Discrete Optimization: Mathematics, Algorithms, and Computation
Poster Session Abstracts
Wednesday, August 28, 2024

The Madness of Multiple Entries in March Madness
Jeff Decary, University of Connecticut

This paper explores multi-entry strategies for betting pools related to single-elimination tournaments. In such betting pools, participants select winners of games, and their respective score is a weighted sum of the number of correct selections. Most betting pools have a top-heavy payoff structure, so the paper focuses on strategies that maximize the expected score of the best-performing entry. There is no known closed-formula expression for the estimation of this metric, so the paper investigates the challenges associated with the estimation and the optimization of multi-entry solutions. We present an exact dynamic programming approach for calculating the maximum expected score of any given fixed solution, which is exponential in the number of entries. We explore the structural properties of the problem to develop several solution techniques. In particular, by extracting insights from the solutions produced by one of our algorithms, we design a simple yet effective problem-specific heuristic that was the best-performing technique in our experiments, which were based on real-world data extracted from recent March Madness tournaments. In particular, our results show that the best 100-entry solution identified by our heuristic had a 2.2% likelihood of winning a $1 million prize in a real-world betting pool.

A Topological Approach to Simple Descriptions of Convex Hulls of Sets Defined by Three Quadrics
Alex Dunbar, Emory University

We study the convex hull of a set S defined by three quadratic inequalities. A simple way of generating inequalities valid on S is to take nonnegative linear combinations of the defining inequalities of S. We call such inequalities aggregations. We introduce a new technique relating aggregations to properties of the spectral curve, i.e. the curve defined by the vanishing of the determinant polynomial, and utilizing known spectral sequences (Agrachev and Lerario, 2012). We find new families beyond those identified in (Dey, Muñoz, and Serrano, 2022; Blekherman, Dey, and Sun, 2024), where the convex hull is defined by aggregations. We also prove a characterization of the emptiness of the projective variety defined by 3 homogeneous quadratics in terms of the spectral curve generalizing results of (Agrachev, 1988).

Using Machine Learning to Motivate Advancements in the Theory of Discrete Curvature on Polytopes
Jillian Eddy, University of California, Davis

By applying discrete curvature metrics to polytopes, we can characterize the geometry of their associated graphs and control desirable characteristics like their diameter. Through the use of online graph databases and randomly generated polytopal data, we produced thousands of graphs and data corresponding to their graph-theoretical features. A unique aspect of our data collection is the computation of the effective resistance curvature at each node of every graph. Utilizing our generated data, we constructed various machine learning classification models to predict if a graph has everywhere positive curvature or not. These models have proved incredibly accurate, and saliency analysis of these models allows us to identify and prioritize the key graph features that influence curvature predictions.

Our utilization of machine learning provided experimental results that can now inform theoretical developments. Via another discrete curvature method developed by Forman, we have characterized which 3-dimensional polytopes are everywhere positive, and that there are finitely many such objects. We hope to use our experimental data to motivate analogous theorems for effective resistance curvature, as well as mimic our computational experiments in higher dimensions to motivate further results in either discrete curvature notion.

Solving Optimization Problems Over Stable Matchings Using Minimum Cuts
Chengyue He, Columbia University

Stability is a fundamental concept in matching markets problems when we do not only wish to optimize a global objective function, but we also care that the output solution is fair at the level of individual agents. We present how the optimization problems on stable matchings relate to minimum cut problems. Our results can be applied to some real-world situations like matching siblings to schools and two-stage stable matchings.

On Sparse Canonical Correlation Analysis
Yongchun Li, The University of Tennessee, Knoxville

The classical Canonical Correlation Analysis (CCA) identifies the correlations between two sets of multivariate variables based on their covariance, which has been widely applied in diverse fields such as computer vision, natural language processing, and speech analysis. Despite its popularity, CCA can encounter challenges in explaining correlations between two variable sets within high-dimensional data contexts. Thus, this paper studies Sparse Canonical Correlation Analysis (SCCA) that enhances the interpretability of CCA. We first show that SCCA generalizes three well-known sparse optimization problems, sparse PCA, sparse SVD, and sparse regression, which are all classified as NP-hard problems. This result motivates us to develop strong formulations and efficient algorithms. Our main contributions include (i) the introduction of a combinatorial formulation that captures the essence of SCCA and allows the development of approximation algorithms; (ii) the derivation of an equivalent mixed-integer semidefinite programming model that facilitates a specialized branch-and-cut algorithm with analytical cuts; and (iii) the establishment of the complexity results for two low-rank special cases of SCCA.The effectiveness of our proposed formulations and algorithms is validated through numerical experiments.

On the Congruency-Constrained Matroid Base
Siyue Liu, Carnegie Mellon University

Consider a matroid where all elements are labeled with an integer. We are interested in finding a base where the sum of the labels is congruent to g mod m. We show that this problem is fixed parameter tractable if we parametrize by m when m is either the product of two primes or a prime power. The algorithm can be generalized to all moduli and, in fact, to all abelian groups if a classic additive combinatorics conjecture by Schrijver and Seymour holds true. We also discuss the optimization version of the problem.

Integer Points in Arbitrary Convex Cones: The Case of the PSD and SOC Cones
Brittney Marsters, University of California, Davis

The integer points in a rational, pointed, polyhedral cone are finitely generated by the Hilbert basis. We extend this classical notion of finite generation of lattice points in cones to more general classes of cones by introducing the action of a finitely generated group. Thus, the semigroup associated with non-polyhedral can sometimes be finitely generated in this way. We show this is true for the cone of positive semidefinite matrices and the second-order cone. Both cones have a finite generating set of integer points, similar in spirit to Hilbert bases, when also considering the action of a finitely generated group.

Convex relaxation for the generalized maximum-entropy sampling problem
Gabriel Ponte, University of Michigan / Federal University of Rio de Janeiro

The generalized maximum-entropy sampling problem (GMESP) is to select an order-s principal submatrix from an order-n covariance matrix, to maximize the product of its t greatest eigenvalues, $0 < t \leq s < n$. Introduced more than 25 years ago, GMESP is a natural generalization of two fundamental problems in statistical design theory: (i) maximum-entropy sampling problem (MESP); (ii) binary D-optimality (D-Opt). In the general case, it can be motivated by a selection problem in the context of principal component analysis (PCA).
We introduce the first convex-optimization based relaxation for GMESP, study its behavior, compare it to an earlier spectral bound, and demonstrate its use in a branch-and-bound scheme. We find that such an approach is practical when s-t is very small.

An ""unsigned"" stochastic lot-sizing problem with an application to battery management
SOSSOU EDOU Rose, École nationale des ponts et chaussées

We consider a capacitated stochastic lot-sizing problem with unsigned orders, demand, and prices. This problem is motivated by a collaboration with EDF, the French electricity provider, aiming to accurately model the optimal management of an electric battery in the ""intraday"" market.
Our main contribution shows that in the special case where orders can be of arbitrary magnitude, the optimal value varies affinely with the initial inventory and an optimal policy can be described by a closed-form expression. This result can be relevant in practice, e.g., it provides bounds. Another contribution addresses the case where prices form a signed martingale, for which we also prove the existence of a closed-form expression of the optimal policy.
On the practical side, we develop an algorithm based on dynamic programming to compute asymptotically optimal policies for the general case. Experiments on industrial instances demonstrate excellent performance.
Joint work with Frédéric Meunier."

Slicing Polyhedra
Antonio Torres, University of California Davis

Given a polyhedron and a plane that intersects it, many interesting questions arise about this intersection. For example, questions about the maximum values for volume, diameter, etc. In this work, we will focus on the combinatorial properties of these "slices," particularly analyzing the behavior of the number of vertices. We will present some results and put forward some conjectures.

Capacitated Network Bargaining Games: Stability and Structure
Lucy Verberk, Eindhoven University of Technology

Capacitated network bargaining games are popular combinatorial games that involve the structure of matchings in graphs. We show that it is always possible to stabilize unit-weight instances of this problem (that is, ensure that they admit a stable outcome) via capacity-reduction and edge-removal operations, without decreasing the total value that the players can get.

Furthermore, for general weighted instances, we show that computing a minimum amount of vertex-capacity to reduce to make an instance stable is a polynomial-time solvable problem. We then exploit this to give approximation results for the NP-hard problem of stabilizing a graph via edge-removal operations.

Our work extends and generalizes previous results in the literature that dealt with a unit-capacity version of the problem, using several new arguments. In particular, while previous results mainly used combinatorial techniques, we here rely on polyhedral arguments and, more specifically, on the notion of circuits of a polytope.