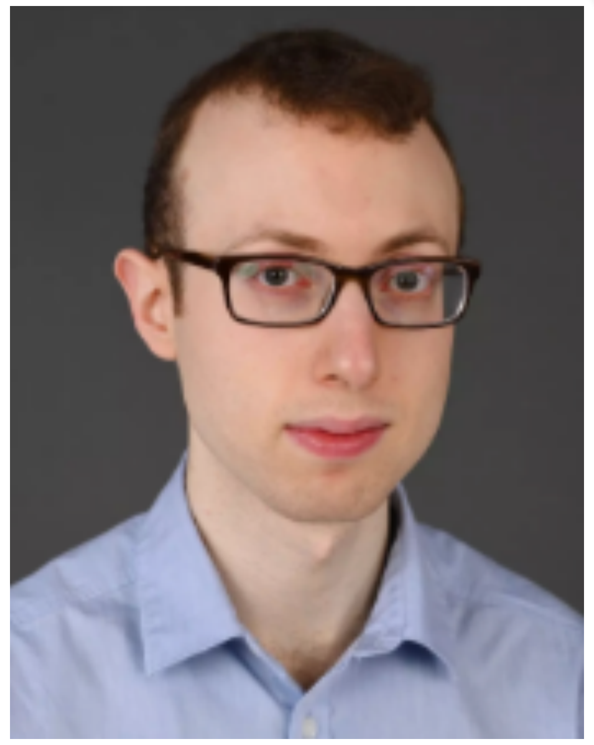
that started at




Jesús A. De Loera


Marie-Charlotte Brandenburg


Mauricio Velasco


Jared Miller


Matteo Tacchi

B. I. (= Before ICERM)

Back to Busemann and Petty: problem solved using intersection bodies

B. I. (= Before ICERM)

Back to Busemann and Petty: problem solved using intersection bodies

→ encode the volume of the central slices of a convex body

B. I. (= Before ICERM)

ETH zürich

Back to Busemann and Petty: problem solved using intersection bodies

encode the volume of the central slices of a convex body

Home > Beiträge zur Algebra und Geometrie / Contributions to Algebra and Geometry

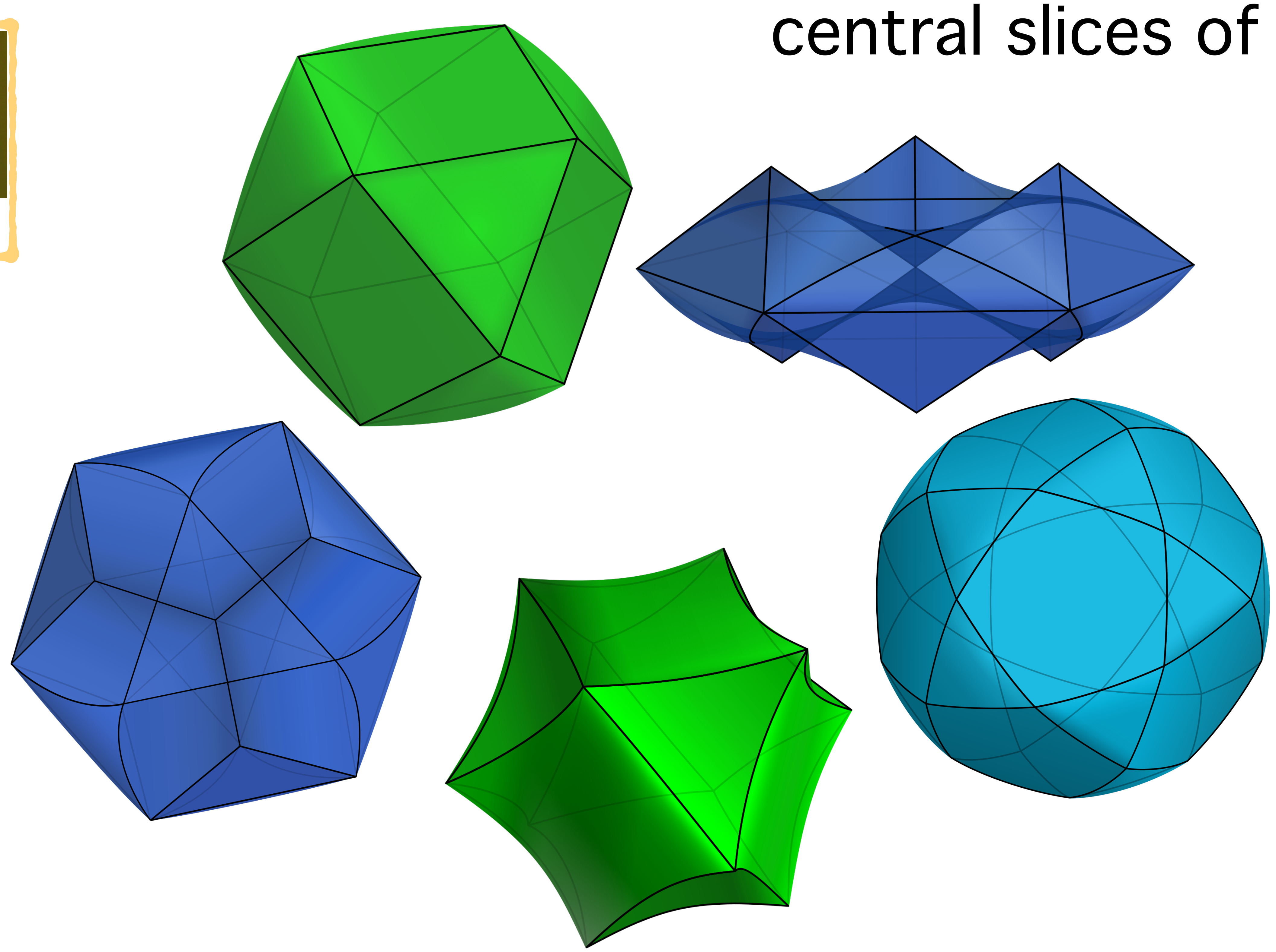
Intersection bodies of polytopes

Katalin Berlow, Marie-Charlotte Brandenburg, Chiara Meroni & Isabelle Shankar

Home > Journal of Algebraic Combinatorics > Article

Intersection bodies of polytopes: translations and convexity

Marie-Charlotte Brandenburg & Chiara Meroni



D. I. (= During ICERM)

Back to Busemann and Petty: problem solved using intersection bodies

→ encode the volume of the central slices of a convex body

D. I. (= During ICERM)

ETH zürich

Back to Busemann and Petty: problem solved using intersection bodies

→ encode the volume of the ~~central~~ slices of a convex body

I want all of them!!!

D. I. (= During ICERM)

ETH zürich

Back to Busemann and Petty: problem solved using intersection bodies

encode the volume of the ~~central~~ slices of a convex body

I want all of them!!!

arXiv > math > arXiv:2304.14239

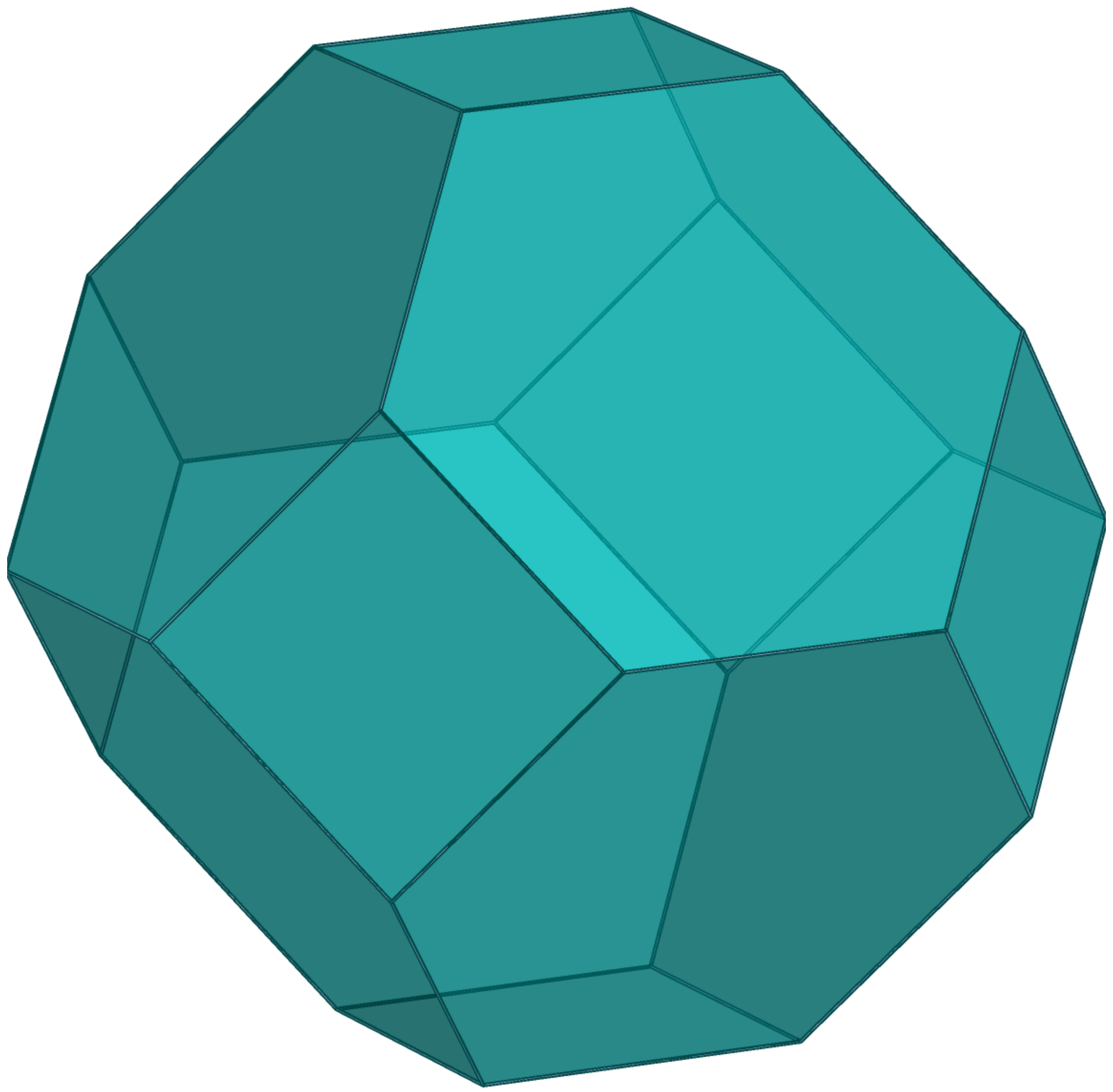
The Best Ways to Slice a Polytope

Marie-Charlotte Brandenburg, Jesús A. De Loera, Chiara Meroni

to appear in Mathematics of Computation

Goal: find the "best" slice of P

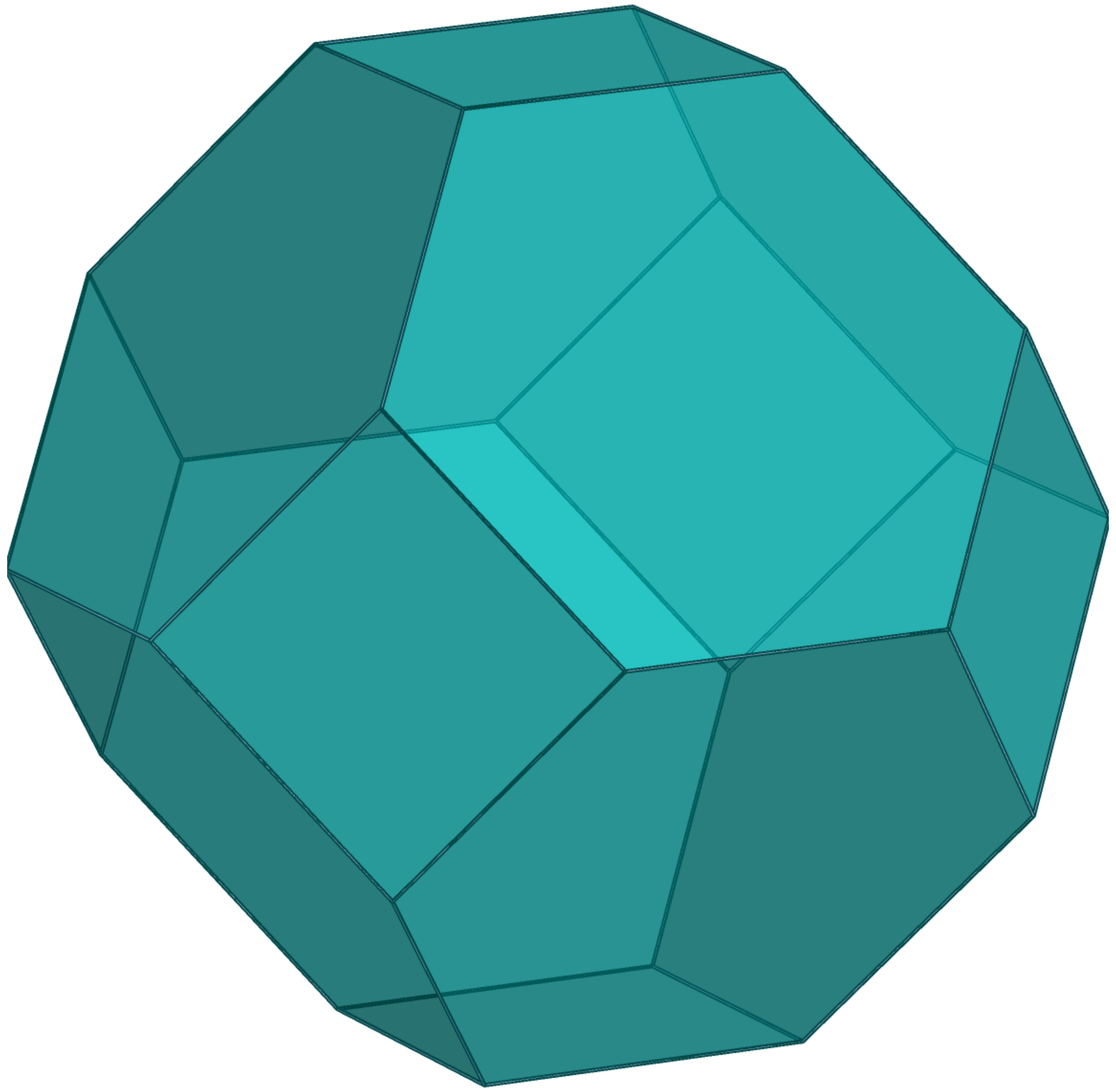
$P =$ permutahedron



Goal: find the “best” slice of P

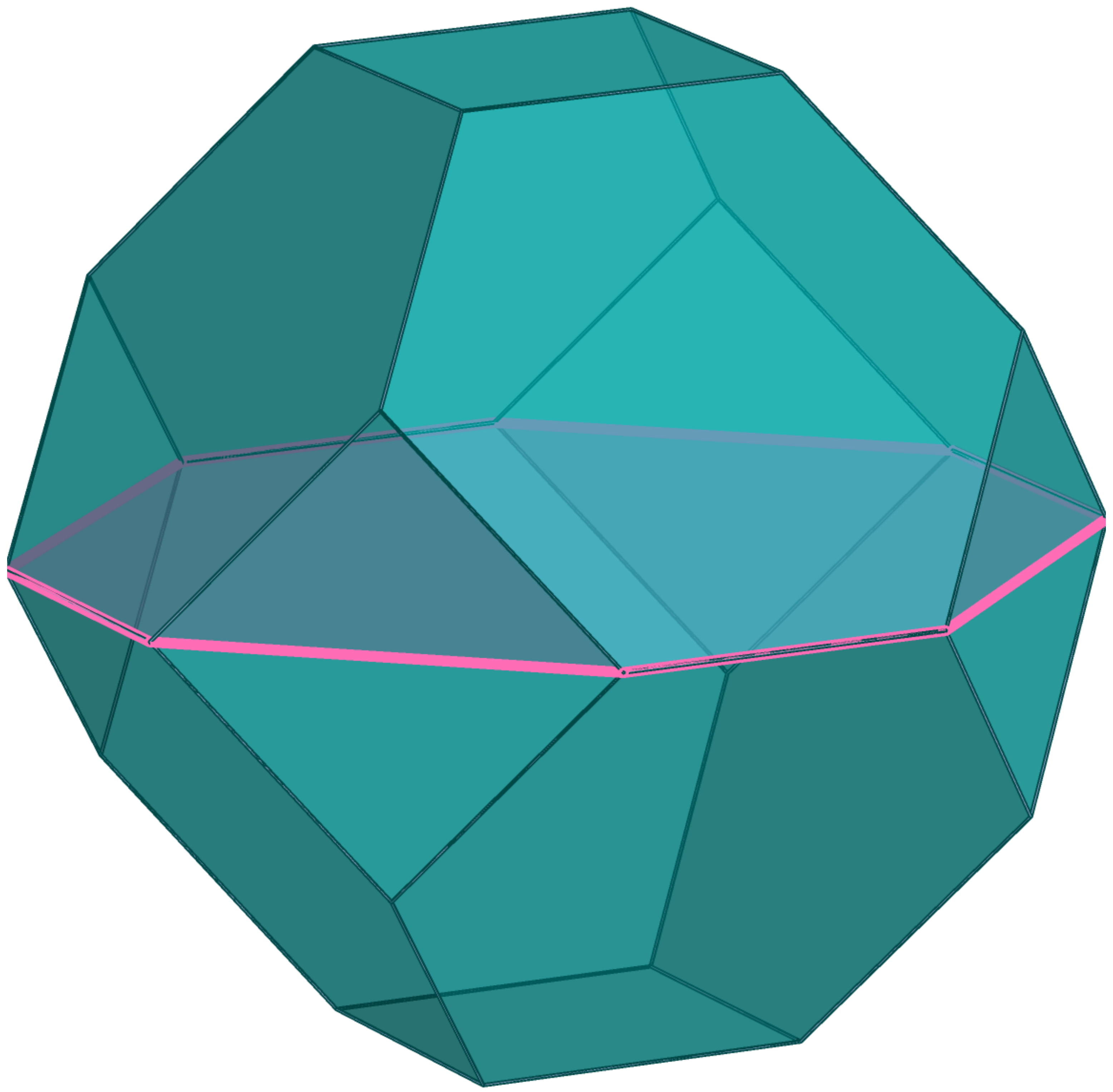
$P =$ permutahedron

best = largest volume

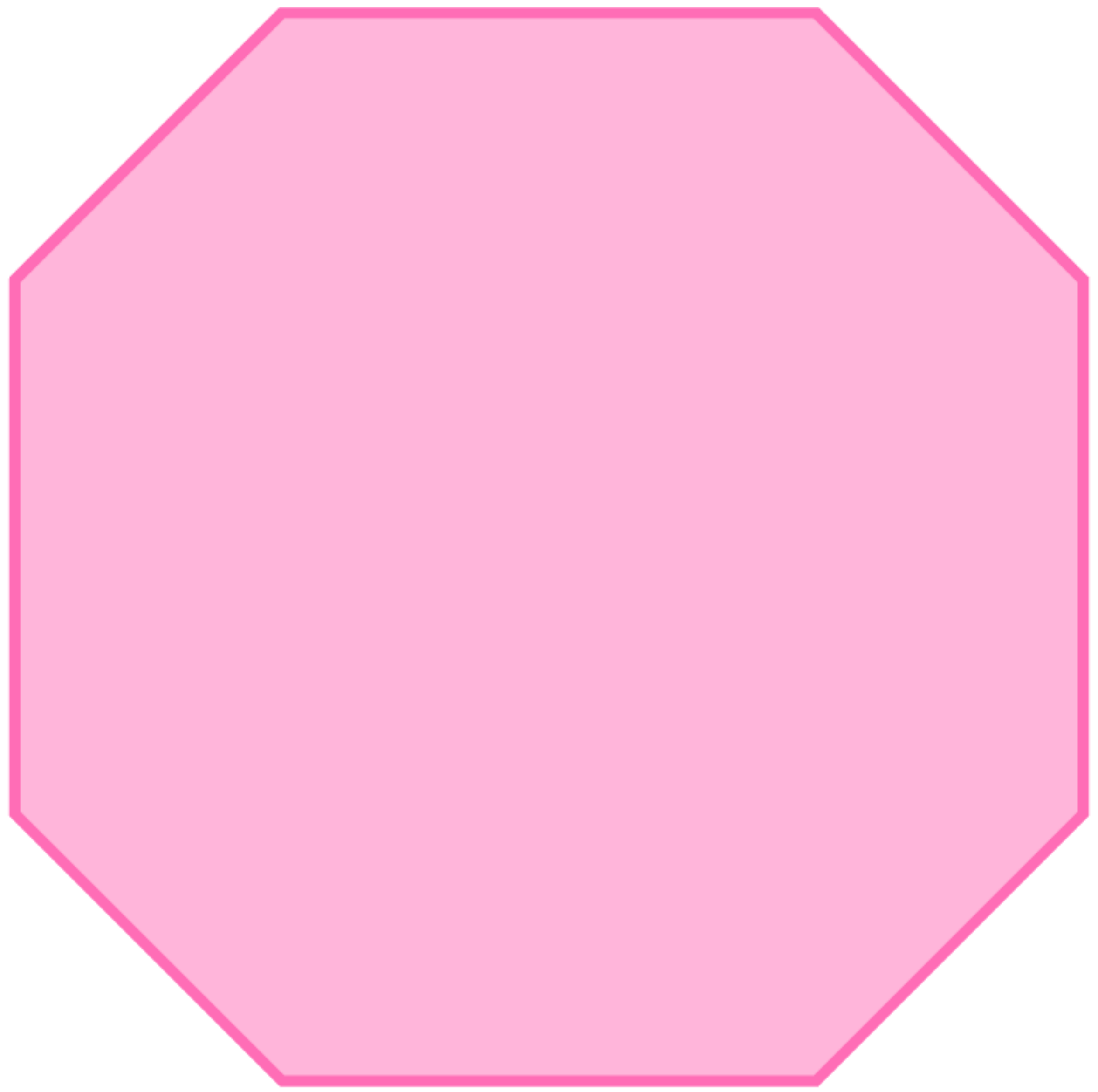


Goal: find the "best" slice of P

P = permutahedron



best = largest volume

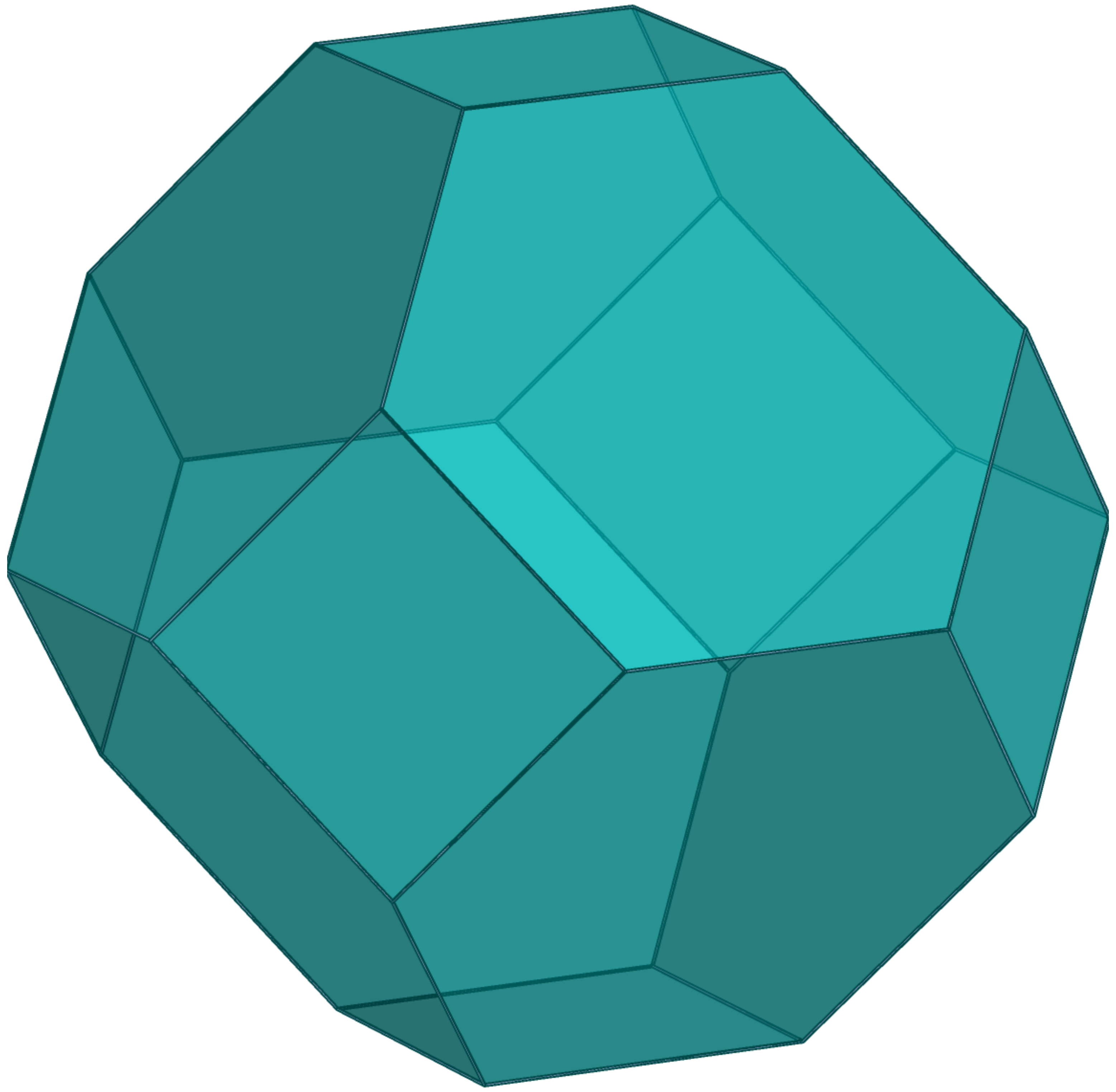


Goal: find the “best” slice of P

ETH zürich

P = permutahedron

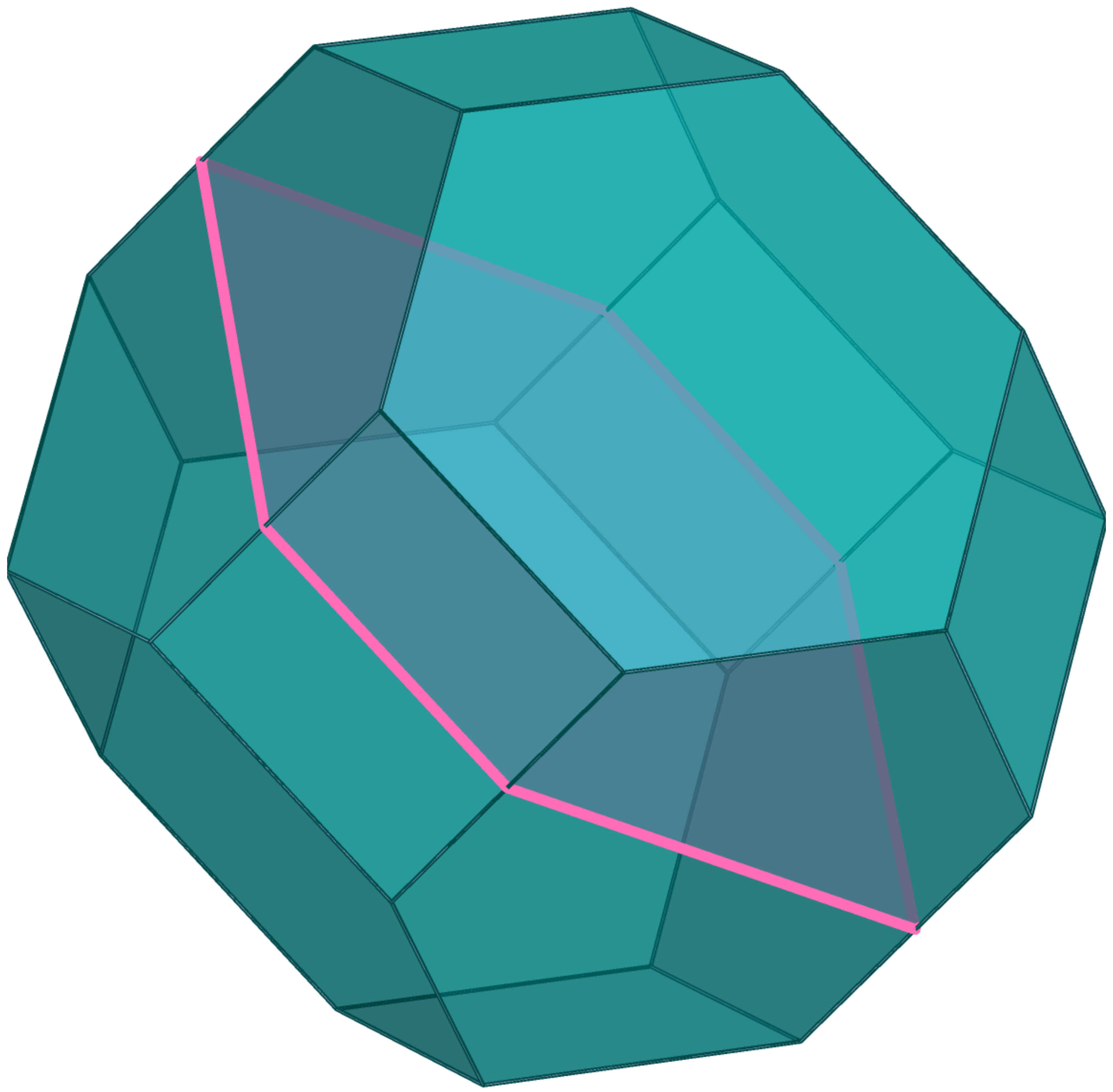
best = smallest volume



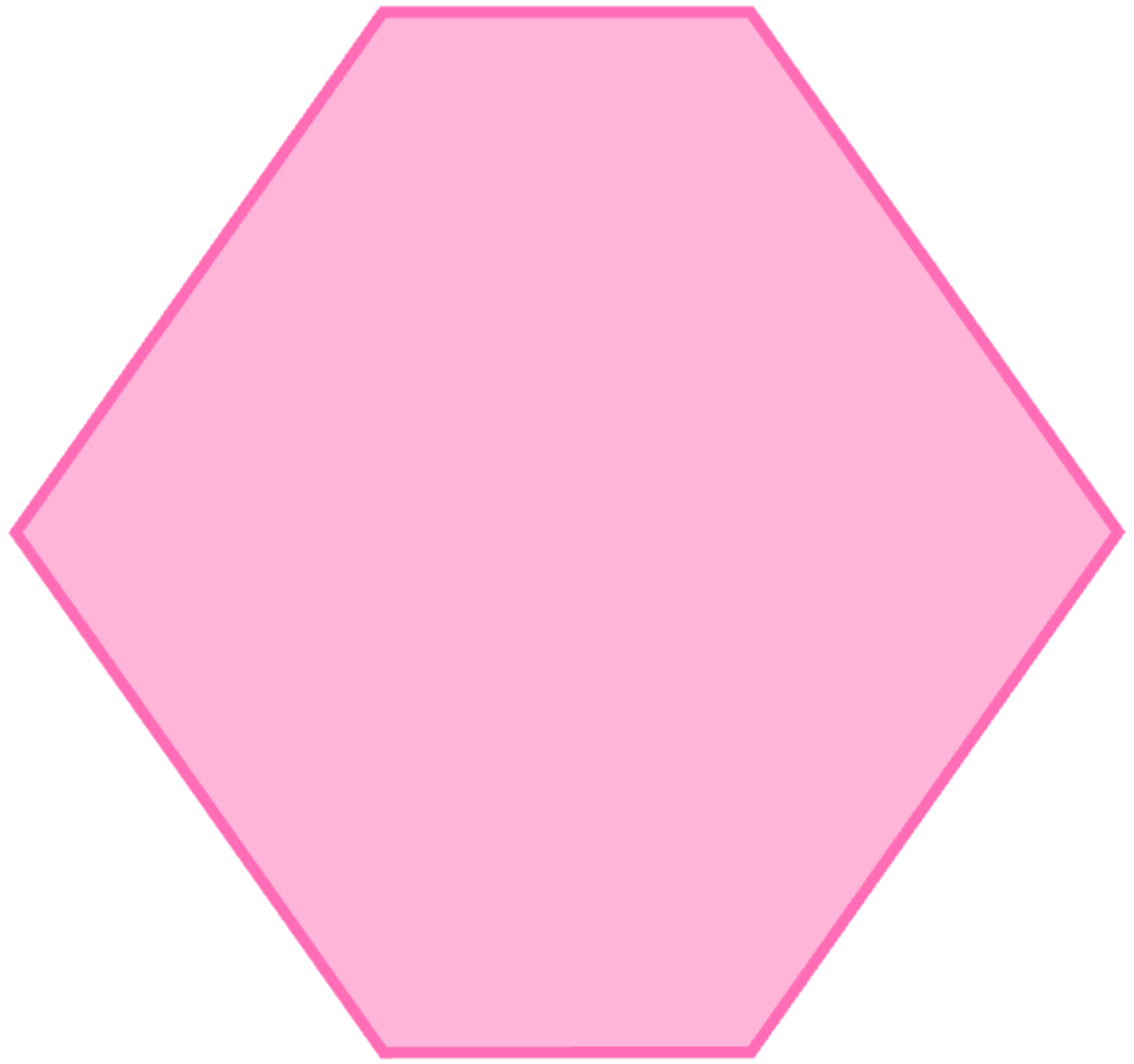
Goal: find the "best" slice of P

ETH zürich

P = permutahedron



best = smallest volume

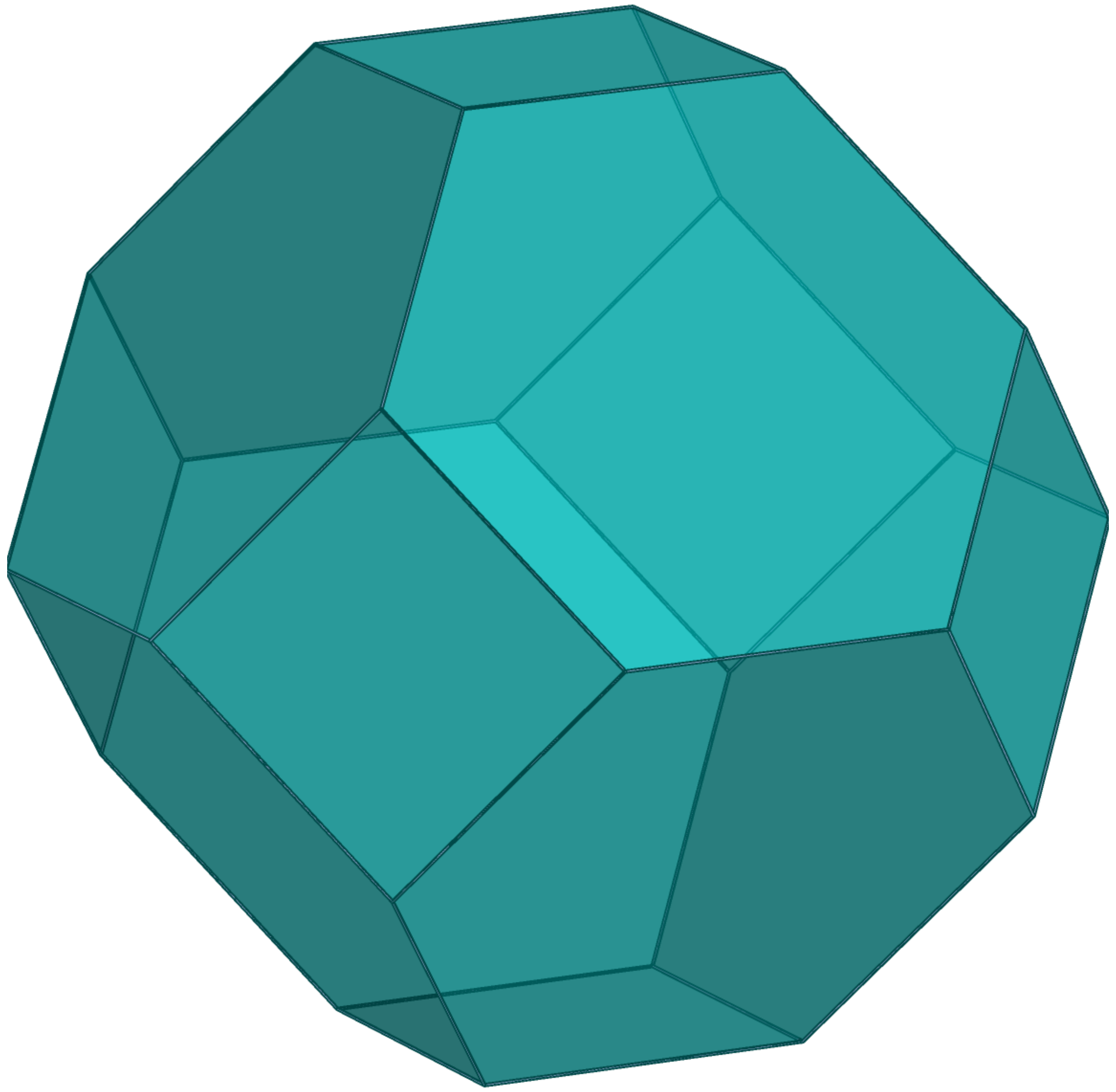


Goal: find the “best” slice of P

ETH zürich

P = permutahedron

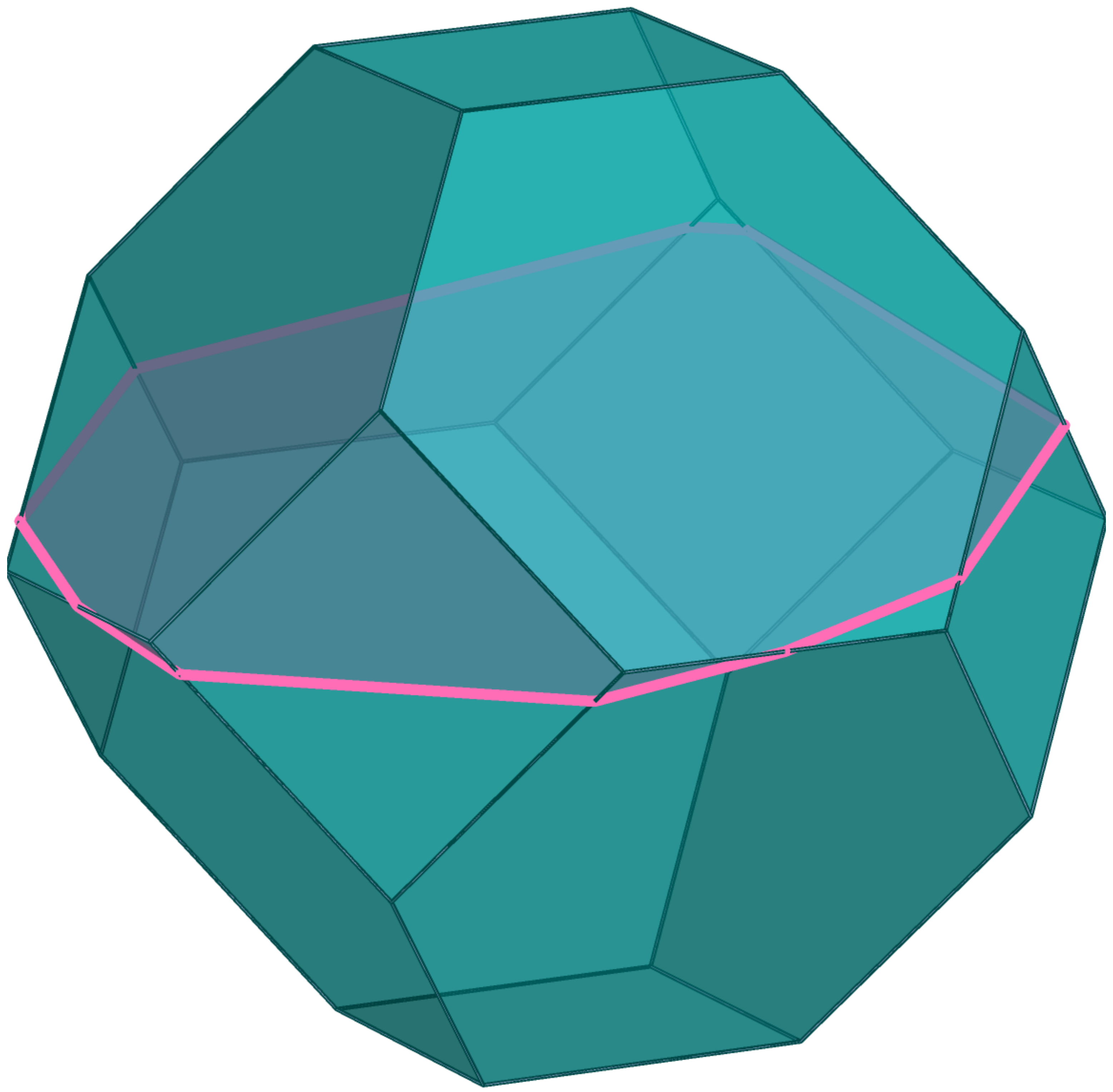
best = largest #vertices



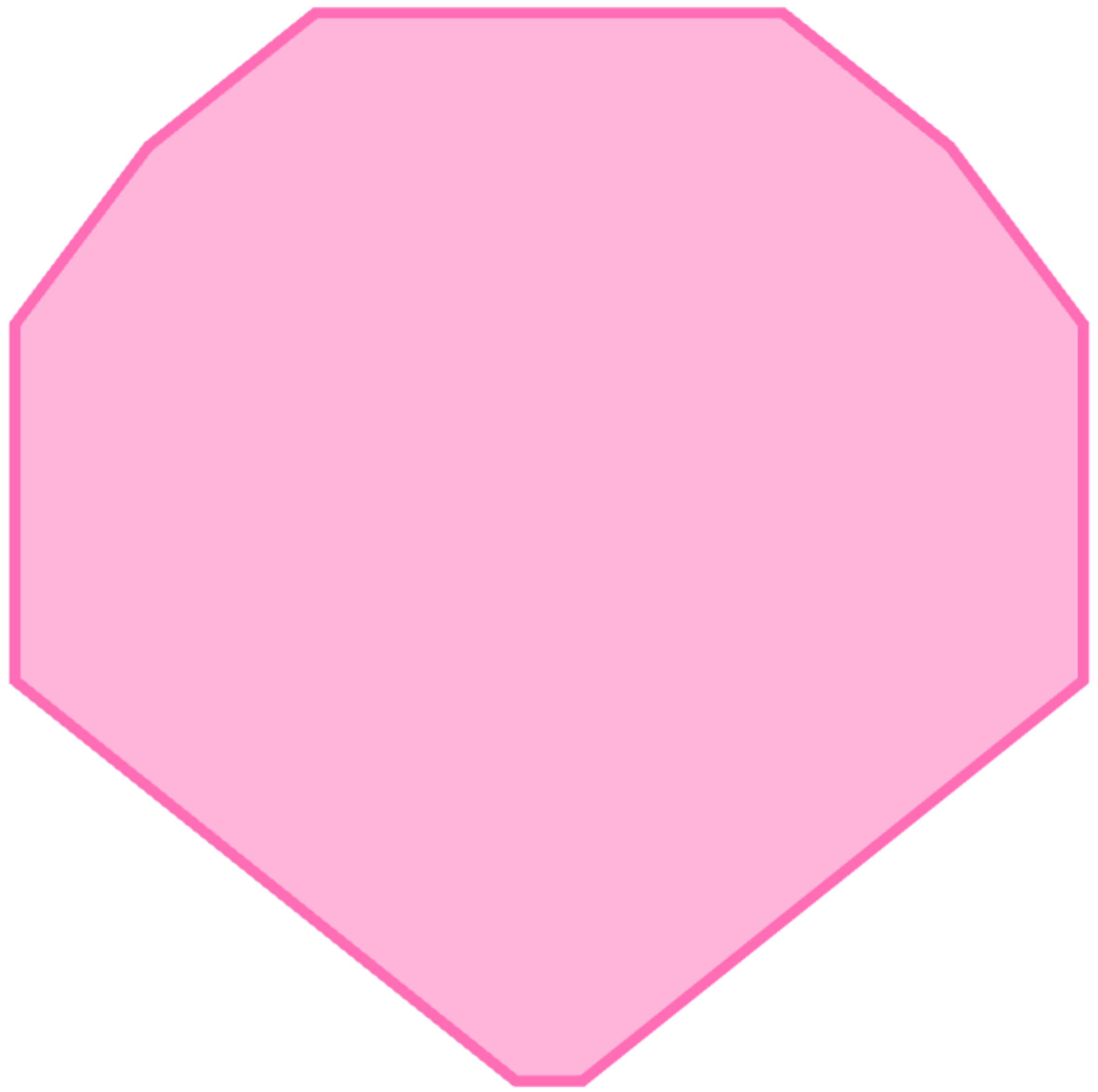
Goal: find the "best" slice of P

ETH zürich

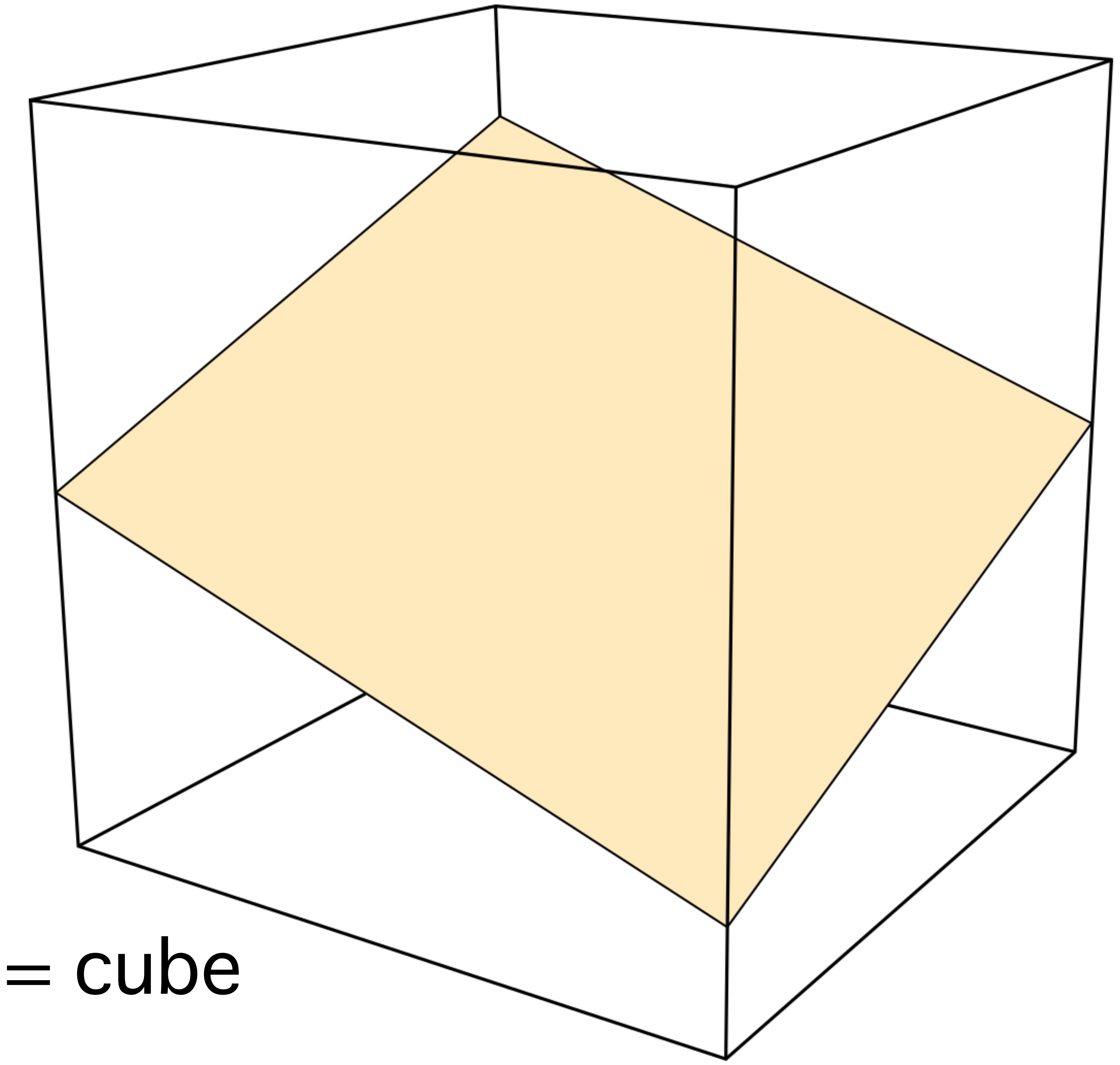
$P =$ permutahedron



best = largest #vertices



Main idea

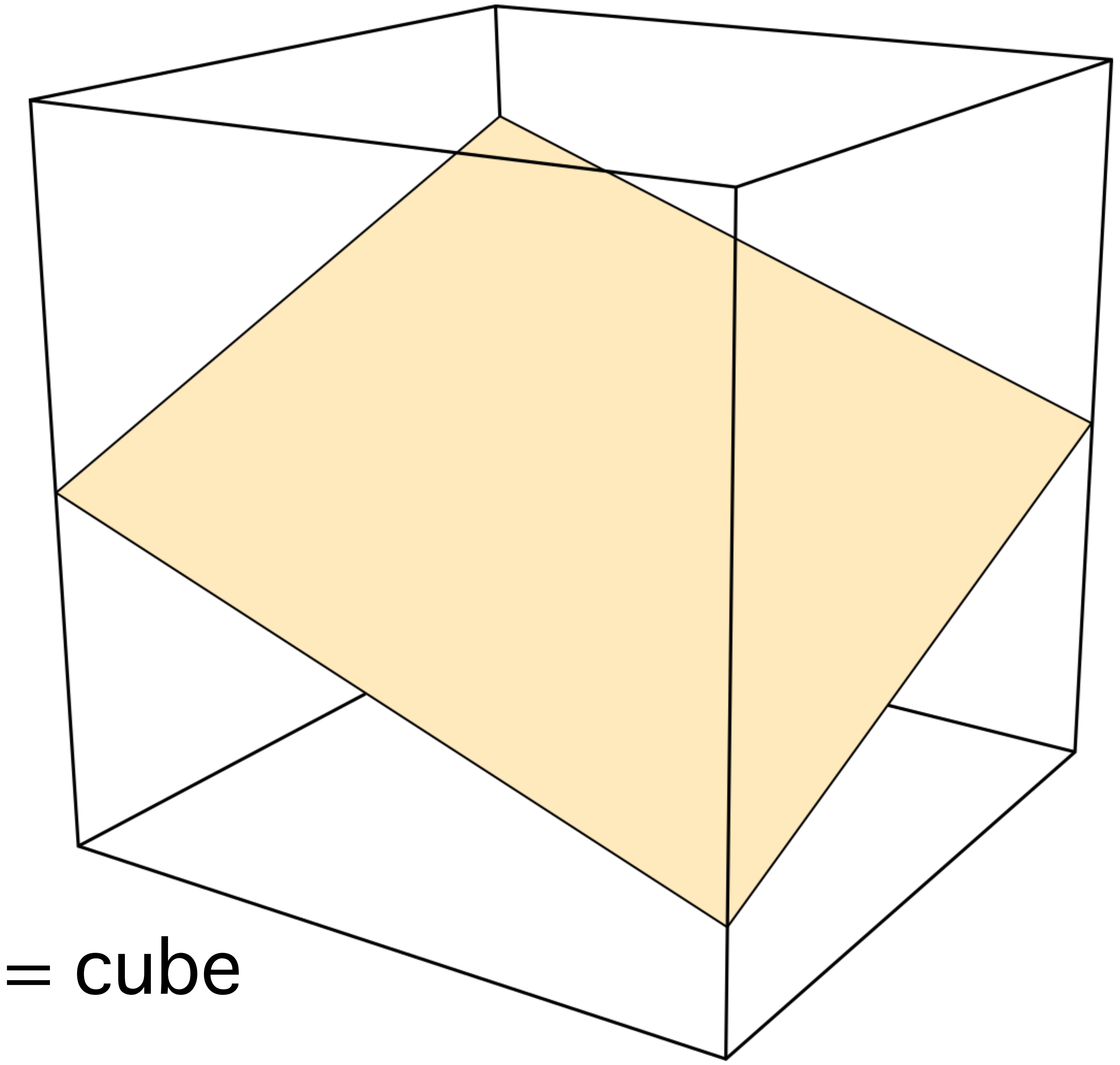


$P =$ cube

Small perturbations (generically) preserve the combinatorial type

Main idea

ETH zürich



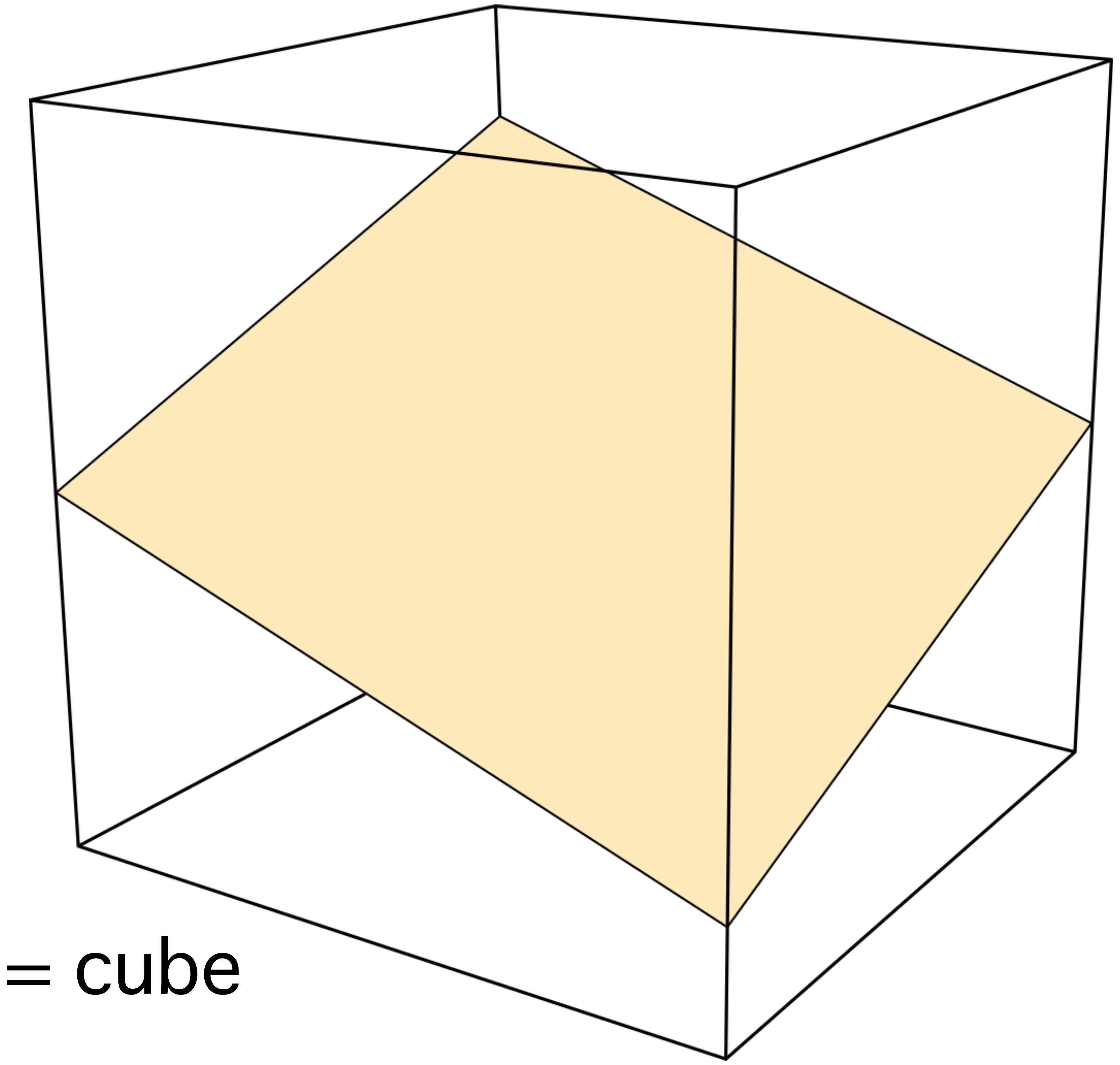
$P =$ cube

Small perturbations (generically) preserve the combinatorial type

The combinatorial type (possibly) changes when the hyperplane crosses a vertex

Main idea

ETH zürich



$P =$ cube

Small perturbations (generically) preserve the combinatorial type

The combinatorial type (possibly) changes when the hyperplane crosses a vertex

Strategy: group the slices with the same combinatorial type

Hyperplane arrangements

Construction: slices with the same combinatorial type belong to the same cell of a certain (combination of) hyperplane arrangement(s)

Hyperplane arrangements

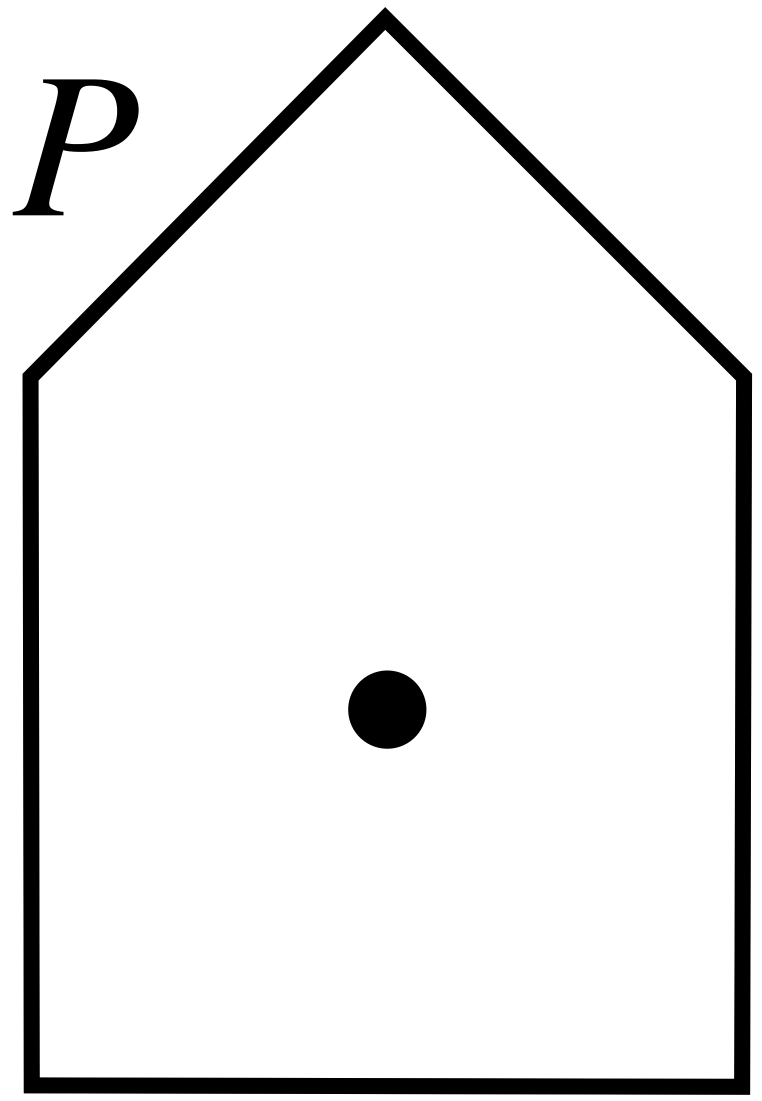
Construction: slices with the same combinatorial type belong to the same cell of a certain (combination of) hyperplane arrangement(s)

	Hyperplane arrangement	Reference object
↻	Central arrangement Cocircuit arrangement	Intersection body Oriented matroid
↗	Parallel arrangement Sweep arrangement	Fiber polytope Sweep polytope

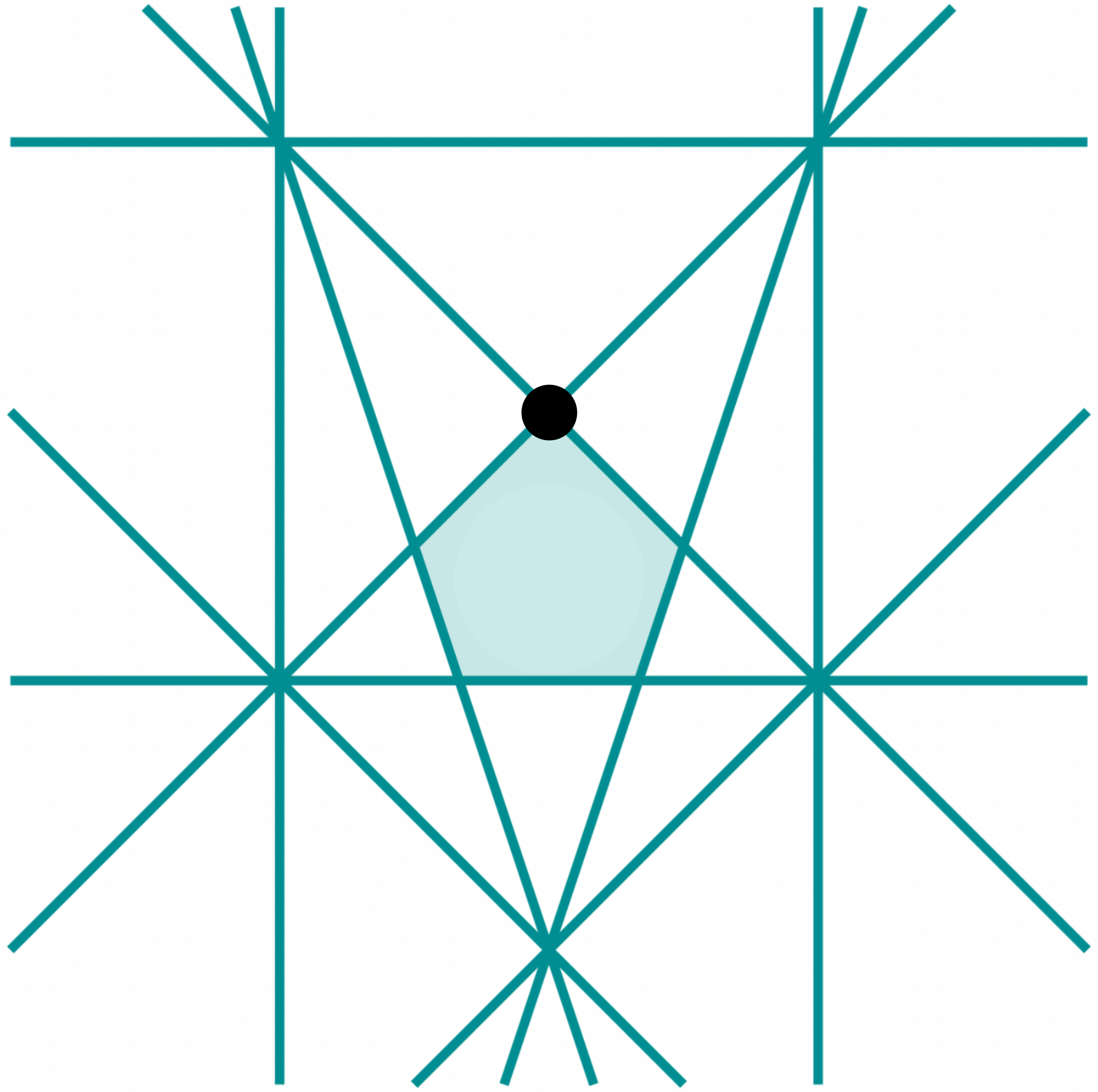
Hyperplane arrangements

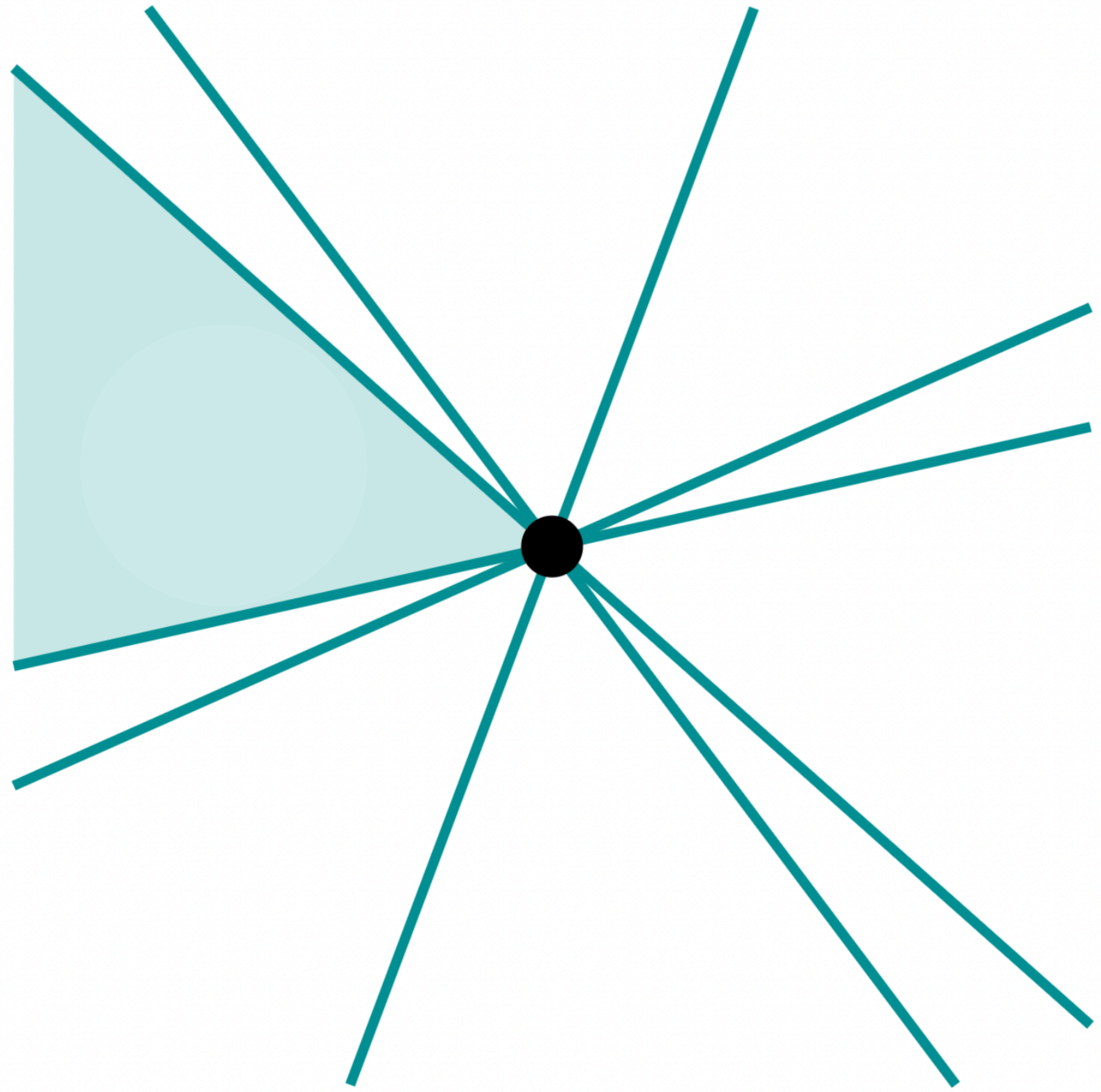
ETH zürich

Construction: slices with the same combinatorial type belong to the same cell of a certain (combination of) hyperplane arrangement(s)



Cocircuit arrangement



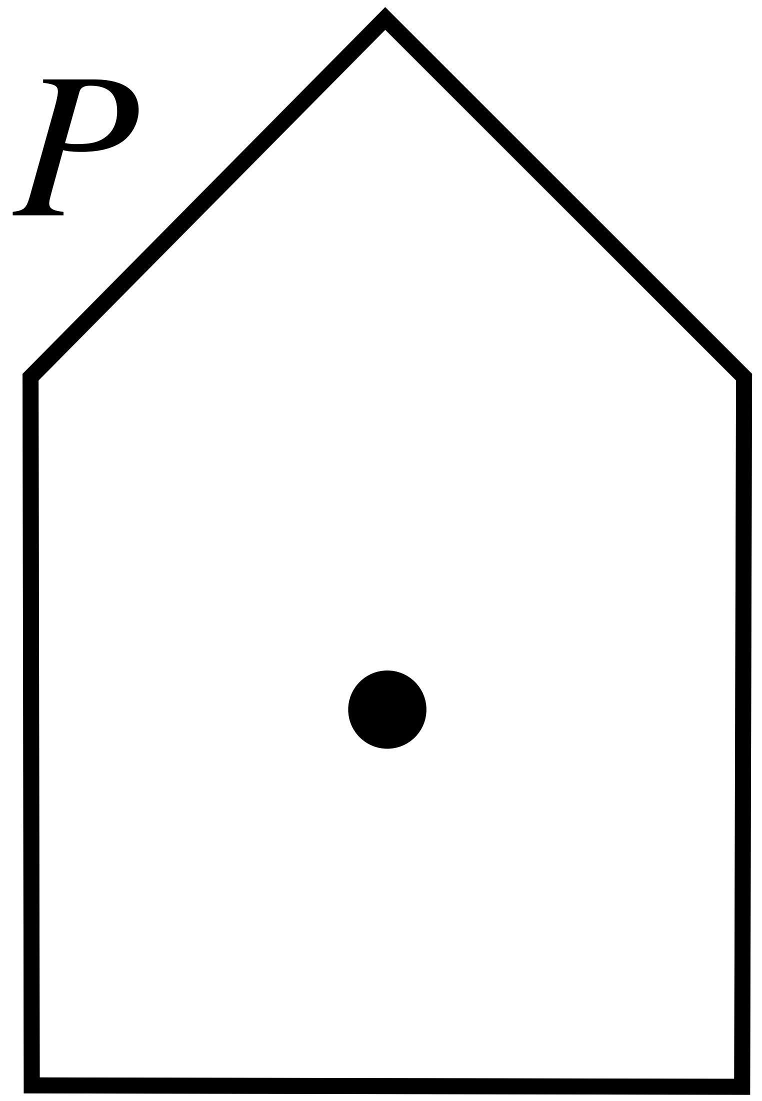


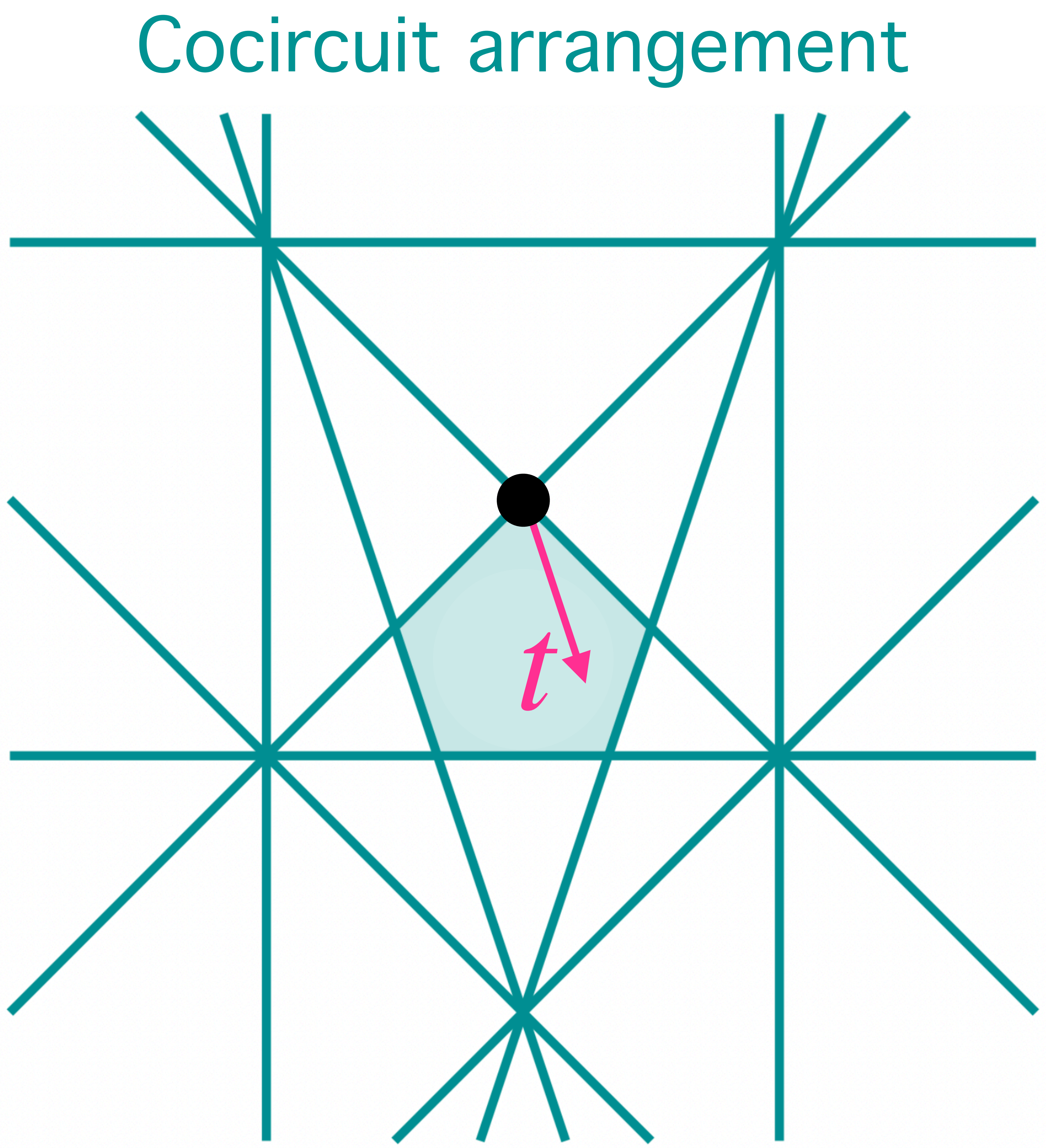
Central arrangement

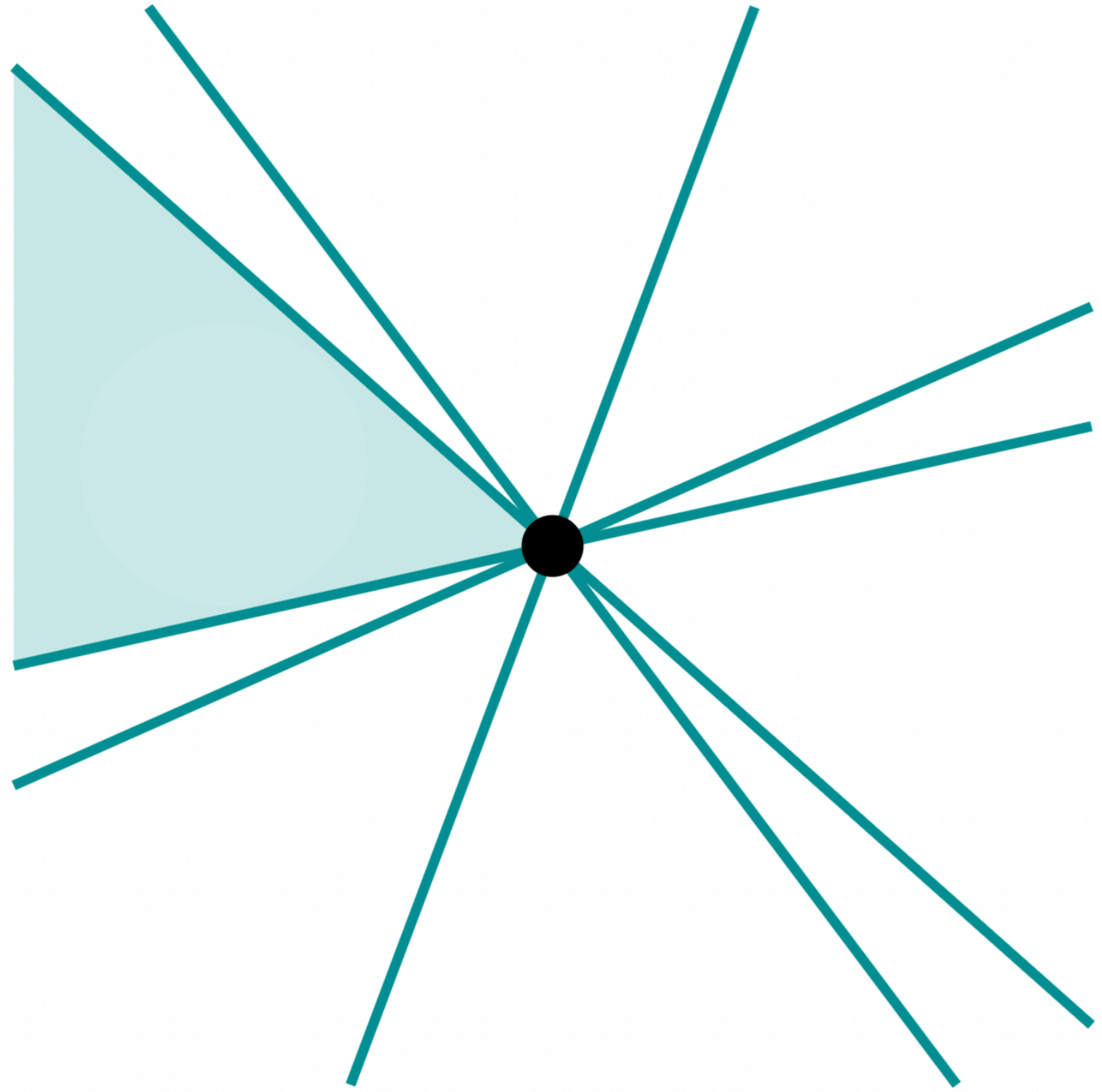
Hyperplane arrangements

ETH zürich

Construction: slices with the same combinatorial type belong to the same cell of a certain (combination of) hyperplane arrangement(s)



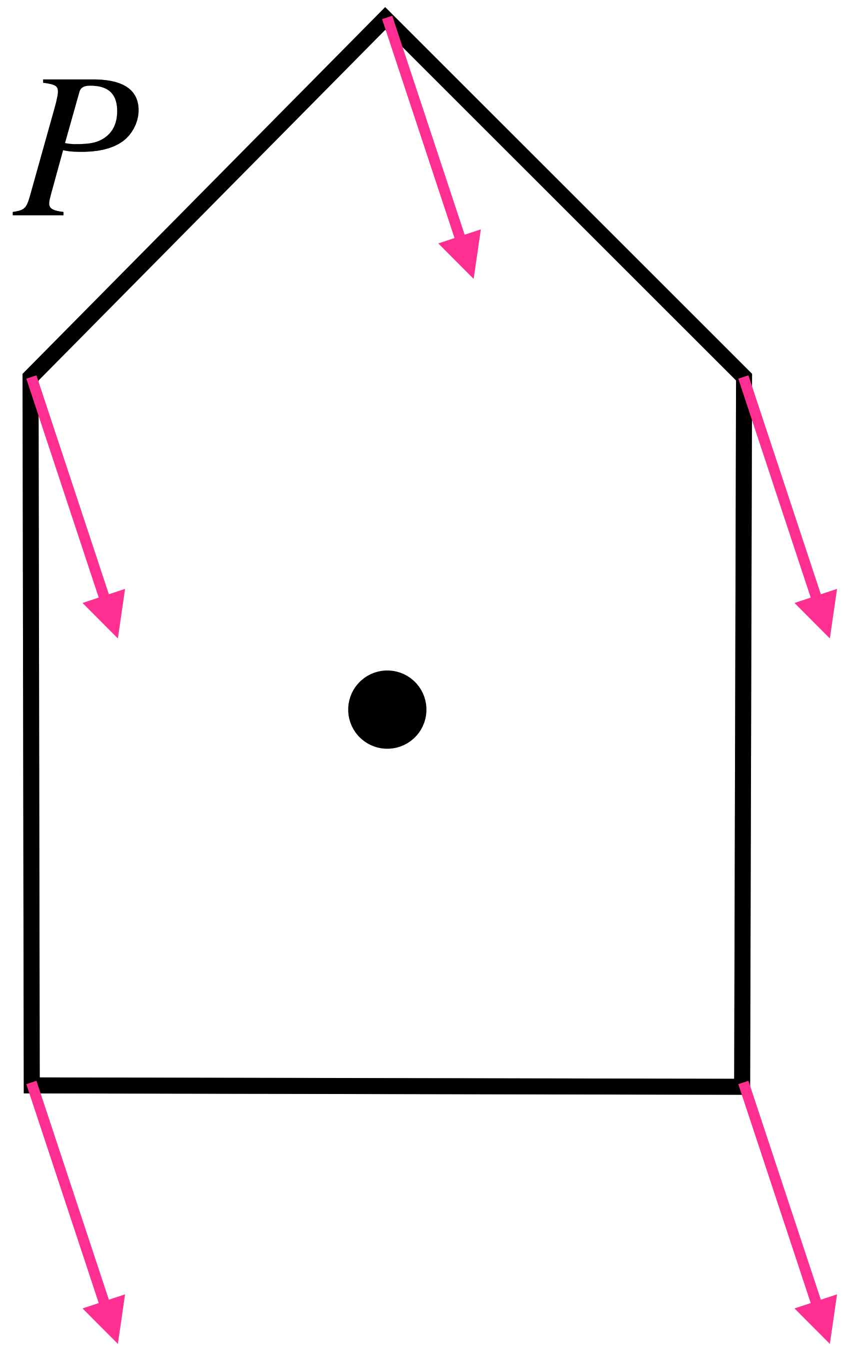


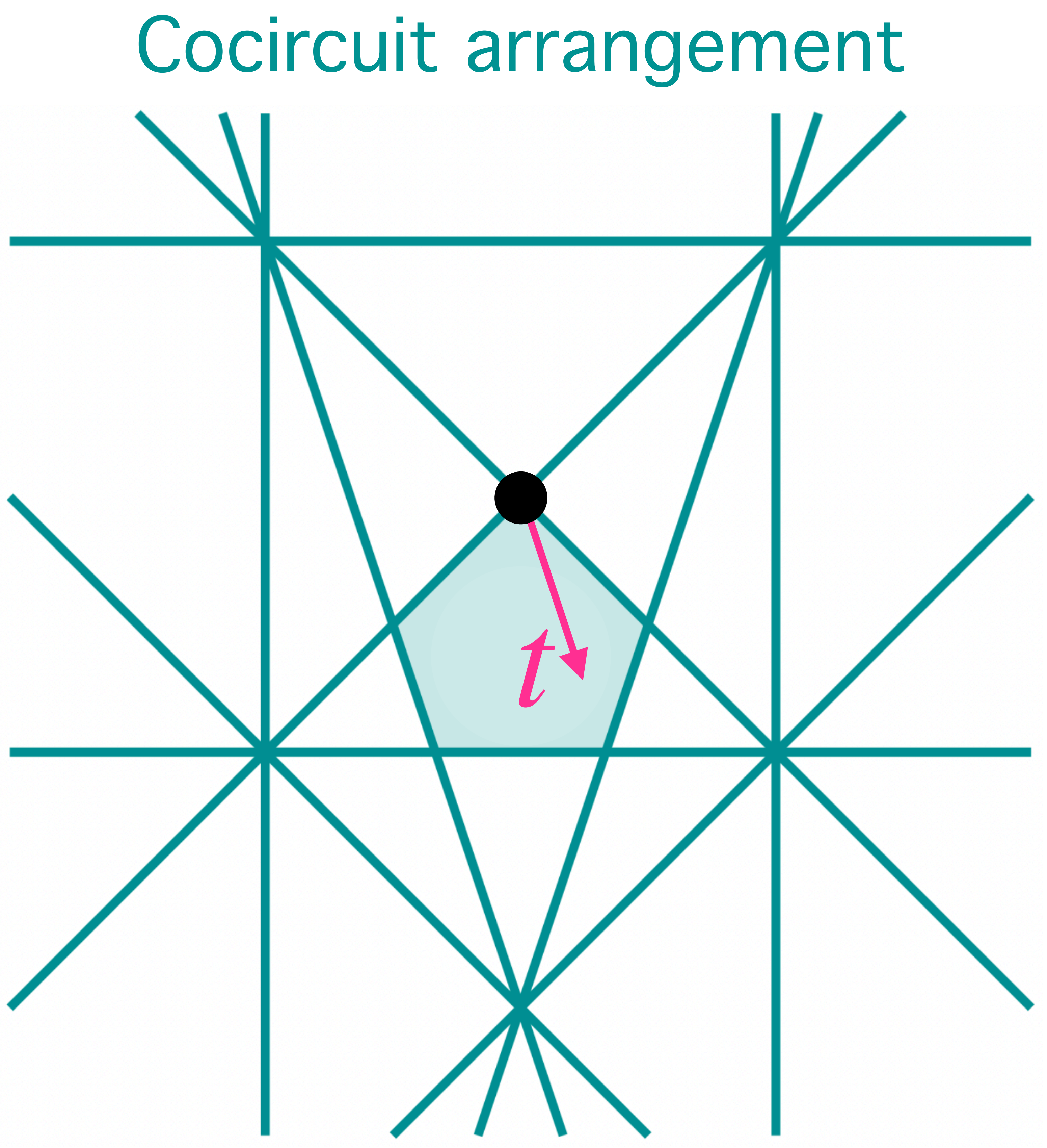


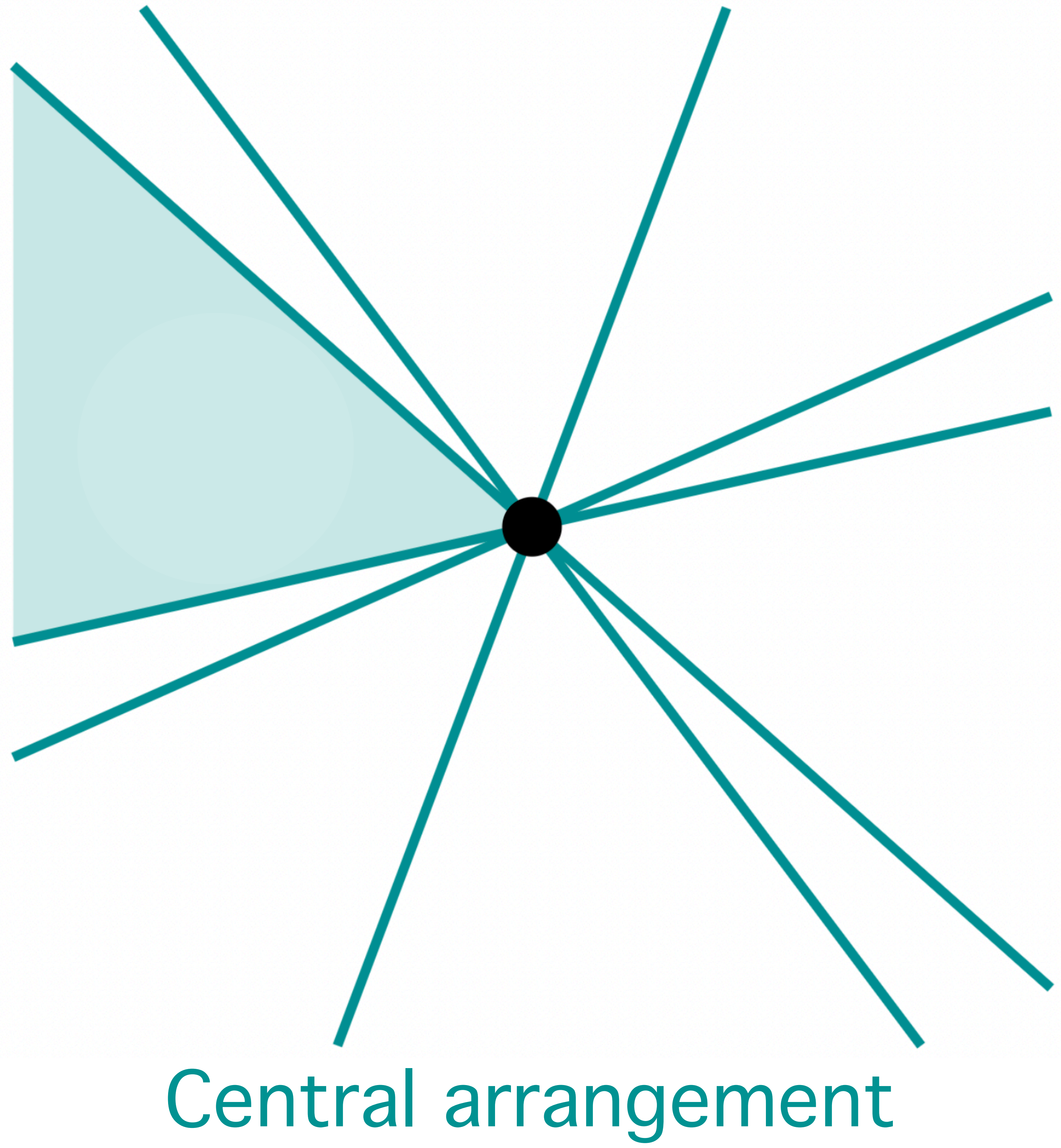
Hyperplane arrangements

ETH zürich

Construction: slices with the same combinatorial type belong to the same cell of a certain (combination of) hyperplane arrangement(s)



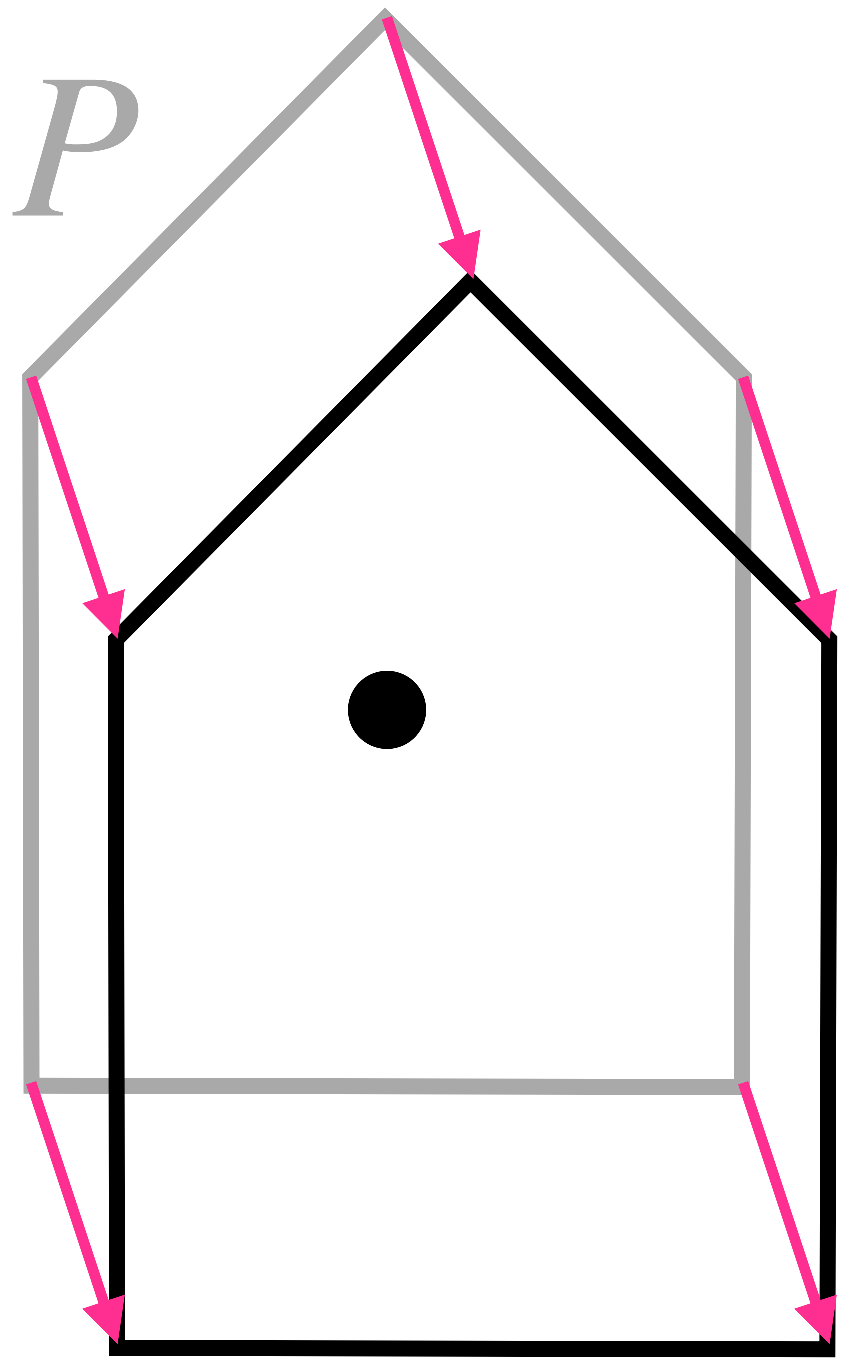




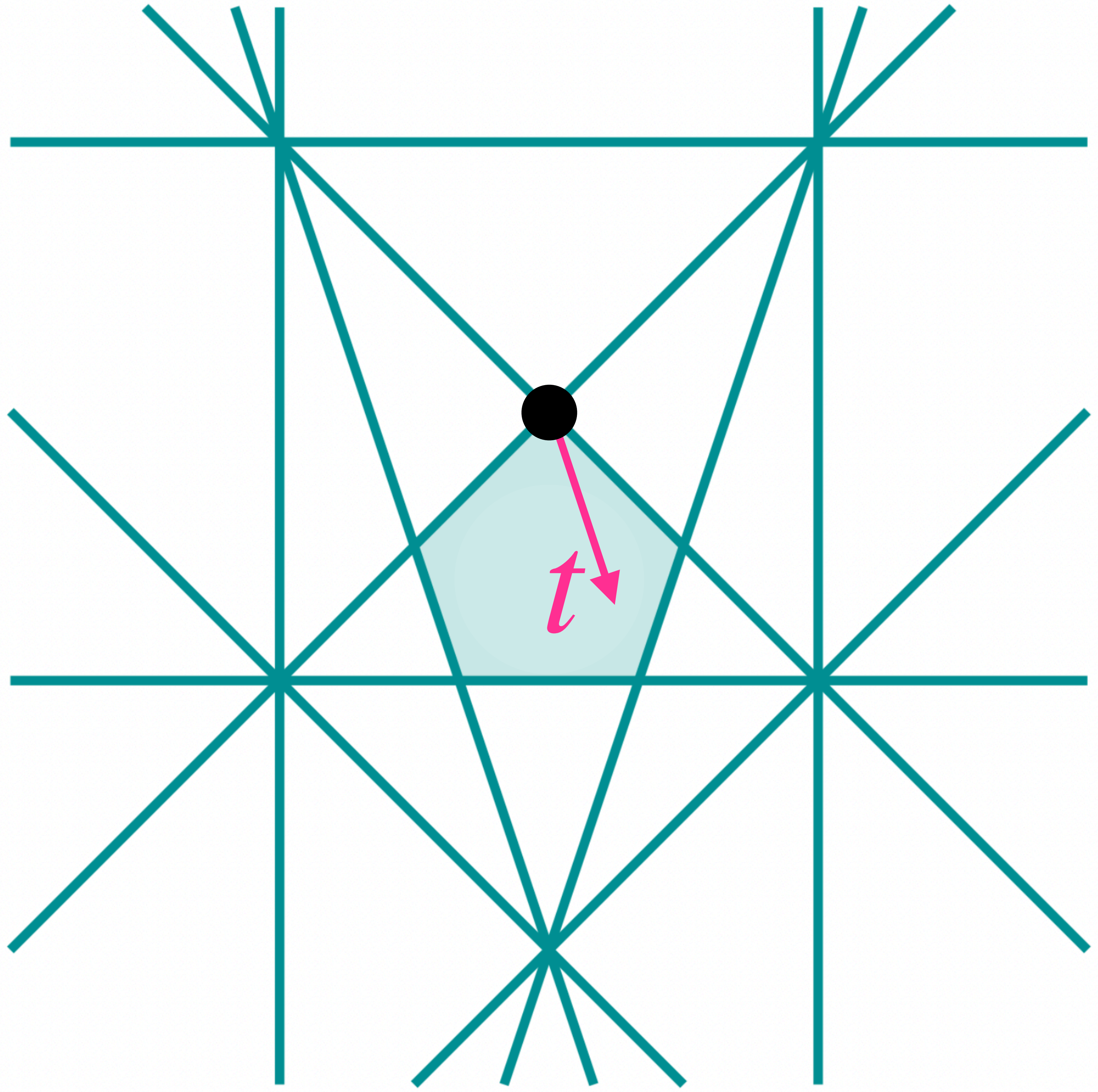
Hyperplane arrangements

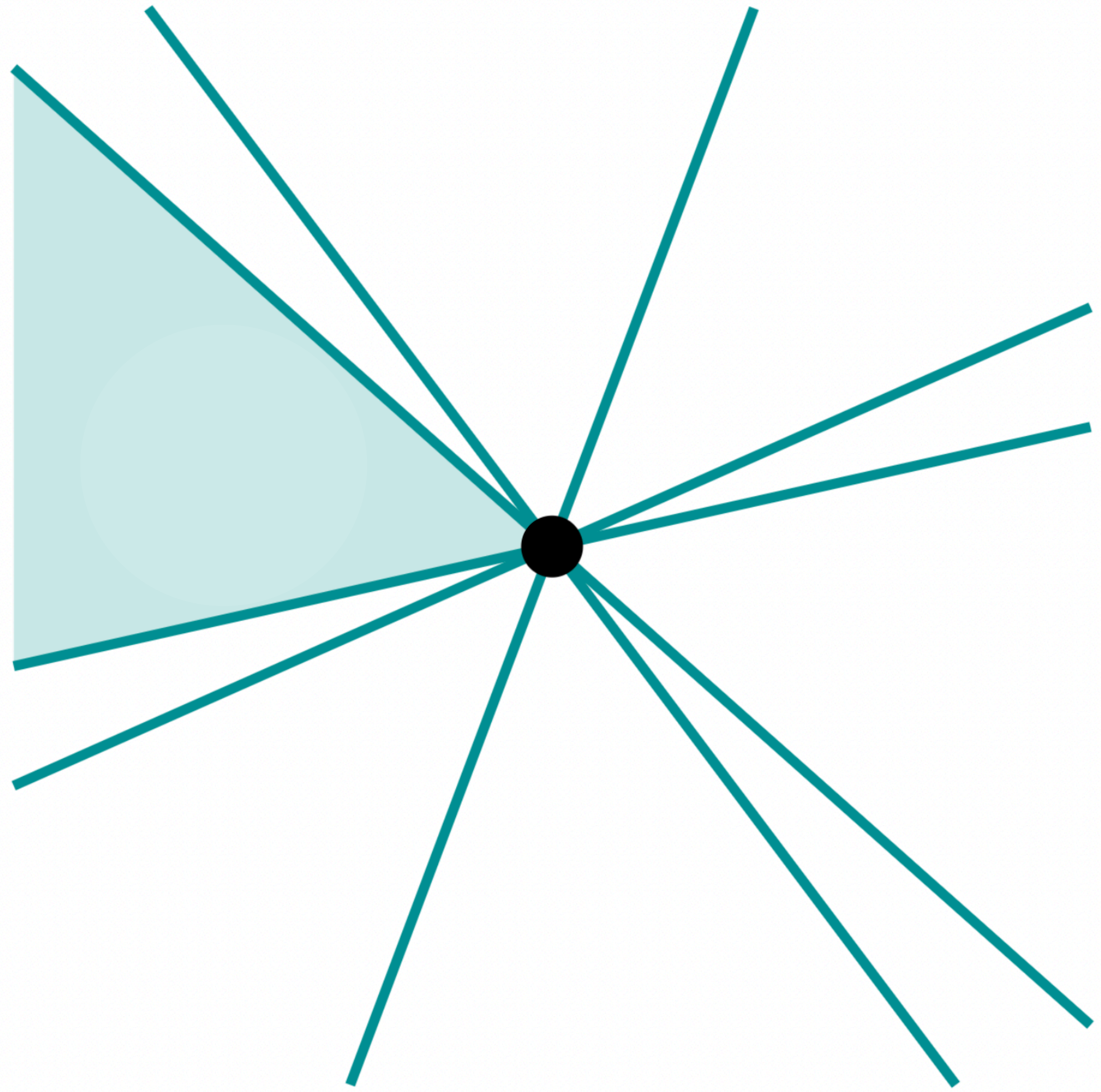
ETH zürich

Construction: slices with the same combinatorial type belong to the same cell of a certain (combination of) hyperplane arrangement(s)



Cocircuit arrangement



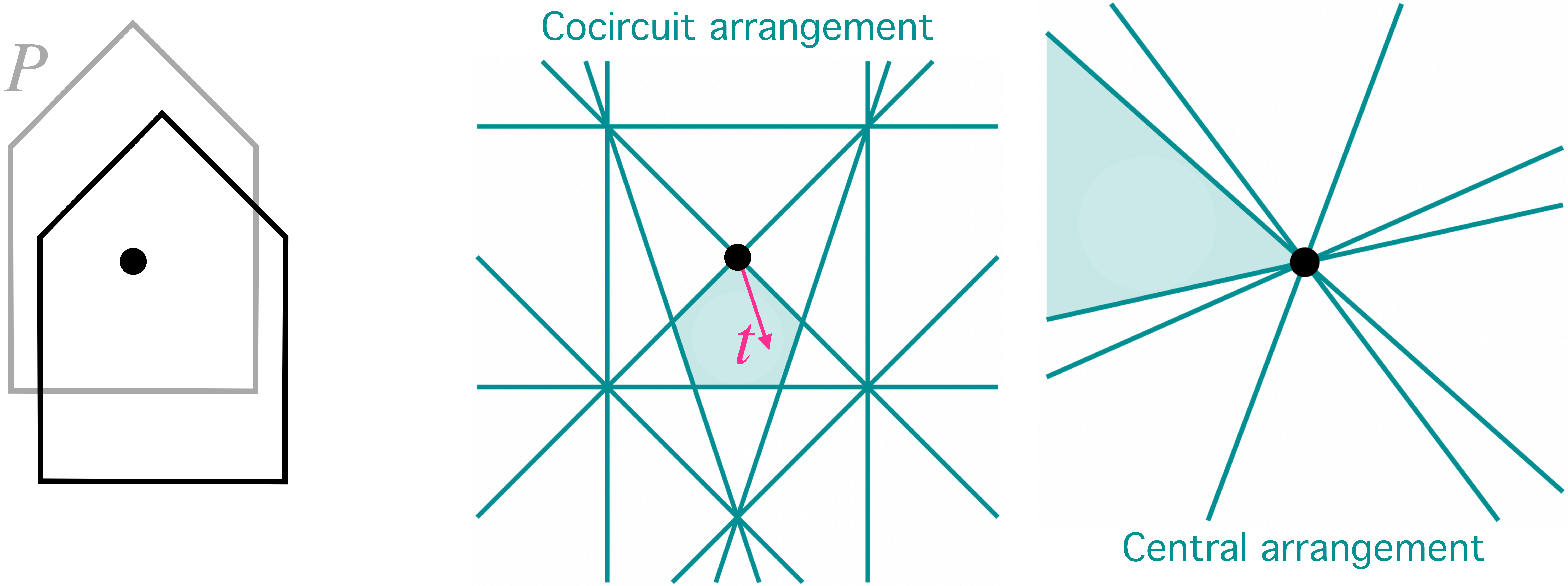


Central arrangement

Hyperplane arrangements

ETH zürich

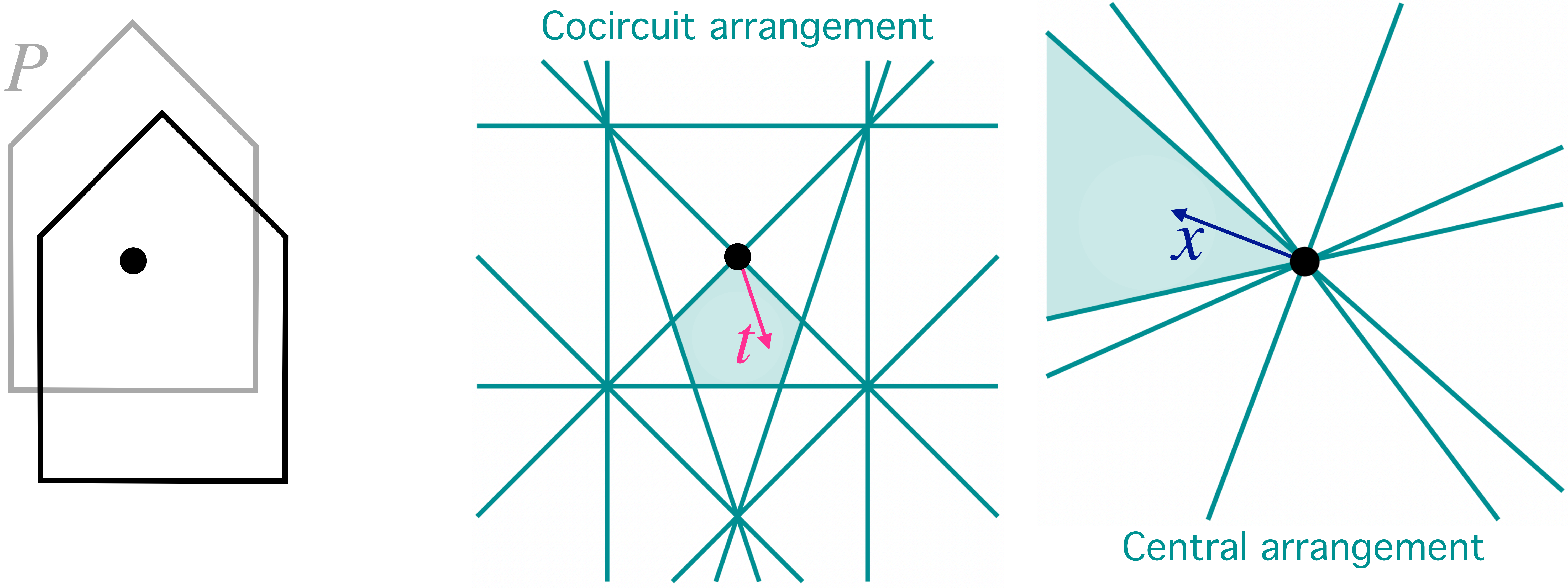
Construction: slices with the same combinatorial type belong to the same cell of a certain (combination of) hyperplane arrangement(s)



Hyperplane arrangements

ETH zürich

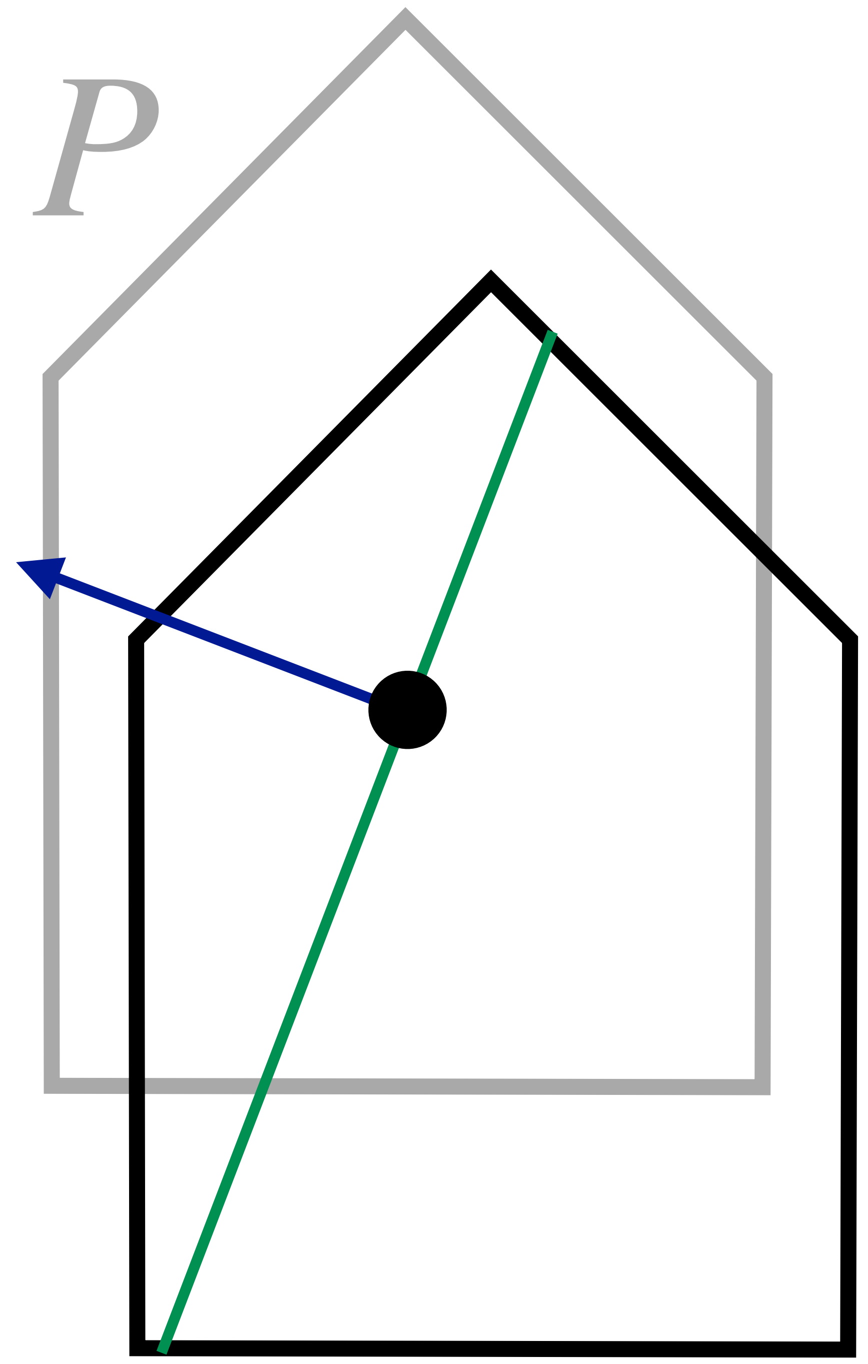
Construction: slices with the same combinatorial type belong to the same cell of a certain (combination of) hyperplane arrangement(s)



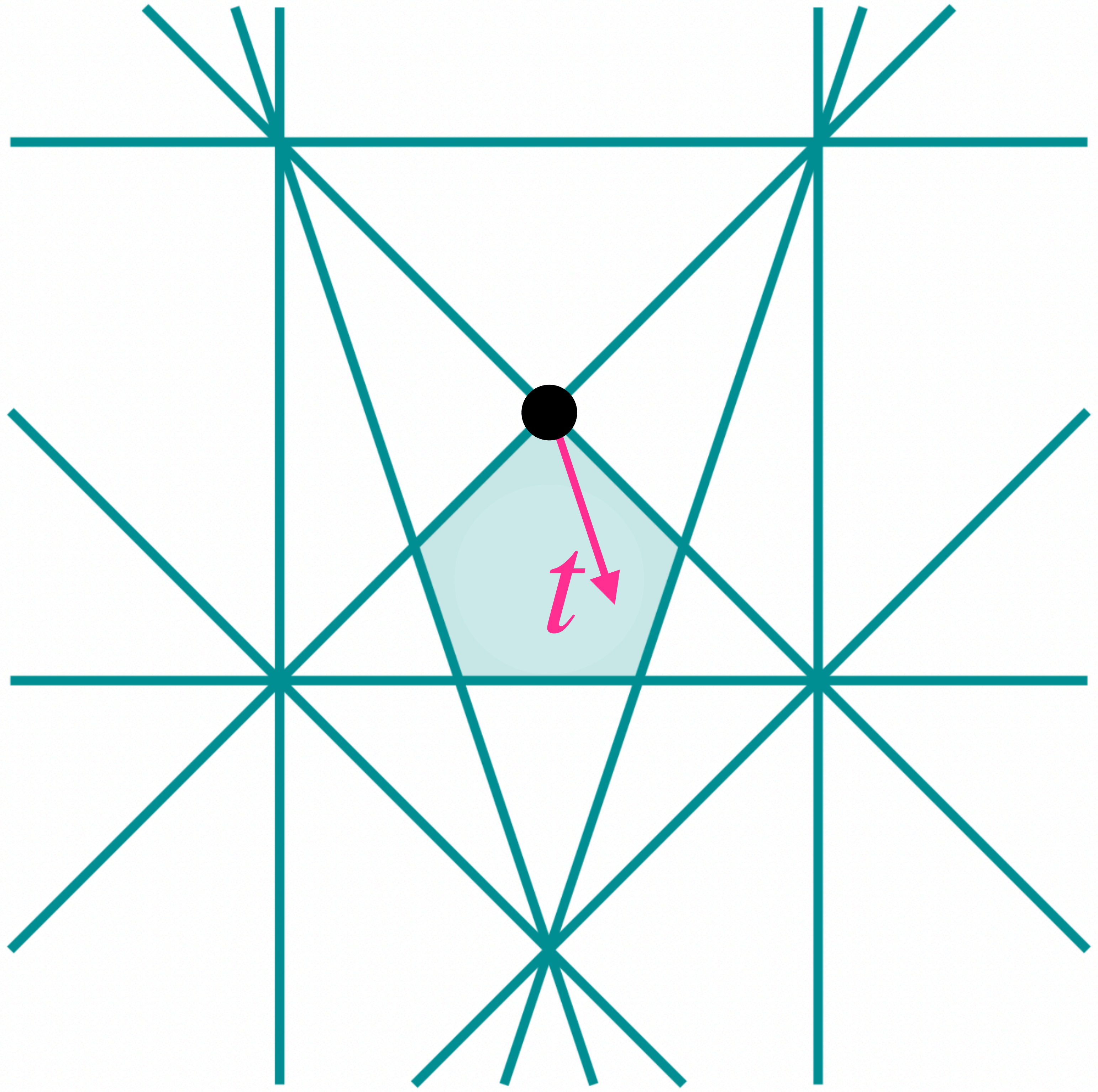
Hyperplane arrangements

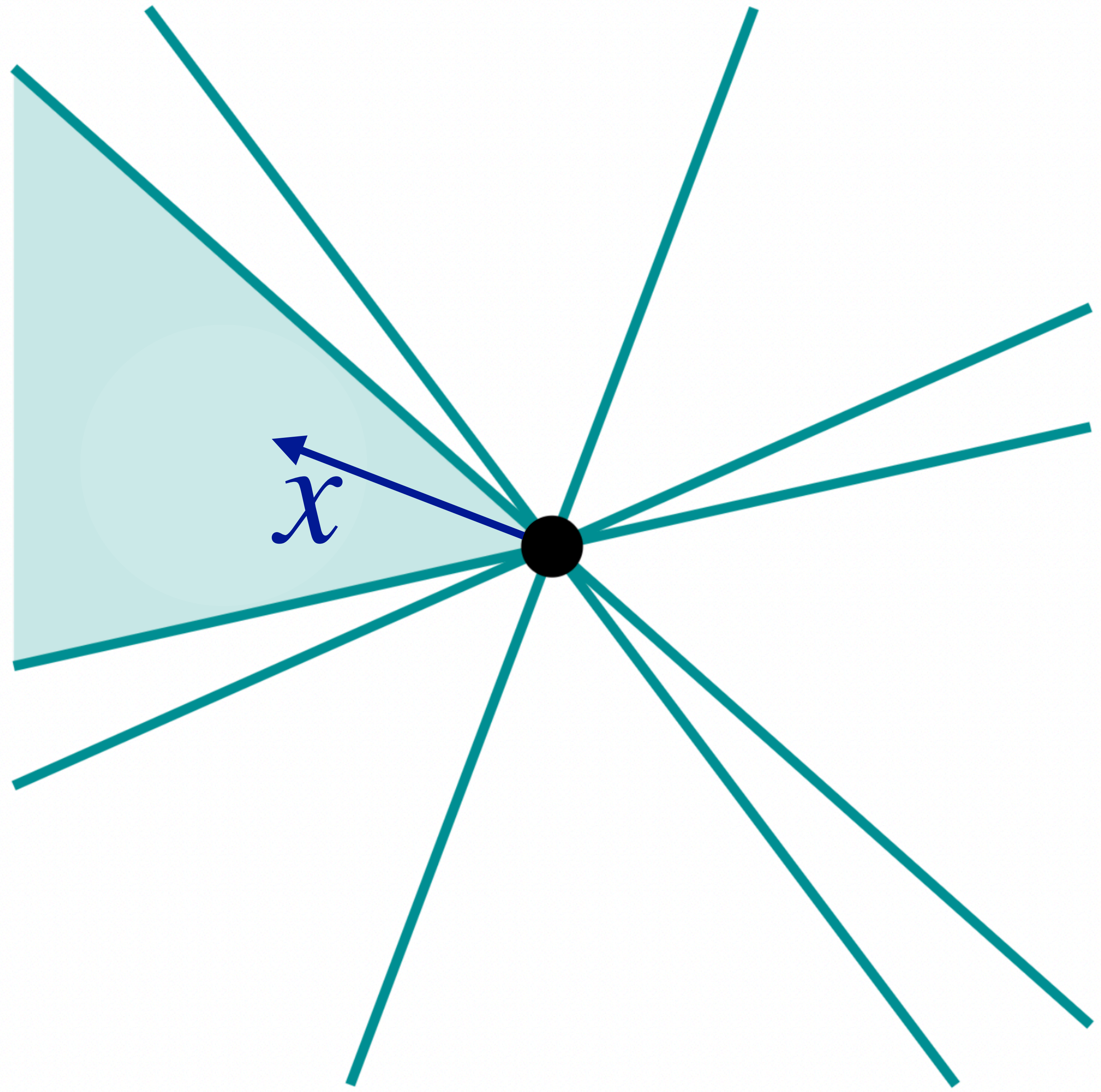
ETH zürich

Construction: slices with the same combinatorial type belong to the same cell of a certain (combination of) hyperplane arrangement(s)



Cocircuit arrangement



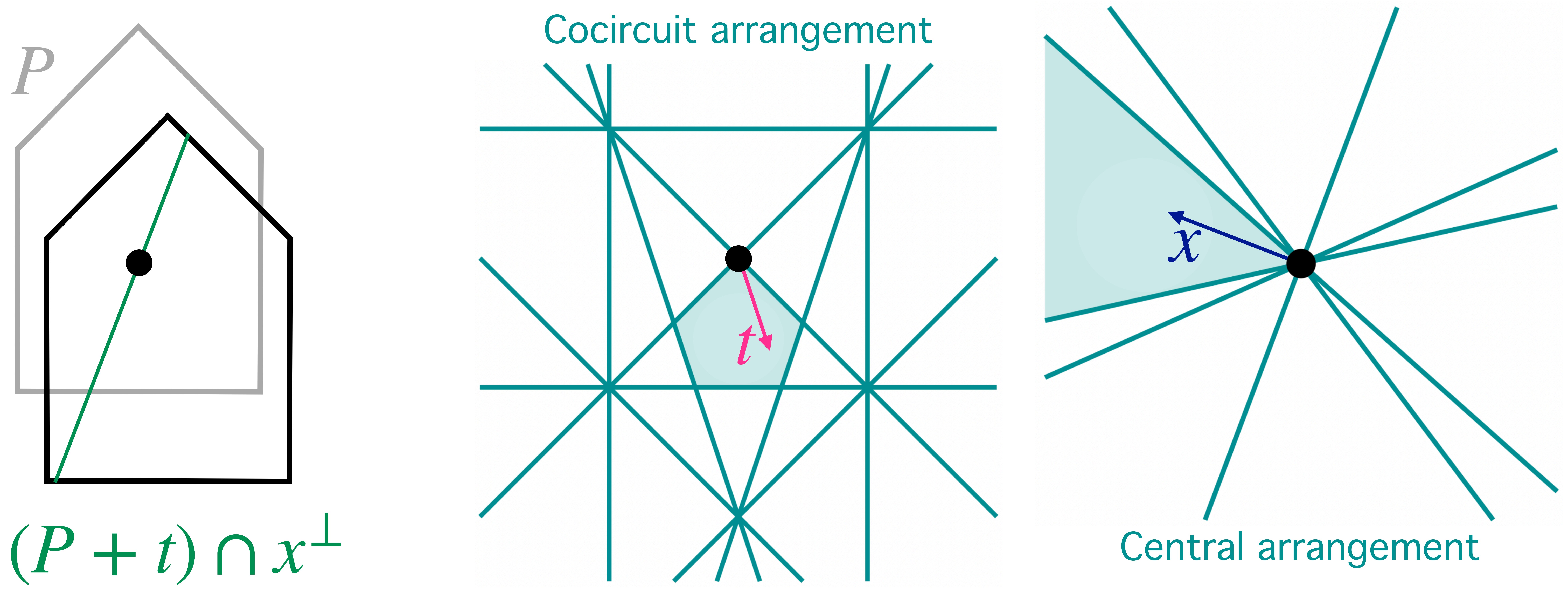


Central arrangement

Hyperplane arrangements

ETH zürich

Construction: slices with the same combinatorial type belong to the same cell of a certain (combination of) hyperplane arrangement(s)



Let's optimize!

ETH zürich

Theorem [B,DL,M]: Let $P \subset \mathbb{R}^d$ be a polytope.
For fixed d, we can find the slice $P \cap H$ with

- $\max f_k(P \cap H)$
- $\max \sum_{\substack{F \subset P, \\ F \cap H \neq 0}} \omega(F)$
- $\max \mathrm{vol}(P \cap H)$
- $\min \mathrm{vol}(P \cap H)$ (through a fixed point)
- $\max \int_{P \cap H} f(x)\, \mathrm{d}x$
- ...

in polynomial time.

Let's optimize!

ETH zürich

Theorem [B,DL,M]: Let $P \subset \mathbb{R}^d$ be a polytope.
For fixed d, we can find the slice $P \cap H$ with

- $\max f_k(P \cap H)$
- $\max \sum_{F \subset P, \\ F \cap H \neq 0} \omega(F)$
- $\max \operatorname{vol}(P \cap H)$
- $\min \operatorname{vol}(P \cap H)$ (through a fixed point)
- $\max \int_{P \cap H} f(x)\, \mathrm{d}x$
- ...

in polynomial time.

or projection $\pi_H(P)$,
or half-space $P \cap H^+$

Let's optimize!

ETH zürich

Theorem [B,DL,M]: Let $P \subset \mathbb{R}^d$ be a polytope.
For fixed d, we can find the slice $P \cap H$ with

- $\max f_k(P \cap H)$
- $\max \sum_{\substack{F \subset P,\\ F \cap H \neq 0}} \omega(F)$
- $\max \operatorname{vol}(P \cap H)$
- $\min \operatorname{vol}(P \cap H)$ (through a fixed point)
- $\max \int_{P \cap H} f(x)\, \mathrm{d}x$
- ...

in polynomial time.

or projection $\pi_H(P)$,
or half-space $P \cap H^+$

Cannot hope for more, in general

Let's optimize!

ETH zürich

Theorem [B,DL,M]: Let $P \subset \mathbb{R}^d$ be a polytope.
For fixed d, we can find the slice $P \cap H$ with

- $\max f_k(P \cap H)$
- $\max \sum_{\substack{F \subset P, \\ F \cap H \neq 0}} \omega(F)$
- $\max \operatorname{vol}(P \cap H)$
- $\min \operatorname{vol}(P \cap H)$ (through a fixed point)
- $\max \int_{P \cap H} f(x) \, \mathrm{d}x$
- ...

in polynomial time.

or projection $\pi_H(P)$,
or half-space $P \cap H^+$

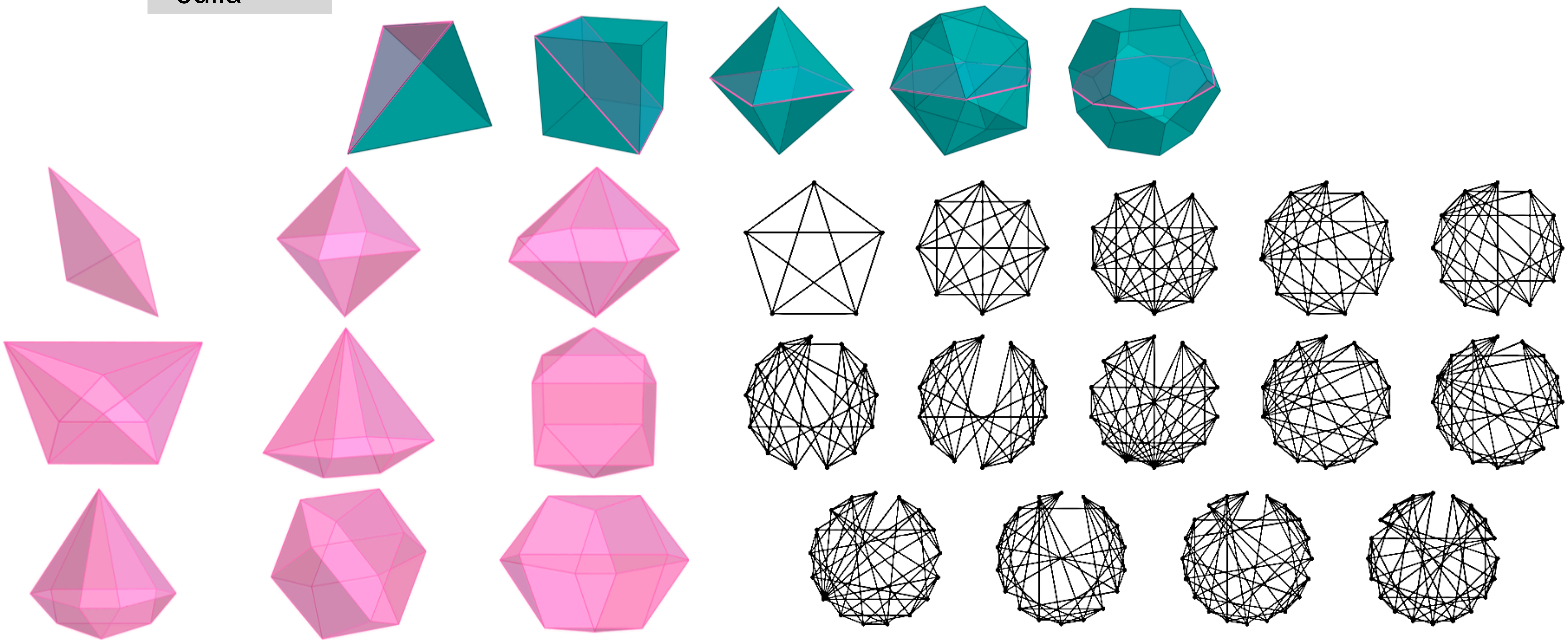
Cannot hope for more, in general

Theorem [B,DL,M]:
It is #P-hard to compute the volume of the slice with largest volume.

Theory and practice

- SageMath
- Julia

https://mathrepo.mis.mpg.de/BestSlicePolytopes/



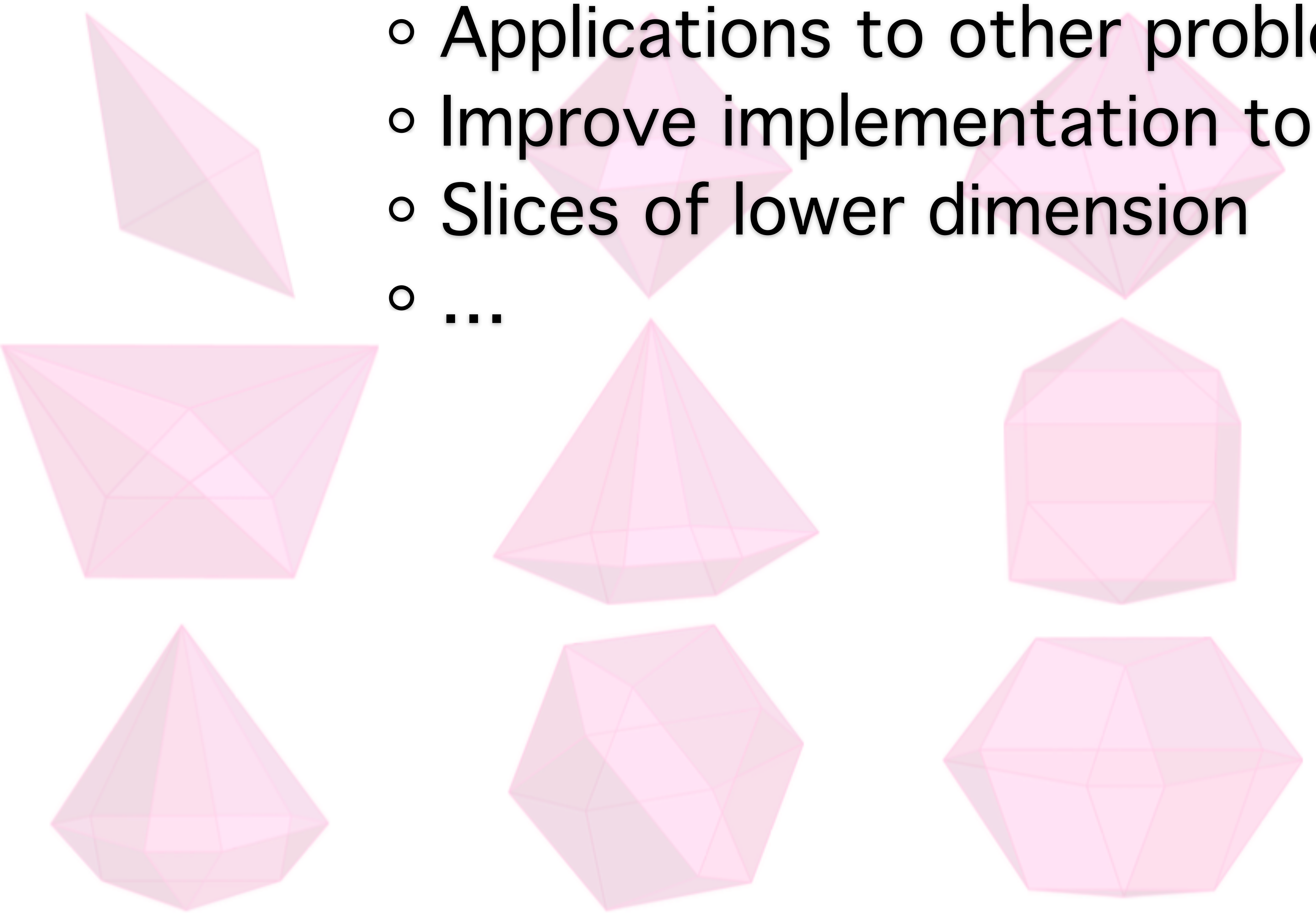
Theory and practice

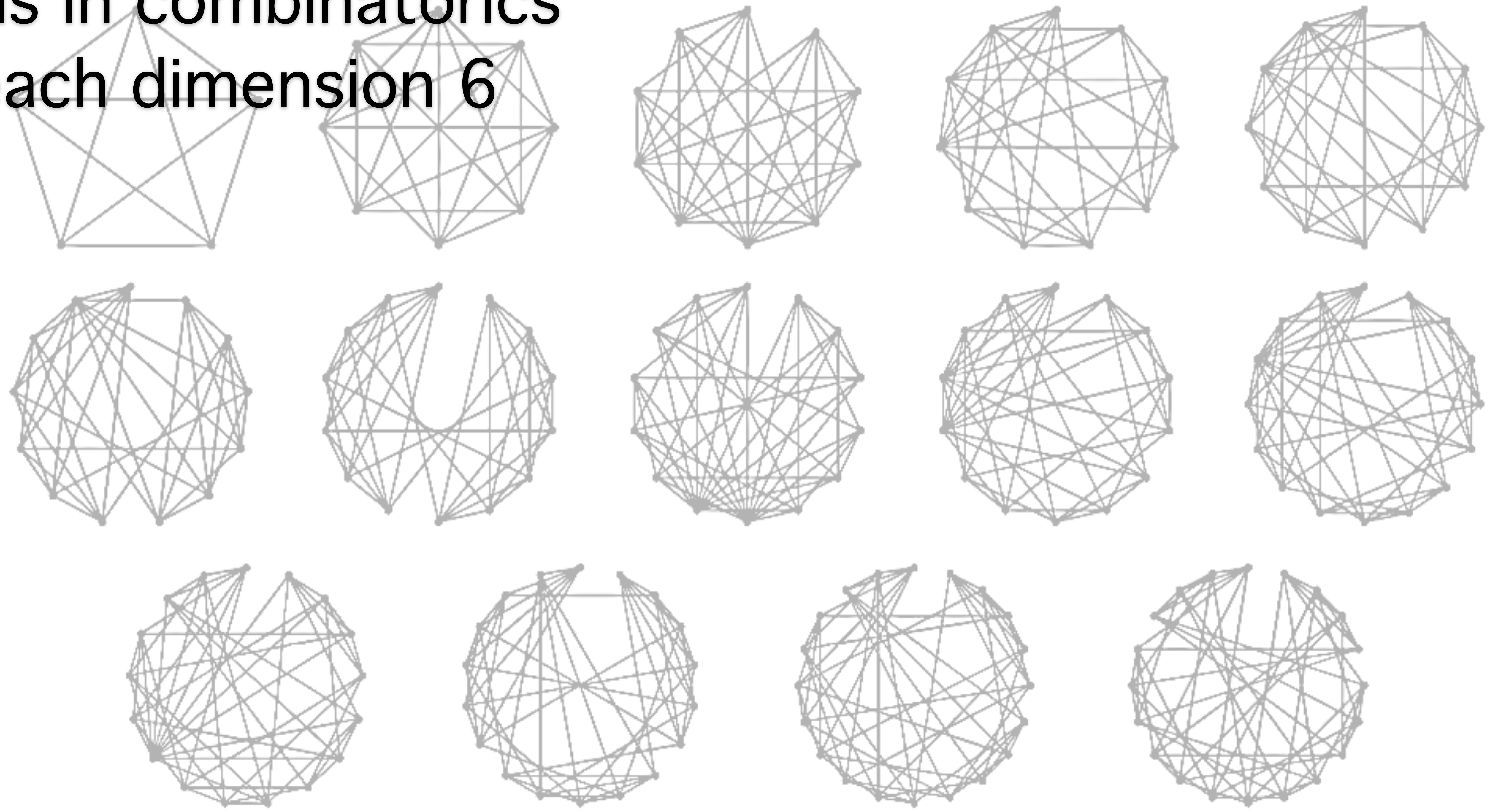
ETH zürich

- SageMath
- Julia

https://mathrepo.mis.mpg.de/BestSlicePolytopes/

- Conjecture on the behaviour of slices from a discrete perspective
- Combinatorics of slices for special families of polytopes
- Applications to other problems in combinatorics
- Improve implementation to reach dimension 6
- Slices of lower dimension
- ...





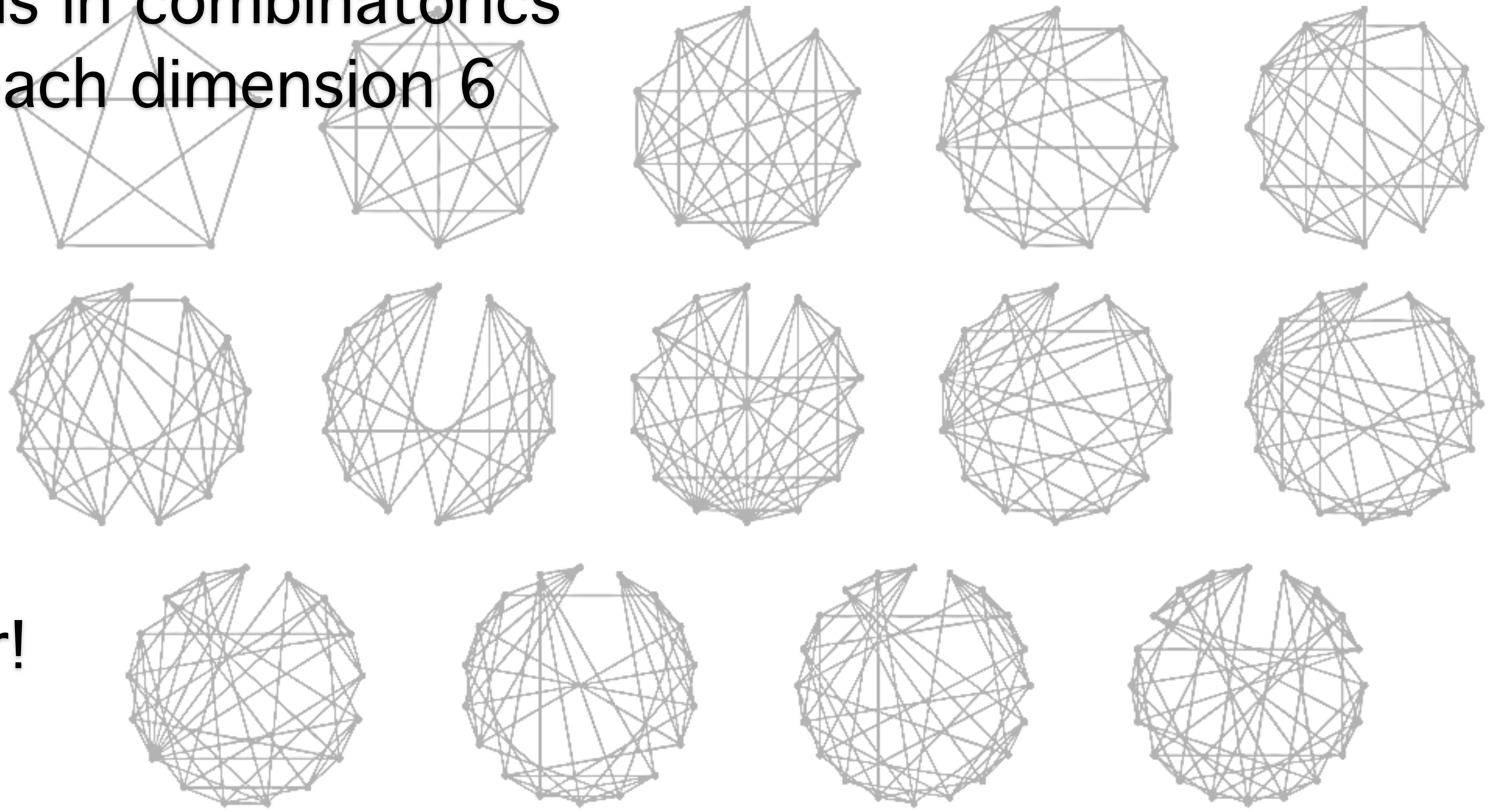
Theory and practice

ETH zürich

- SageMath
- Julia

https://mathrepo.mis.mpg.de/BestSlicePolytopes/

- Conjecture on the behaviour of slices from a discrete perspective
- Combinatorics of slices for special families of polytopes
- Applications to other problems in combinatorics
- Improve implementation to reach dimension 6
- Slices of lower dimension
- ...



Interested? Curious?

- ✦ Talk to me!
- ✦ Talk to Jesús!
- ✦ Check out Antonio's poster!

A. I. (= After ICERM)

ETH zürich

A. I. (= After ICERM)

ETH zürich

Hey! There are also NON-discrete objects!

arXiv > math > arXiv:2403.04438

Maximizing Slice-Volumes of Semialgebraic Sets using Sum-of-Squares Programming

Jared Miller, Chiara Meroni, Matteo Tacchi, Mauricio Velasco

Recompile 12 87%

ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO

A. I. (= After ICERM)

ETH zürich

Hey! There are also NON-discrete objects!

arXiv > math > arXiv:2403.04438

Maximizing Slice-Volumes of Semialgebraic Sets using Sum-of-Squares Programming

Jared Miller, Chiara Meroni, Matteo Tacchi, Mauricio Velasco

Recompile 12 87%

ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO

PROS: $\{x \in \mathbb{R}^d \mid g_i(x) \geq 0$ for $i = 1, \ldots, N\}$

CONS: Only an approximation
In $\mathbb{R}^3$ it is already out of reach

KEY: SDP
Adapted Lasserre hierarchies

A. I. (= After ICERM)

ETH zürich

Hey! There are also NON-discrete objects!

arXiv > math > arXiv:2403.04438

Maximizing Slice-Volumes of Semialgebraic Sets using Sum-of-Squares Programming

Jared Miller, Chiara Meroni, Matteo Tacchi, Mauricio Velasco

PROS: $\{x \in \mathbb{R}^d \mid g_i(x) \geq 0$ for $i = 1,\ldots,N\}$

CONS: Only an approximation

In $\mathbb{R}^3$ it is already out of reach

KEY: SDP

Adapted Lasserre hierarchies

Recompile 12 87%

ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO

B-P problem and Bourgain's conjecture: optimization over all convex bodies

A. I. (= After ICERM)

ETH zürich

Hey! There are also NON-discrete objects!

arXiv > math > arXiv:2403.04438

Maximizing Slice-Volumes of Semialgebraic Sets using Sum-of-Squares Programming

Jared Miller, Chiara Meroni, Matteo Tacchi, Mauricio Velasco

PROS: $\{x \in \mathbb{R}^d \mid g_i(x) \geq 0$ for $i = 1,\dots,N\}$

CONS: Only an approximation
In $\mathbb{R}^3$ it is already out of reach

KEY: SDP
Adapted Lasserre hierarchies

Recompile 12 87%

ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO

B-P problem and Bourgain's conjecture:
optimization over all convex bodies

How can we do this?

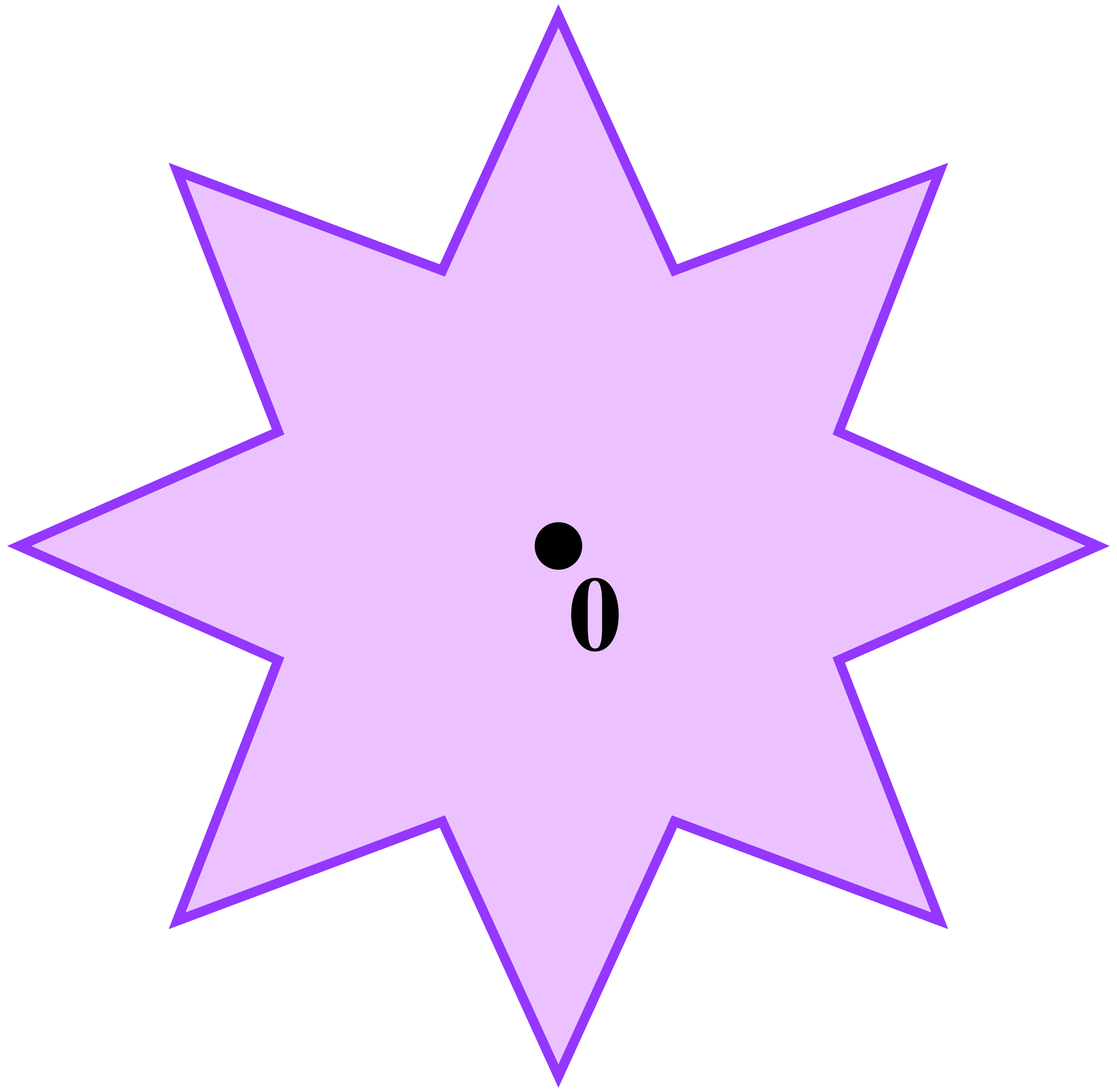
Star-bodies

ETH zürich

Recompile 12 87%

ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

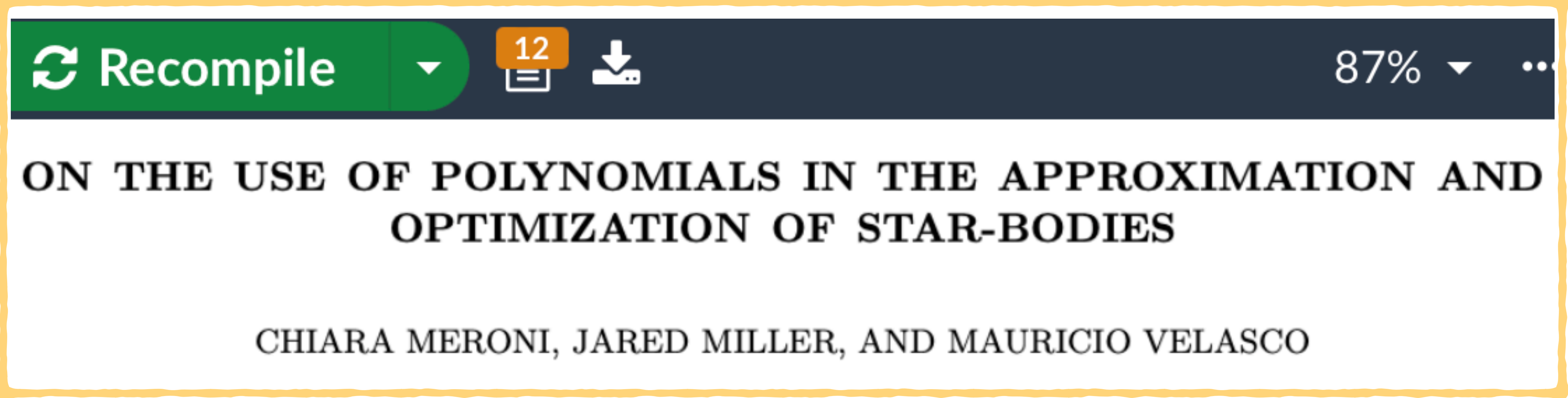
CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO



Assumptions: compact, $\mathbf{0}$ in the interior

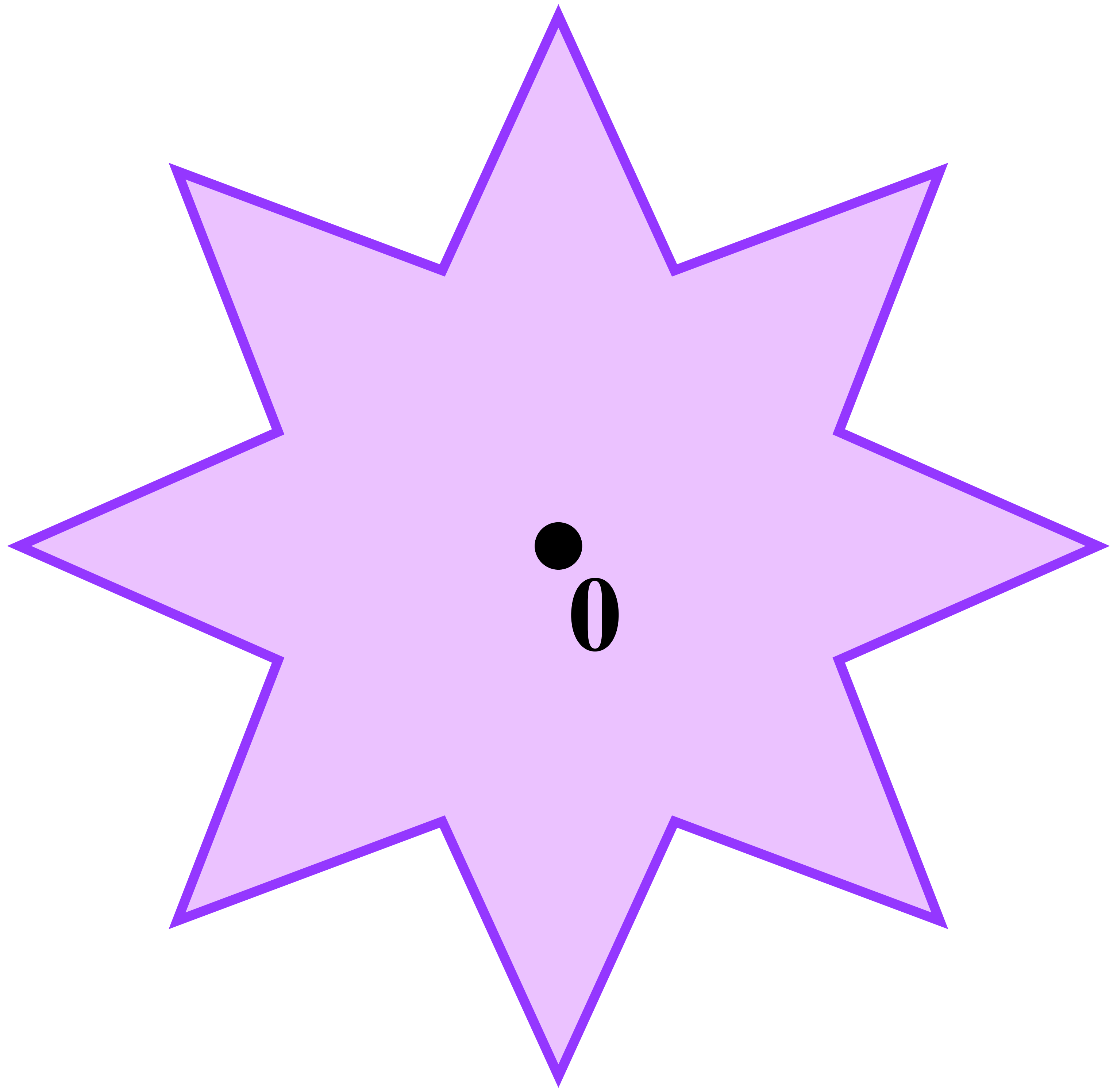
Star-bodies

ETH zürich


Recompile 12 87%

ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

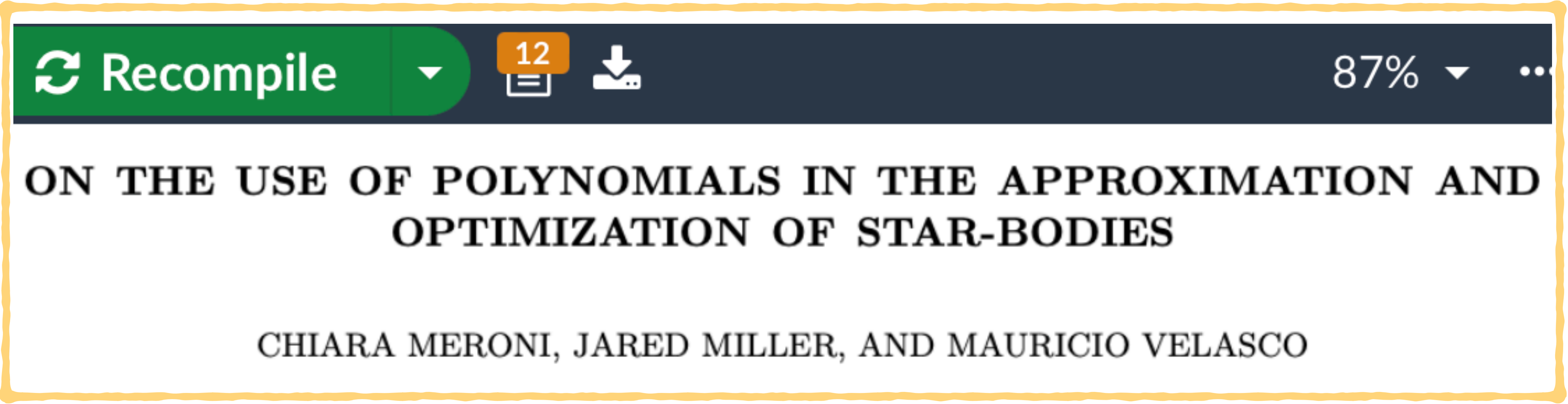
CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO



Polytopes

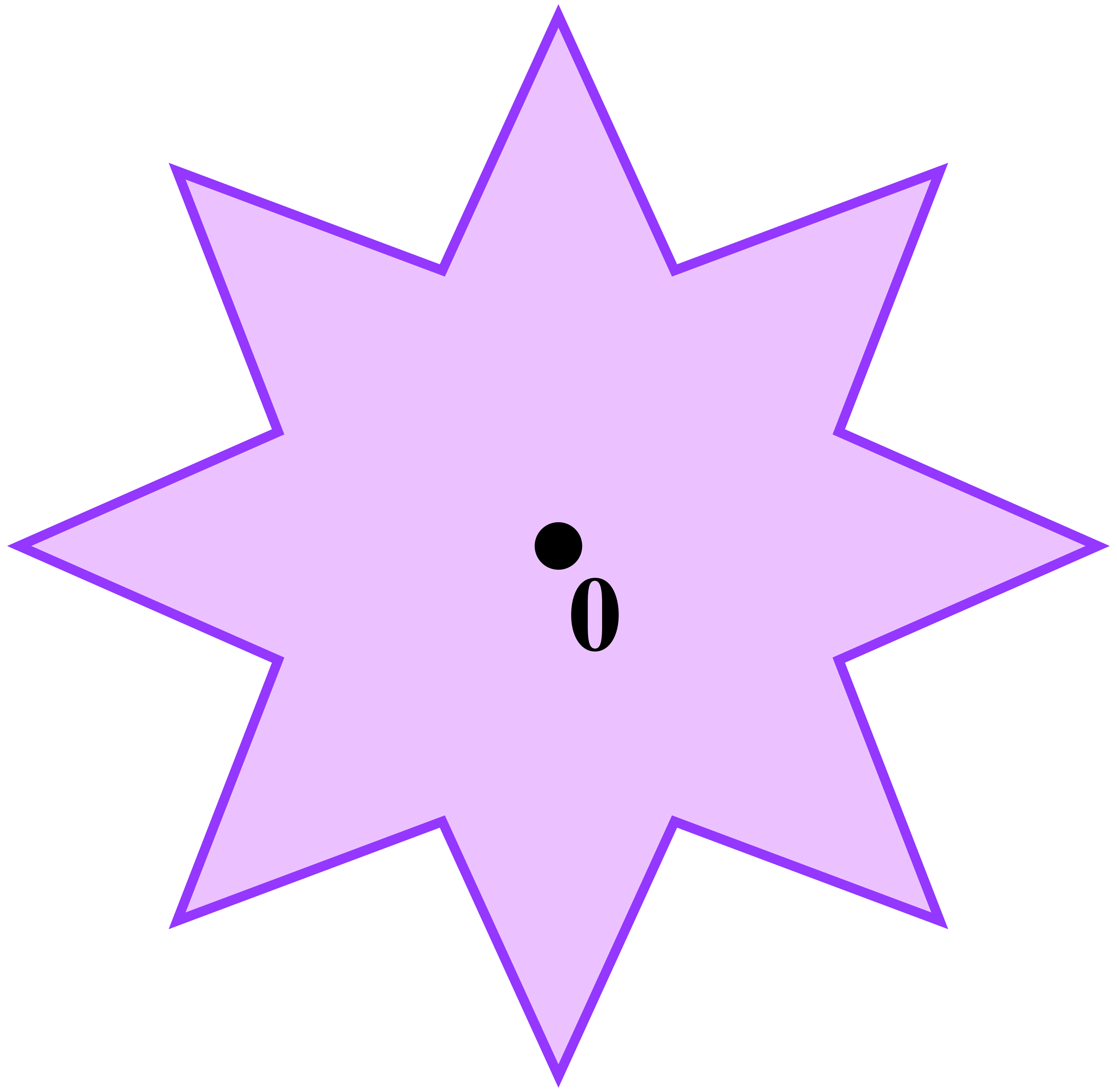
Assumptions: compact, $\mathbf{0}$ in the interior

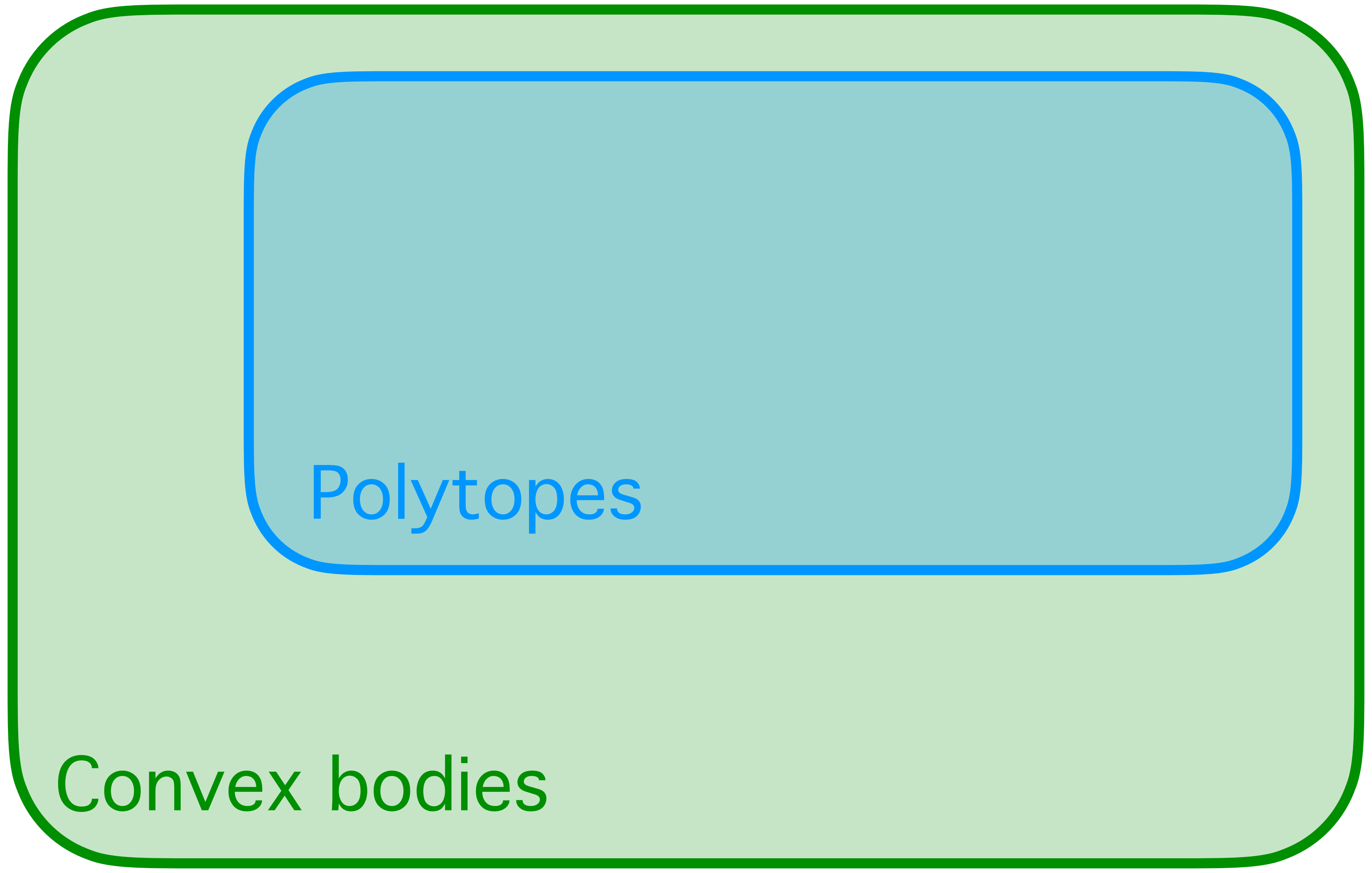
Star-bodies



ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

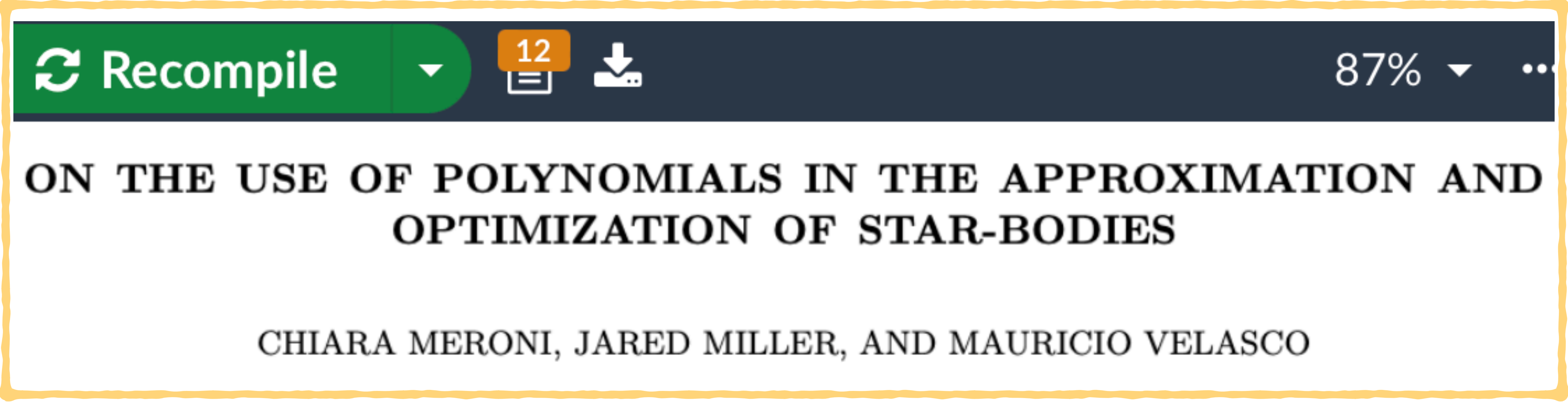
CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO





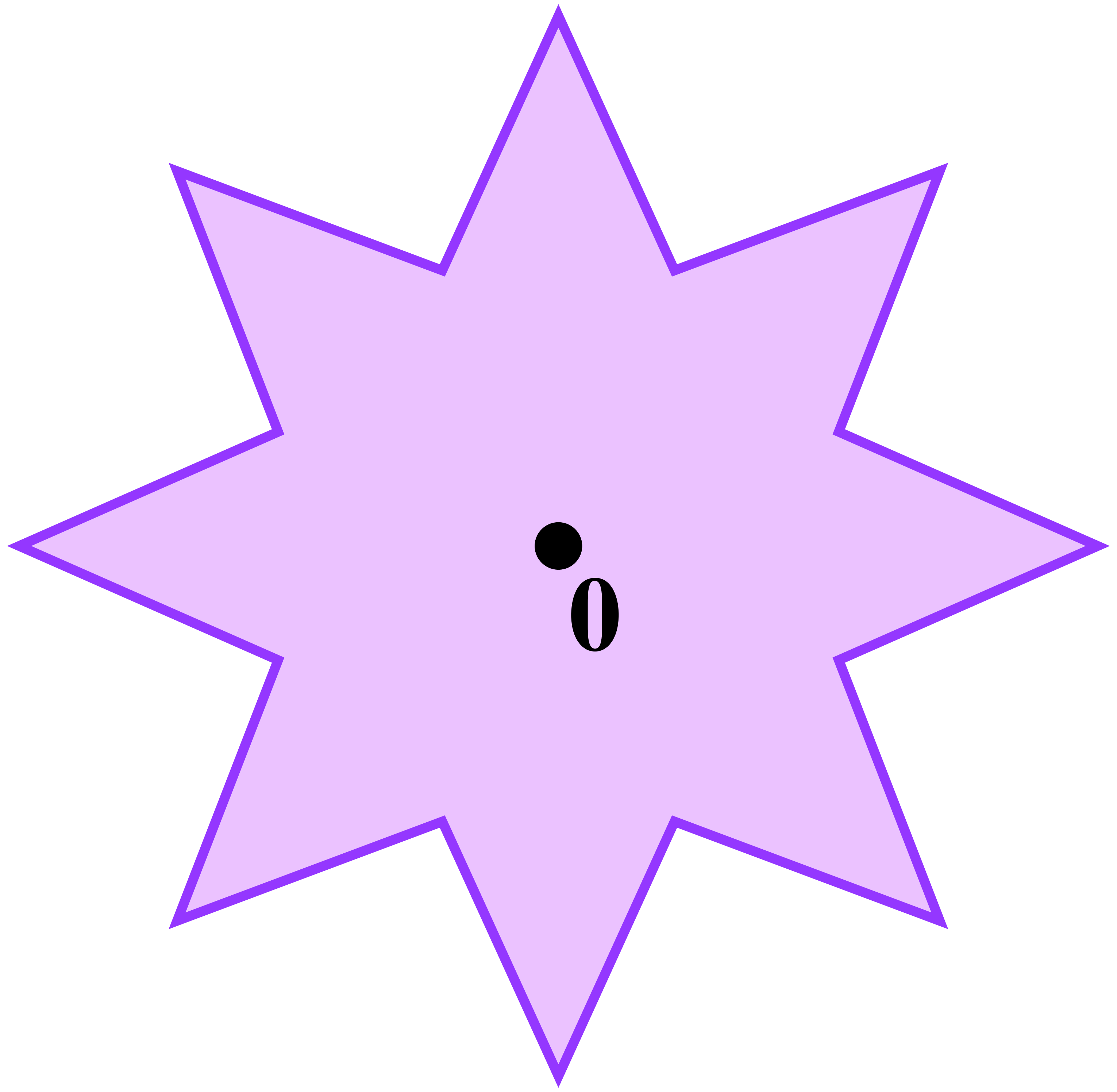
Assumptions: compact, $\mathbf{0}$ in the interior

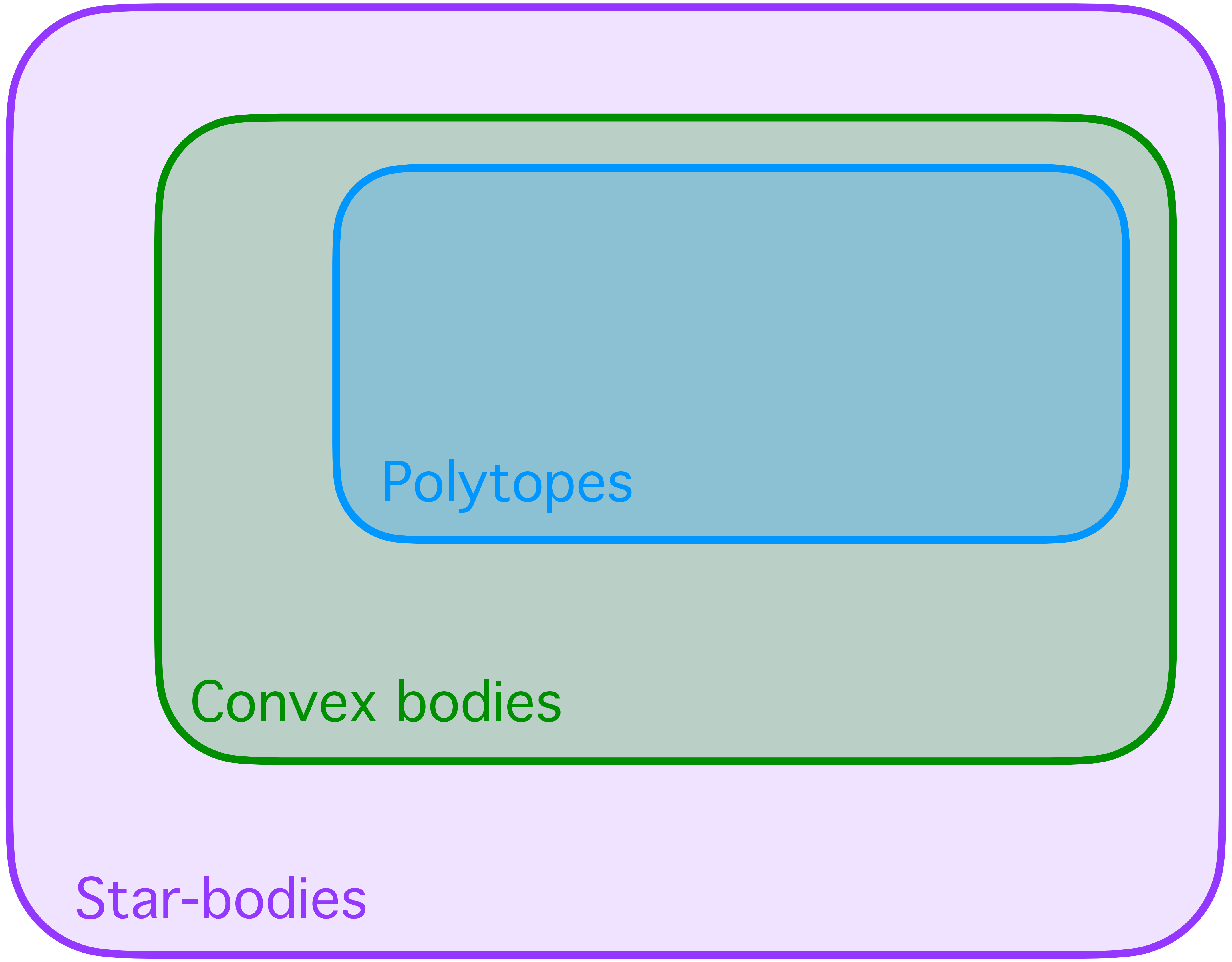
Star–bodies



ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO

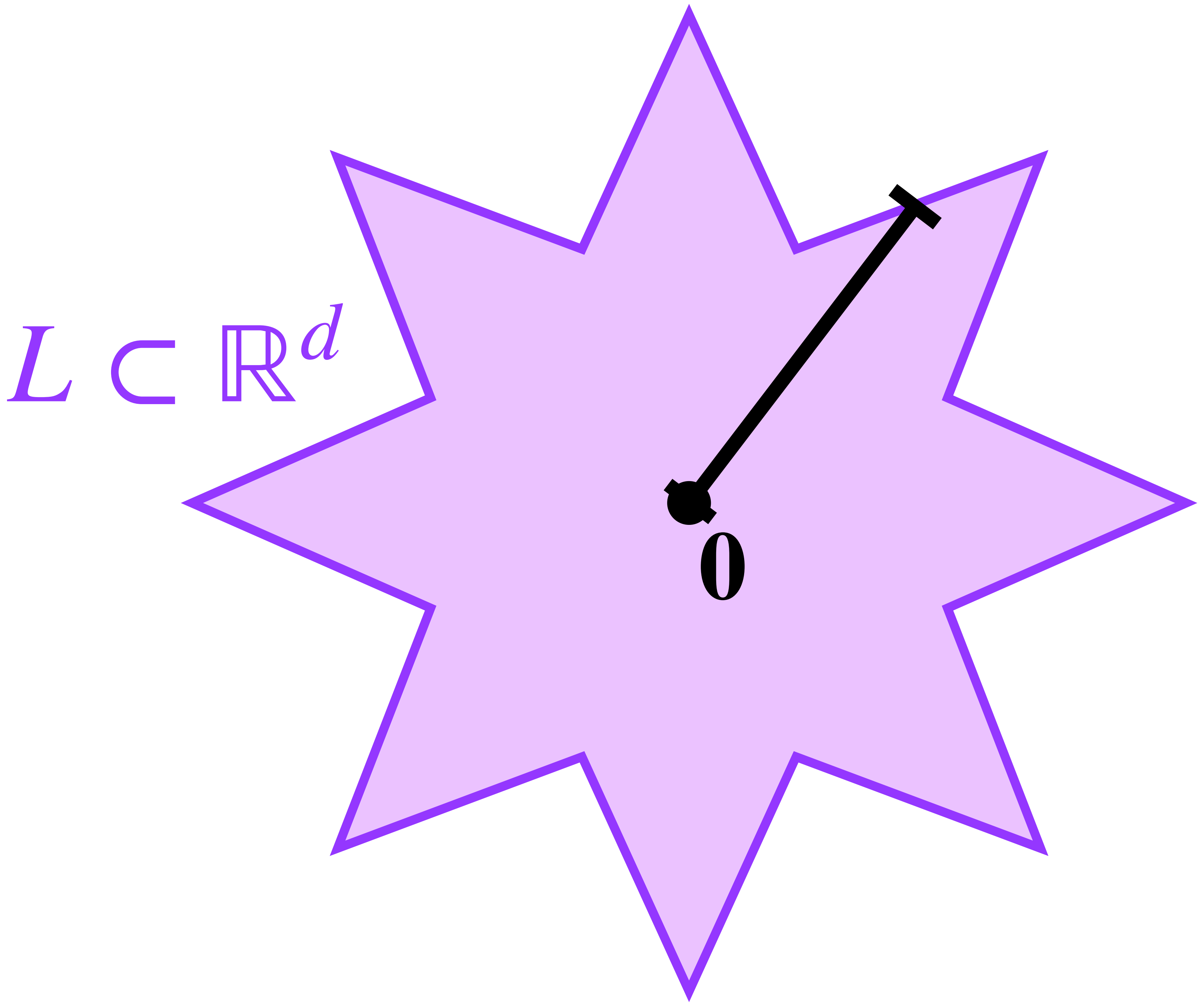




Assumptions: compact, $\mathbf{0}$ in the interior

Radial & gauge functions

Radial function: $\rho_L(x) = \max\{\lambda \in \mathbb{R}_{>0} \mid \lambda x \in L\}$, for all $x \in S^{d-1}$

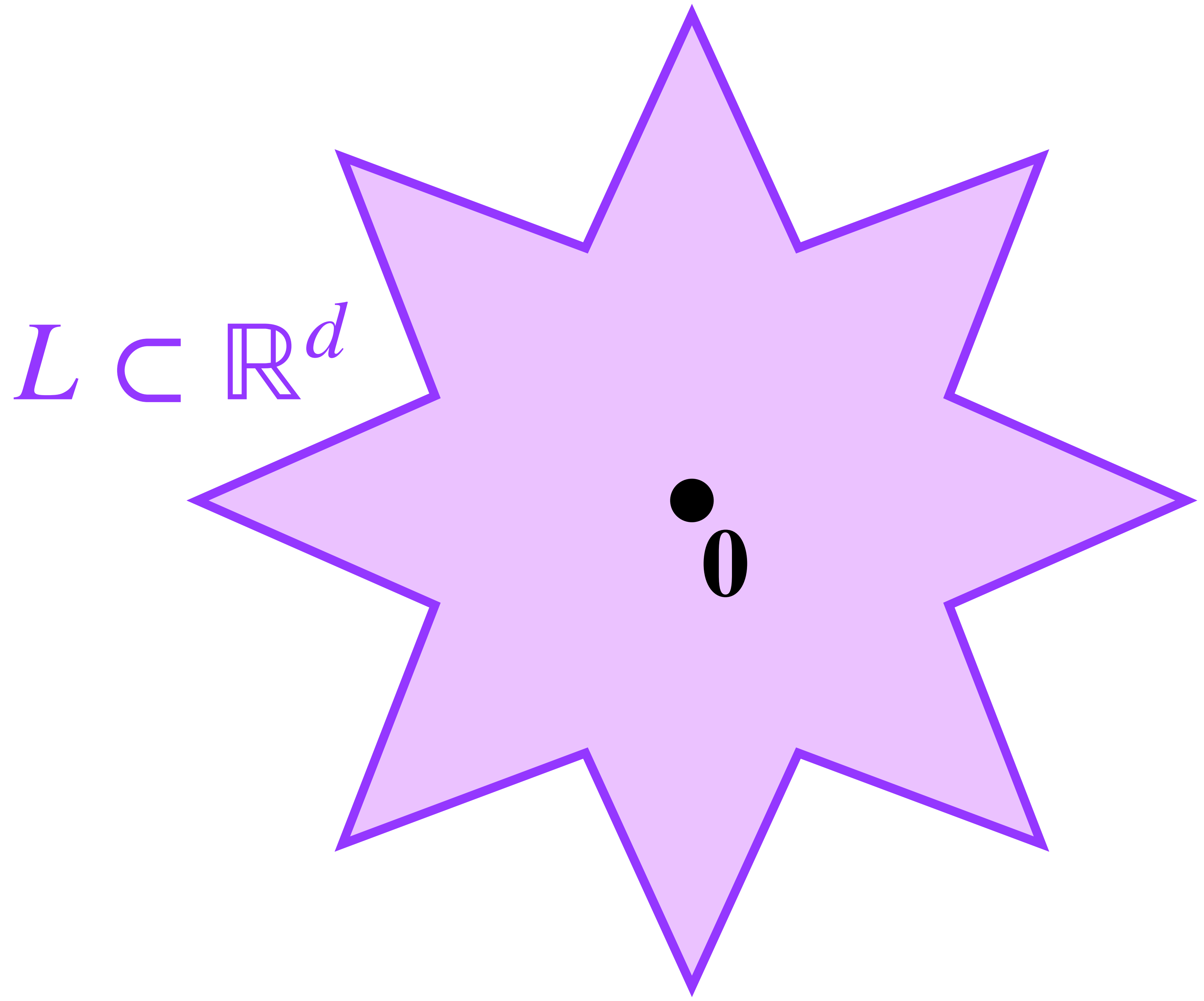


Radial & gauge functions

ETH zürich

Radial function: $\rho_L(x) = \max\{\lambda \in \mathbb{R}_{>0} \mid \lambda x \in L\}$, for all $x \in S^{d-1}$

Gauge function: $\gamma_L(x) = \dfrac{1}{\rho_L(x)}$, for all $x \in S^{d-1}$

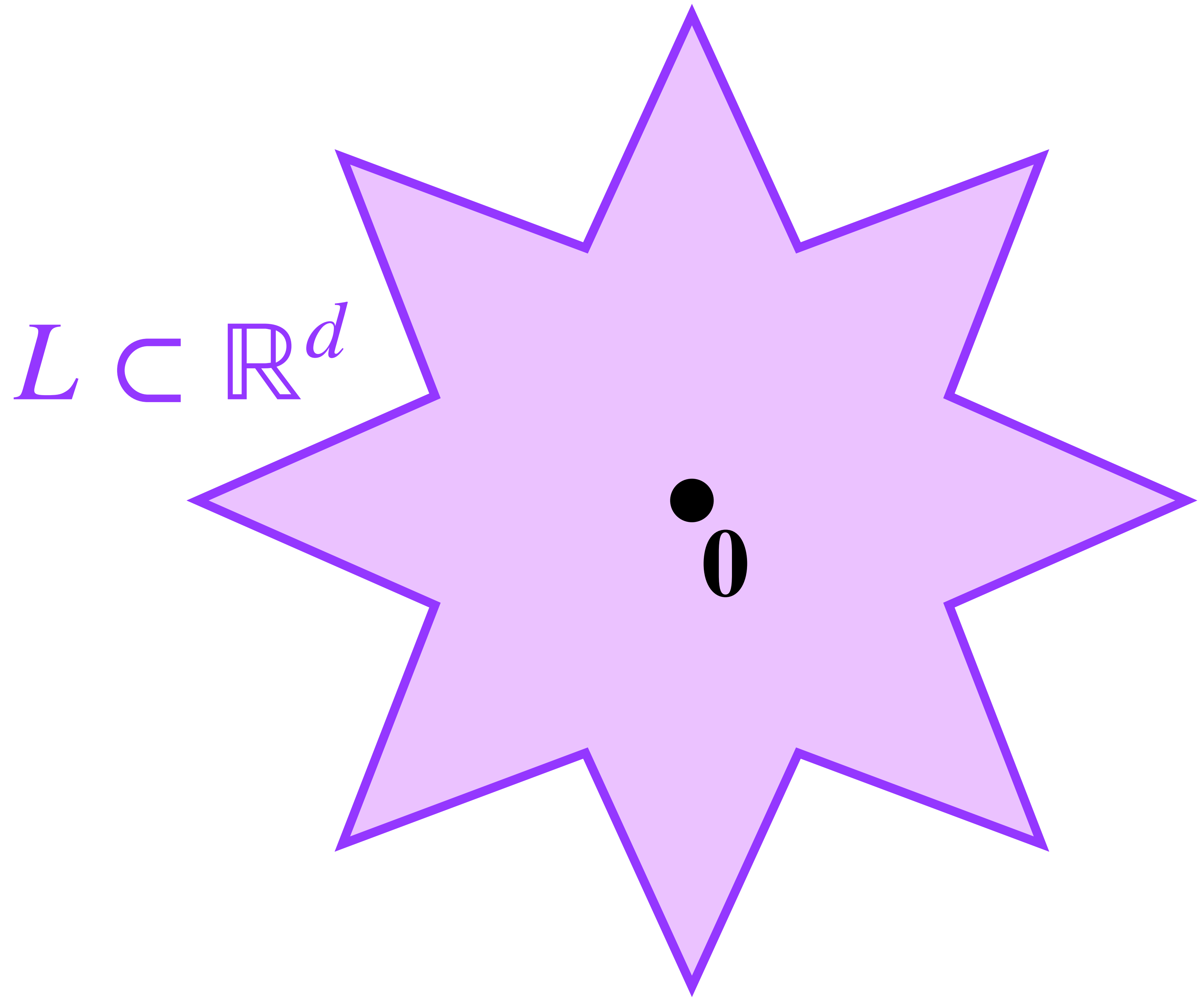


Radial & gauge functions

ETH zürich

Radial function: $\rho_L(x) = \max\{\lambda \in \mathbb{R}_{>0} \mid \lambda x \in L\}$, for all $x \in S^{d-1}$

Gauge function: $\gamma_L(x) = \dfrac{1}{\rho_L(x)}$, for all $x \in S^{d-1}$



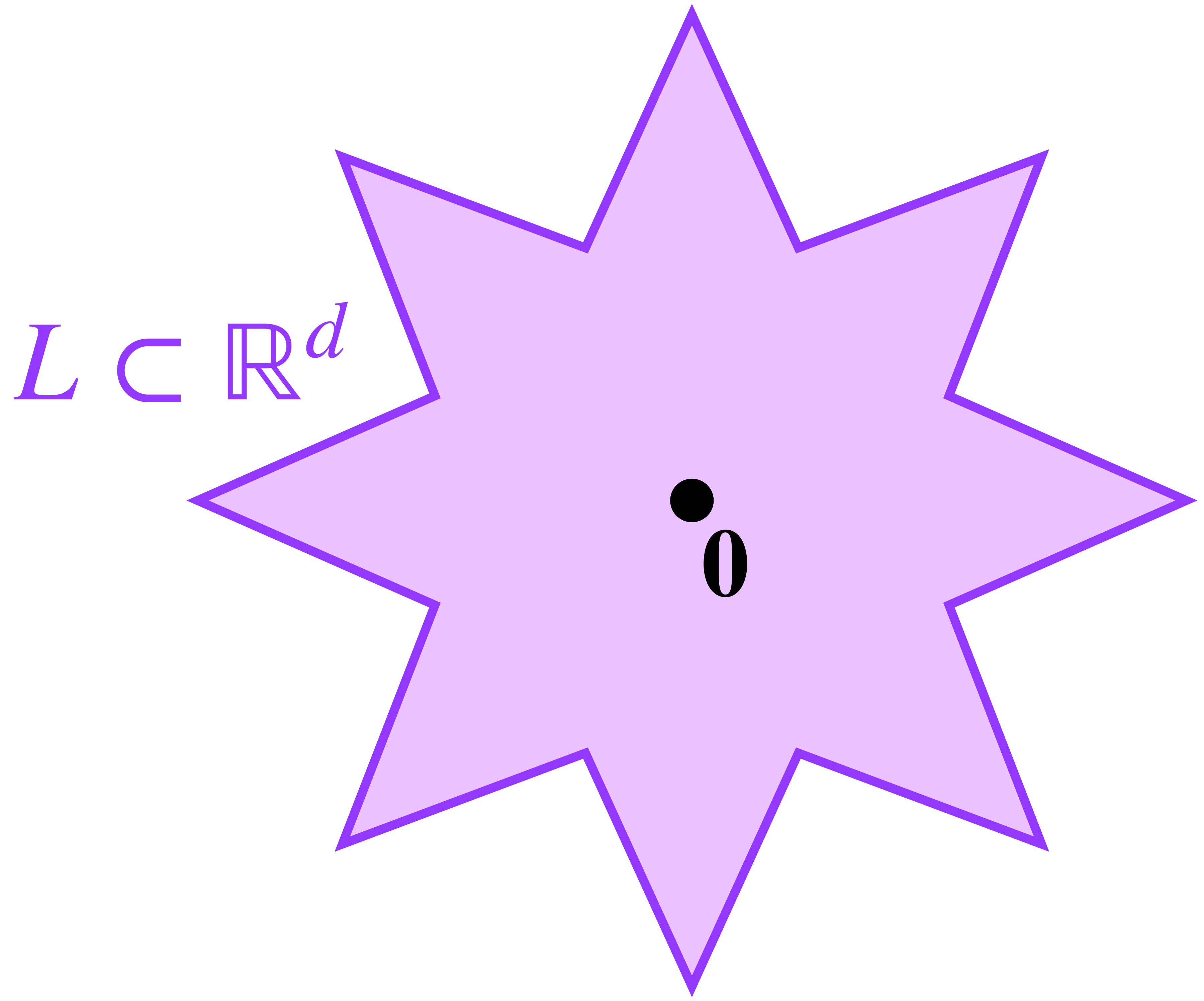
L can be uniquely identified with ρ_L or γ_L

Radial & gauge functions

ETH zürich

Radial function: $\rho_L(x) = \max\{\lambda \in \mathbb{R}_{>0} \mid \lambda x \in L\}$, for all $x \in S^{d-1}$

Gauge function: $\gamma_L(x) = \dfrac{1}{\rho_L(x)}$, for all $x \in S^{d-1}$



L can be uniquely identified with ρ_L or γ_L

For $P = \{x \in \mathbb{R}^d \mid Ax \leq 1\}$ we have

$$\gamma_P(x) = \max_i \, A_i \cdot x \, , \quad \rho_P(x) = \min_i \left\{ \frac{1}{A_i \cdot x} \mid A_i \cdot x > 0 \right\}$$

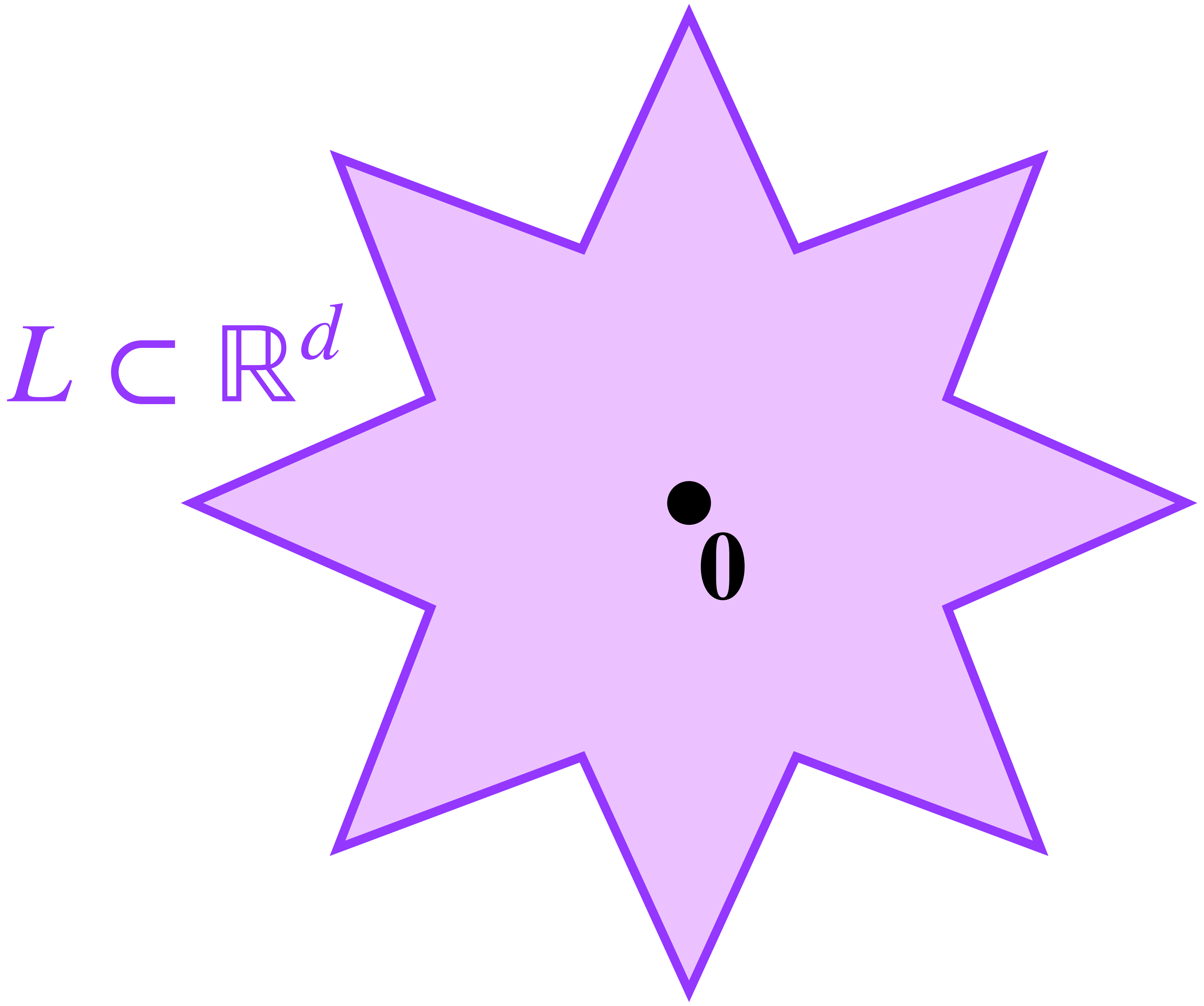
Radial & gauge functions

ETH zürich

Radial function: $\rho_L(x) = \max\{\lambda \in \mathbb{R}_{>0} \mid \lambda x \in L\}$, for all $x \in S^{d-1}$

Gauge function: $\gamma_L(x) = \dfrac{1}{\rho_L(x)}$, for all $x \in S^{d-1}$

can be wild!



L can be uniquely identified with ρ_L or γ_L

For $P = \{x \in \mathbb{R}^d \mid Ax \leq 1\}$ we have

$$\gamma_P(x) = \max_i \; A_i \cdot x \; , \qquad \rho_P(x) = \min_i \left\{ \frac{1}{A_i \cdot x} \;\middle|\; A_i \cdot x > 0 \right\}$$

Polyradial & polygauge bodies

ETH zürich

Polyradial body: a star-body whose radial function is polynomial
(Polygauge) (gauge)

$$\left\{ \text{Polyradial star-bodies} \right\} \longleftrightarrow \left\{ \text{positive polynomials on } S^{d-1} \right\}$$

$$\left\{ \text{Polygauge star-bodies} \right\} \longleftrightarrow \left\{ \text{positive polynomials on } S^{d-1} \right\}$$

Polyradial & polygauge bodies

ETH zürich

Polyradial body: a star-body whose radial function is polynomial

(Polygauge) (gauge)

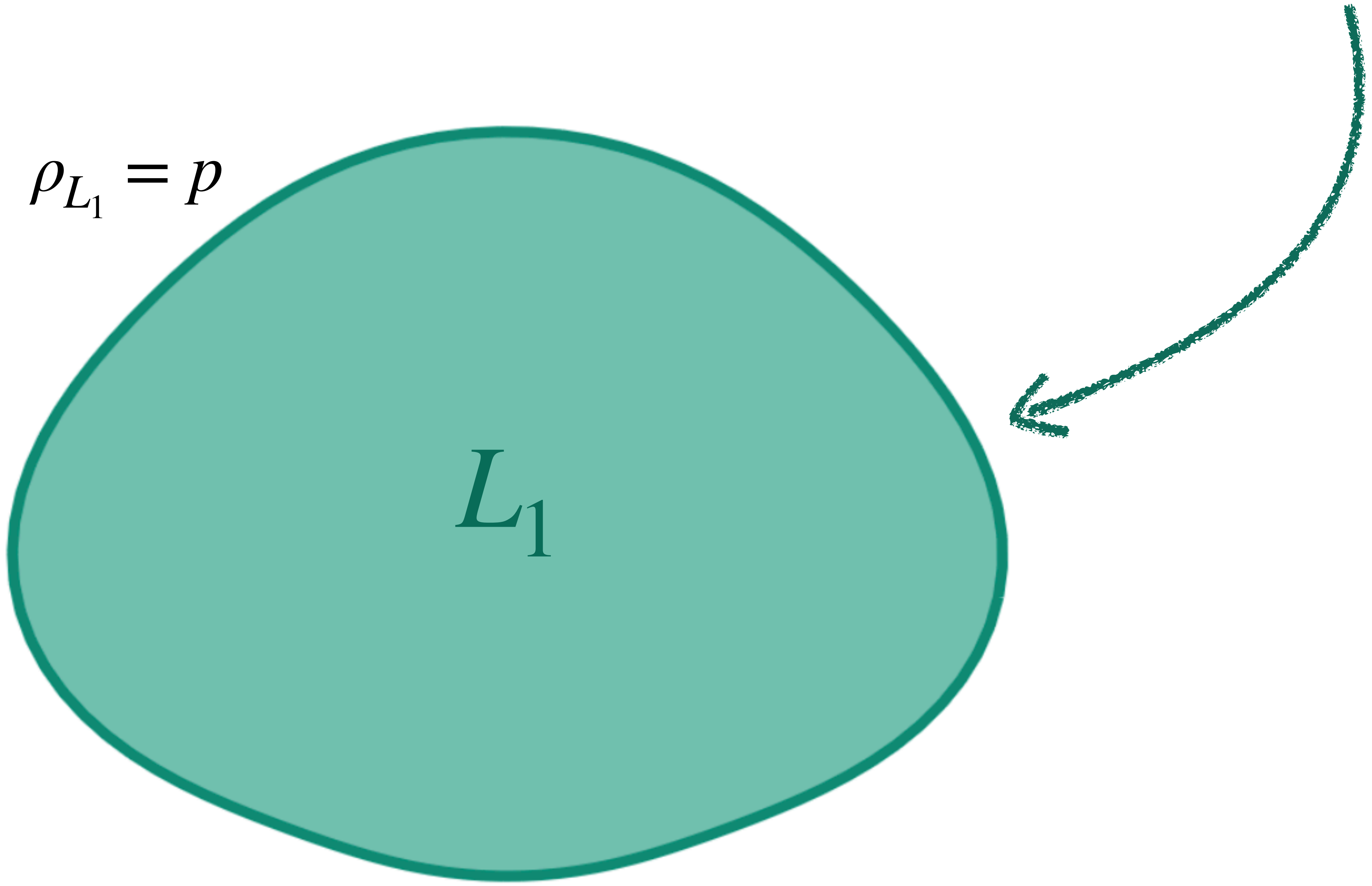
$$p = 32x^6 + 32y + 128 = 32((\cos\theta)^6 + \sin\theta + 4)$$

Polyradial & polygauge bodies

ETH zürich

Polyradial body: a star-body whose radial function is polynomial
(Polygauge) (gauge)

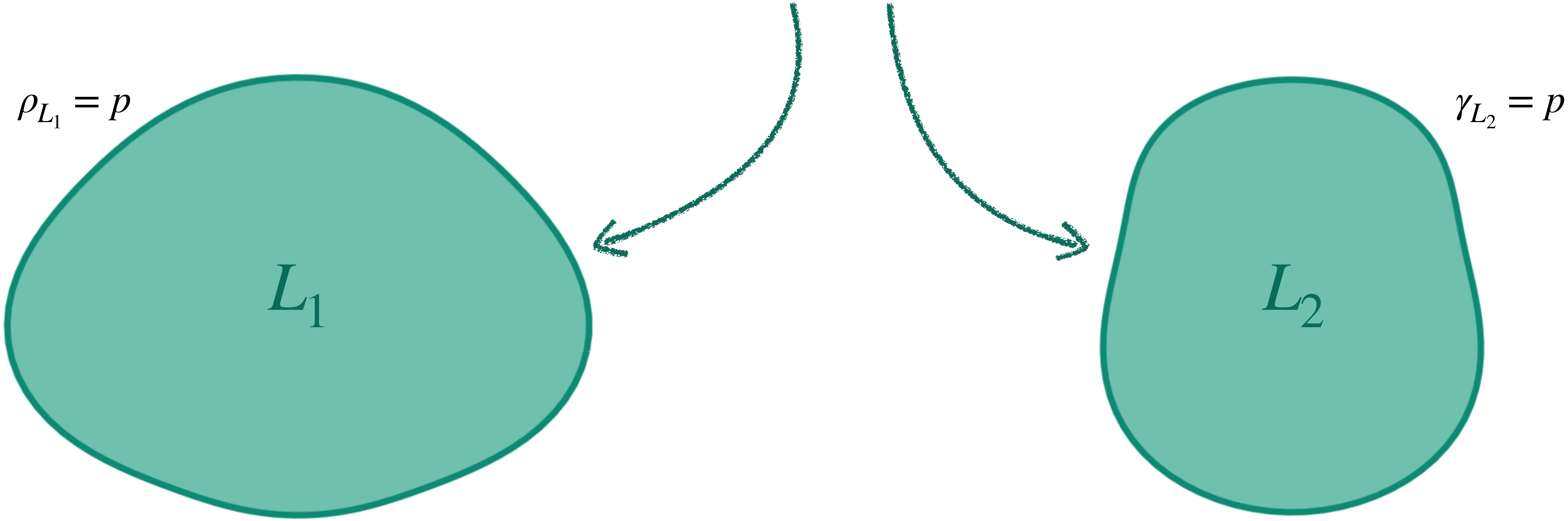
$$p = 32x^6 + 32y + 128 = 32((\cos\theta)^6 + \sin\theta + 4)$$



Polyradial & polygauge bodies

Polyradial body: a star-body whose radial function is polynomial
(Polygauge) (gauge)

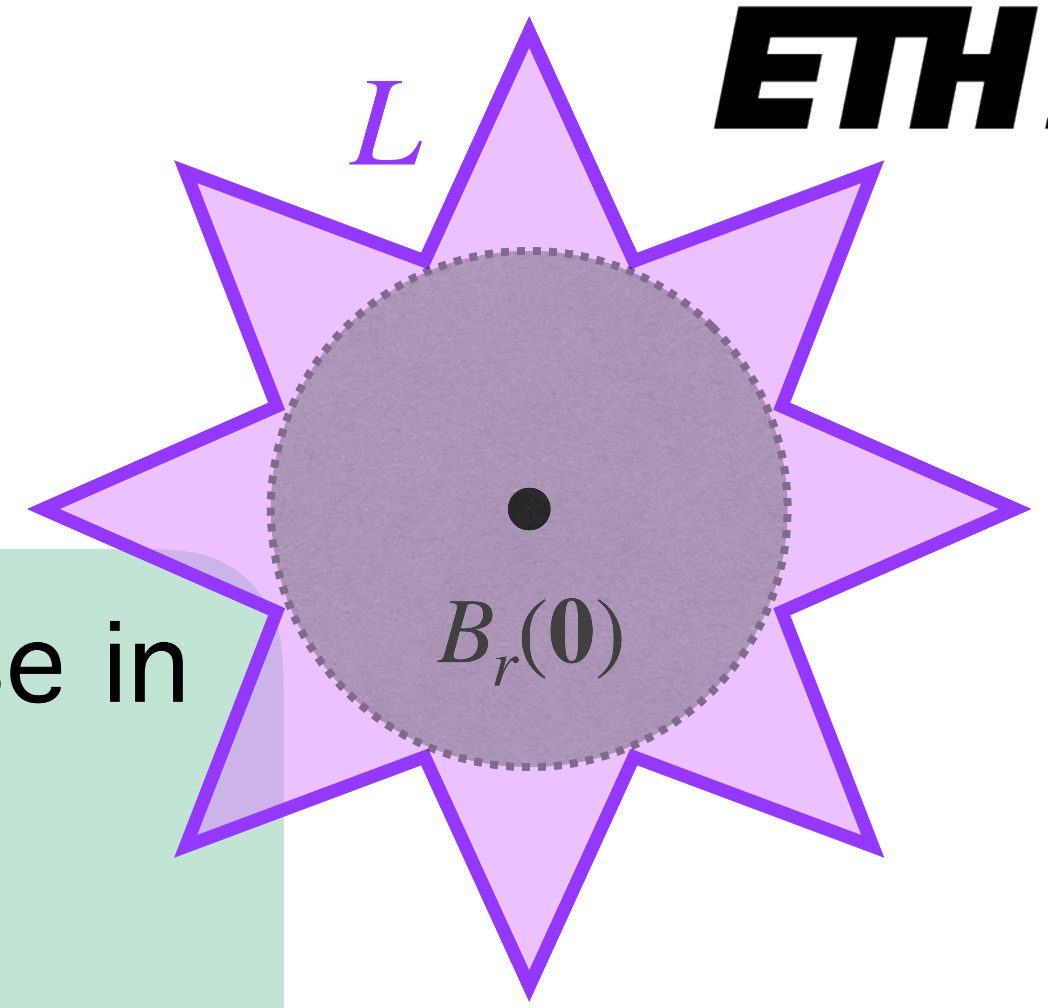
$$p = 32x^6 + 32y + 128 = 32((\cos\theta)^6 + \sin\theta + 4)$$



Density

How strict is this assumption?

Density



How strict is this assumption?

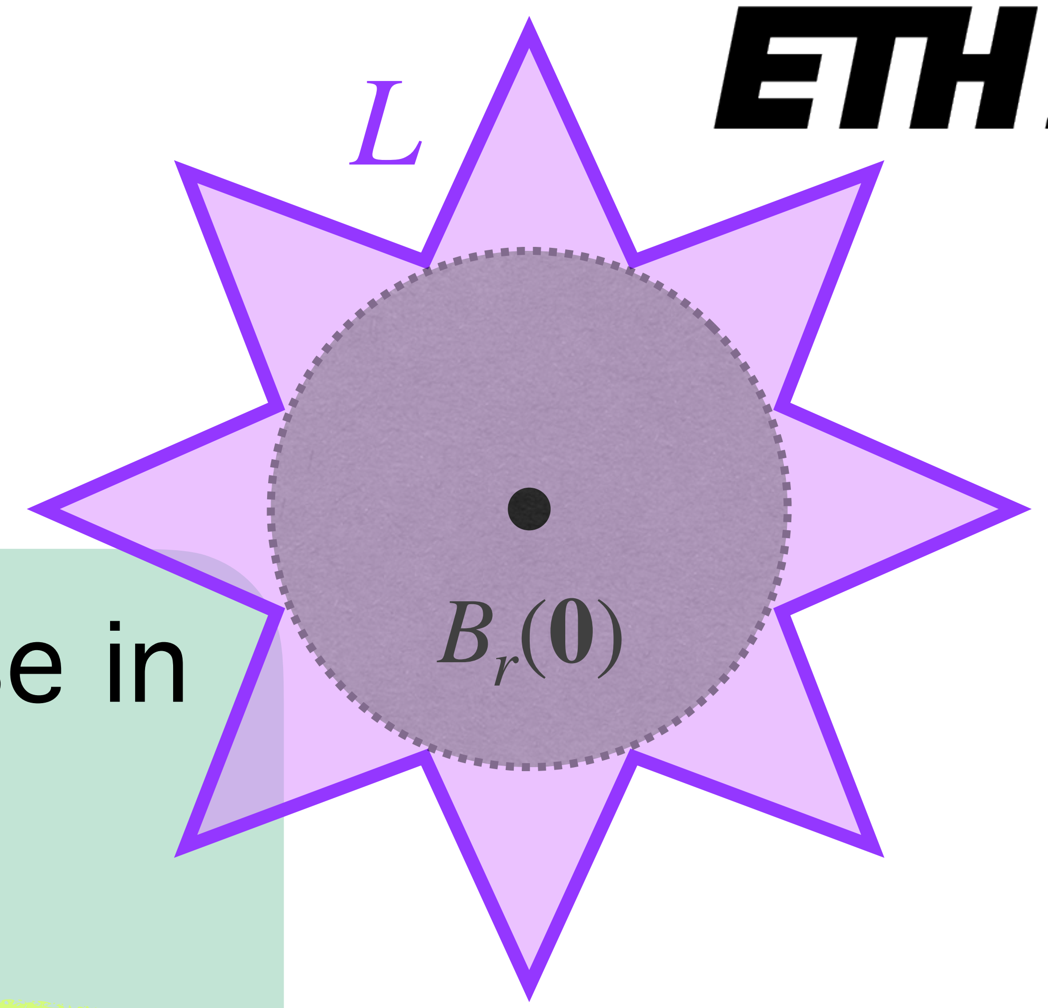
Theorem [M,M,V]: The set of polyradial/polygauge bodies is dense in the set of star-bodies with continuous radial/gauge function.

If $B_r(\mathbf{0}) \subset L$, then the smallest distance between L and

- a polygauge body of degree k is in $\mathcal{O}\left(\frac{1}{r\sqrt{k}}\right)$,
- a polyradial body of degree k is in $\mathcal{O}\left(\frac{1}{rm\sqrt{k}}\right)$, where $m = \inf \left|\frac{1}{\rho_L(x)}\right|^2$.

Density

How strict is this assumption?



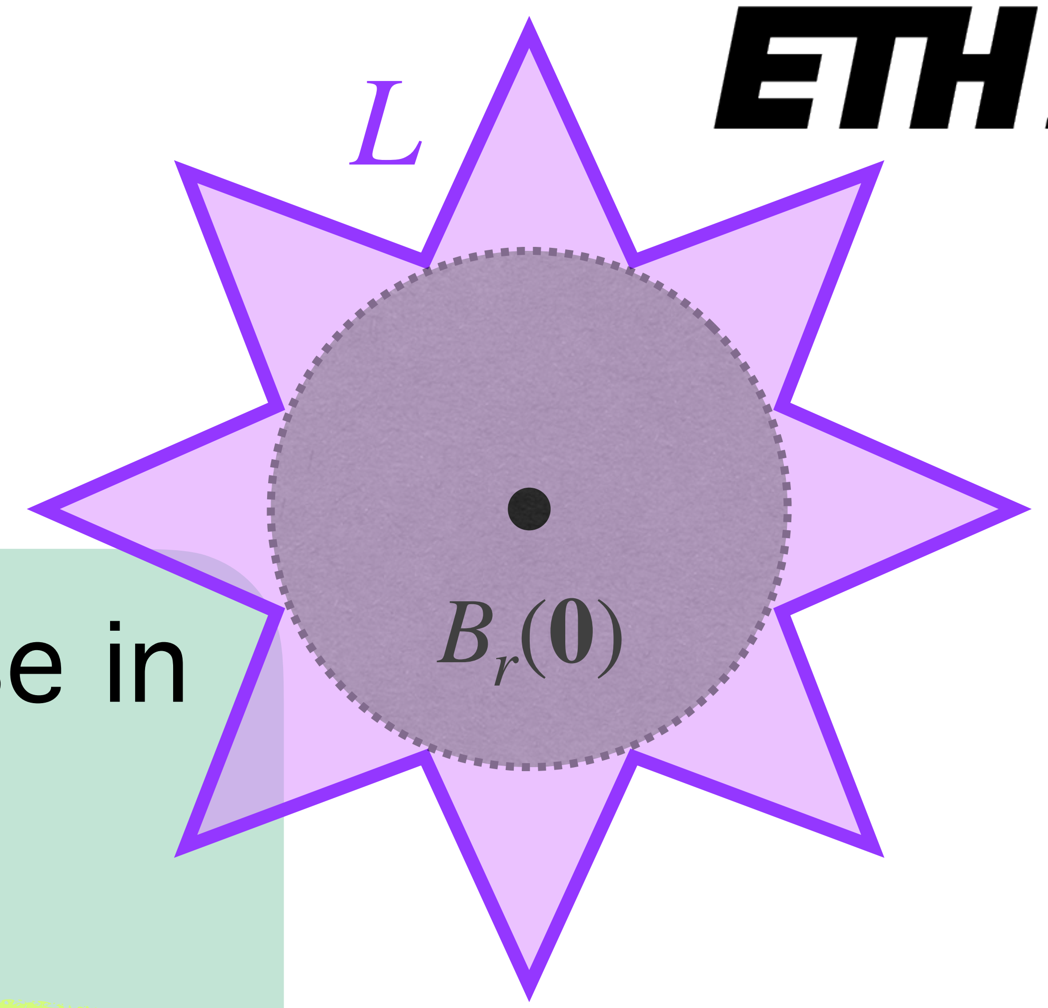
Theorem [M,M,V]: The set of polyradial/polygauge bodies is dense in the set of star-bodies with continuous radial/gauge function.

If $B_r(\mathbf{0}) \subset L$, then the smallest distance between L and

- a polygauge body of degree k is in $\mathcal{O}\left(\frac{1}{r\sqrt{k}}\right)$,
- a polyradial body of degree k is in $\mathcal{O}\left(\frac{1}{rm\sqrt{k}}\right)$, where $m = \inf \left|\frac{1}{\rho_L(x)}\right|^2$.

Spherical harmonics
Funk-Hecke formula
Fourier series
Convolutions

Density



How strict is this assumption?

Theorem [M,M,V]: The set of polyradial/polygauge bodies is dense in the set of star-bodies with continuous radial/gauge function.

If $B_r(\mathbf{0}) \subset L$, then the smallest distance between L and

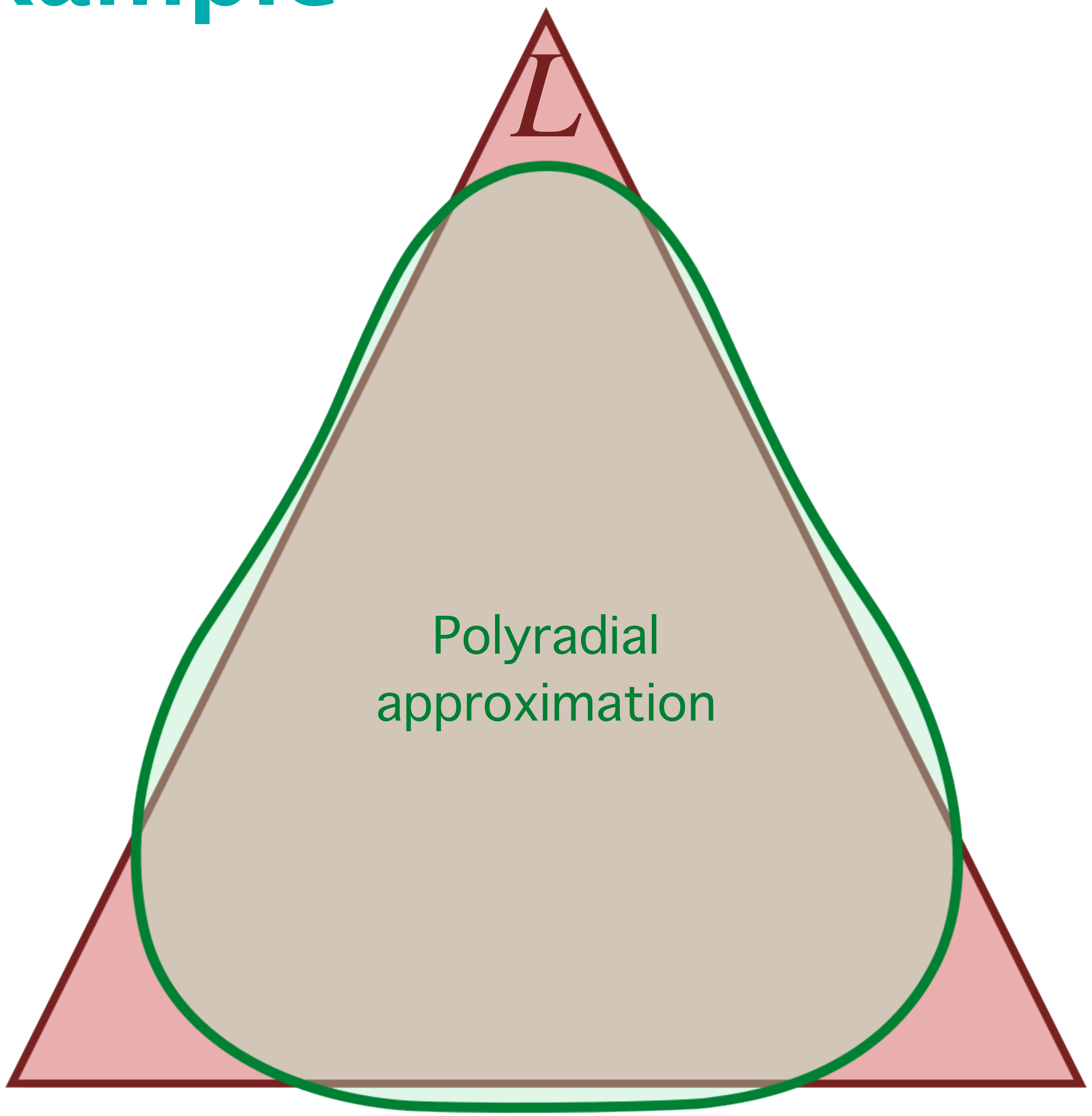
- a polygauge body of degree k is in $\mathcal{O}\left(\frac{1}{r\sqrt{k}}\right)$,
- a polyradial body of degree k is in $\mathcal{O}\left(\frac{1}{rm\sqrt{k}}\right)$, where $m = \inf \left|\frac{1}{\rho_L(x)}\right|^2$.

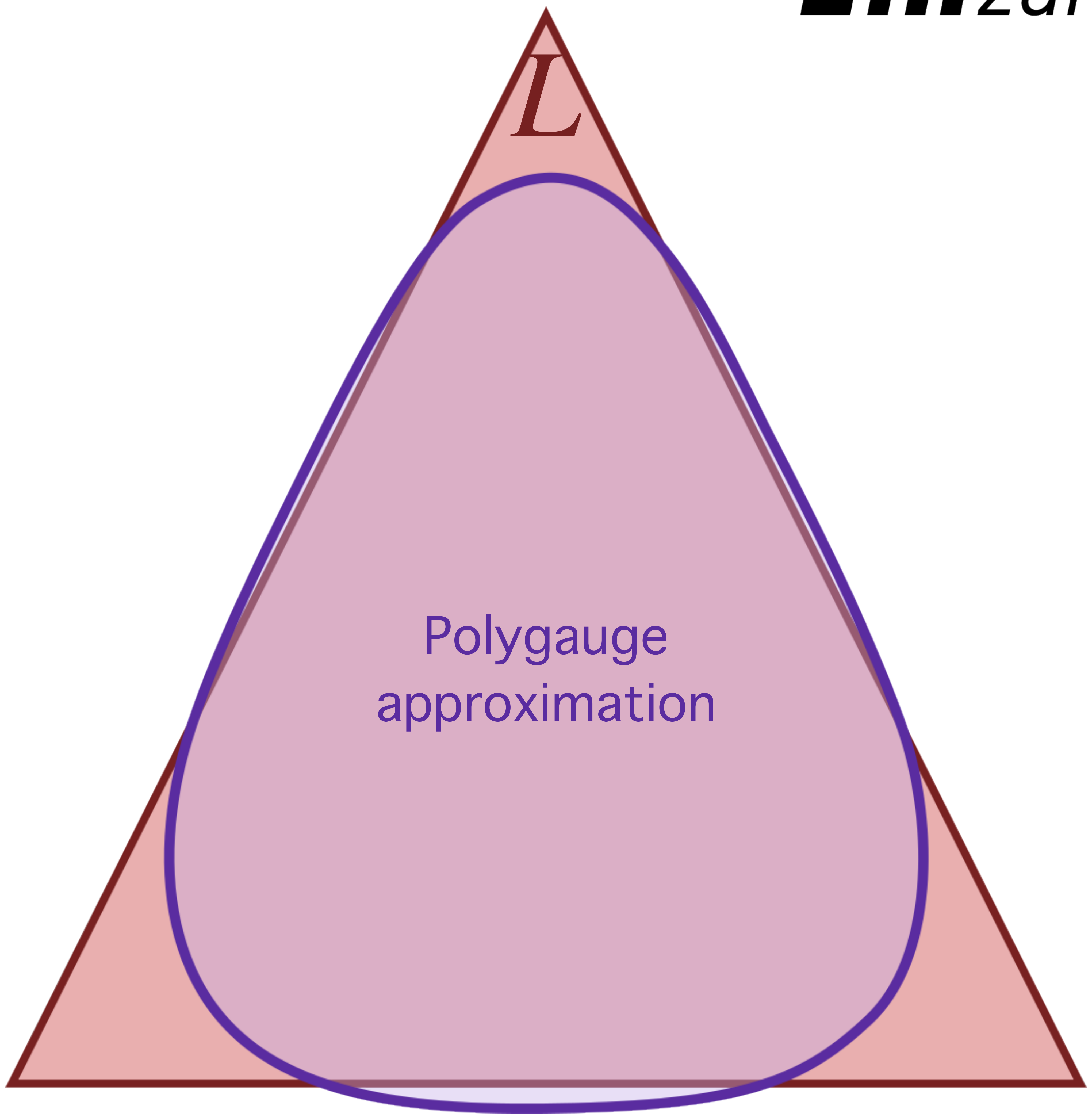
Spherical harmonics
Funk-Hecke formula
Fourier series
Convolutions

Moreover: if L is convex, it's easy to make the approximating polygauge body convex too.

Example

ETH zürich

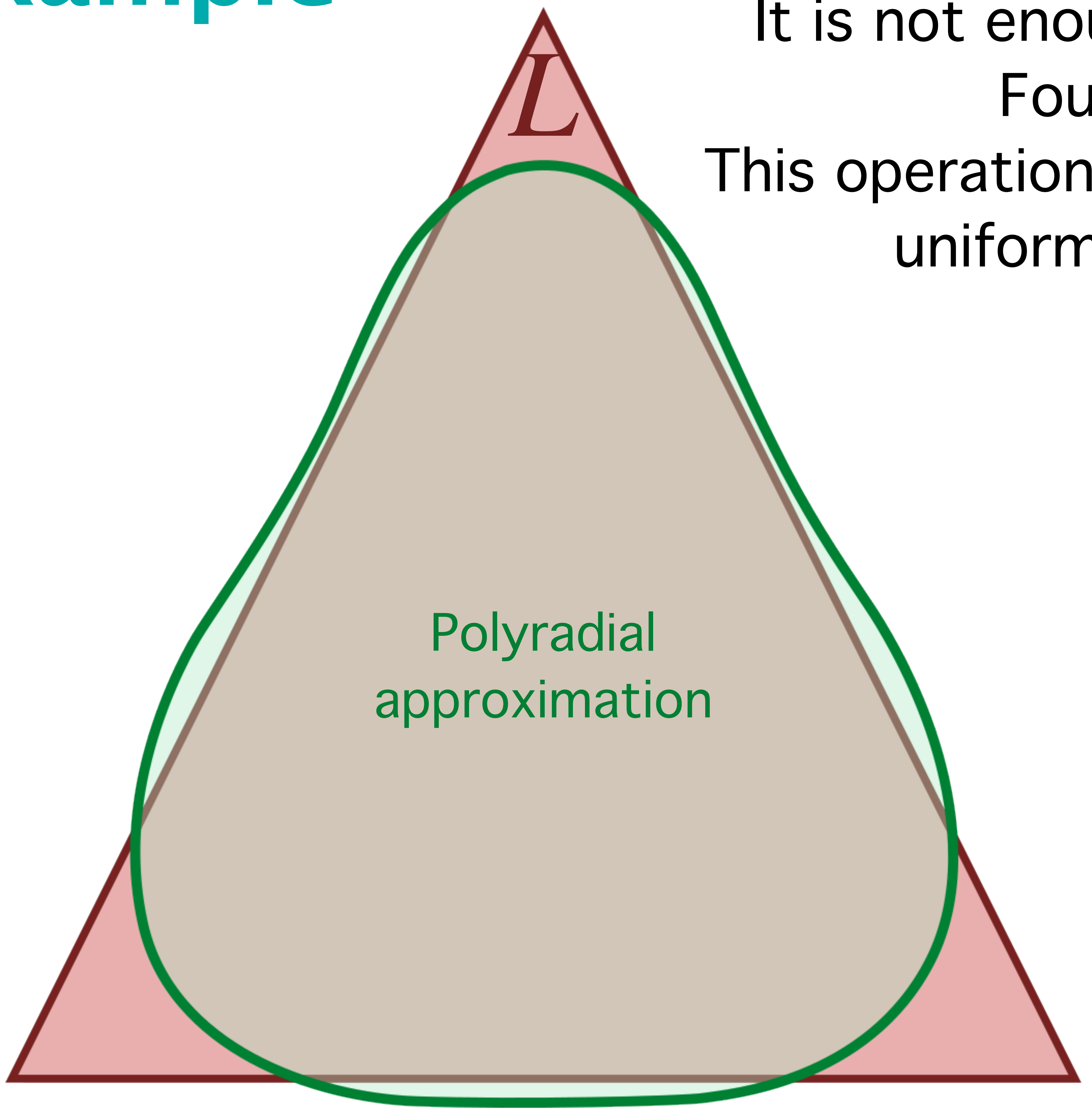


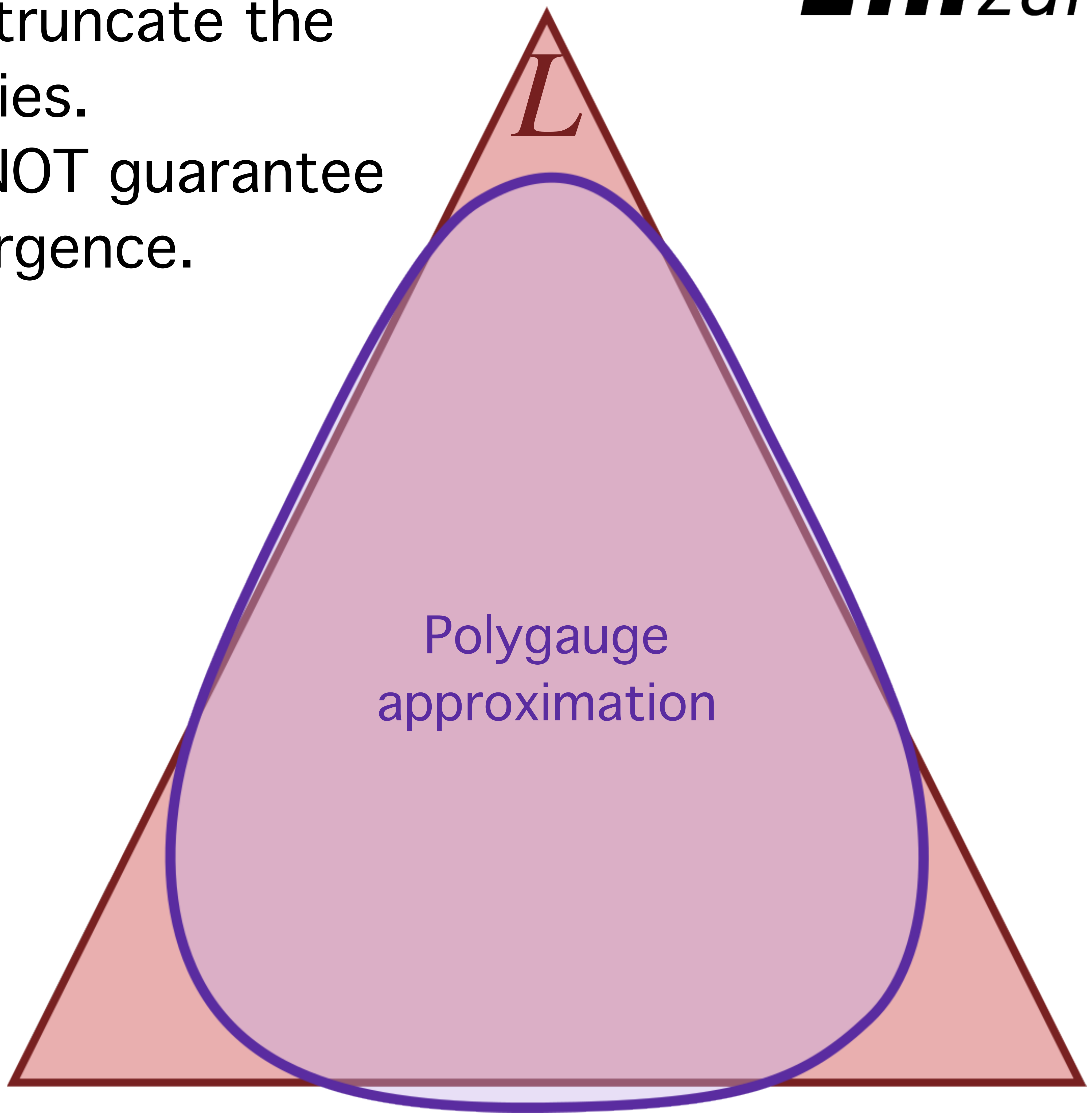


Example

ETH zürich

Note:
It is not enough to truncate the Fourier series.
This operation does NOT guarantee uniform convergence.





Okay but why?

ETH zürich

Back to the slice volume...
(central)

$$\mathrm{vol}(L \cap x^{\perp}) = \mathscr{R}(\rho_L(x))$$

Okay but why?

ETH zürich

Back to the slice volume...
(central)

~ Radon transform

$$\mathrm{vol}(L \cap x^{\perp}) = \mathscr{R}(\rho_L(x))$$

Okay but why?

ETH zürich

Back to the slice volume...
(central)

$\sim$ Radon transform

$$\mathrm{vol}(L \cap x^{\perp}) = \mathscr{R}(\rho_L(x))$$

It is a positive polynomial
in $x \in S^{d-1}$ if L is polyradial!

Okay but why?

ETH zürich

Back to the slice volume...
(central)

$\sim$ Radon transform

$$\mathrm{vol}(L \cap x^{\perp}) = \mathscr{R}(\rho_L(x))$$

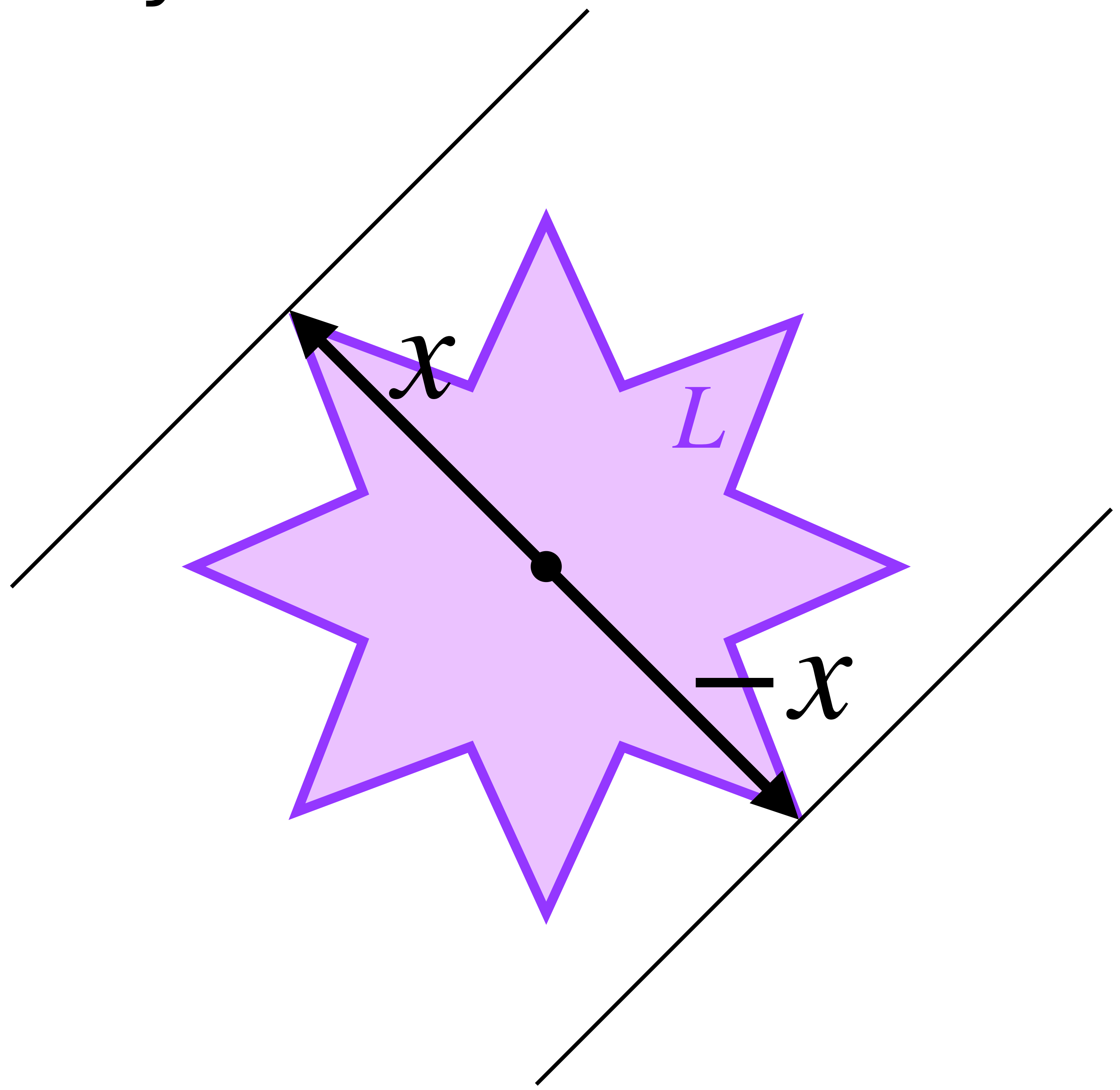
It is a positive polynomial in $x \in S^{d-1}$ if L is polyradial!

Finding the slice with maximal (minimal) volume is reduced to a sum-of-squares optimization

More optimization…

With the gauge function we can express the width of a body:

$$w(L) = \max_{x \in S^{d-1}} \gamma_{(\mathrm{conv}\, L)^\circ}(x) + \gamma_{(\mathrm{conv}\, L)^\circ}(-x)$$

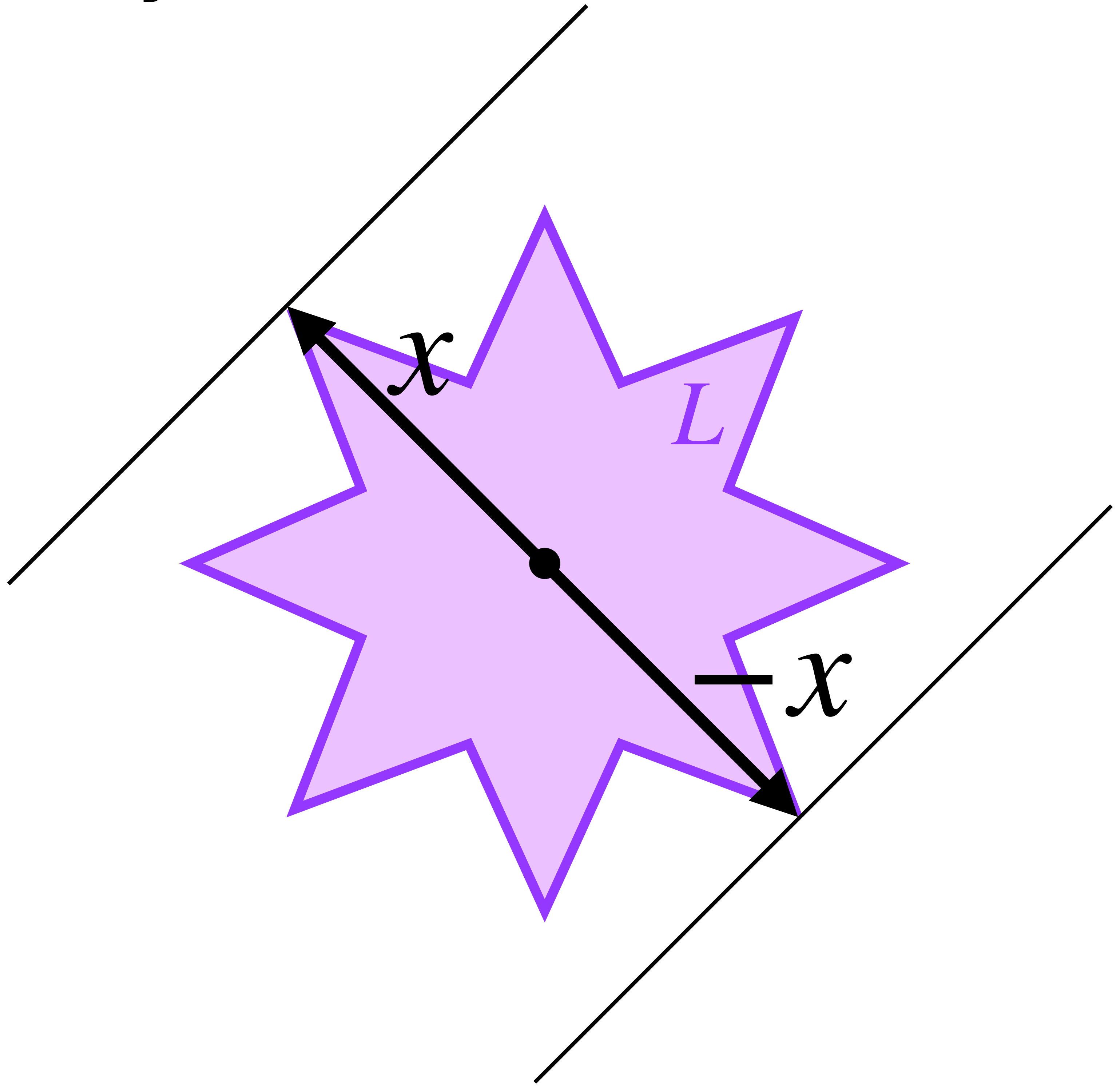


More optimization…

ETH zürich

With the gauge function we can express the width of a body:

$$w(L) = \max_{x \in S^{d-1}} \gamma_{(\mathrm{conv}\, L)^\circ}(x) + \gamma_{(\mathrm{conv}\, L)^\circ}(-x)$$



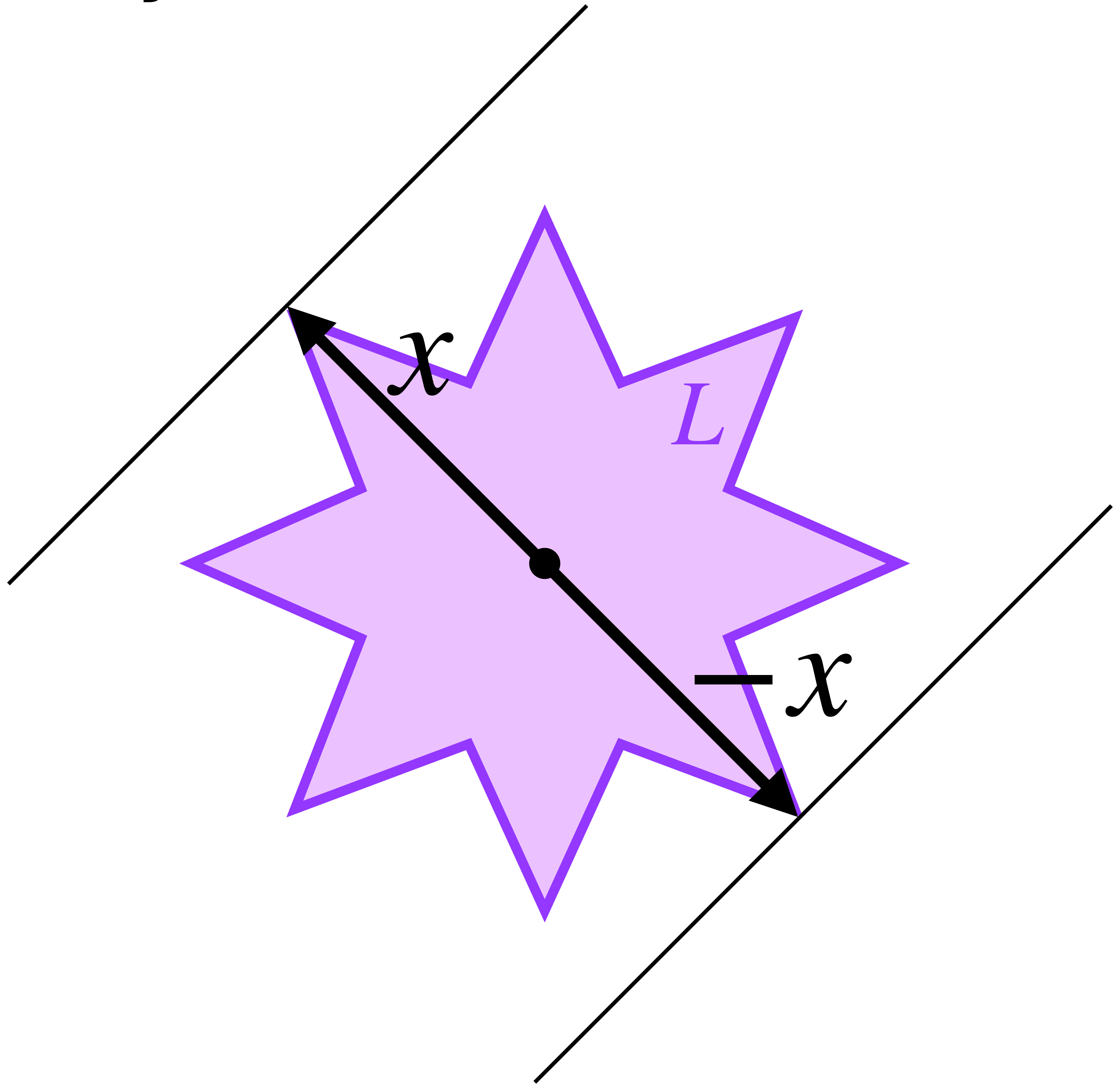
Meissner: among all convex bodies with constant unit width, what is the smallest possible volume?

More optimization…

ETH zürich

With the gauge function we can express the width of a body:

$$w(L) = \max_{x \in S^{d-1}} \gamma_{(\mathrm{conv}\, L)^\circ}(x) + \gamma_{(\mathrm{conv}\, L)^\circ}(-x)$$



Meissner: among all convex bodies with constant unit width, what is the smallest possible volume?

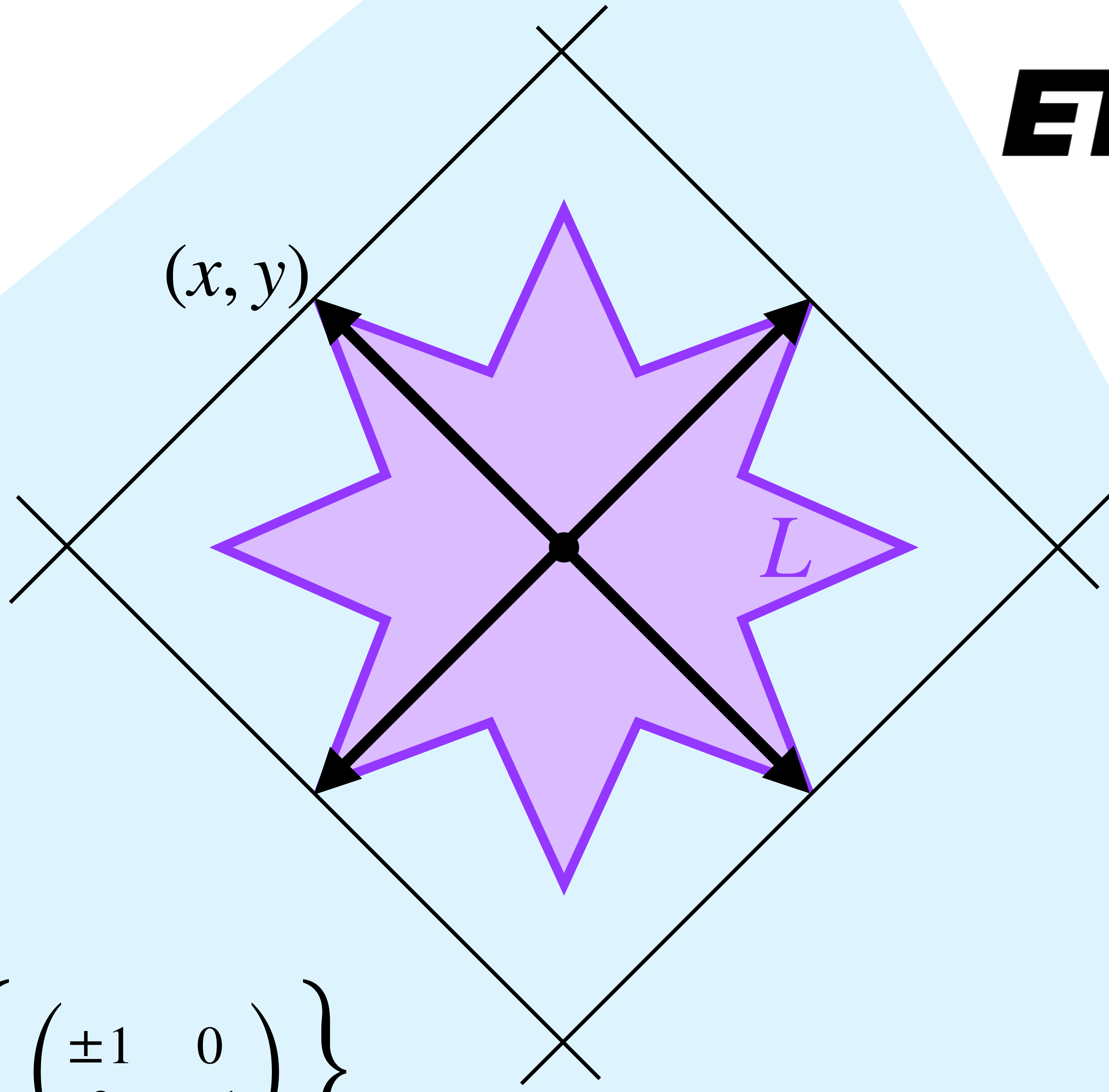
Volume and width constraints can be formulated in terms of the gauge function

G-width

Define the G-width now, for a group $G \leq O(d)$:

$$w_G(L) = \max_{x \in S^{d-1}} \frac{1}{\operatorname{vol} G} \int_G \gamma_{(\operatorname{conv} L)^\circ}(Gx)\, \mathrm{d}\omega_G$$

$$w(L) = w_{\{Id,-Id\}}(L)$$



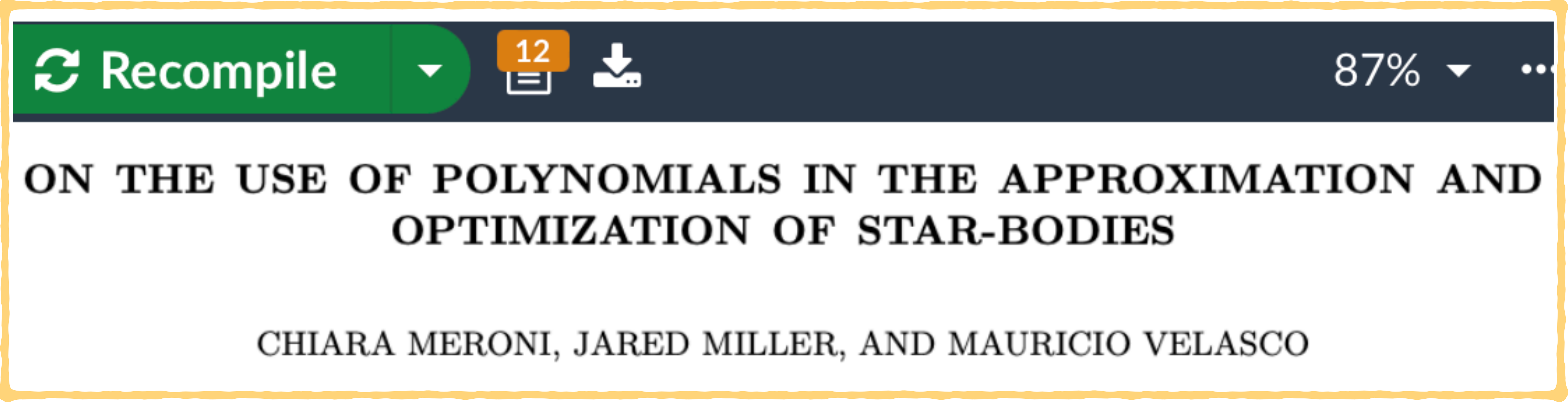
$$G = \left\{ \begin{pmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{pmatrix} \right\}$$

$$w_G(L) = \max_{(x,y) \in S^1} \left(\gamma_{(\operatorname{conv} L)^\circ}(x, y) + \gamma_{(\operatorname{conv} L)^\circ}(x, -y) + \gamma_{(\operatorname{conv} L)^\circ}(-x, -y) + \gamma_{(\operatorname{conv} L)^\circ}(-x, y) \right)$$

G-Meissner: among all convex bodies with constant unit G-width, what is the smallest possible volume?

Summary

ETH zürich

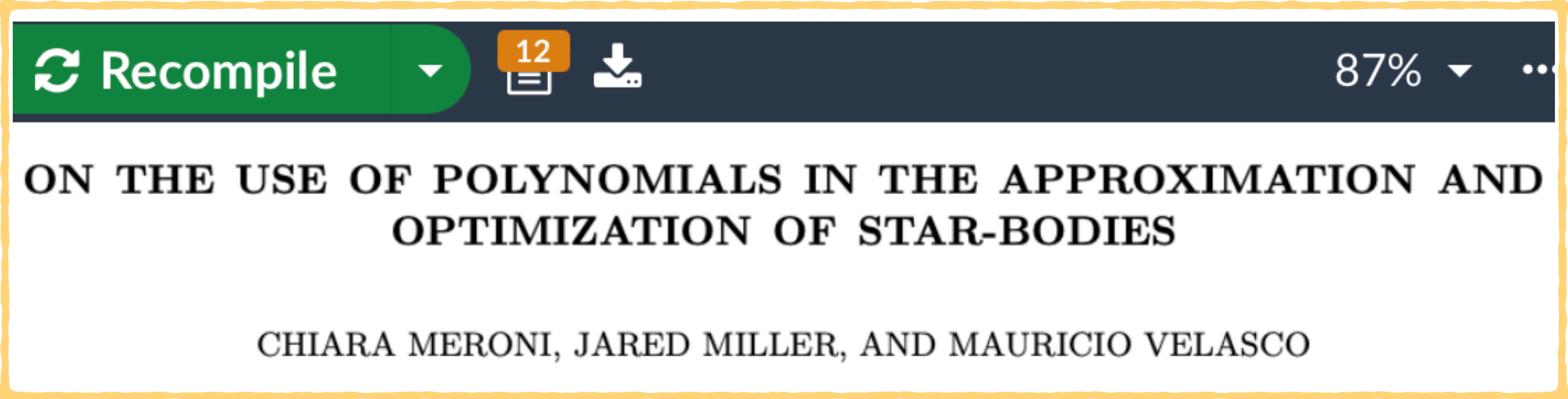


ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO

Summary

ETH zürich



ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO

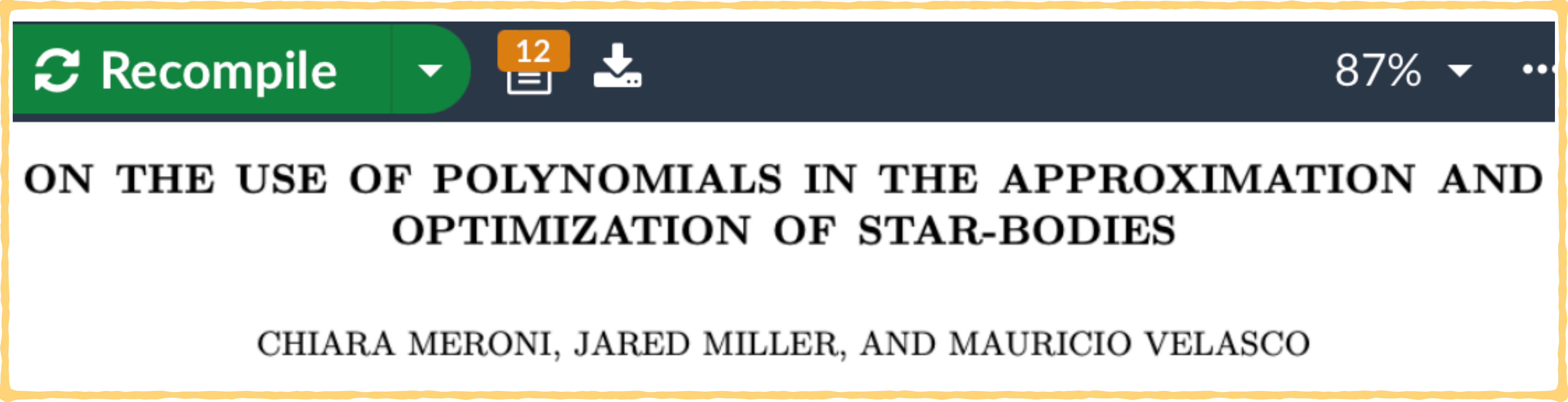
PROS:
- Exact for polyradial/polygauge bodies
- Optimization over the set of convex/star-bodies/polytopes
- Works probably in $\mathbb{R}^3$

CONS:
- For polytopes just an approximation
- In $\mathbb{R}^4$ it is probably out of reach

WHY NOT ON ARXIV: Implementation is a work-in-progress!

Summary

ETH zürich



Recompile 12 87%

ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO

PROS: Exact for polyradial/polygauge bodies
Optimization over the set of convex/star-bodies/polytopes
Works probably in $\mathbb{R}^3$

CONS: For polytopes just an approximation
In $\mathbb{R}^4$ it is probably out of reach

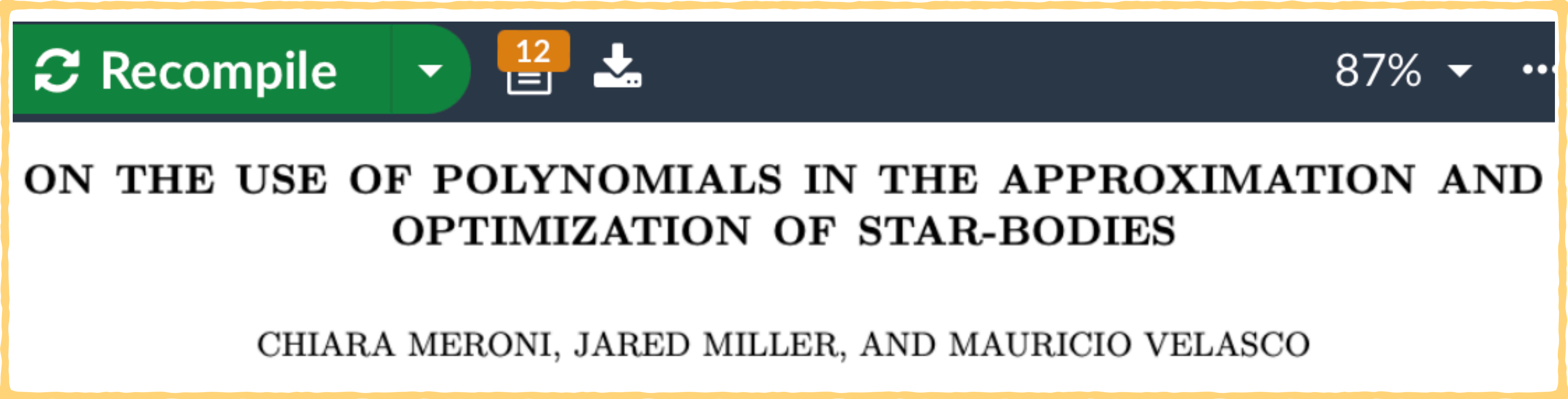
!!!

WHY NOT ON ARXIV: Implementation is a work-in-progress!

Do you have an optimization problem that fits in this framework?
Come talk to me!

Summary

ETH zürich



ON THE USE OF POLYNOMIALS IN THE APPROXIMATION AND OPTIMIZATION OF STAR-BODIES

CHIARA MERONI, JARED MILLER, AND MAURICIO VELASCO

PROS:
- Exact for polyradial/polygauge bodies
- Optimization over the set of convex/star-bodies/polytopes
- Works probably in $\mathbb{R}^3$

CONS:
- For polytopes just an approximation
- In $\mathbb{R}^4$ it is probably out of reach

WHY NOT ON ARXIV: Implementation is a work-in-progress!

!!!

Do you have an optimization problem that fits in this framework?
Come talk to me!

Thank you!
Thanks ICERM!