CONSTRUCTING AND CLASSIFYING THE SPACE OF SMALL INTEGER WEIGHING MATRICES

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DEFINITIONS

Weighing Matrix

$$W(n,w) = \{M \mid \{0,+,-\}^{n\times n}\ MM^{+} = wI\}$$

Conference Matrix

$$C(n) = W(n, n-1) \ \text{(up to column reordering)}$$

Hadamard Matrix

$$H(n) = W(n,n)$$

INTEGER WEIGHING MATRIX

Integer Weighing matrix

$$IW(n,w) = \{M \mid M \in Z^{n\times n} \; MM^{+} = wI\}$$

Partial Integer Weighing Matrix

$$\mathrm{P}IW(m,n,w) = \{P \mid P \in Z^{m\times n} \quad PP^{+} = wI\}$$

Circulant Weighing Matrix

$$C(n,w) = \{M \mid M \in W(n,k) \;\; M \; is \; ciculant\}$$

W →IW →W

$\boldsymbol{A_{1,1}}$	$\boldsymbol{A_{1,2}}$	…	$\boldsymbol{A_{1,4}}$	$\boldsymbol{A_{1,r}}$
$\boldsymbol{A_{2,1}}$	$\boldsymbol{A_{2,2}}$		$\boldsymbol{A_{2,4}}$	$\boldsymbol{A_{2,r}}$
…			..	…
…			..	..
$\boldsymbol{A_{r,1}}$	$\boldsymbol{A_{r,2}}$	…	$\boldsymbol{A_{r,4}}$	$\boldsymbol{A_{r,r}}$

$$M \in W(r * n, k) \rightarrow \boldsymbol{A_{i,j}} \in \boldsymbol{C}(\boldsymbol{n}, \boldsymbol{k})$$

$$A_{i,j} \rightarrow (a_{i,j} \in Z) \rightarrow M \in IW(r,k)$$

IW(N,K) EXISTENCE THEOREMS

$a, b, c, d \in Z$

- $w = a^2 \Rightarrow \forall n: \exists\, IW(n, w)$
- $w = a^2 + b^2 \Rightarrow \forall n: \exists\, IW(2n, w)$
- $w = a^2 + b^2 + c^2 \Rightarrow \forall n: \exists\, IW(4n, w)$ *(WITT)*
- $w = a^2 + b^2 + c^2 + d^2 \Rightarrow \forall n: \exists\, IW(4n, w)$

- $\exists\, IW(2n+1, w) \Rightarrow w = a^2$
- $\exists\, IW(4n+2, w) \Rightarrow w = a^2 + b^2$

ANTISYMMETRIC IW(N,W)

- n=2q+1 ➔ No Solution
- $w = a^2 \Rightarrow \ \forall n: \exists\, IW(2n, w)$
- $w = a^2 + b^2 \Rightarrow \ \forall n: \ \exists\, IW(4n, w)$
- $\forall n, w: \ \exists\, IW(8n, w)$

- $\exists\, IW(4n + 2, w) \Rightarrow \exists a \in Z : \ w = a^2$
- $\exists\, IW(8n + 4, w) \Rightarrow \exists a, b \in Z : \ w = a^2 + b^2$

SYMMETRIC IW

- $\exists a \in Z : w = a^2 \Rightarrow \ \forall n: \exists\, IW(n, w)$
- $\exists a, b \in Z : w = a^2 + b^2 \Rightarrow \ \forall n: \ \exists\, IW(2n, w)$
- $\exists a, b, c \in Z : w = a^2 + b^2 + c^2 \ \Rightarrow \ \forall n: \ \exists\, IW(4n, w)$
- $\exists a, b, c, d \in Z : w = a^2 + b^2 + c^2 + d^2 \ \Rightarrow \ \forall n: \ \exists\, IW(8n, w)$

- $\exists\, IW(2n+1, w) \Rightarrow \exists a \in Z : \ w = a^2$
- $\exists\, IW(4n+2, w) \Rightarrow \exists a, b \in Z : w = a^2 + b^2$
- ***Conjecture*** $\exists\, \boldsymbol{IW(8n+4, w) \Rightarrow \exists a, b, c \in Z : w = a^2 + b^2 + c^2}$

HADAMARD EQUIVALENCE

Two P*IW* (*m, n, k*) matrices are *Hadamard Equivalent* (or H-equivalent) if one can be obtained from the other using a sequence of the following operations:

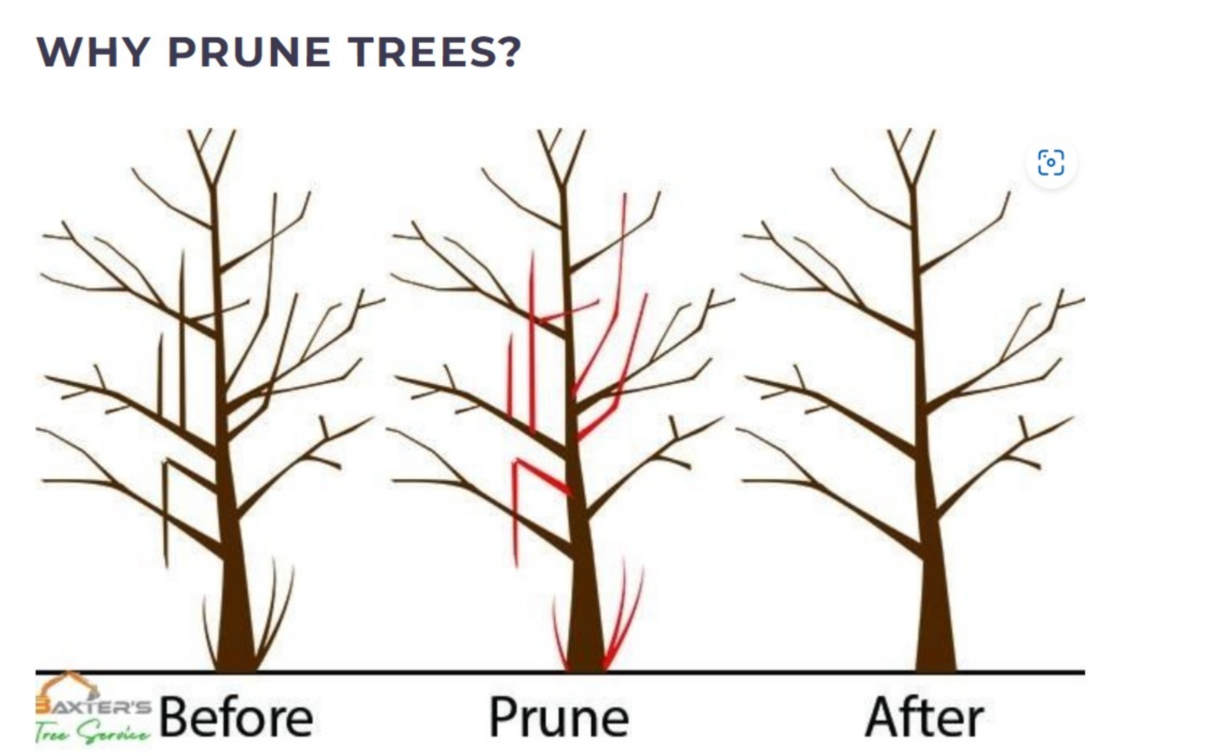
(1) Multiplying a row or a column by -1, and

(2) Swapping two rows or columns.

MINIMALITY CONDITION

- Fixing Rows and Column signs
- RowLex – Lexicographic ordering of the rows

Keep trimming



PARTIAL MATRIX COMPLETION

- Complete a partial matrix

  1. NewLine is orthogonal to previous lines ?
  2. New partial matrix is MINIMAL ?

  If both are OK than add the line and continue to build.

  else – try the next line.

.

DYNAMIC PROGRAMMING

CheckLine(Partial_Matrix,Line)

- If $PM^{+} L = 0$ return TRUE

else return FALSE

Complete(PM,line,index)

```
for all newLine from Index to LastIndex {
    PMtemp=PM.addLine(newline)
    if (PMtemp.isMinimal())
        if ChekLine(PM,newline)
            PM.addLine(newline)
            if (line = size)
                print (PM)
            else
                complete(PM,line+1,index+1)
}
```

Build IW(size,w)

```
vectors = NSOKS(w)
newLines=SetSIgns(Permute(vectors))
for all vectors
    PM=vector
    Complete(PM,1,vectorindex)
end
```

RESULTS

- 44 (49) classes of IW (7,25)
- Efficient Search (IW(7,25) ran for few minutes on a PC)
- Improved NSOKS algorithm (All possible NSOKS(100000,200))

BLOCK DIAGONAL STRUCTURE

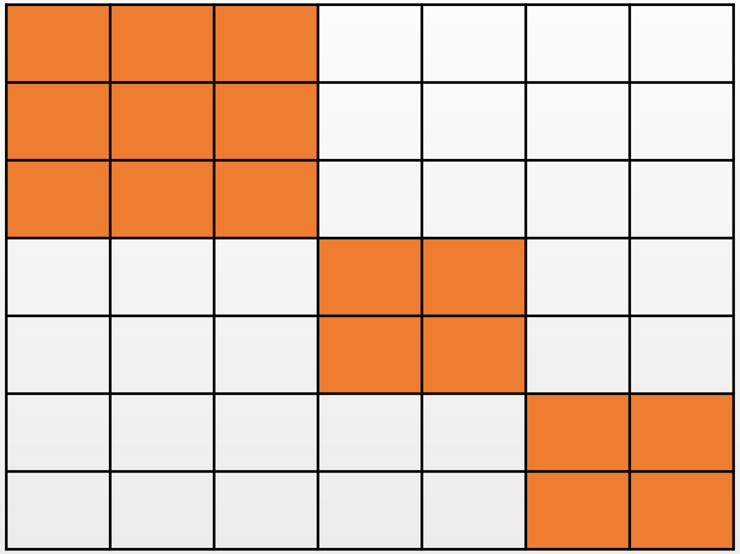


FIGURE 1. The full TH classification of $IW(7,25)$

Type	Multiplicity
$7A$	1
$5A \oplus B$	1
$3A \oplus 2B$	1
$A \oplus 3B$	1
$3A \oplus C$	2
$A \oplus B \oplus C$	2
$2A \oplus D$	2
$B \oplus D$	2
$A \oplus E$	13
F	19

N=1,2,3,4

$$A_1 = [5].$$

$$B_1 = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}.$$

$$\emptyset.$$

$$C_1 = \left[\begin{array}{cc|cc} 1 & 4 & 2 & -2 \\ 4 & 1 & -2 & 2 \\ \hline 2 & -2 & 4 & 1 \\ -2 & 2 & 1 & 4 \end{array}\right], C_2 = \left[\begin{array}{cc|cc} 1 & 4 & 2 & -2 \\ 4 & -1 & -2 & -2 \\ \hline 2 & -2 & 4 & 1 \\ 2 & 2 & -1 & 4 \end{array}\right]$$

N=5

$$D_1 = \left[\begin{array}{c|cccc} 3 & -2 & -2 & -2 & -2 \\ \hline 2 & -2 & 0 & 1 & 4 \\ 2 & 4 & -2 & 0 & 1 \\ 2 & 1 & 4 & -2 & 0 \\ 2 & 0 & 1 & 4 & -2 \end{array}\right], D_2 = \left[\begin{array}{c|cccc} 3 & 2 & 2 & 2 & 2 \\ \hline 2 & 3 & -2 & -2 & -2 \\ 2 & -2 & 3 & -2 & -2 \\ 2 & -2 & -2 & 3 & -2 \\ 2 & -2 & -2 & -2 & 3 \end{array}\right]$$

N=6

$$\left[\begin{array}{ccc|ccc} -4 & 1 & 0 & 2 & 0 & 2 \\ 0 & -4 & 1 & 2 & 2 & 0 \\ 1 & 0 & -4 & 0 & 2 & 2 \\ \hline -2 & -2 & 0 & -4 & 0 & 1 \\ 0 & -2 & -2 & 1 & -4 & 0 \\ -2 & 0 & -2 & 0 & 1 & -4 \end{array}\right], \begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 \\ 2 & 1 & -4 & -2 & 0 & 0 \\ 2 & -4 & 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & -4 & 1 & -2 \\ 0 & 0 & 1 & -2 & -2 & 4 \\ -1 & 2 & 0 & 0 & 4 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 \\ 2 & 0 & -3 & -2 & 2 & 2 \\ 2 & -2 & -2 & 0 & -2 & -3 \\ 1 & -4 & 2 & 0 & 0 & 2 \\ 0 & 0 & 2 & -4 & 1 & -2 \\ 0 & -1 & 0 & 2 & 4 & -2 \end{bmatrix}, \begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 \\ 2 & 0 & -3 & -2 & 2 & 2 \\ 2 & -3 & 0 & -2 & -2 & -2 \\ 1 & -2 & -2 & 4 & 0 & 0 \\ 0 & 2 & -2 & 0 & 1 & -4 \\ 0 & 2 & -2 & 0 & -4 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 \\ 2 & 0 & -4 & 0 & 2 & 1 \\ 2 & -4 & 0 & 0 & -1 & -2 \\ 1 & 0 & 0 & -4 & -2 & 2 \\ 0 & 2 & -1 & -2 & 0 & -4 \\ 0 & 1 & -2 & 2 & -4 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 & 2 & 1 & 1 & 1 \\ 3 & -1 & -1 & 1 & -2 & -3 \\ 2 & -1 & -3 & -1 & 3 & 1 \\ 1 & 1 & -1 & -3 & -3 & 2 \\ 1 & -2 & 3 & -3 & 1 & -1 \\ 1 & -3 & 1 & 2 & -1 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 3 & 2 & 1 & 1 & 1 \\ 3 & -1 & -1 & 1 & -2 & -3 \\ 2 & -3 & 1 & -1 & -1 & 3 \\ 1 & 2 & -3 & -3 & -1 & 1 \\ 1 & -1 & 1 & -3 & 3 & -2 \\ 1 & -1 & -3 & 2 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 3 & 2 & 1 & 1 & 1 \\ 3 & -2 & 1 & -1 & -1 & -3 \\ 2 & -3 & -1 & 1 & 1 & 3 \\ 1 & 1 & -1 & -3 & -3 & 2 \\ 1 & 1 & -3 & 3 & -2 & -1 \\ -1 & -1 & 3 & 2 & -3 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 3 & 3 & 2 & 1 & 1 & 1 \\ 3 & -2 & -3 & 1 & 1 & 1 \\ 1 & 1 & -1 & 2 & -3 & -3 \\ 2 & -3 & 3 & -1 & -1 & -1 \\ 1 & 1 & -1 & -3 & 2 & -3 \\ -1 & -1 & 1 & 3 & 3 & -2 \end{bmatrix}, \begin{bmatrix} 3 & 3 & 2 & 1 & 1 & 1 \\ 3 & -3 & 1 & 1 & -1 & -2 \\ 2 & 1 & -3 & -1 & -3 & 1 \\ 1 & 1 & -1 & -3 & 2 & -3 \\ 1 & -1 & -3 & 2 & 3 & 1 \\ -1 & 2 & -1 & 3 & -1 & -3 \end{bmatrix}, \begin{bmatrix} 3 & 3 & 2 & 1 & 1 & 1 \\ 3 & -3 & 1 & 1 & -1 & -2 \\ 2 & -1 & -1 & -3 & -1 & 3 \\ 1 & 2 & -3 & 1 & -3 & -1 \\ 1 & 1 & -1 & -3 & 2 & -3 \\ 1 & -1 & -3 & 2 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 2 & 2 & 2 & 0 \\ 2 & 2 & 0 & -2 & -3 & 2 \\ 2 & 0 & -2 & -3 & 2 & -2 \\ 2 & -2 & -3 & 2 & 0 & 2 \\ 2 & -3 & 2 & 0 & -2 & -2 \\ 0 & -2 & 2 & -2 & 2 & 3 \end{bmatrix},$$

$$\begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 \\ 2 & -2 & -2 & 0 & 3 & 2 \\ 2 & -2 & -2 & 0 & -2 & -3 \\ 1 & 0 & 0 & -4 & -2 & 2 \\ 0 & 3 & -2 & -2 & 2 & -2 \\ 0 & 2 & -3 & 2 & -2 & 2 \end{bmatrix}$$

N=7

$$
\begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 & 0 \\ 2 & 0 & -3 & -2 & 2 & 2 & 0 \\ 2 & -4 & 0 & 0 & 0 & -2 & 1 \\ 1 & 0 & -2 & 0 & -4 & 0 & -2 \\ 0 & 2 & -2 & 0 & 1 & -4 & 0 \\ 0 & 1 & 0 & -2 & -2 & 0 & 4 \\ 0 & 0 & 2 & -4 & 0 & -1 & -2 \end{bmatrix},
\begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 & 0 \\ 2 & -1 & -1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -3 & 0 & 1 & -2 & -3 \\ 1 & 0 & -3 & 2 & -1 & 3 & 1 \\ 1 & -1 & -1 & 0 & -3 & -3 & 2 \\ 1 & -3 & 1 & 0 & -2 & 1 & -3 \\ 1 & -3 & 0 & 2 & 3 & -1 & 1 \end{bmatrix},
\left[\begin{array}{c|ccc|ccc} 4 & 2 & 2 & 1 & 0 & 0 & 0 \\ \hline 1 & -4 & 1 & 2 & 1 & 1 & 1 \\ 2 & -1 & -1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -4 & 2 & 1 & 1 & 1 \\ \hline 1 & -1 & -1 & 0 & -3 & -3 & 2 \\ 1 & -1 & -1 & 0 & -3 & 2 & -3 \\ 1 & -1 & -1 & 0 & 2 & -3 & -3 \end{array}\right],
\begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 & 0 \\ 2 & -1 & -2 & -2 & 2 & 2 & 2 \\ 2 & -2 & -1 & -2 & -2 & -2 & -2 \\ 1 & -2 & -2 & 4 & 0 & 0 & 0 \\ 0 & 2 & -2 & 0 & 3 & -2 & -2 \\ 0 & 2 & -2 & 0 & -2 & 3 & -2 \\ 0 & 2 & -2 & 0 & -2 & -2 & 3 \end{bmatrix},
$$

$$
\begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 & 0 \\ 2 & -1 & -2 & -2 & 2 & 2 & 2 \\ 2 & -2 & -2 & 0 & 0 & -2 & -3 \\ 0 & 2 & 0 & -4 & -1 & -2 & 0 \\ 1 & -2 & 0 & 0 & -4 & 0 & 2 \\ 0 & 2 & -2 & 0 & -2 & 3 & -2 \\ 0 & 2 & -3 & 2 & 0 & -2 & 2 \end{bmatrix},
\begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 & 0 \\ 2 & -1 & -3 & 0 & 3 & 1 & 1 \\ 1 & 1 & -1 & -4 & -1 & 1 & -2 \\ 1 & 0 & -3 & 2 & -3 & -1 & -1 \\ 1 & -1 & 0 & -2 & -1 & -3 & 3 \\ 1 & -3 & 1 & 0 & 1 & -2 & -3 \\ 1 & -3 & 1 & 0 & -2 & 3 & 1 \end{bmatrix},
\begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 & 0 \\ 2 & -1 & -3 & 0 & 3 & 1 & 1 \\ 1 & 1 & -1 & -4 & -2 & 1 & 1 \\ 1 & 0 & -3 & 2 & -3 & -1 & -1 \\ 1 & -1 & 0 & -2 & 1 & -3 & -3 \\ 1 & -3 & 1 & 0 & -1 & 3 & -2 \\ 1 & -3 & 1 & 0 & -1 & -2 & 3 \end{bmatrix},
\begin{bmatrix} 4 & 2 & 2 & 1 & 0 & 0 & 0 \\ 2 & -1 & -3 & 0 & 3 & 1 & 1 \\ 1 & 1 & -3 & 0 & -3 & 1 & -2 \\ 1 & -1 & 1 & -4 & -1 & 2 & 1 \\ 1 & -1 & 0 & -2 & 1 & -3 & -3 \\ 1 & -1 & -1 & 0 & -2 & -3 & 3 \\ 1 & -4 & 1 & 2 & -1 & 1 & -1 \end{bmatrix},
$$

N=7

$$\begin{bmatrix} 4 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & -2 & -2 & -2 & -2 & -2 \\ 1 & -2 & 4 & -1 & -1 & -1 & -1 \\ 1 & -2 & -1 & 4 & -1 & -1 & -1 \\ 1 & -2 & -1 & -1 & 4 & -1 & -1 \\ 1 & -2 & -1 & -1 & -1 & 4 & -1 \\ 1 & -2 & -1 & -1 & -1 & -1 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & -2 & -2 & -2 & -2 & -2 \\ 1 & -2 & 4 & -1 & -1 & -1 & -1 \\ 1 & -2 & -1 & 3 & 1 & 0 & -3 \\ 1 & -2 & -1 & 1 & 0 & -3 & 3 \\ 1 & -2 & -1 & 0 & -3 & 3 & 1 \\ 1 & -2 & -1 & -3 & 3 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & -1 & 2 & -2 & -2 & -2 & -2 \\ 1 & 2 & -4 & -1 & -1 & -1 & -1 \\ 1 & -2 & -1 & 3 & 1 & 0 & -3 \\ 1 & -2 & -1 & 1 & 0 & -3 & 3 \\ 1 & -2 & -1 & 0 & -3 & 3 & 1 \\ 1 & -2 & -1 & -3 & 3 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & -1 & 2 & -2 & -2 & -2 & -2 \\ 1 & 2 & -4 & -1 & -1 & -1 & -1 \\ 1 & -2 & -1 & 4 & -1 & -1 & -1 \\ 1 & -2 & -1 & -1 & 4 & -1 & -1 \\ 1 & -2 & -1 & -1 & -1 & 4 & -1 \\ 1 & -2 & -1 & -1 & -1 & -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & -3 & 2 & 0 & 0 & -2 & -2 \\ 1 & 2 & 0 & -3 & -3 & -1 & -1 \\ 1 & 0 & -3 & 3 & -2 & -1 & -1 \\ 1 & 0 & -3 & -2 & 3 & -1 & -1 \\ 1 & -2 & -1 & -1 & -1 & 4 & -1 \\ 1 & -2 & -1 & -1 & -1 & -1 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & -3 & 2 & 0 & 0 & -2 & -2 \\ 1 & 2 & 0 & -3 & -3 & -1 & -1 \\ 1 & 0 & -3 & 2 & -1 & 1 & -3 \\ 1 & 0 & -3 & -1 & 2 & -3 & 1 \\ 1 & -2 & -1 & 1 & -3 & 0 & 3 \\ 1 & -2 & -1 & -3 & 1 & 3 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & -3 & 2 & 0 & 0 & -2 & -2 \\ 1 & 2 & -1 & -1 & -3 & 0 & -3 \\ 1 & 0 & -1 & -1 & -2 & -3 & 3 \\ 1 & 0 & -3 & -2 & 3 & -1 & -1 \\ 1 & -2 & 0 & -3 & -1 & 3 & 1 \\ 1 & -2 & -3 & 3 & -1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 & 2 & 1 & 1 & 1 & 0 \\ 3 & -1 & -2 & 1 & 0 & -3 & 1 \\ 2 & -2 & 1 & -2 & -2 & 2 & 2 \\ 1 & 1 & -2 & 0 & -3 & 1 & -3 \\ 1 & 0 & -2 & -3 & 3 & 1 & -1 \\ 1 & -3 & 2 & 1 & 1 & 0 & -3 \\ 0 & -1 & -2 & 3 & 1 & 3 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 3 & 3 & 2 & 1 & 1 & 1 & 0 \\ 3 & -1 & 0 & -1 & -2 & -3 & 1 \\ 2 & 0 & -3 & -2 & 0 & 2 & -2 \\ 1 & -1 & -2 & 1 & 3 & 0 & 3 \\ 1 & -2 & 0 & 3 & 1 & -1 & -3 \\ 1 & -3 & 2 & 0 & -1 & 3 & 1 \\ 0 & 1 & -2 & 3 & -3 & 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 & 3 & 1 & 0 & -3 & -2 \\ -3 & -1 & 2 & 1 & -1 & 3 & 0 \\ 3 & 0 & 3 & 1 & 1 & 1 & 2 \\ 0 & -1 & 1 & -3 & 3 & 1 & -2 \\ 1 & -3 & 1 & -2 & -3 & -1 & 0 \\ -1 & 3 & 1 & -3 & -1 & 0 & 2 \\ 2 & 2 & 0 & 0 & -2 & 2 & -3 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 2 & 2 & 2 & 0 & 0 \\ 2 & 2 & -1 & -2 & -2 & 2 & 2 \\ 2 & -1 & 2 & -2 & -2 & -2 & -2 \\ 2 & -2 & -2 & 2 & -1 & 2 & -2 \\ 2 & -2 & -2 & -1 & 2 & -2 & 2 \\ 0 & 2 & -2 & 2 & -2 & -3 & 0 \\ 0 & 2 & -2 & -2 & 2 & 0 & -3 \end{bmatrix}$$

N=7 SPECIAL CASES

$$
,\begin{bmatrix}
4 & 2 & 1 & 1 & 1 & 1 & 1 \\
2 & -1 & 2 & -2 & -2 & -2 & -2 \\
1 & 2 & -4 & -1 & -1 & -1 & -1 \\
1 & -2 & -1 & 4 & -1 & -1 & -1 \\
1 & -2 & -1 & -1 & 4 & -1 & -1 \\
1 & -2 & -1 & -1 & -1 & 4 & -1 \\
1 & -2 & -1 & -1 & -1 & -1 & 4
\end{bmatrix},
$$

$$
\begin{bmatrix}
4 & 2 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & -2 & -2 & -2 & -2 & -2 \\
1 & -2 & 4 & -1 & -1 & -1 & -1 \\
1 & -2 & -1 & 4 & -1 & -1 & -1 \\
1 & -2 & -1 & -1 & 4 & -1 & -1 \\
1 & -2 & -1 & -1 & -1 & 4 & -1 \\
1 & -2 & -1 & -1 & -1 & -1 & 4
\end{bmatrix}
$$

SUMMARY

- Classified IW(25,7)
- Symmetric Conference Matrix (2n) == Real ETF(2n,n)

  Hence e.g. no Real ETF(22,11) ->

  Conference(22) -> W(22,21) -> 21 =a^2+b^2 FALSE

Open Questions

- Build W(35,25) from IW(7,25)
- Generally classify IW(n,w) without "0" s structure
- Generally classify circulant/symmetric IW(n,w)

REFERENCES

- https://arxiv.org/abs/2304.09495v1
- **New weighing matrices via partitioned group actions;** May 2024 Discrete Mathematics 347(5):113908
- **A practical algorithm for completing half-Hadamard matrices using LLL;** 2021 ournal of Algebraic Combinatorics 55(1)