Toward dynamical coupling of neutrino quantum kinetics in relativistic astrophysics

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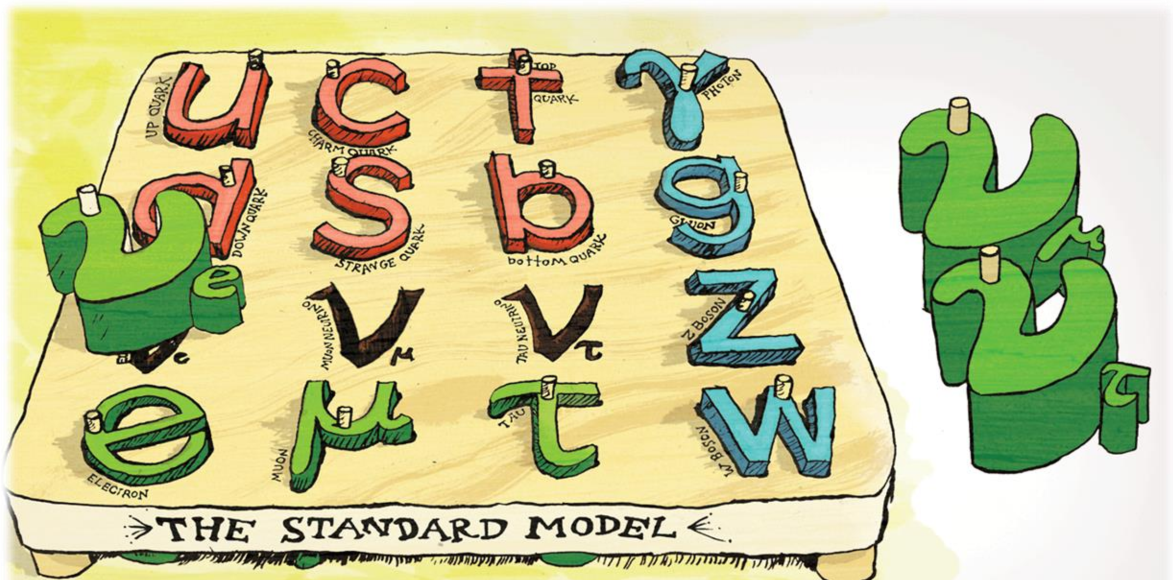
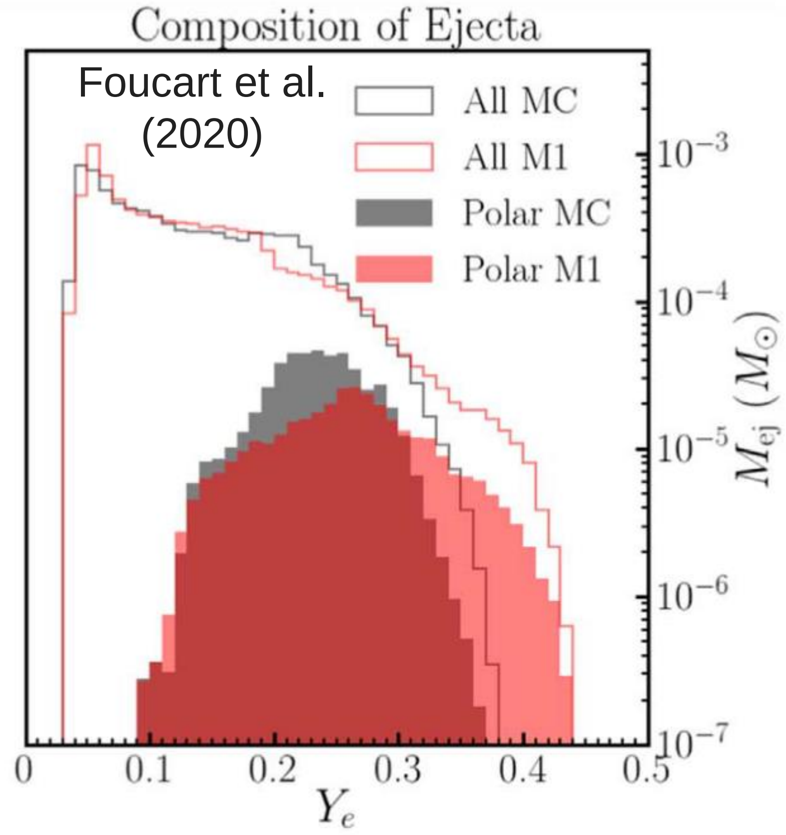
Outline

- **Introduction**: Why should we care about neutrino flavor?
- **Background**: Theory of flavor-changing neutrinos
- **Simulation**: What numerical treatments are appropriate?
  - Leakage / effective models
  - Diffusion / miscidynamics
  - Classical moments / quantum moments
  - Reduced speed of light / reduced coupling
- **Beyond flavor**: neutrino *helicity*

Electron Neutrinos are Special

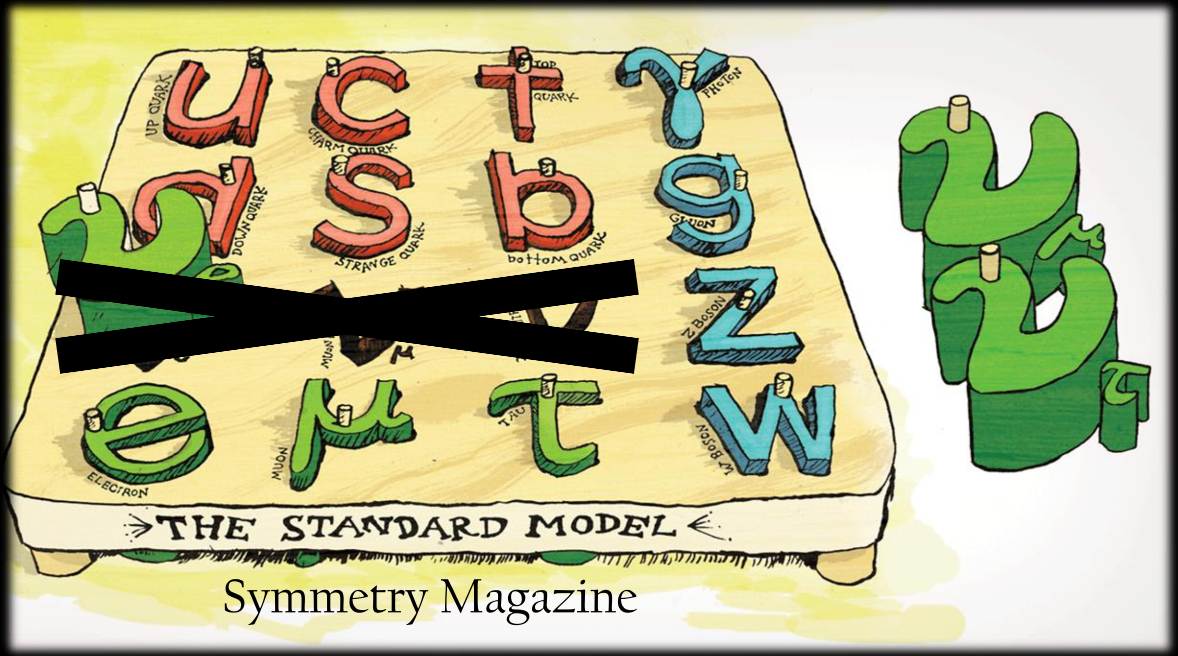


Symmetry Magazine

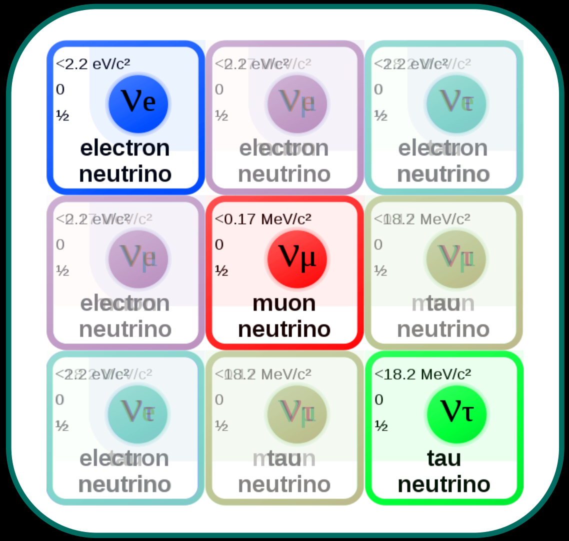


Need accurate neutrino transport to extract physics from observed neutrinos, gravitational waves, and light.

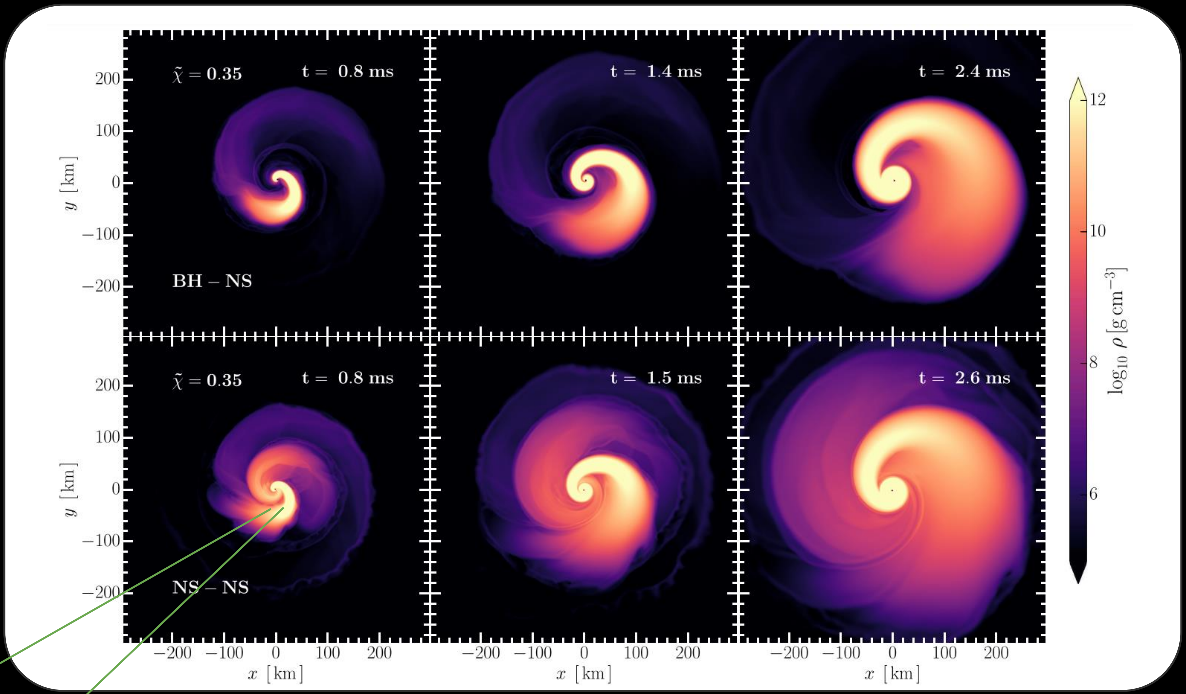
Electron Neutrinos are Special



Symmetry Magazine

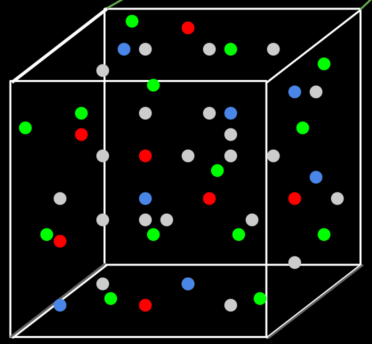


Quantum Neutrino Plasma



(Most et al. (2021))

- proton
- neutron
- electron
- neutrino



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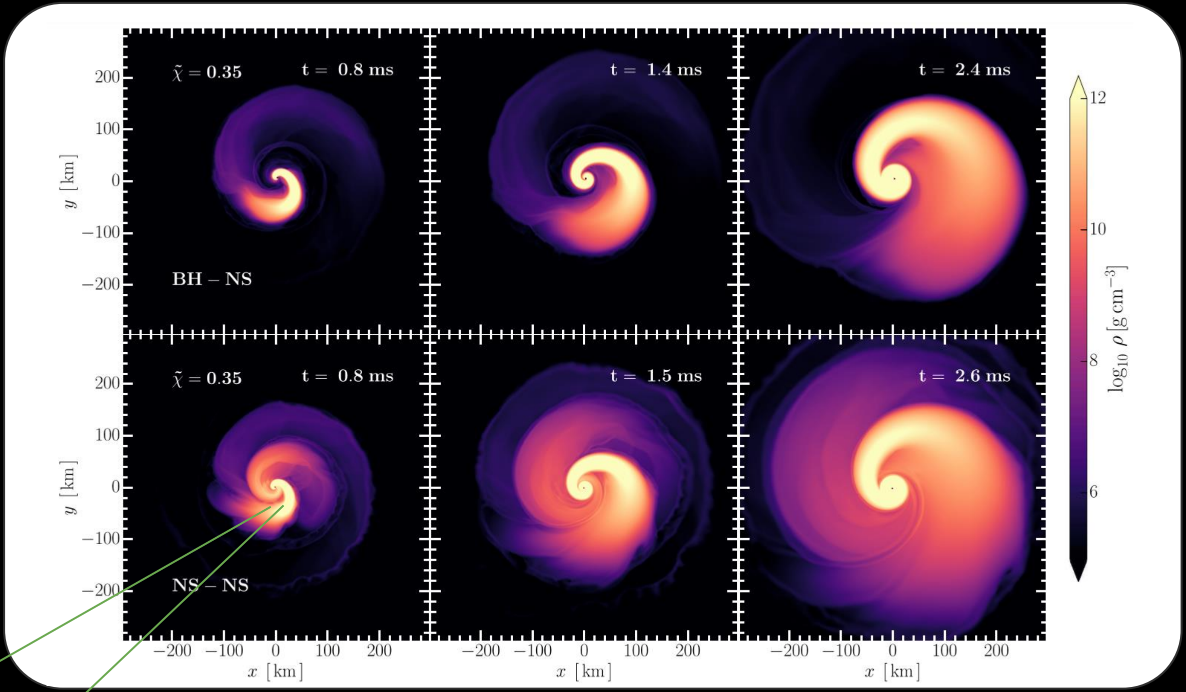
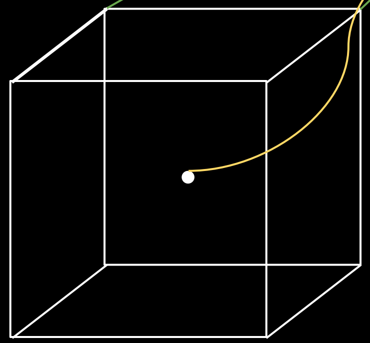
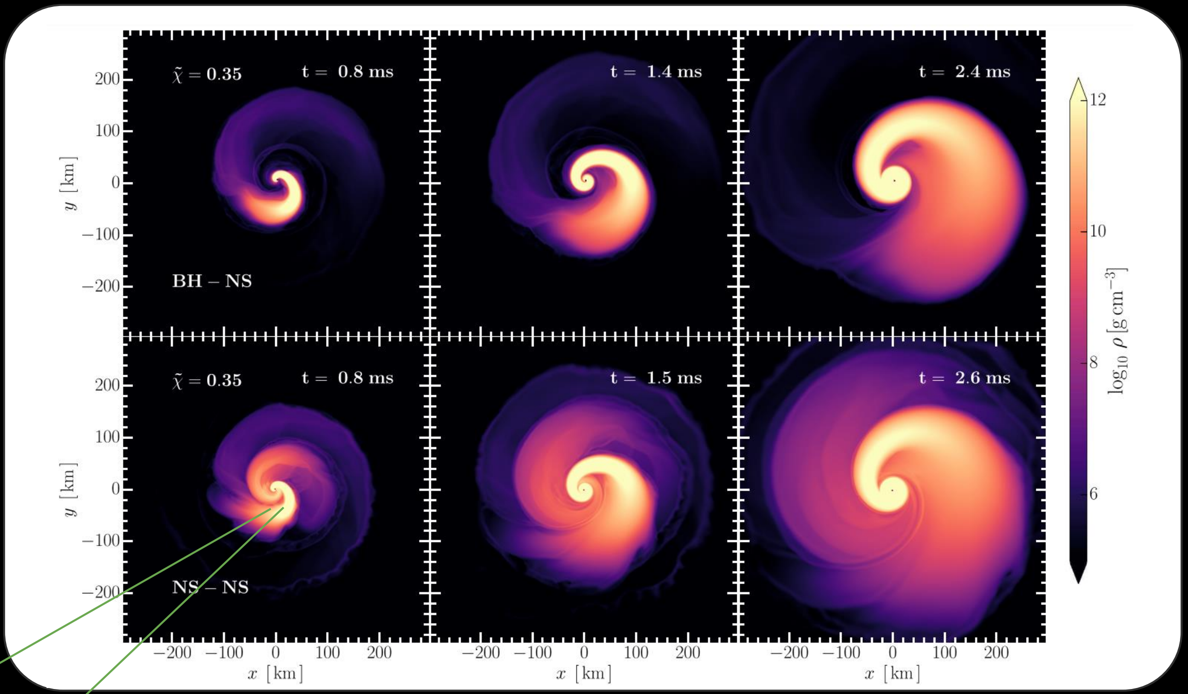


(Most et al. (2021))

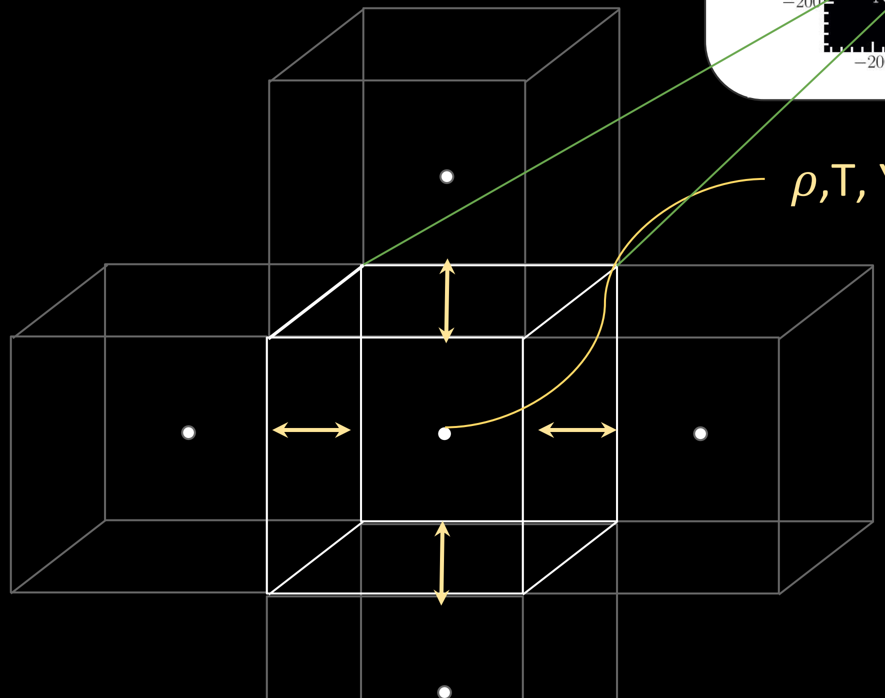
ρ,T, Y_e, v, B, metric



Quantum Neutrino Plasma



(Most et al. (2021))



ρ,T, Y_e, v, B, metric

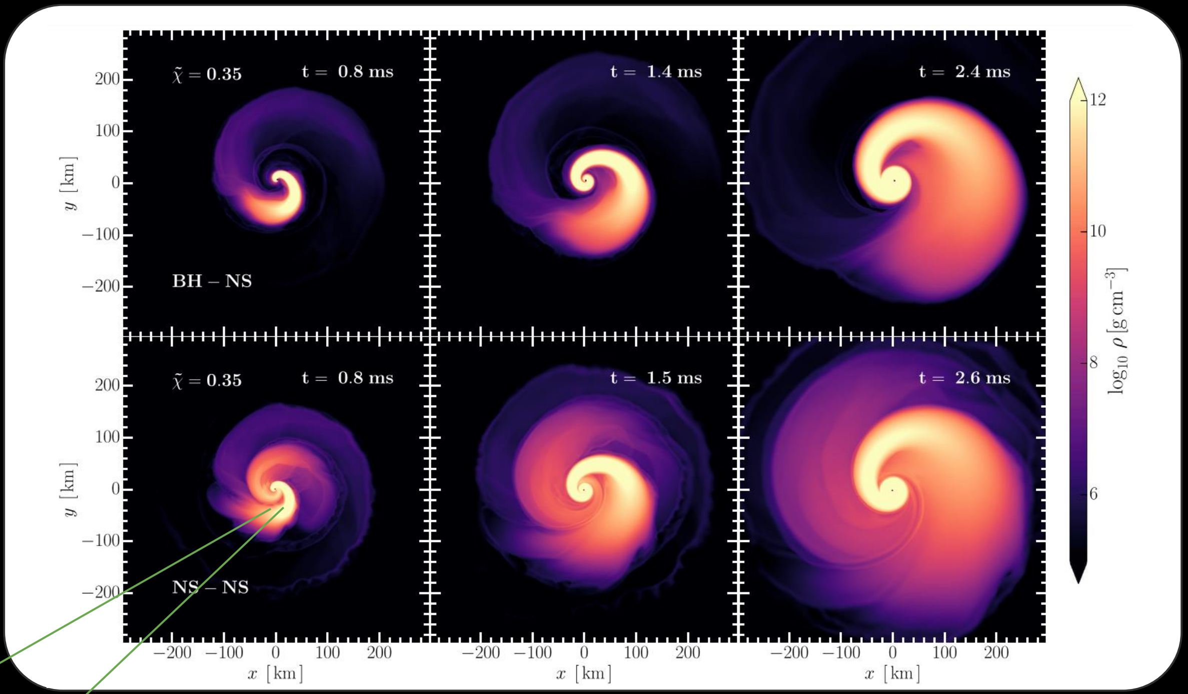
$$D_t \rho = -\rho \vec{\nabla} \cdot \vec{u}$$

$$D_t \vec{u} = -\frac{\vec{\nabla} P}{\rho}$$

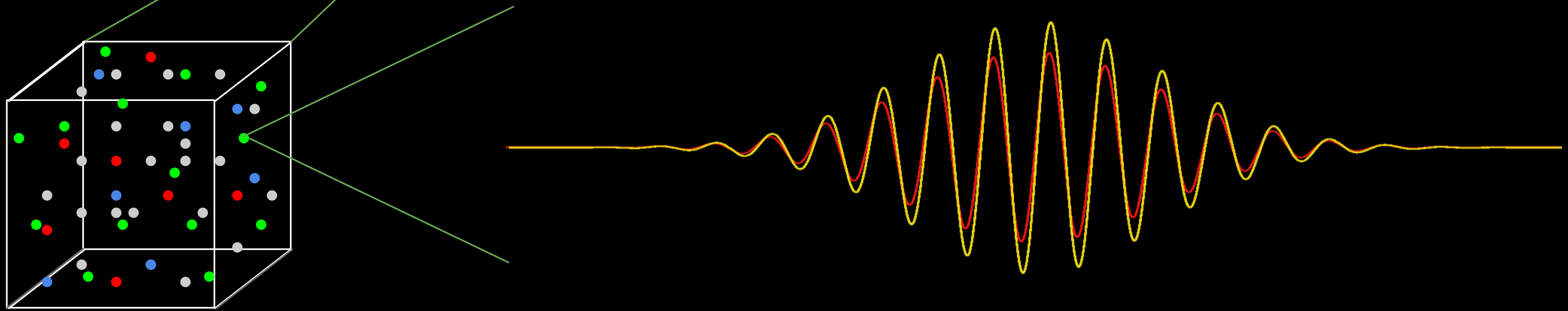
Equation of State

(because in equilibrium)

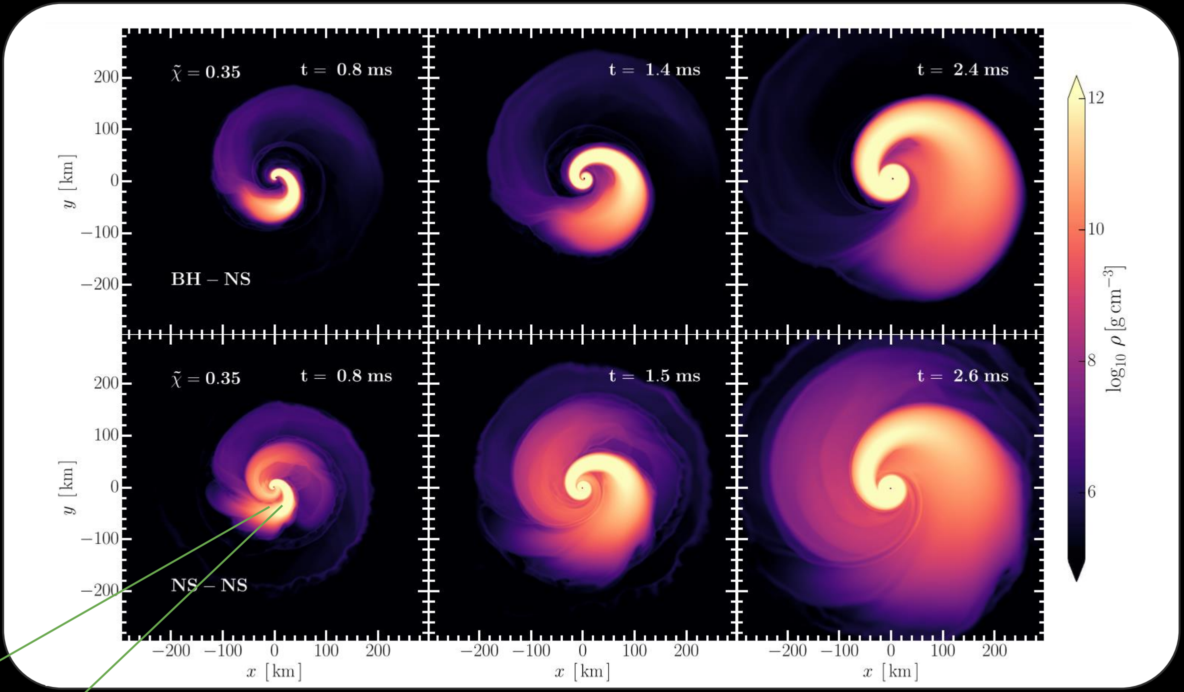
Quantum Neutrino Plasma



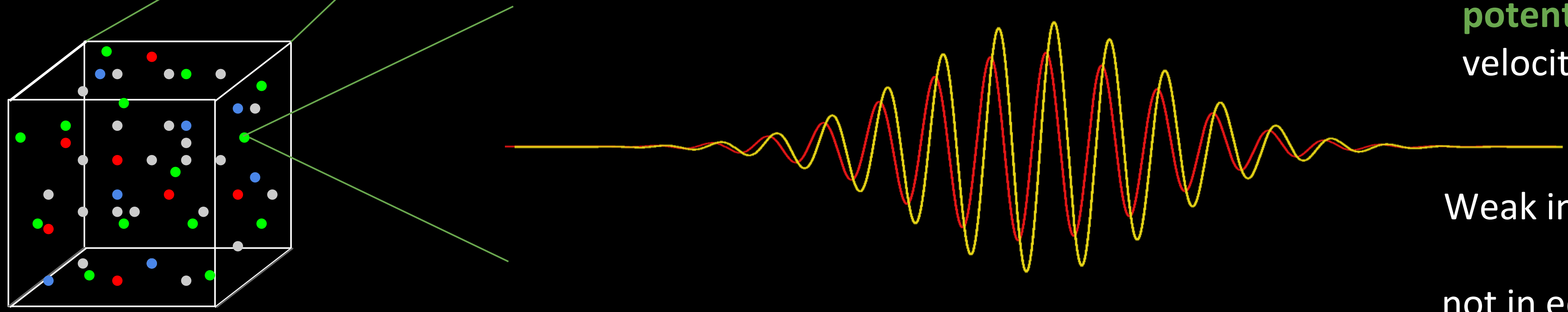
(Most et al. (2021))



Quantum Neutrino Plasma



(Most et al. (2021))



Neutrino **mass** and **potential** affect velocity.

Weak interactions → not in equilibrium.

$$\frac{\partial f_{ab}}{\partial t} + c\boldsymbol{\Omega} \cdot \nabla f_{ab} = \mathcal{C}_{ab} - \frac{i}{\hbar}[\mathcal{H}, f]_{ab}$$

Transport — Collision — Flavor

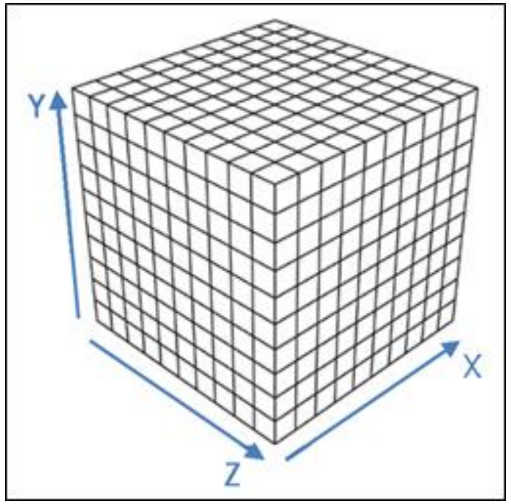
Vlasenko+ (2014)

Volpe (2015)

Blaschke & Cirigliano (2016)

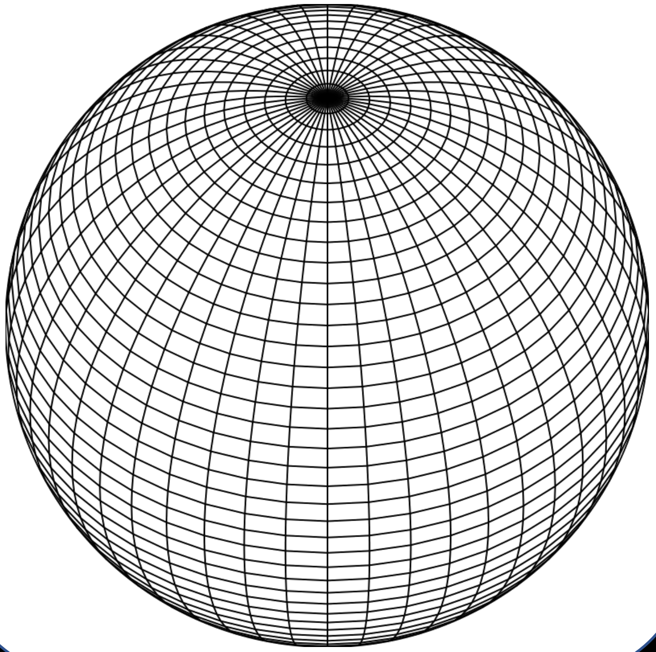
$f_{ab} =$

Position (3D)



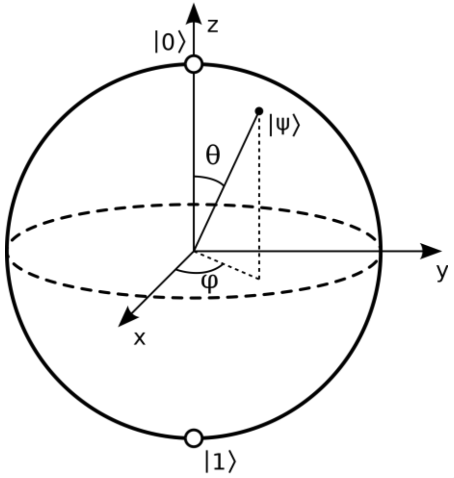
x

Momentum (3D)



x

Flavor
(3x3 matrix)



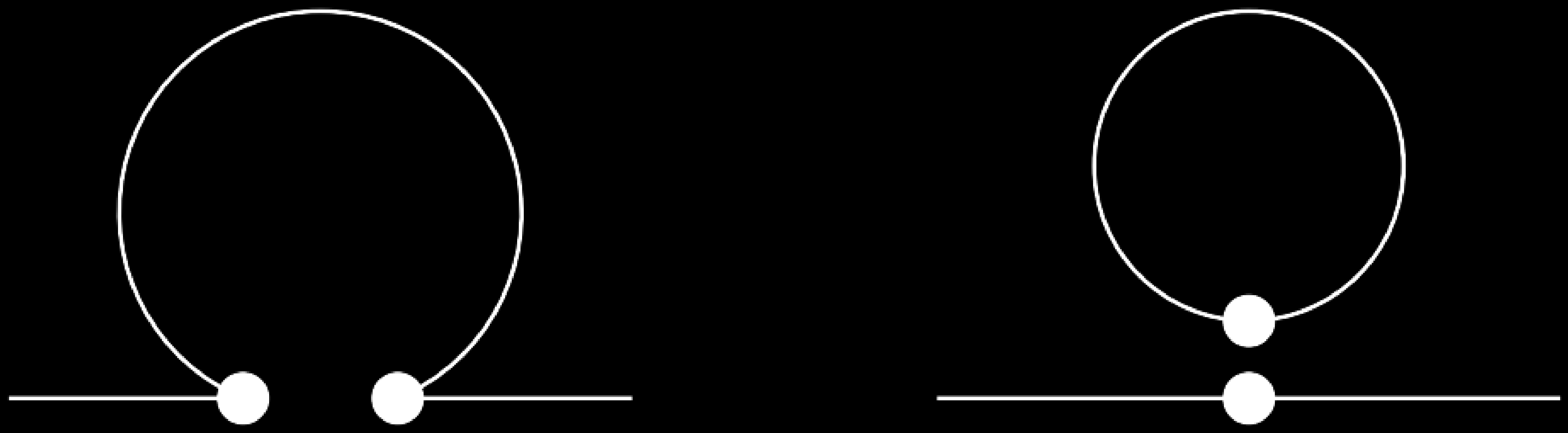
$$\frac{\partial f_{ab}}{\partial t} + c\mathbf{\Omega} \cdot \nabla f_{ab} = \mathcal{C}_{ab} - \frac{i}{\hbar}[\mathcal{H}, f]_{ab}$$

Flavor

Vlasenko+ (2014)

Volpe (2015)

Blaschke & Cirigliano (2016)



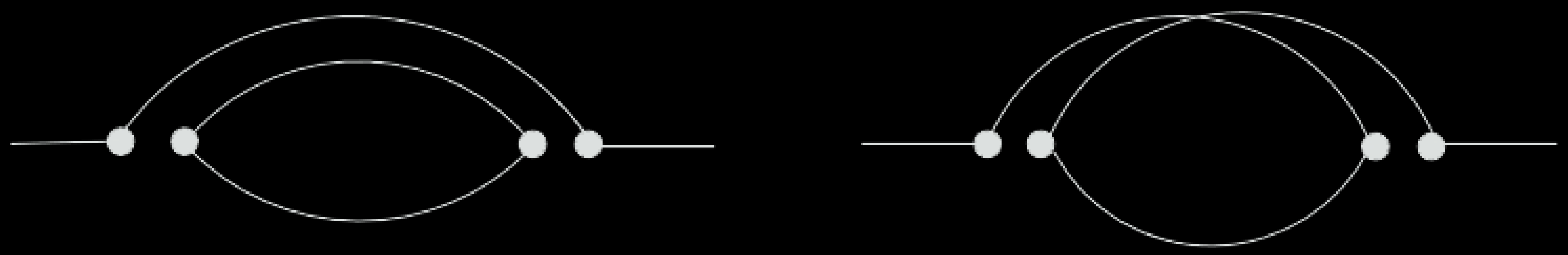
$$\frac{\partial f_{ab}}{\partial t} + c\mathbf{\Omega} \cdot \nabla f_{ab} = \boxed{\mathcal{C}_{ab}} - \frac{i}{\hbar}[\mathcal{H}, f]_{ab}$$

Vlasenko+ (2014)

Volpe (2015)

Blaschke & Cirigliano (2016)

"The Supernova Problem"



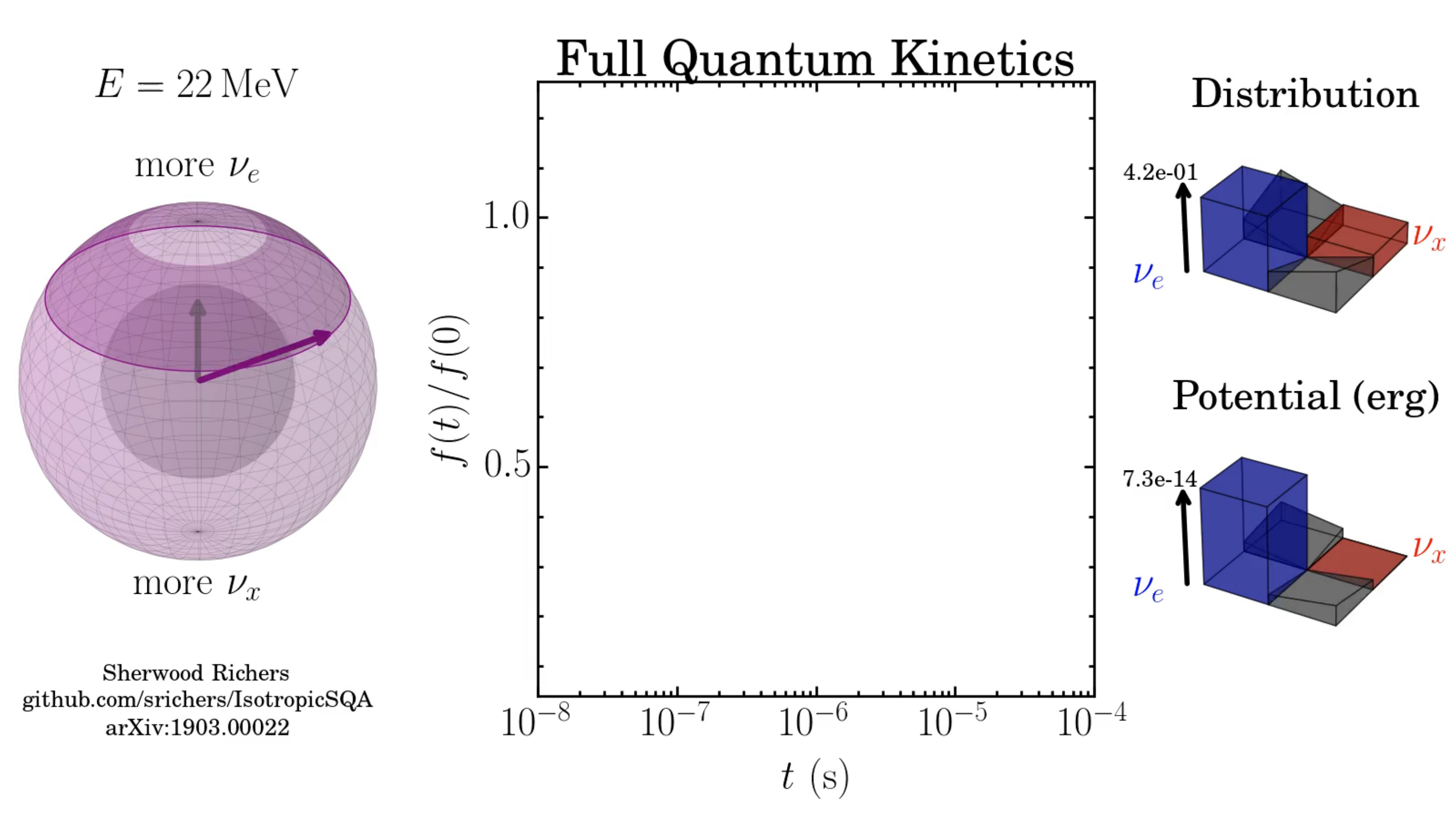
Neutrino Transport Reviews
Bruenn (1985)
Burrows, Reddy, Thompson (2007)
Mezzacappa (2022)

Combining with one-loop effects
Cherry (2012)
Vlasenko (2017)
Vlasenko & McLaughlin (2018)
SR et al. (2019)
Shalgar & Tamborra (2020, 2022)
Johns (2021)
Martin et al. (2021)
Sasaki et al. (2021)

Nagakura (2022)
Hansen et al. (2022)
Johns & Xiong (2022)
Kato & Nagakura (2022)
Padilla-Gay et al. (2022)
Kato, Nagakura, & Zaizen (2023)
Lin & Duan (2023)
Xiong et al. (2023)
...

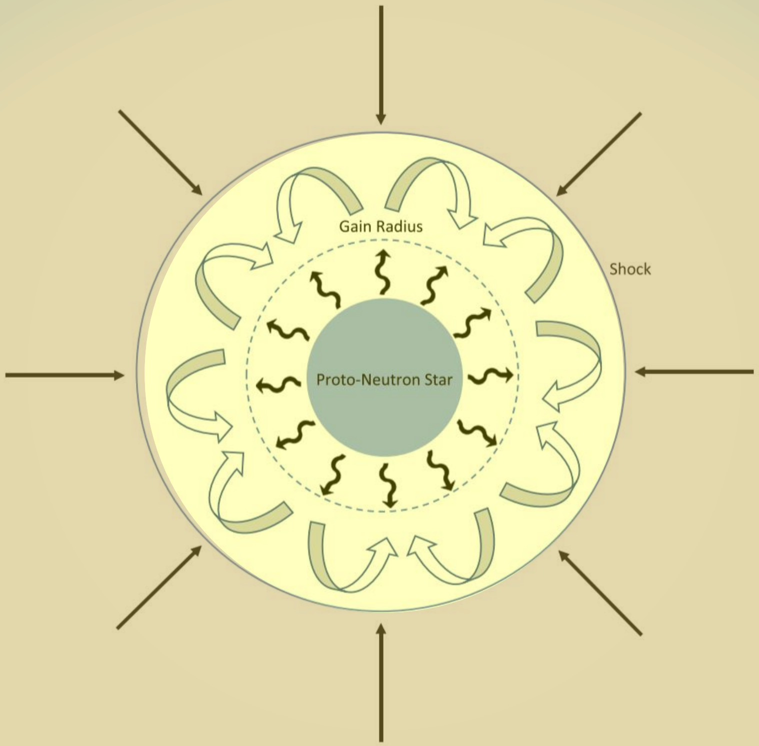
$$\frac{\partial f_{ab}}{\partial t} + c\mathbf{\Omega} \cdot \nabla f_{ab} = \mathcal{C}_{ab} - \frac{i}{\hbar}[\mathcal{H}, f]_{ab}$$

Oscillations and **collisions** are not generally separable

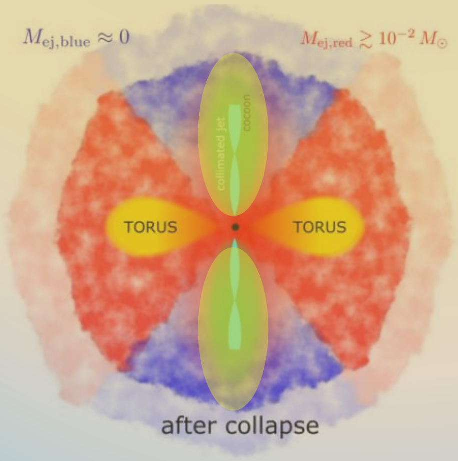


Richers+ (2019)

Flavor Transformation



Mezzacappa (2020)



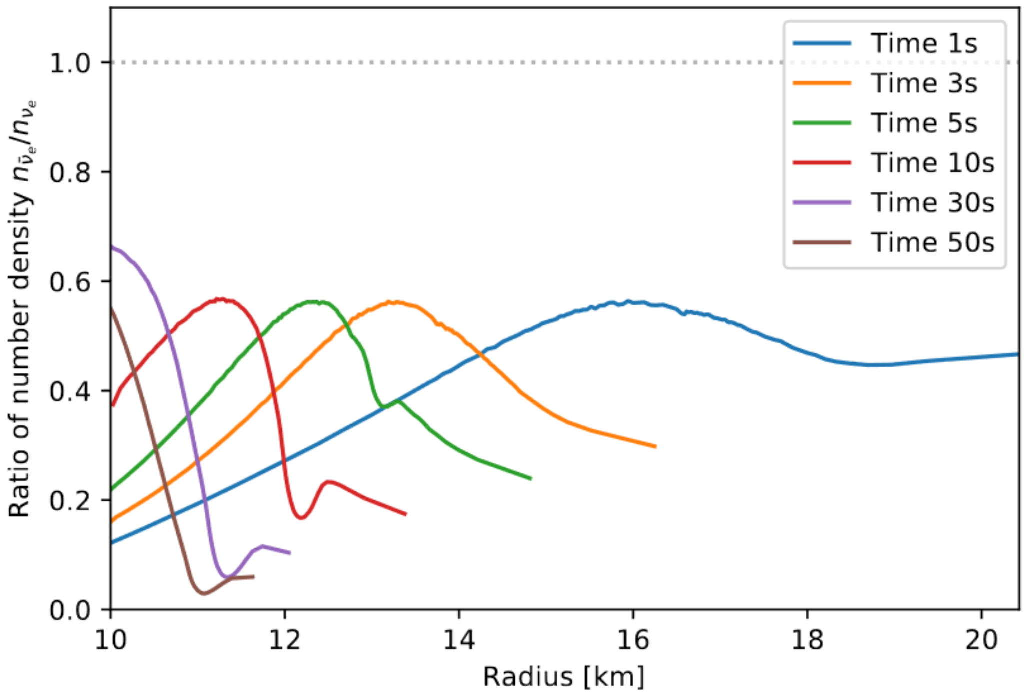
Gill et al. (2019)

- Vacuum (easy)
- MSW (easy)
- Collective Oscillations
- Matter-Neutrino Resonance
- Halo Effect
- Fast Flavor Instability
- Collisional Instability

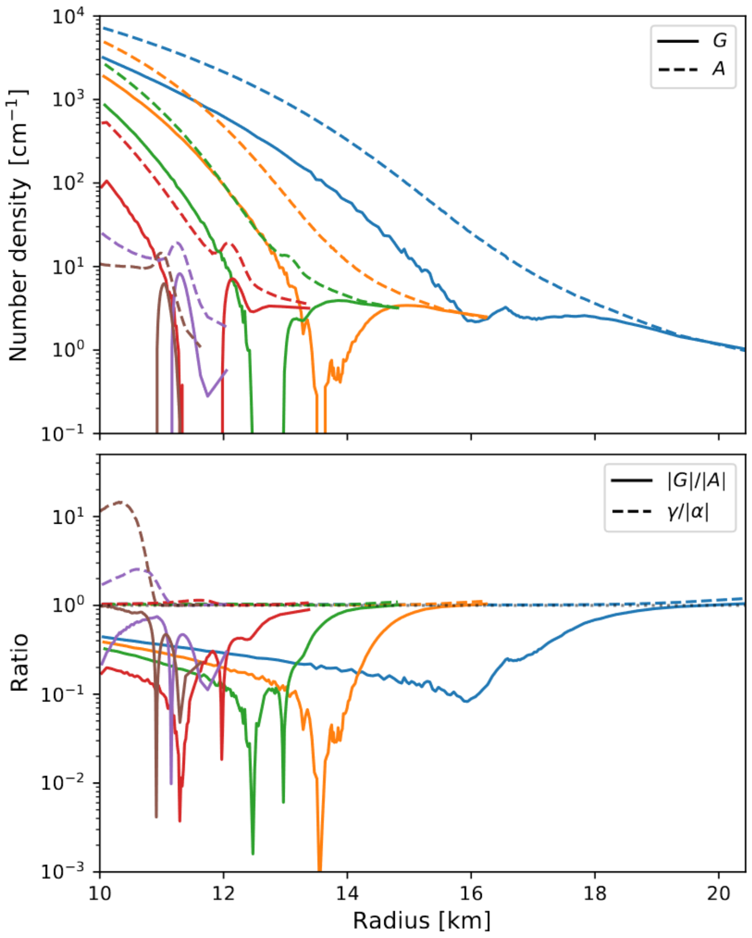
Flurry of recent work: Abbar, Bhattacharyya, Capozzi, Chakraborty, Dasgupta, Duan, Fernandez, Foucart, George, Grohs, Hansen, Johns, Just, Kato, Kneller, Li, Martin, McLaughlin, Morinaga, Nagakura, Padilla-Gay, Raffelt, Roggero, Sasaki, Siegel, Sigl, Shalgar, Tamborra, Wu, Xiong, Zaizen (and many others)

Quick note: no FFI or collisional instability in cooling PNS

Zaizen, SR et al. (in prep)

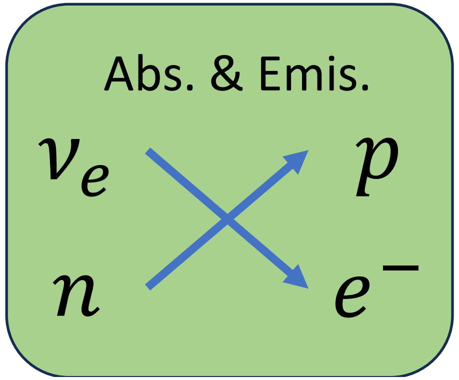


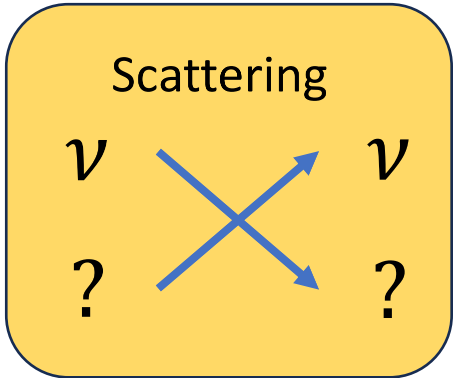
FFI made difficult by low ratio of antineutrinos to neutrinos



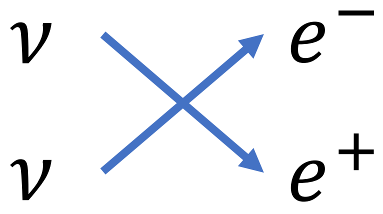
CI suppressed by presence of heavy lepton neutrinos

Collisional Processes

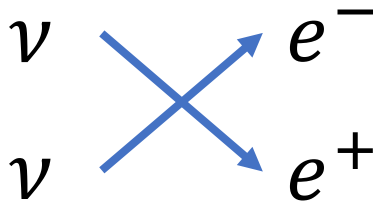




Pair Annihilation
Bremsstrahlung



4-neutrino
Processes



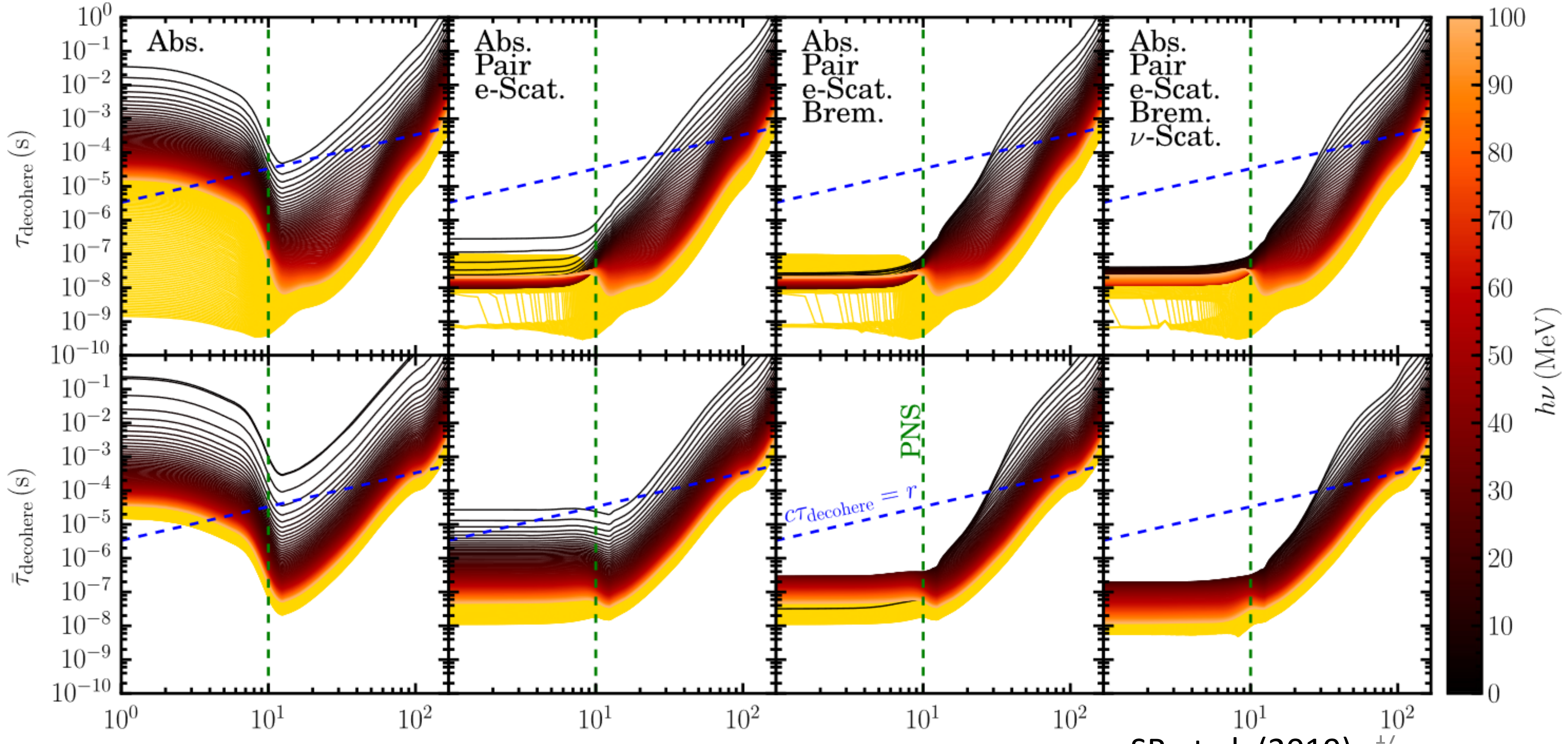
$$\Pi^+_{ab} = \int \frac{d^3\nu_1'}{c^4} \langle R \rangle^+_{ab} f'_{1ab},$$

$$\Pi^-_{ab} = \int \frac{d^3\nu_1'}{c^4} \langle R \rangle^-_{ab} (\delta_{ab} - f'_{1ab}),$$

$$R^+_{(\nu_a)} = \sum_c (1 + \delta_{ac}) \int \frac{d^3\nu_2'}{c^3} \frac{d^3\nu_3'}{c^3} \times r_{(p_1+p_3 \to p+p_2)} (1 - f'_{2cc}) f'_{3cc},$$

$$R^-_{(\nu_a)} = \sum_c (1 + \delta_{ac}) \int \frac{d^3\nu_2'}{c^3} \frac{d^3\nu_3'}{c^3} \times r_{(p+p_2 \to p_1+p_3)} f'_{2cc} (1 - f'_{3cc})$$

Multiple collision processes matter

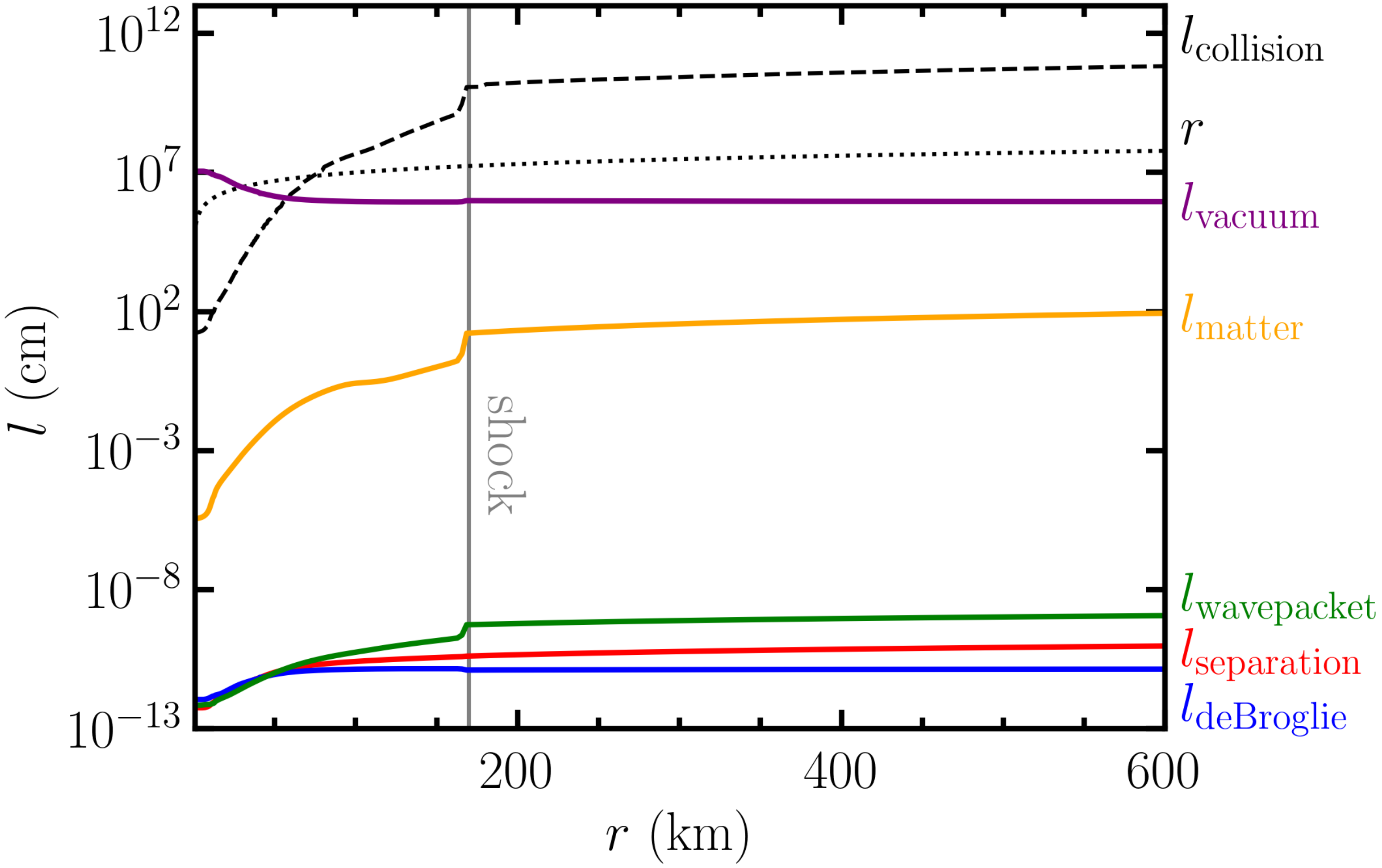


SR et al. (2019)

The Problem

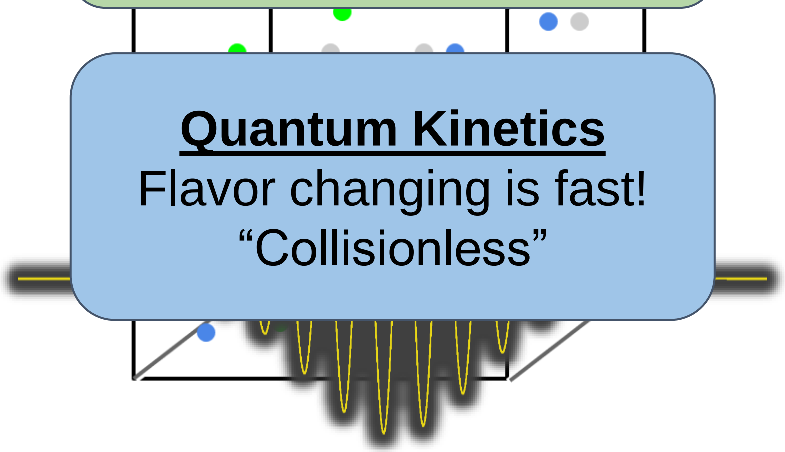
- Neutrino transport is the dominant cost of state-of-the-art simulations of core-collapse supernovae and neutron star mergers
- Neutrino flavor transformation modifies amount of heating, amount of mass ejection, and composition of ejecta
- Neutrino flavor transformation occurs on smaller length/time scales than transport

How hard could it be?



Neutrino Decoupling
size > mean free path

Classical Scattering
“Instantaneous” collisions



Quantum Kinetics
Flavor changing is fast!
“Collisionless”

The Problem

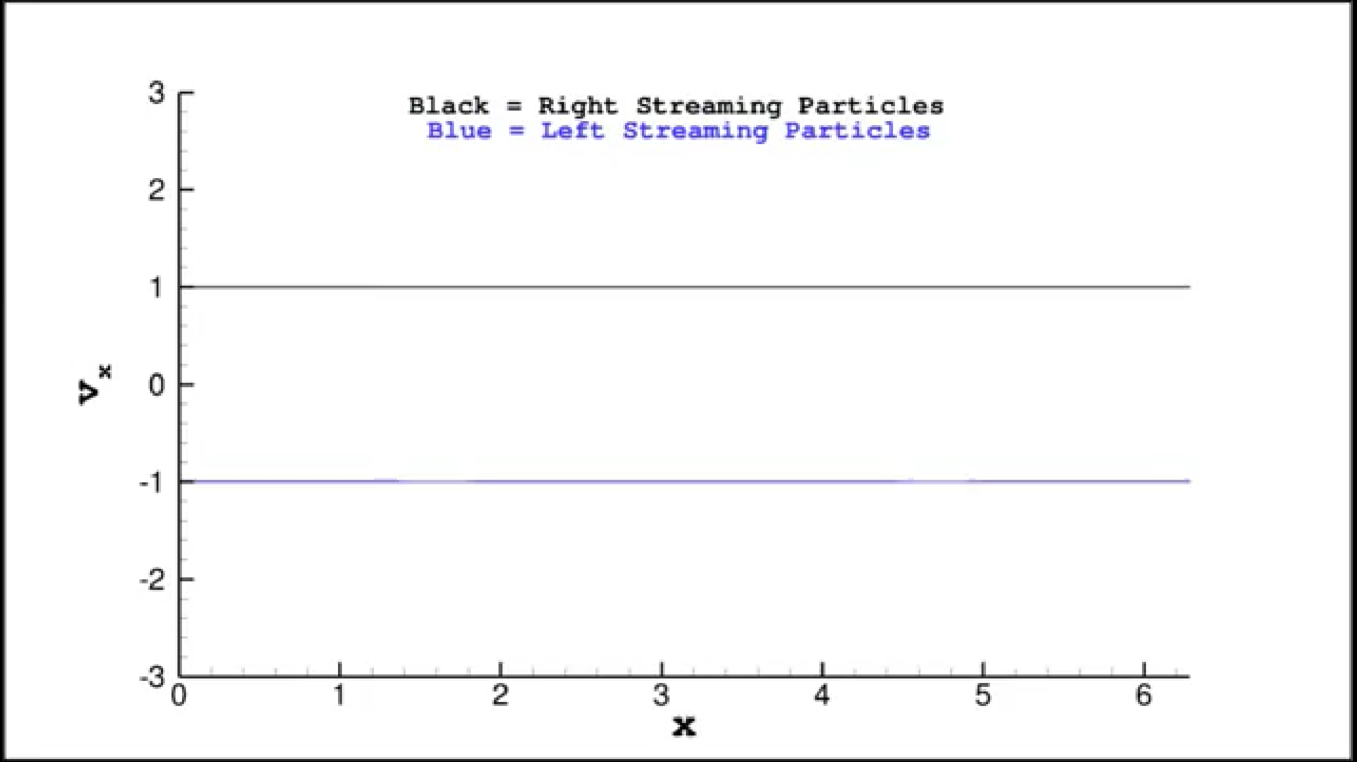
- Neutrino transport is the dominant cost of state-of-the-art simulations of core-collapse supernovae and neutron star mergers

- Neutrino flavor transformation modifies amount of heating, amount of mass ejection, and composition of ejecta

- Neutrino flavor transformation occurs on smaller length/time scales than transport

Focus: Fast Flavor Instability

Aside: Plasma Instabilities

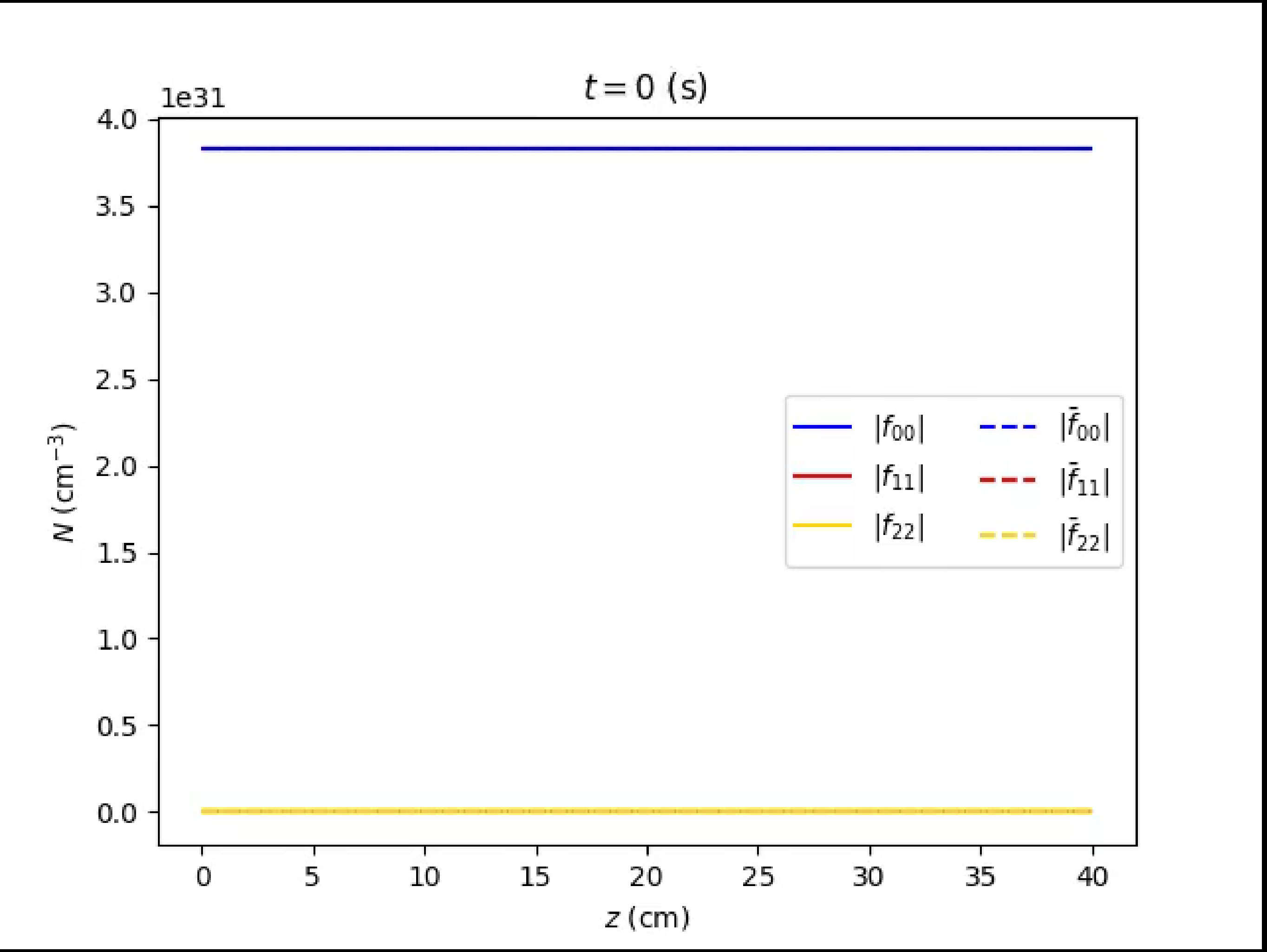


Frans Ebersohn

Because **charged particles** feel potential from other **charged particles**:

1. Perturbation in particle **velocities** induces **electric+magnetic field**
2. **Electric+magnetic field** influences particle **velocities**
3. Particle perturbations grow exponentially

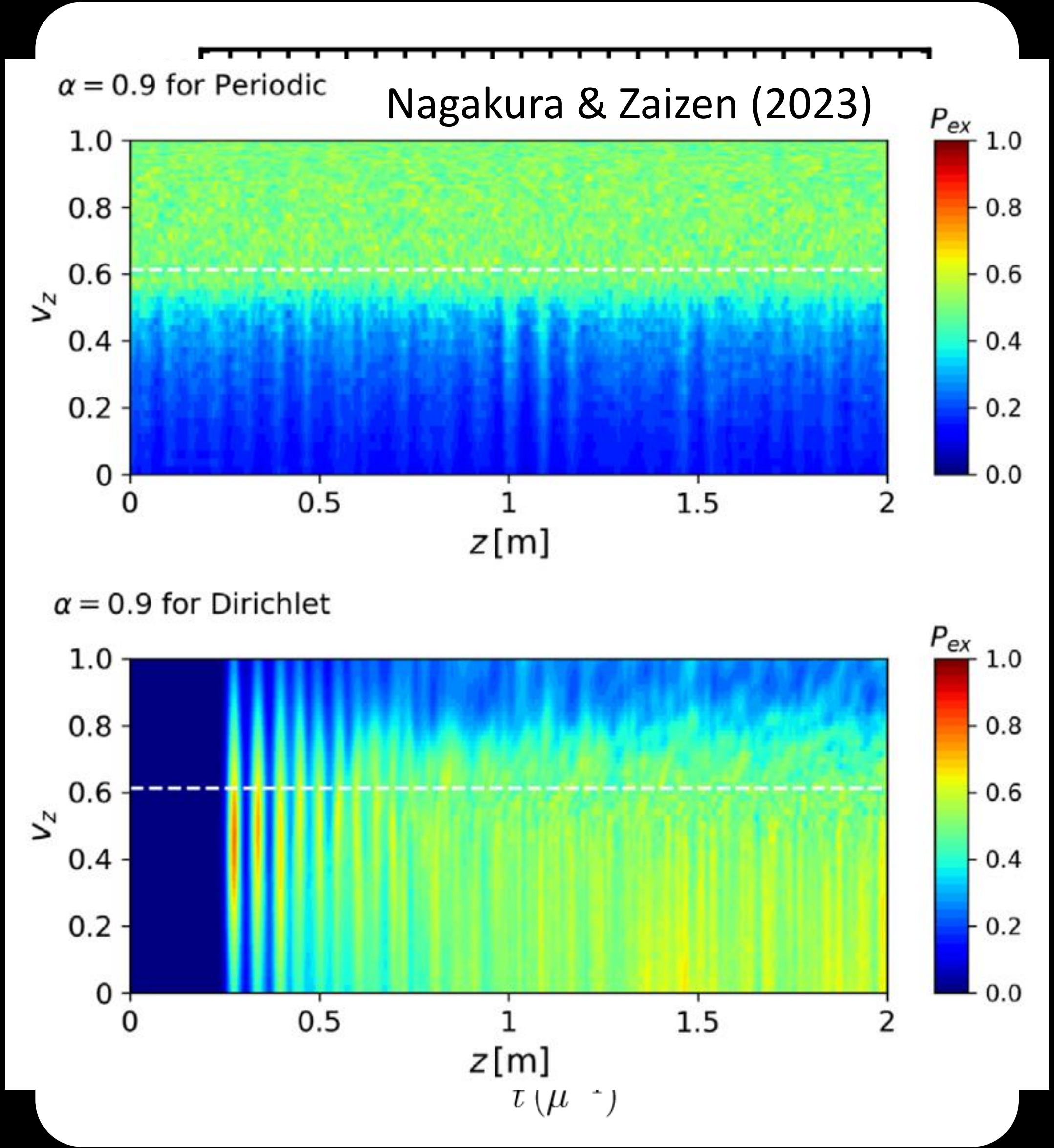
Neutrino Plasma Instabilities



Because **neutrinos** feel potential from other **neutrinos**:

1. Perturbation in particle **flavor** induces **flavor background**
2. **Flavor background** influences particle **flavor**
3. Particle perturbations grow exponentially

General Features of the *local* FFI



SR+ (2022), following many other works

1. Exponential growth of perturbations
   Sawyer (2005), Dasgupta, Sen, Mirizzi, Morinaga, Padilla-Gay, Abbar, Xiong, Wu, Bhattacharyya, Zaizen, George, Duan, Sigl, Capozzi, Shalgar, Raffelt, Chakraborty, Kato ... [many contributions]

2. Complete mixing within “ELN Crossing”, incomplete elsewhere to preserve lepton #
   Bhattacharyya & Dasgupta (2021)

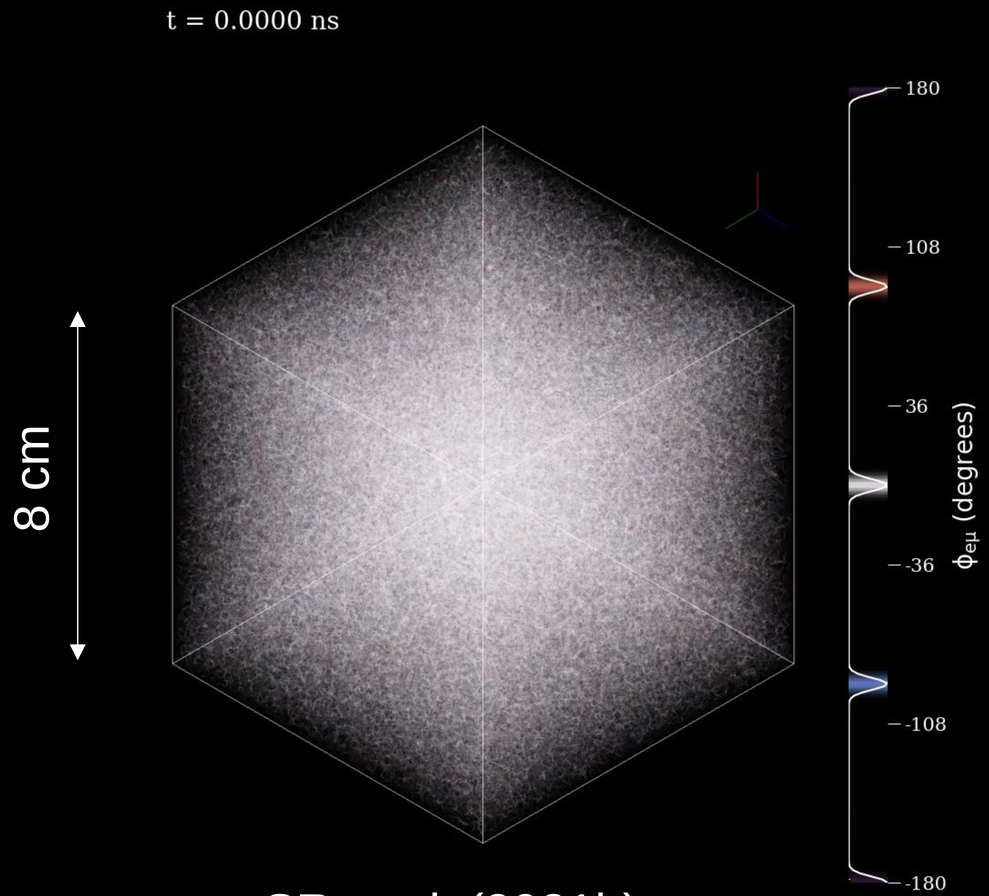
3. Modes spreading to exponential distribution.
   SR et al. (2021)

4. Coherent post-saturation flavor wave
   Duan et al. (2021)

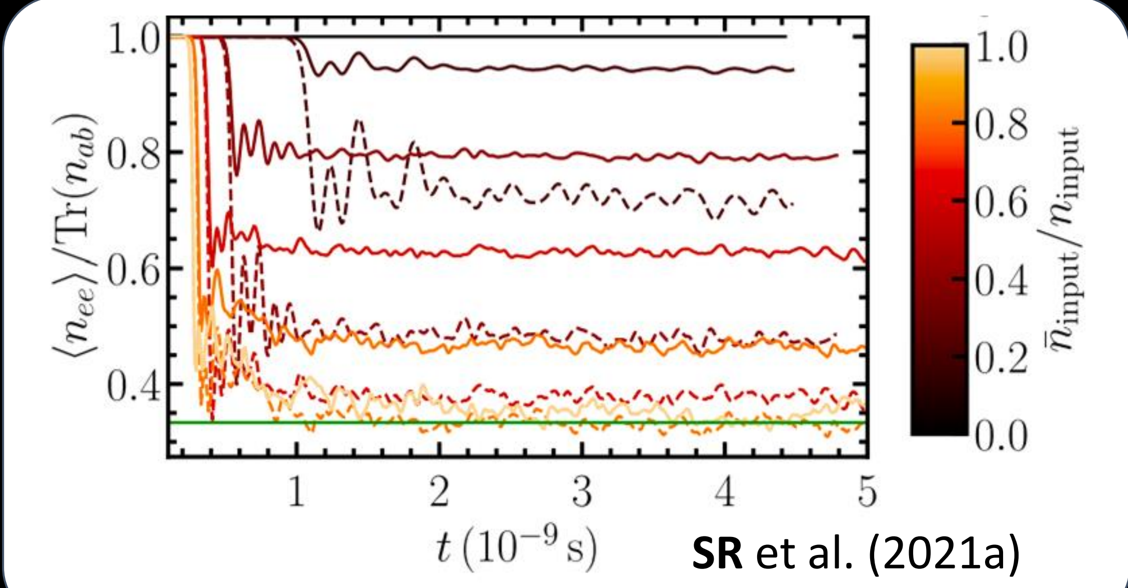
5. Non-trivial interplay with collisions
   Padilla-Gay, Shalgar, Johns, Xiong, Sasaki, Sigl, Tamborra, Hansen, Martin, SR, Azari, Lin, Duan

6. Sensitive to boundary conditions
   Zaizen, Nagakura, Xiong, Wu, Abbar, George, Lin, Bhattacharyya, Cornelius, Shalgar, Tamborra

LOCAL 3D models look like 1D models



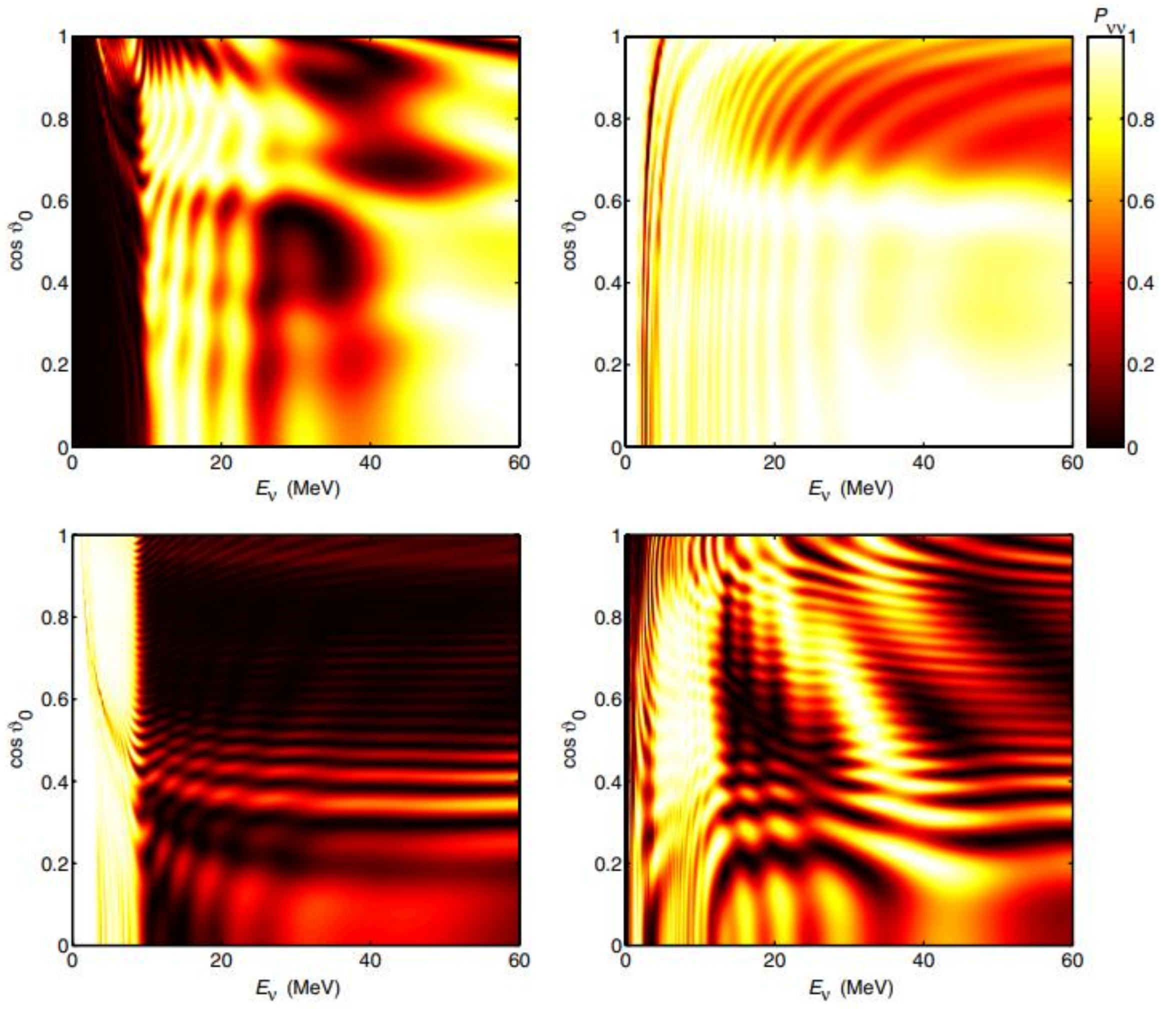
SR et al. (2021b)



SR et al. (2021a)

Amount of flavor transformation depends on the angular distribution.

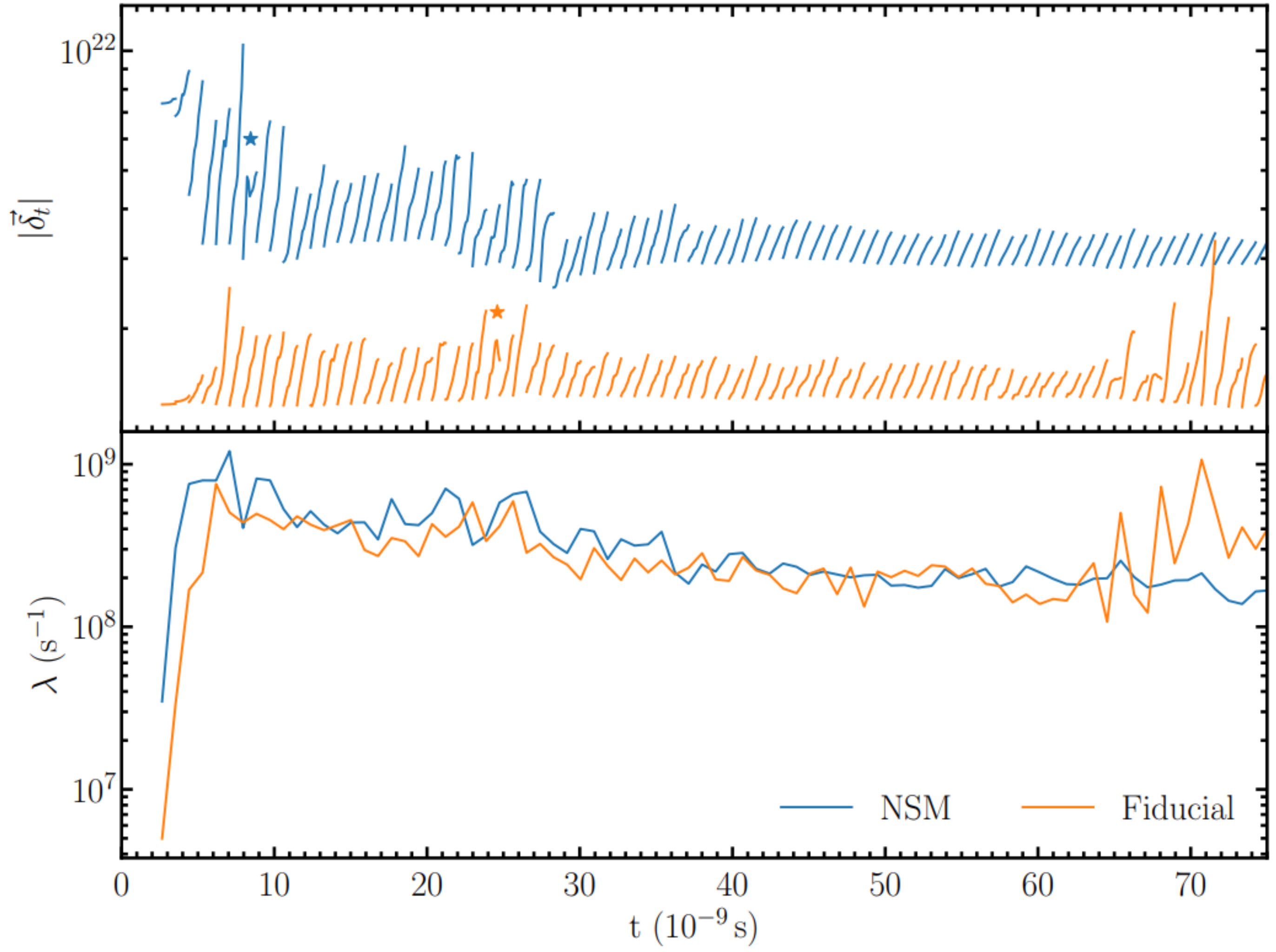
The results are sensitive to resolution



Duan et al. (2006)

- High-resolution 3D NSM simulations: **12.5 meters**
  Kiuchi et al (2023)
- High-resolution 2D flavor transformation: **3 m**
  Nagakura (2023)
- Estimated required resolution: **0.0003 m**

Quantifying the rate of information loss



Erick Urquilla Orellana
arXiv:2401.01936

Lyapunov exponent:

$\lambda \approx 0.4\, ns^{-1} \approx 0.6\ bits/ns$

53 bits in double-precision float
→ 100 nanoseconds

Approaches to Simulating Neutrino Quantum Kinetics

<u>Analytic Approaches</u>: Potential theoretical framework
(but still incomplete)

Hydrodynamics

- Evolve *equilibrium* distribution assuming fast collisional relaxation

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0 \\ \dfrac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \dfrac{\nabla p}{\rho} = \mathbf{g} \\ \dfrac{\partial e}{\partial t} + \mathbf{u} \cdot \nabla e + \dfrac{p}{\rho} \nabla \cdot \mathbf{u} = 0 \end{cases}$$

Miscidynamics

- Evolve *equilibrium* distributions assuming fast relaxation under a Hamiltonian

$$i\left(\partial_t + \hat{\boldsymbol{p}} \cdot \partial_{\boldsymbol{x}}\right) \rho_\nu^{\rm eq}(t, \boldsymbol{x}, \boldsymbol{p}) = i C_\nu^{\rm eq}(t, \boldsymbol{x}, \boldsymbol{p})$$

$$\rho_{\nu,\boldsymbol{p}}^{\rm eq} = \frac{1}{\exp\left[\beta\left(H_{\nu,\boldsymbol{p}}^{\rm eq} - \mu_{\nu,\boldsymbol{p}}\right) + \lambda\left(\delta Q / \delta \overline{\rho_{\nu,\boldsymbol{p}}}\right)\right] + 1}$$

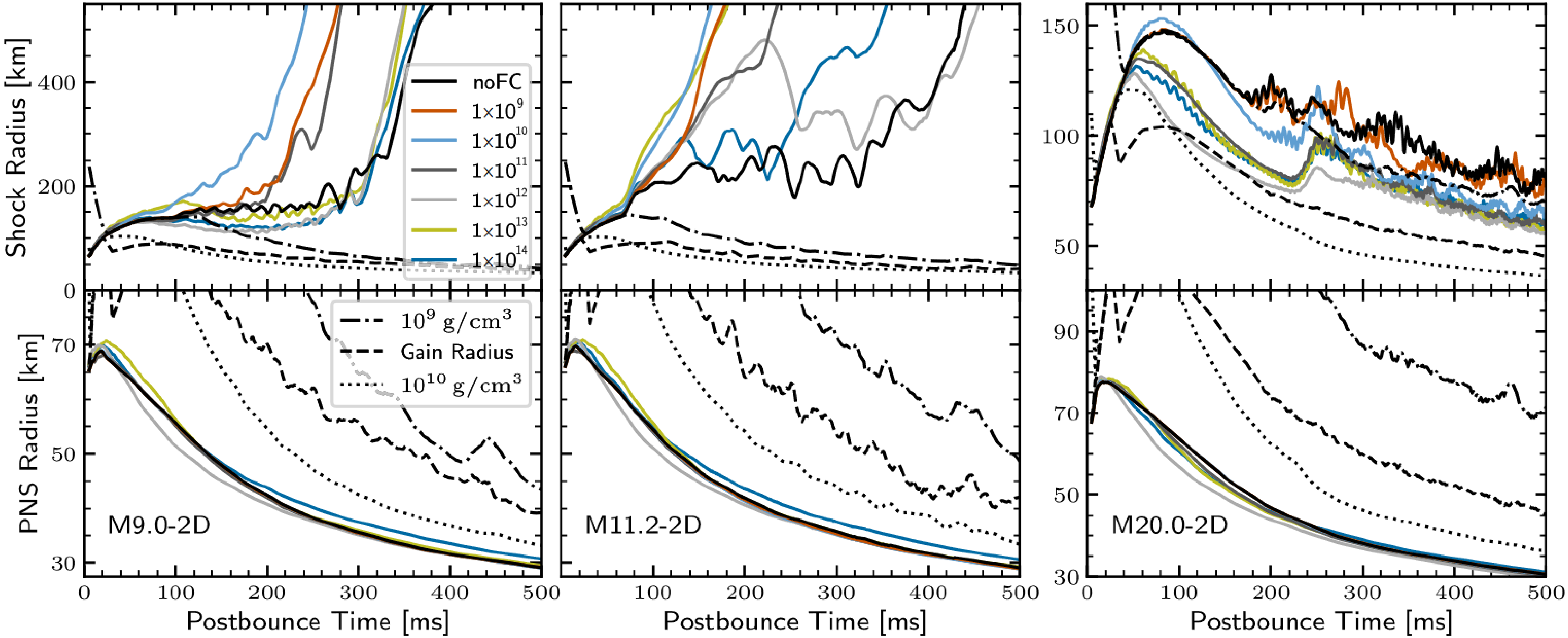
Unknown how to determine Lagrange multipliers efficiently

(Johns 2023)
See also: Padilla-Gay et al. (2022)

<u>Effective Treatments</u>: Bound what *could* happen

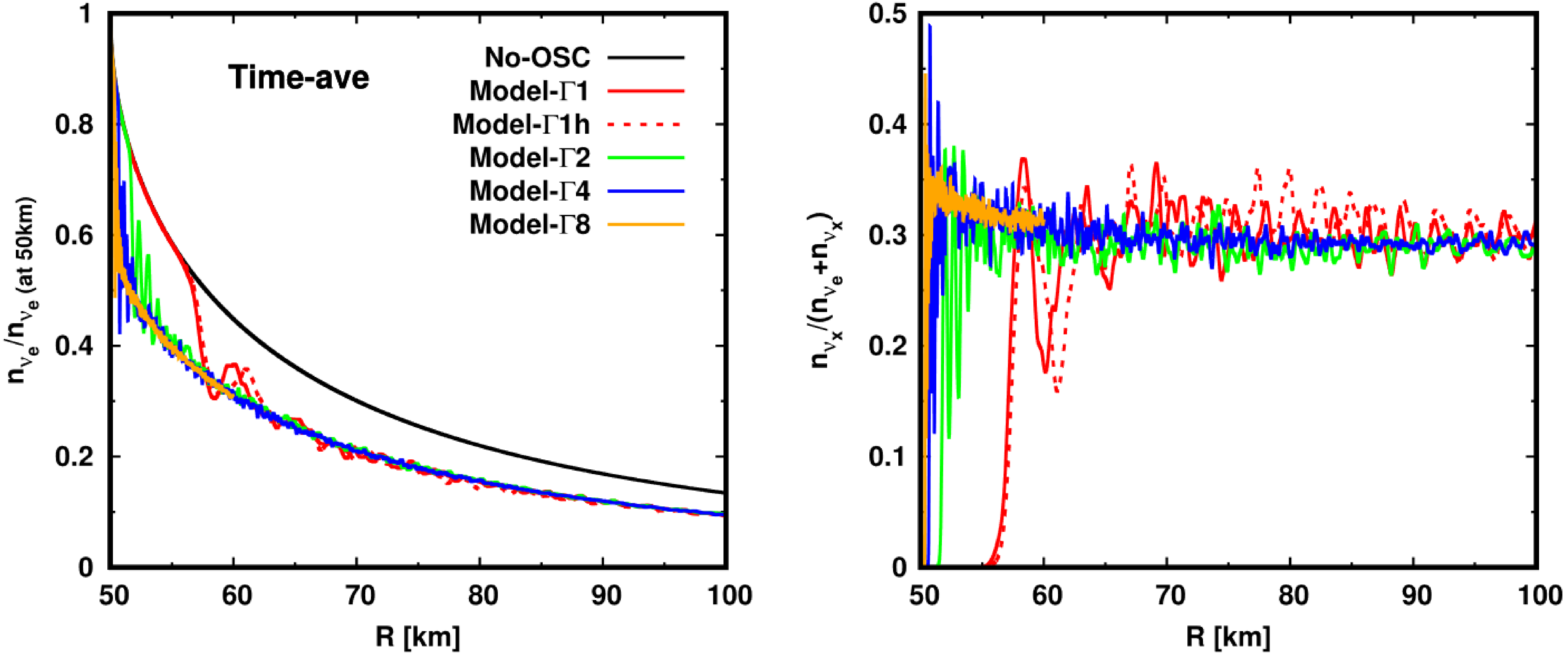
(but uncertain connection to first principles)

Ehring et al. (2023)



Reduced Coupling: Parameterize scale separation

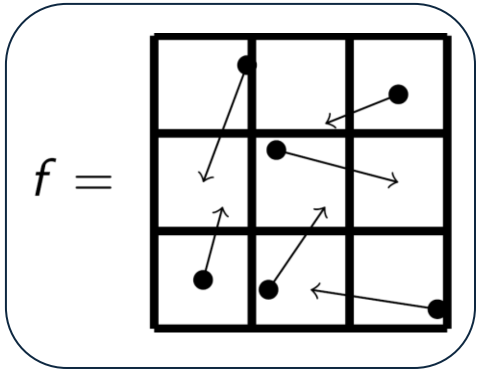
(but neutrino phenomena are riddled with “timescale matching” effects)



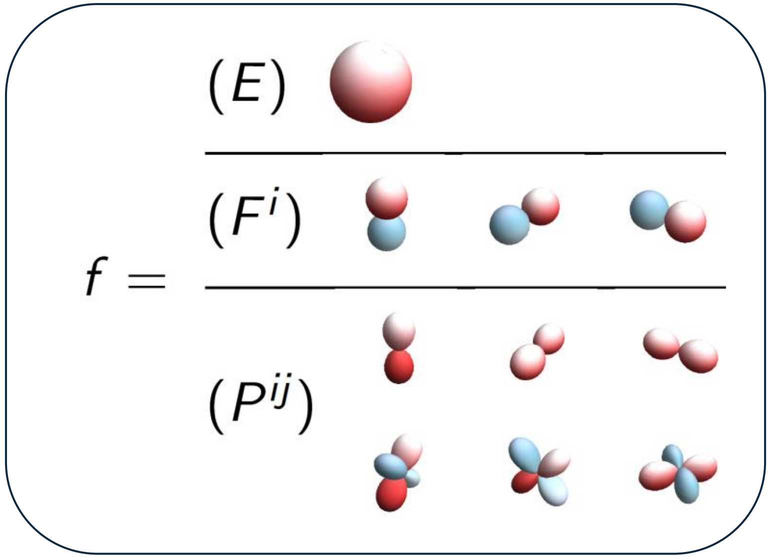
Approximate Methods: Capture the important details

(but they have the same problems as in classical transport)

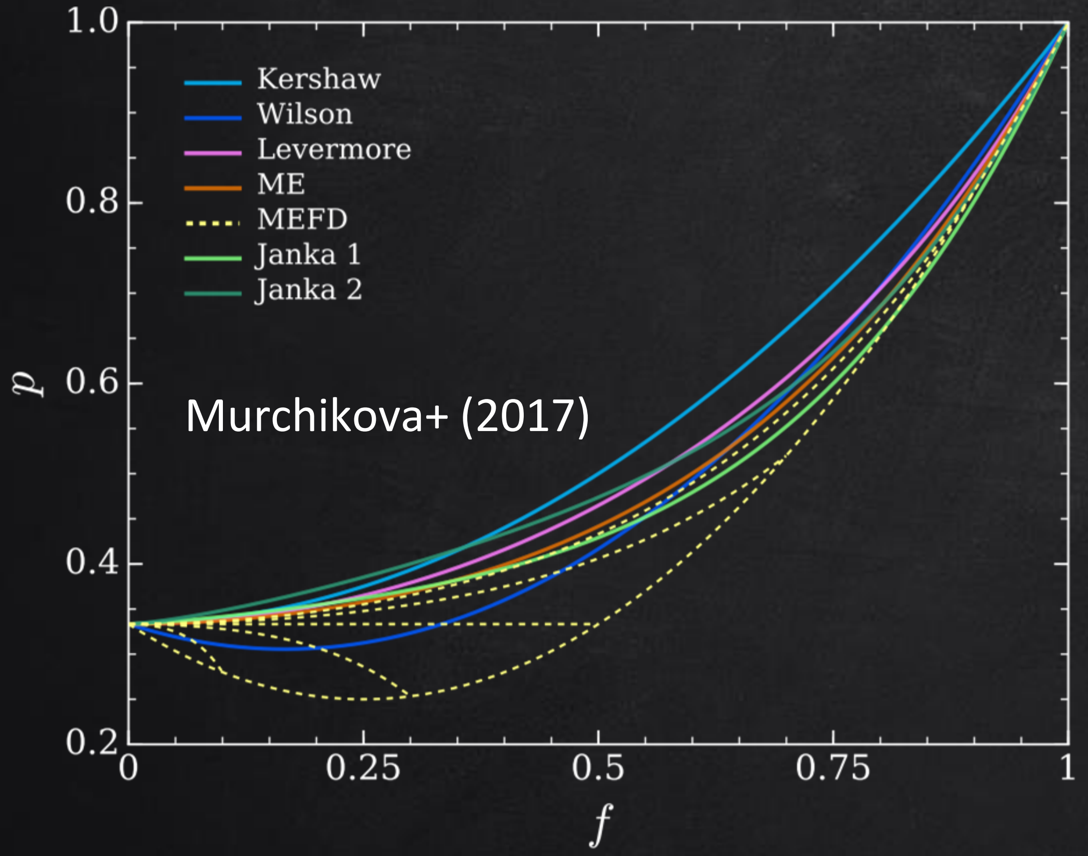
Boltzmann / "Full" QKEs

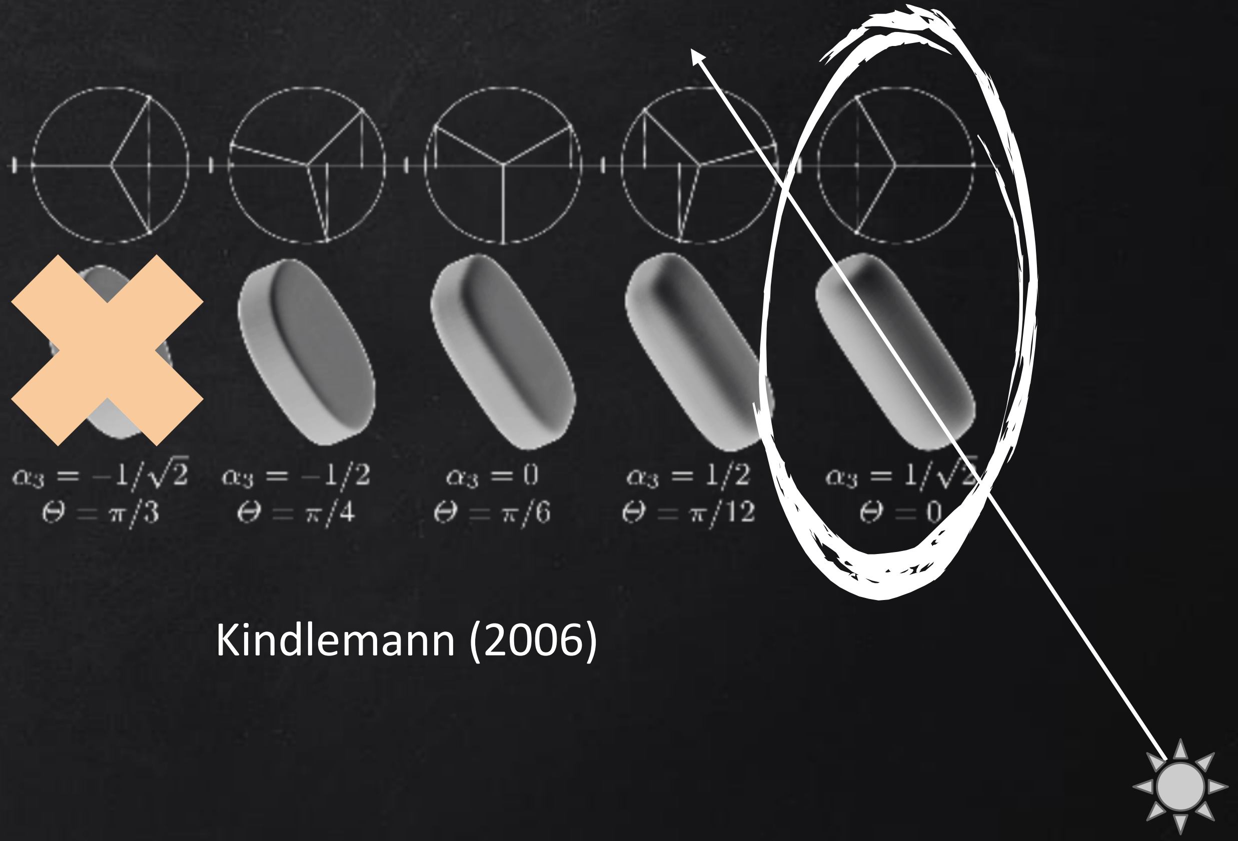


Moment formalism



The Moment Closure



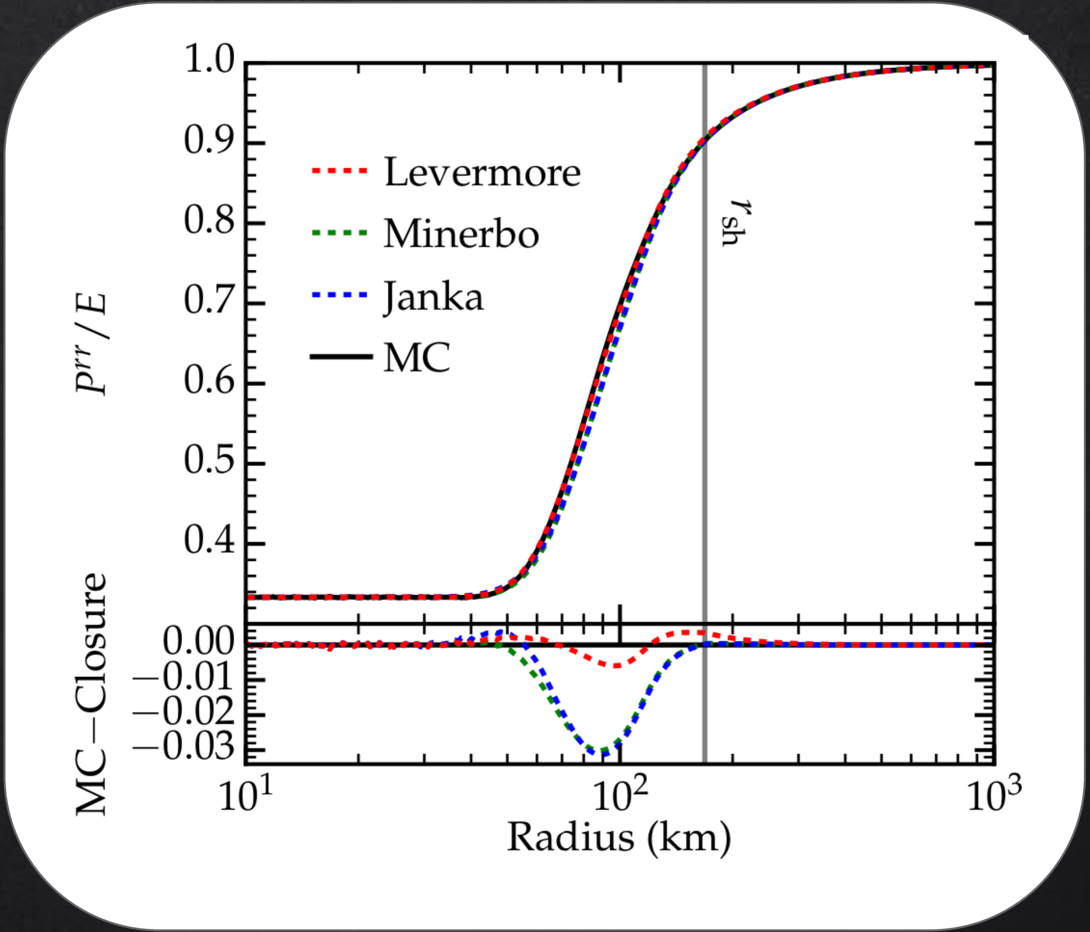


Kindlemann (2006)

Fundamental Closure Assumptions:

- The flux and pressure tensor point in the same direction
- Two eigenvalues are equal
- The pressure tensor can be uniquely determined by the flux and density.

The Moment Closure in 1D



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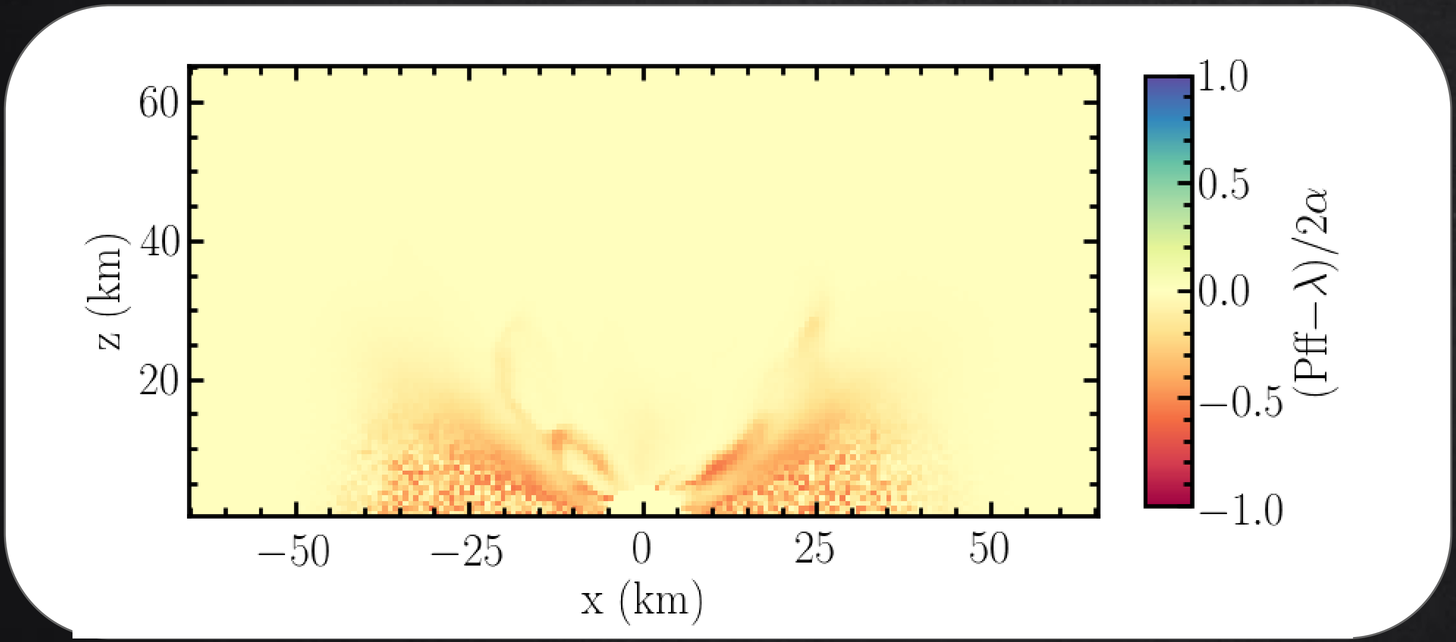
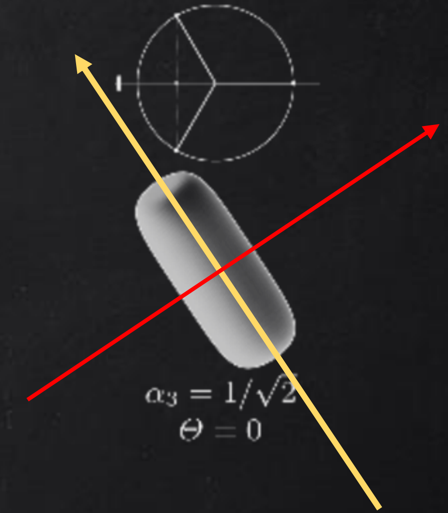
The Moment Closure in 3D

Mergers here, but Iwakami+(2020) reaches similar conclusions in supernovae





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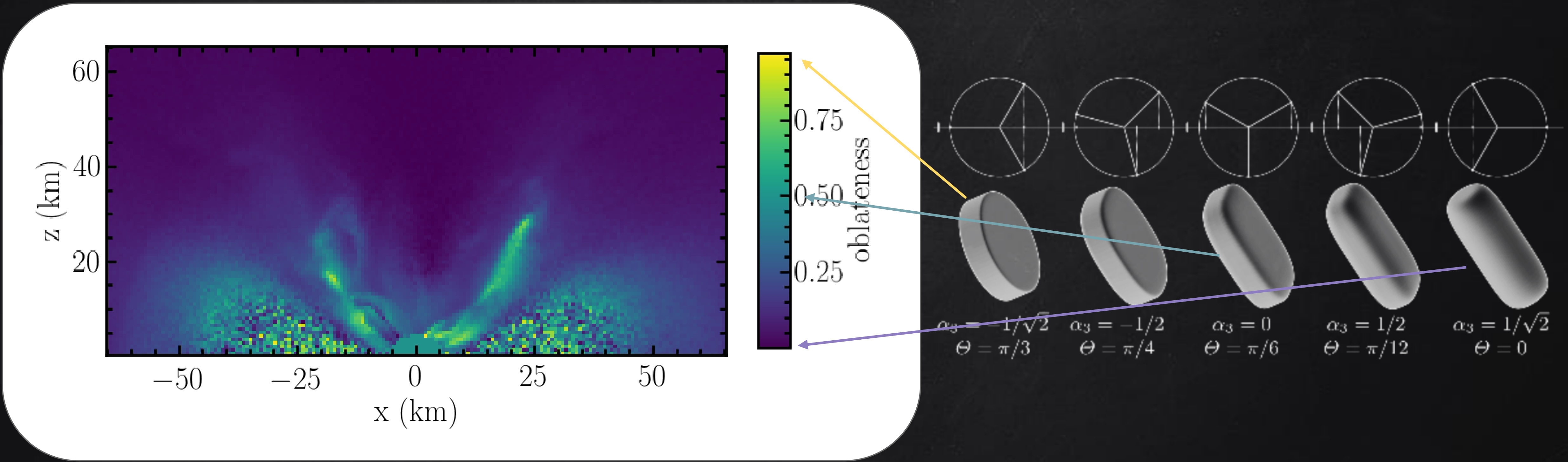
The Moment Closure in 3D

Mergers here, but Iwakami+(2020) reaches similar conclusions in supernovae



Fundamental Closure Assumptions:

- ~~The flux and pressure tensor point in the same direction~~
- ~~Two eigenvalues are equal~~
- The pressure tensor can be uniquely determined by the flux and density.

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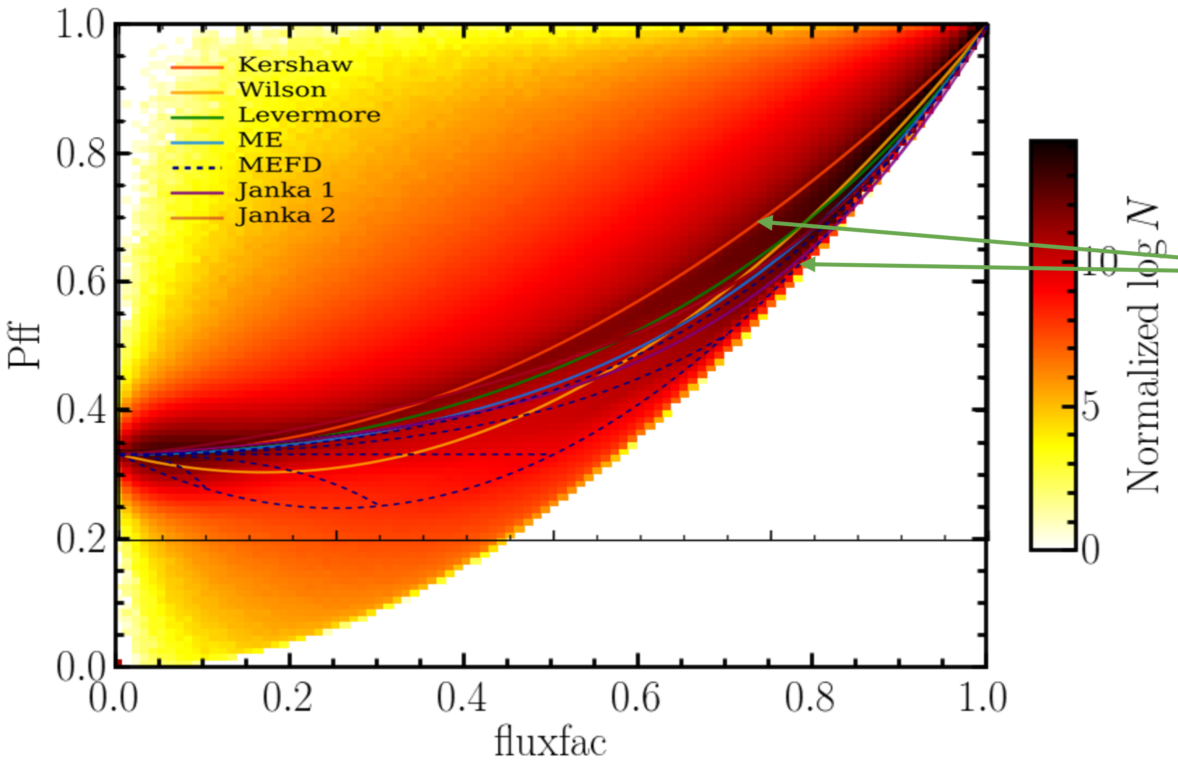


Distinct Branches

No closure fits everything

Fundamental Closure Assumptions:

- ~~The flux and pressure tensor point in the same direction~~
- ~~Two eigenvalues are equal~~
- ~~The pressure tensor can be uniquely determined by the flux and density.~~

Moments: Analytic Stability Analysis

Led by:
Julien Froustey
(NCSU)

Integrate EOMs

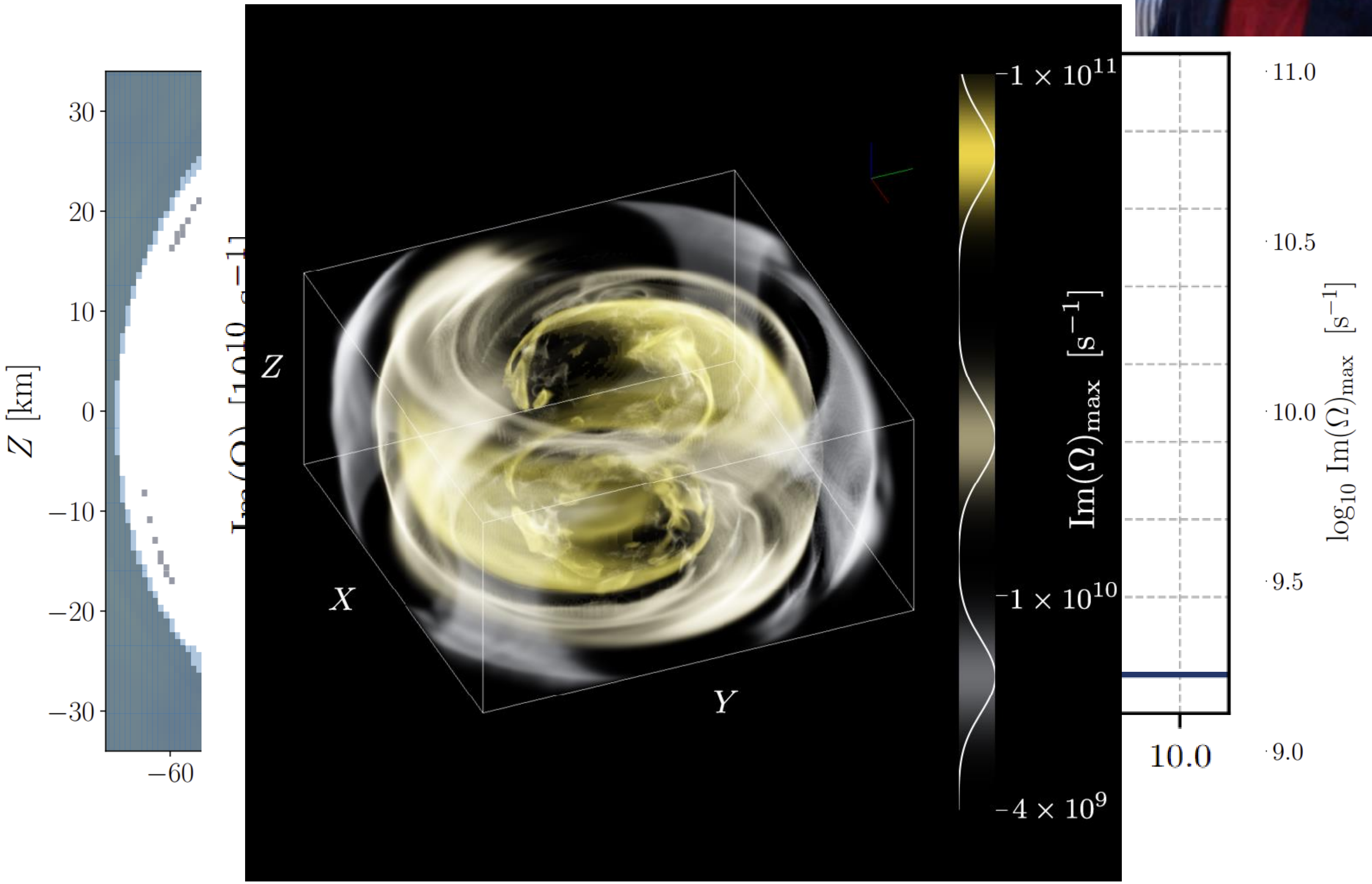
$$\mathrm{i}\left(\frac{\partial N}{\partial t}+\frac{\partial F^j}{\partial x^j}\right)=\sqrt{2}G_F\left[N-\overline{N},N\right]-\sqrt{2}G_F\left[(F-\overline{F})_j,F^j\right]$$

Perturb and linearize

$$N=\begin{pmatrix} N_{ee} & A_{ex}e^{-\mathrm{i}(\Omega t-\mathbf{k}\cdot\mathbf{r})} \\ A_{xe}e^{-\mathrm{i}(\Omega t-\mathbf{k}\cdot\mathbf{r})} & N_{xx}\end{pmatrix}$$

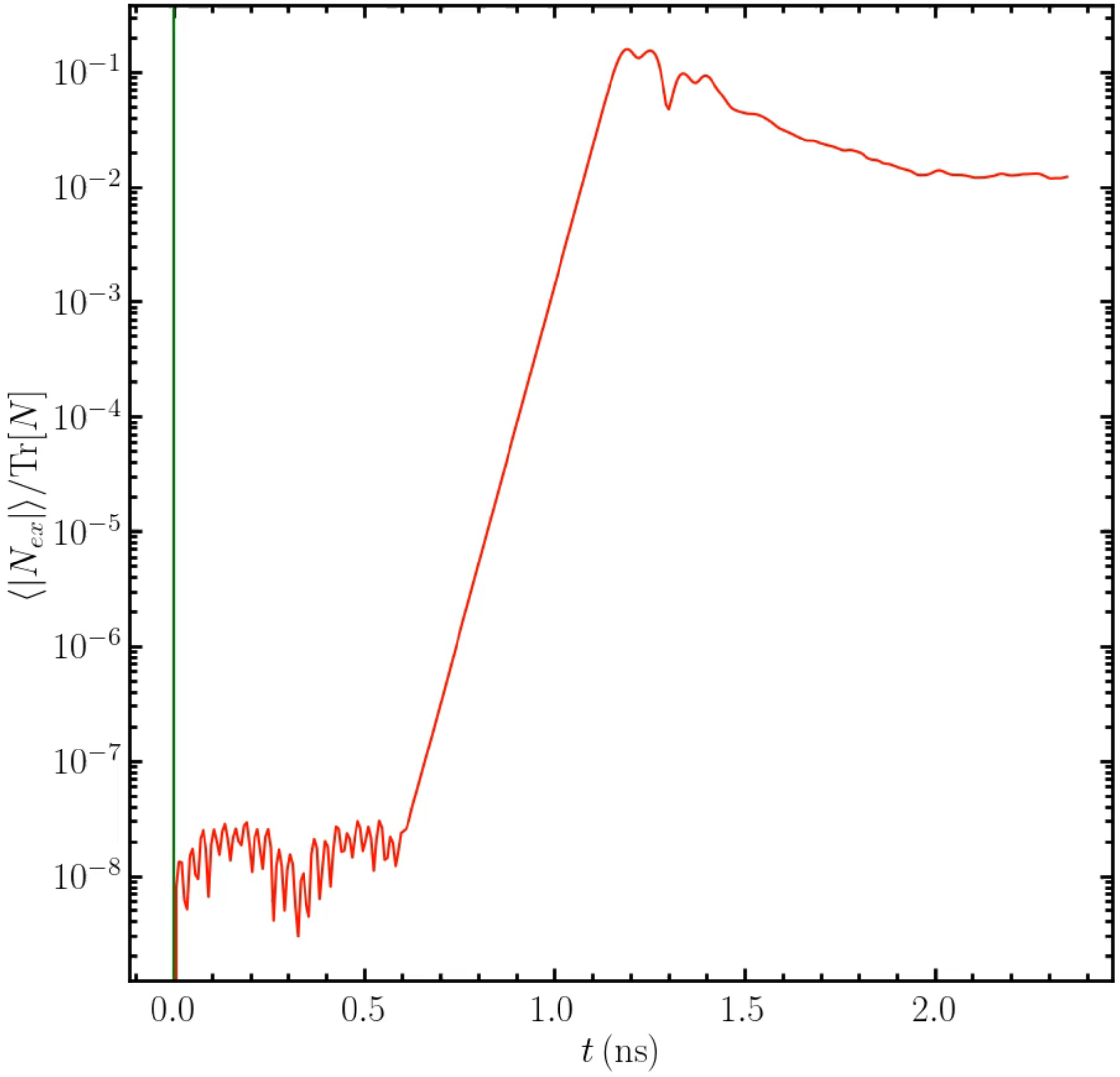
Dispersion Relation

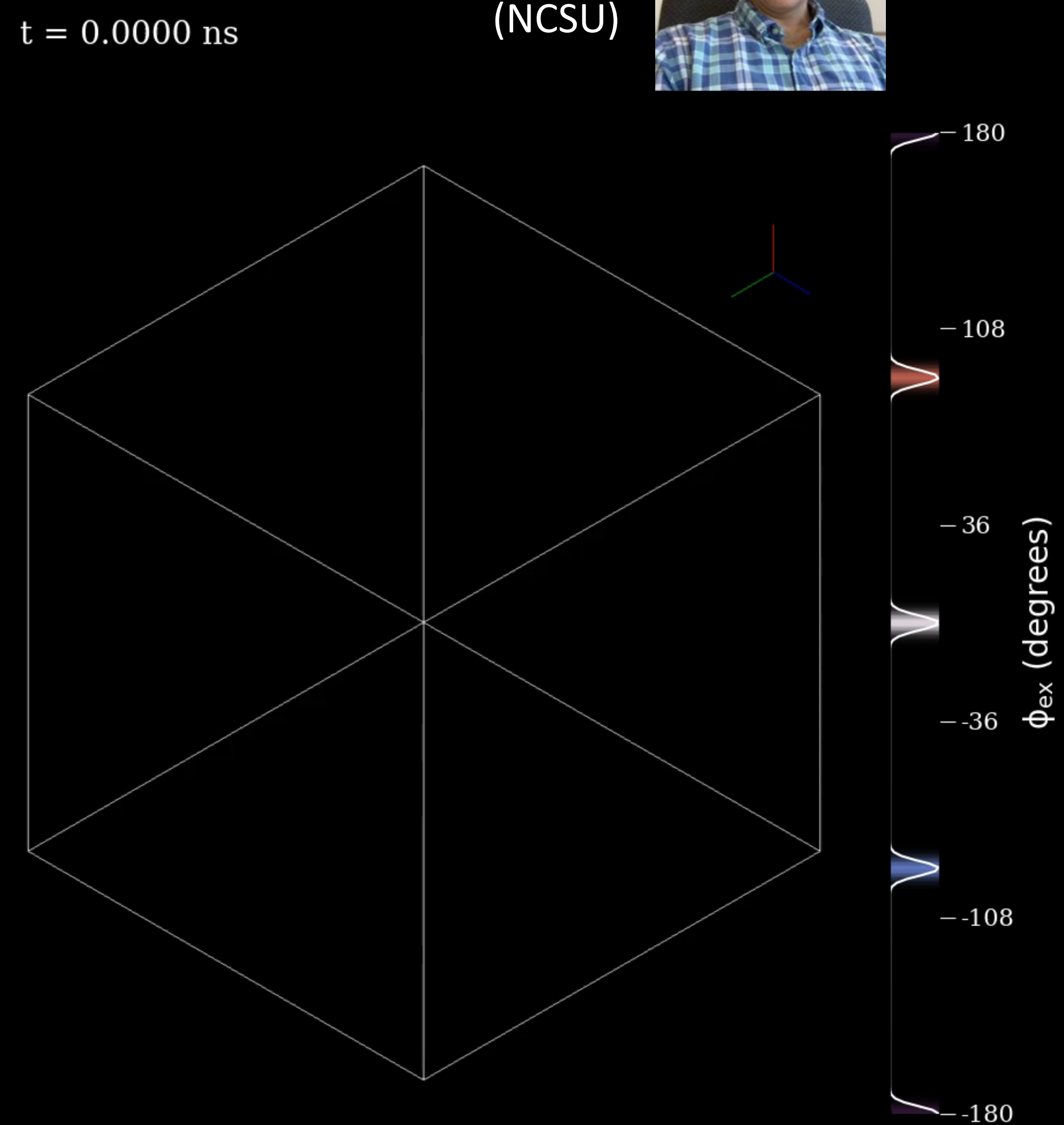
$$\det\left(S_{\mathbf{k}}+\Omega\mathbb{I}\right)=0$$



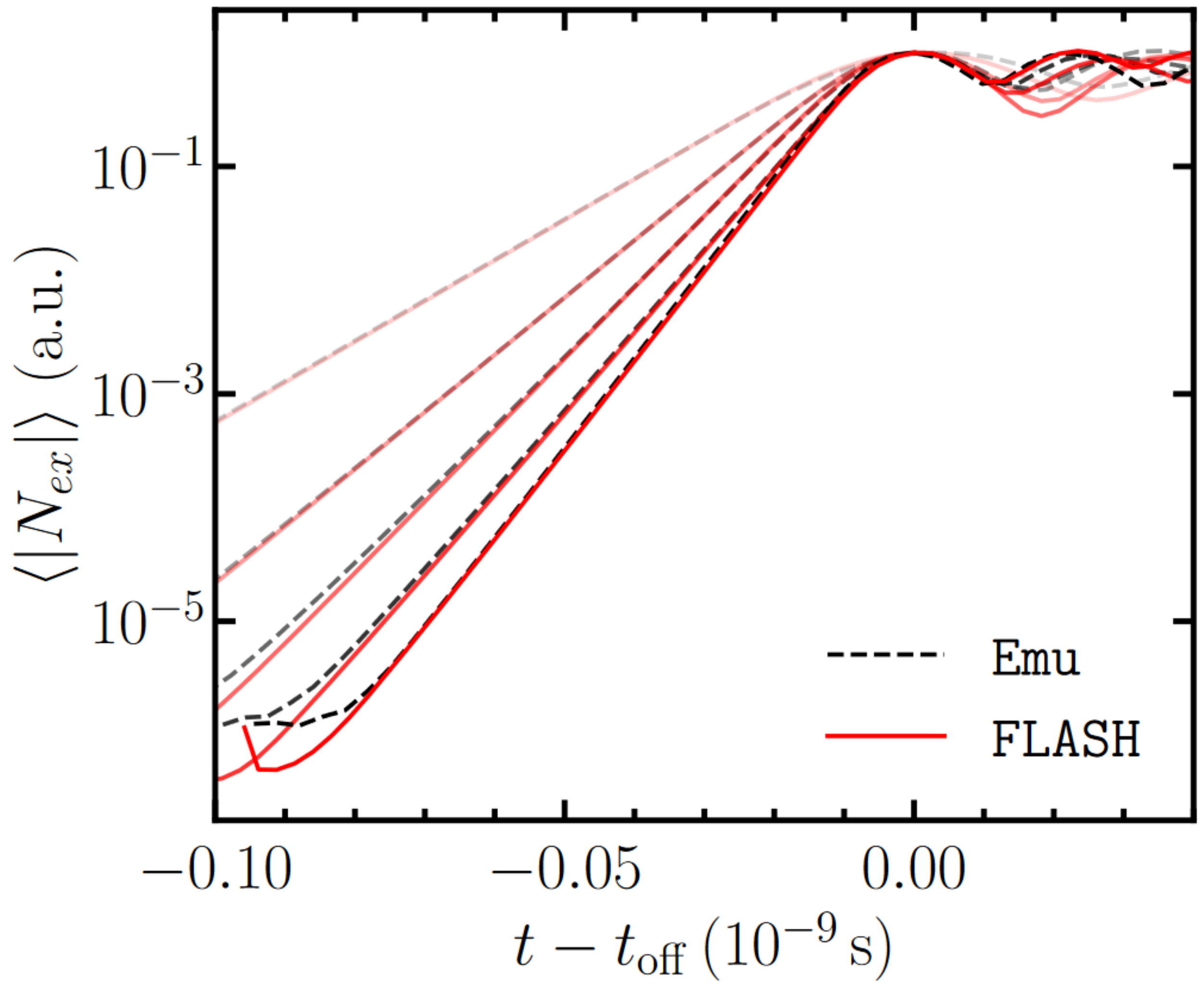
Moments enable fast 3D simulations

Led by:
Evan Grohs
(NCSU)

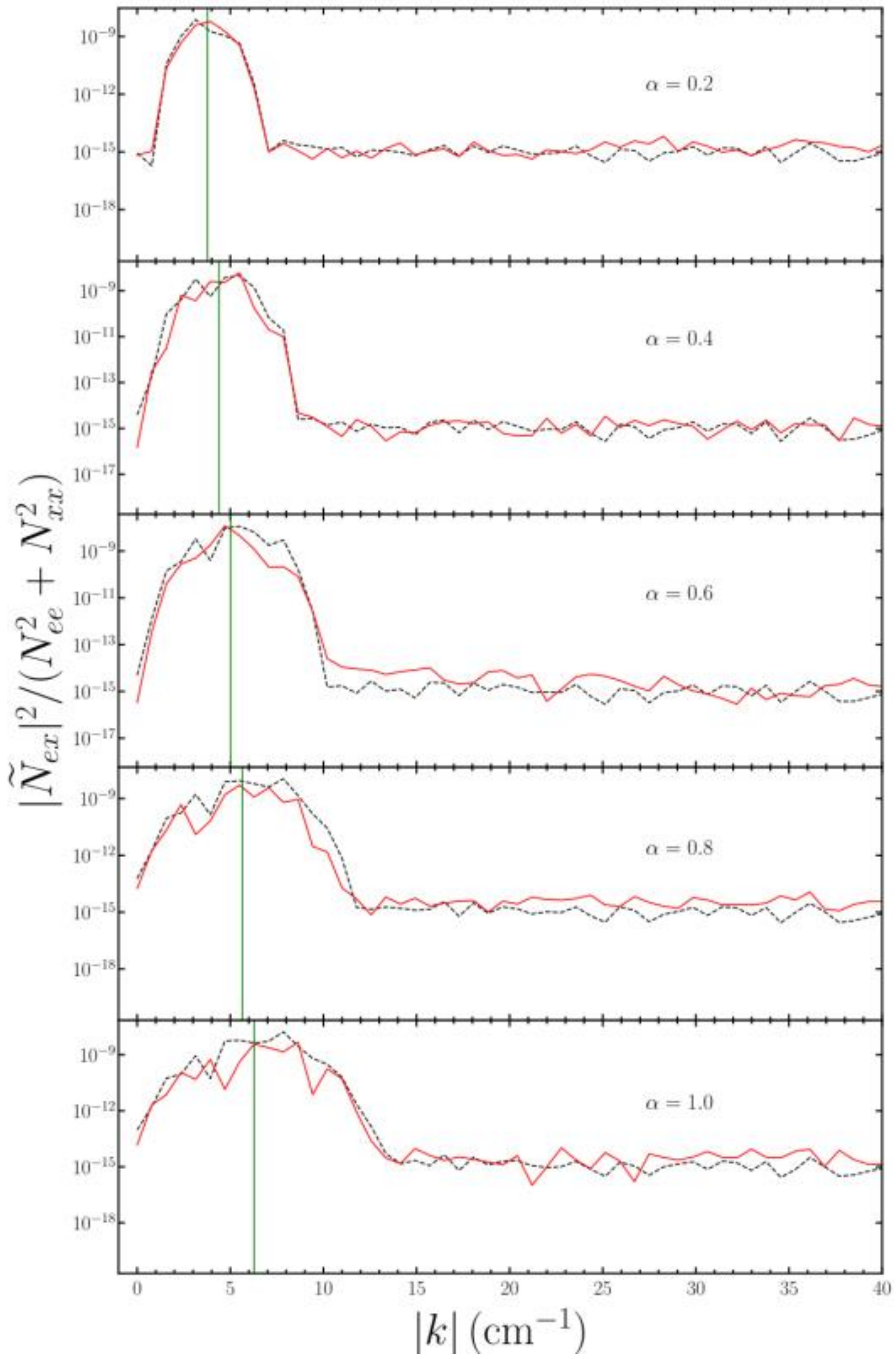




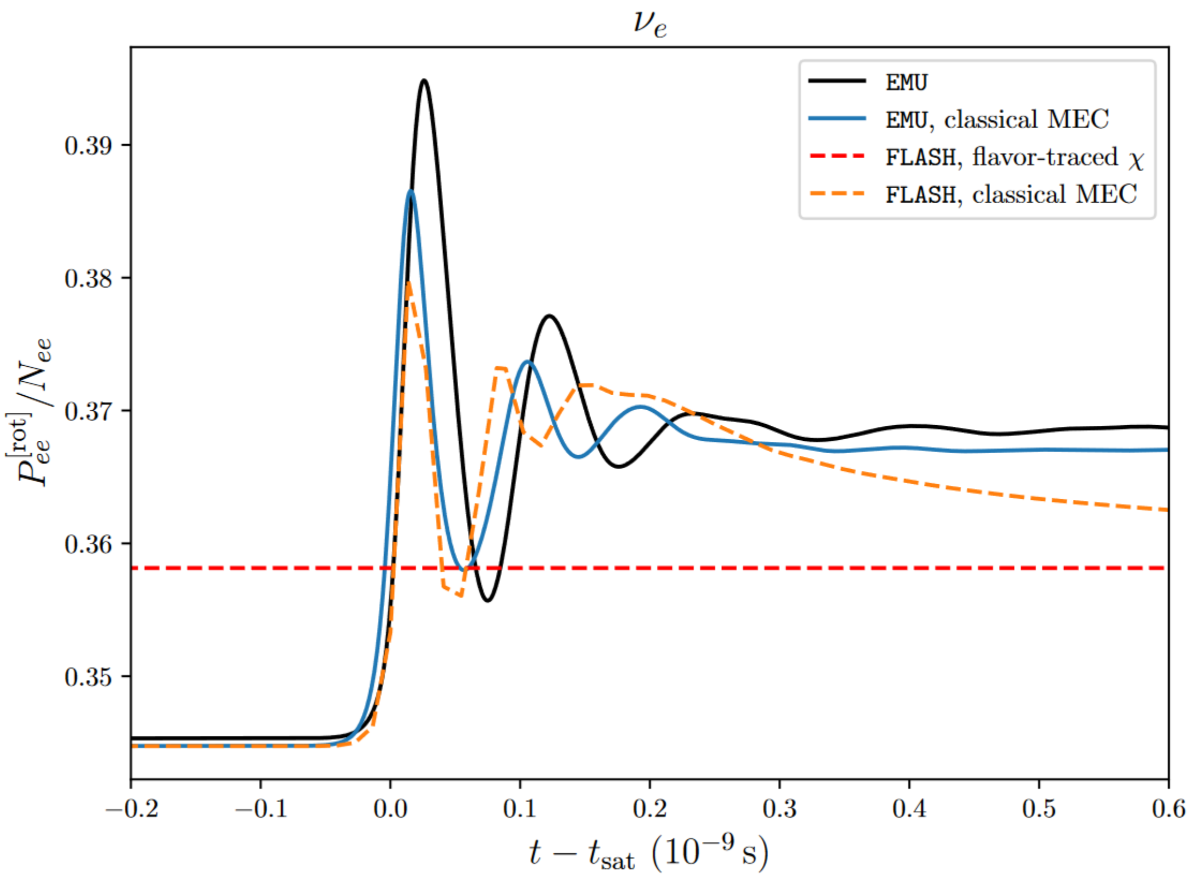
Moments reproduce beam instabilities exactly



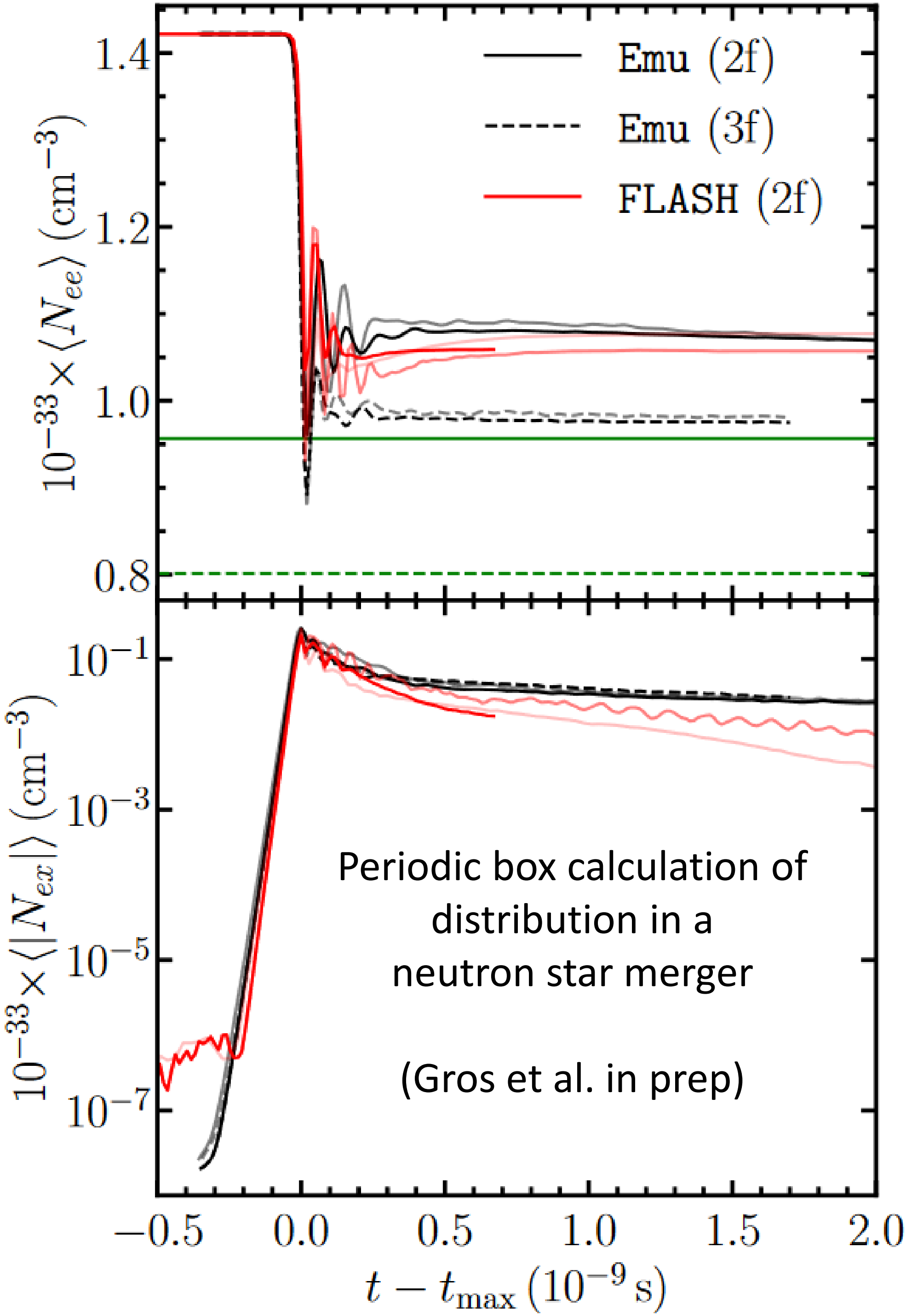
Method enhancements: Grohs et al. (in prep)



Moments predict reasonable outcomes



But the closure is as wrong as in classical transport.



Periodic box calculation of distribution in a neutron star merger

(Gros et al. in prep)

Conclusions

New theoretical frameworks may change the game

Several effective models are proposed or in development

Collisional physics sophistication is growing

Reduced coupling makes global simulations tractable

There are many open lines of research for treating the neutrino QKEs in relativistic astrophysics.

Classical transport inspires development

Moment formalism works unexpectedly well, but the closure is a pain.