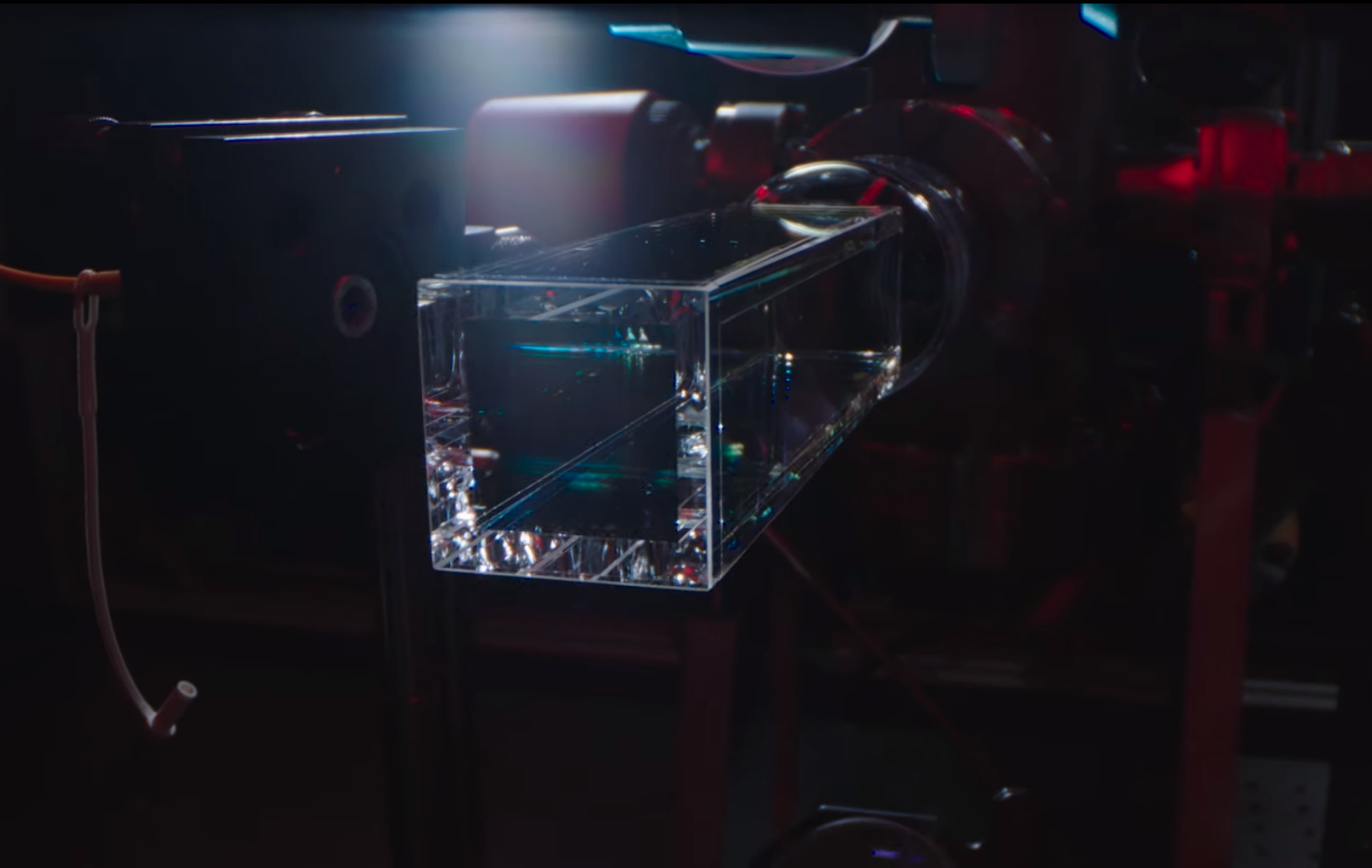
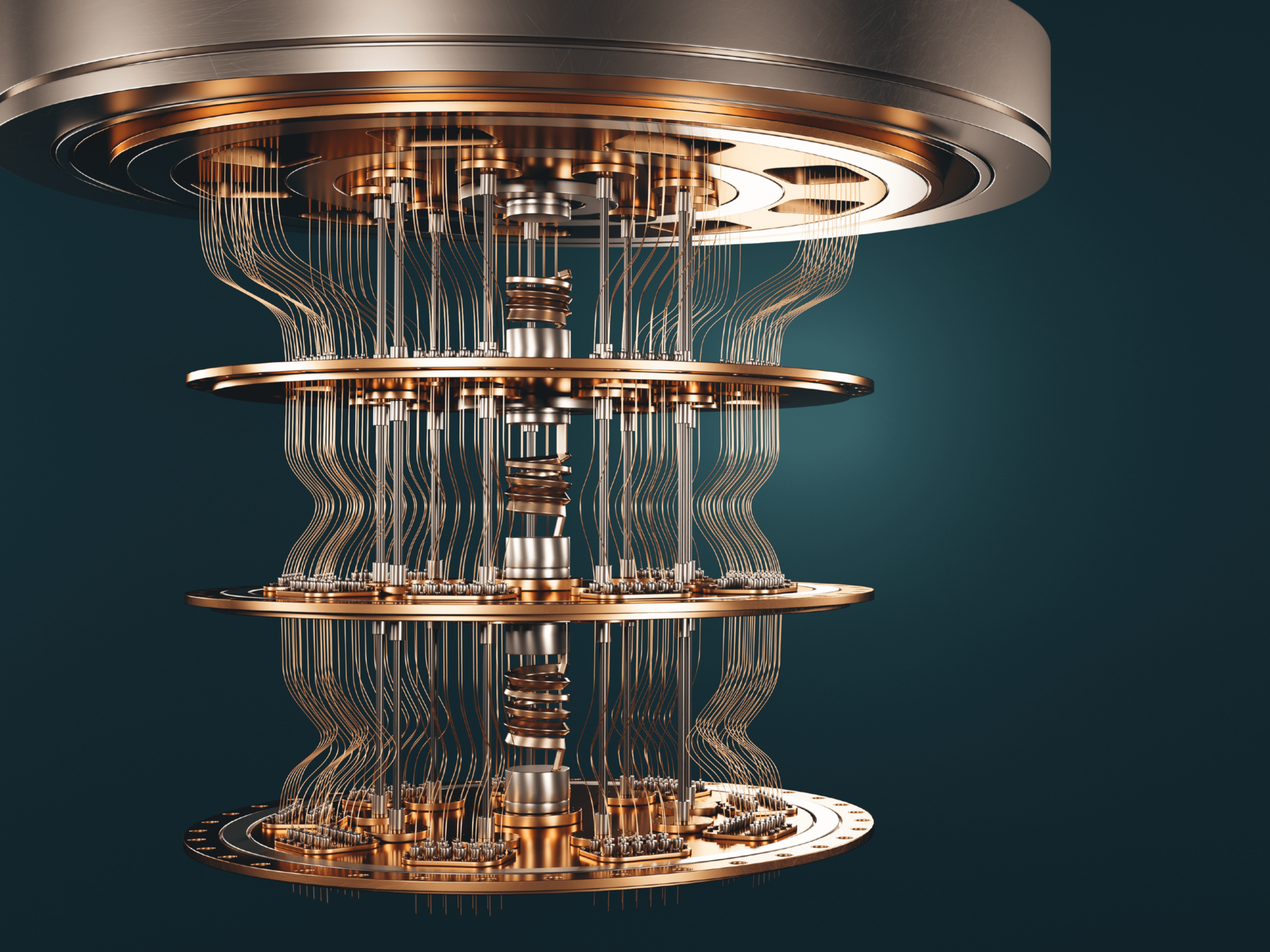




Quantum Computing with Rydberg-atom quantum processors

ICERM @ Brown University

Interacting Particle Systems:

Analysis, Control, Learning and Computation

May 2024

Oliver Tse

Based on joint work with **Robert de Keijzer**, **Servaas Kokkelmans, Luke Visser**

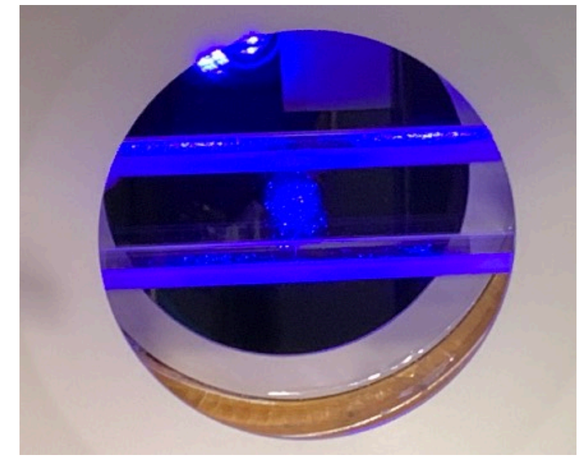
CASA TU/e Eindhoven University of Technology

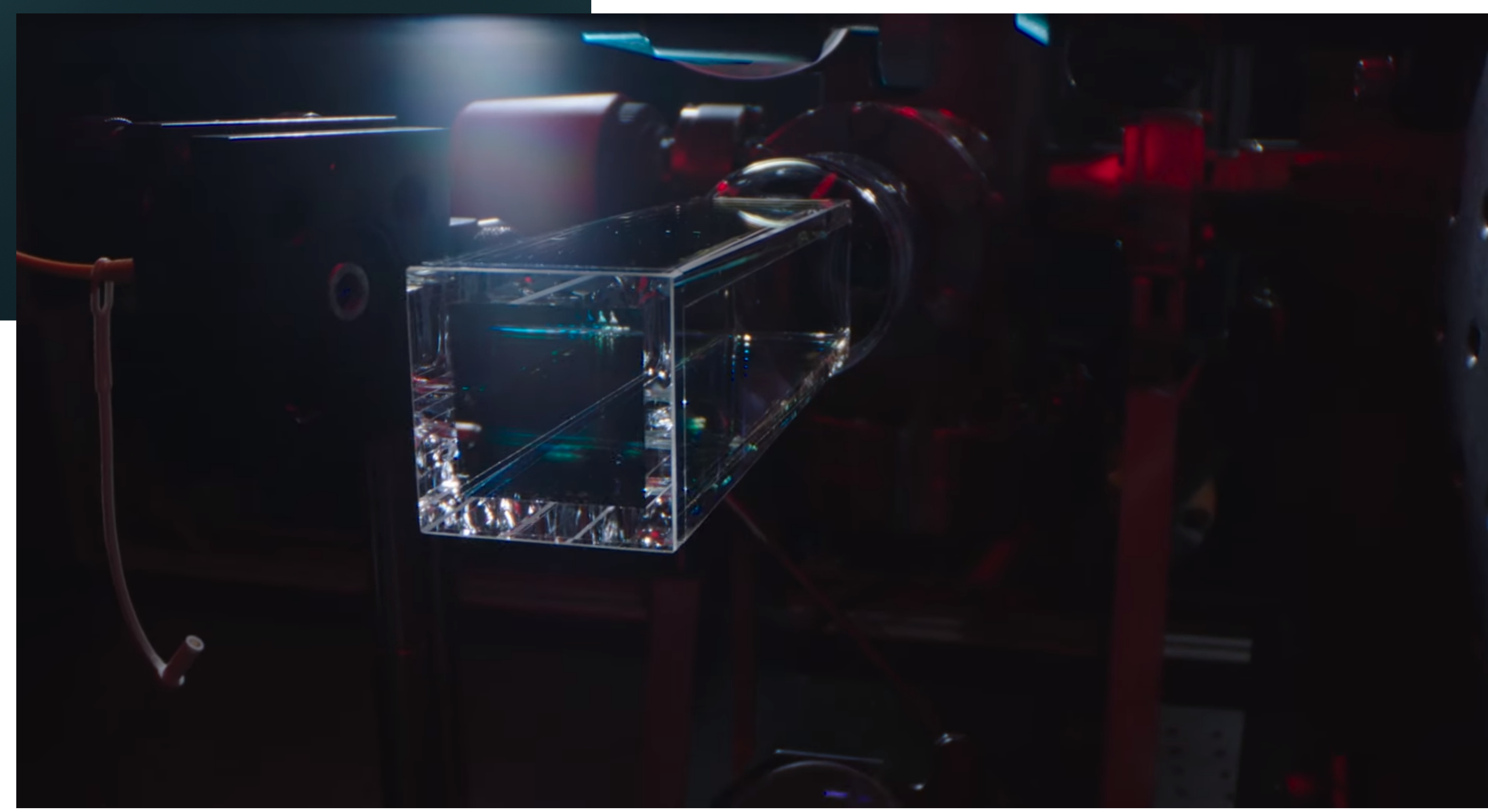
NWO Netherlands Organisation for Scientific Research

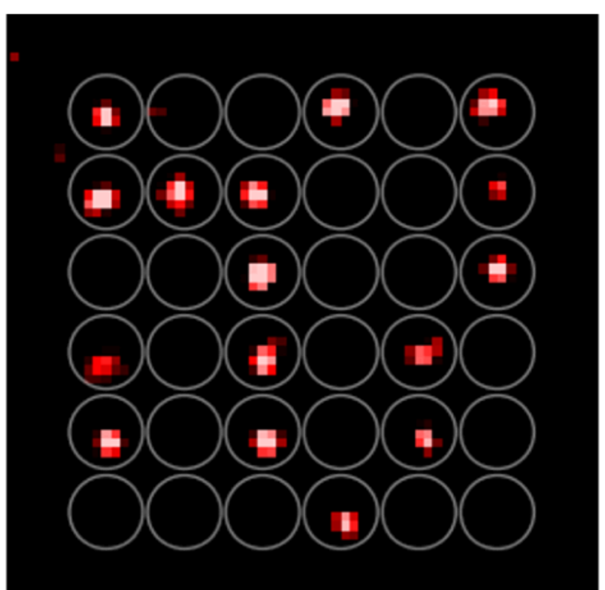
QT/e Center for Quantum Materials and Technology Eindhoven TU/e

EHCI Eindhoven Hendrik Casimir Institute TU/e

The Rydberg-Atom Quantum Computer







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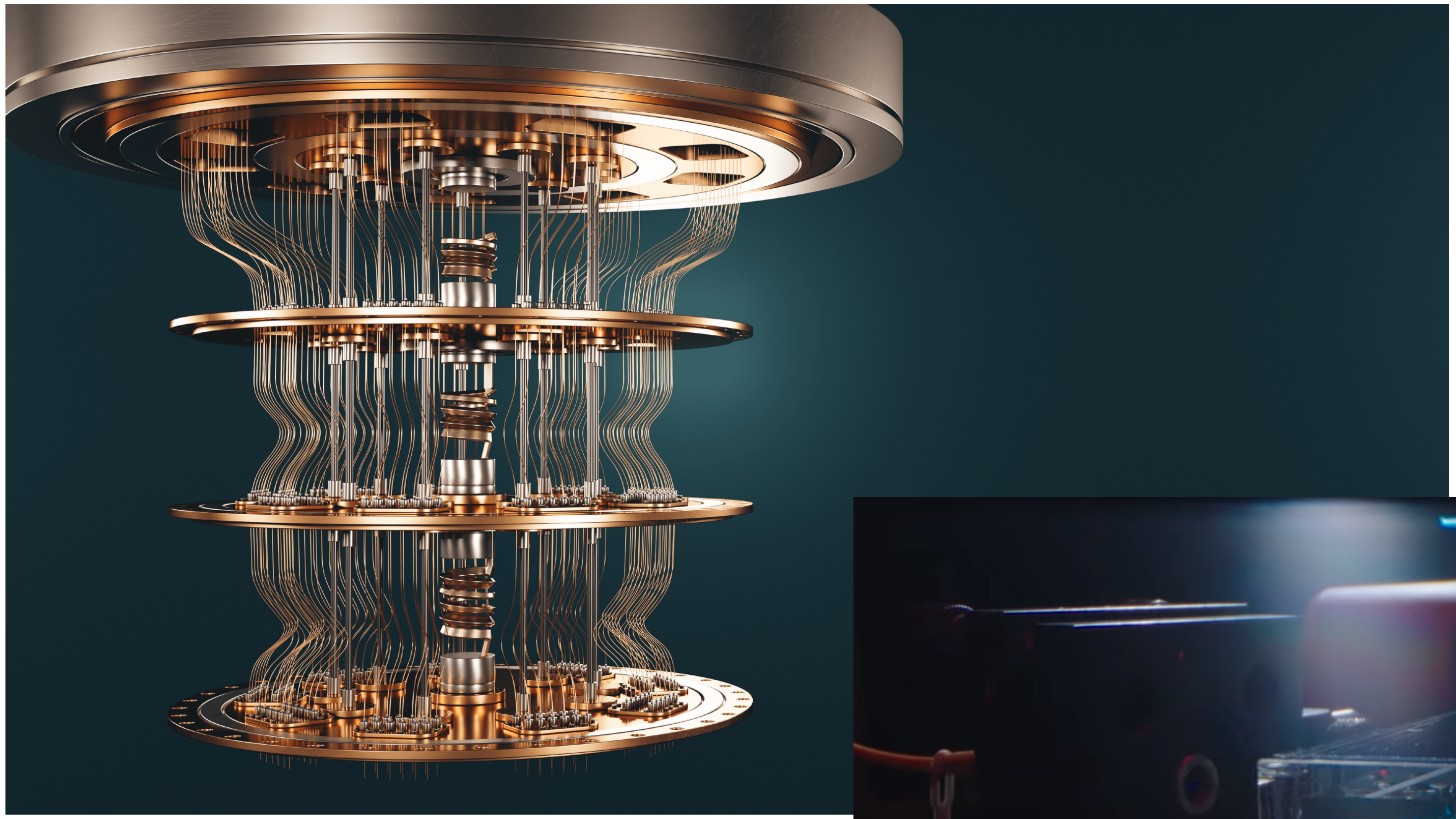
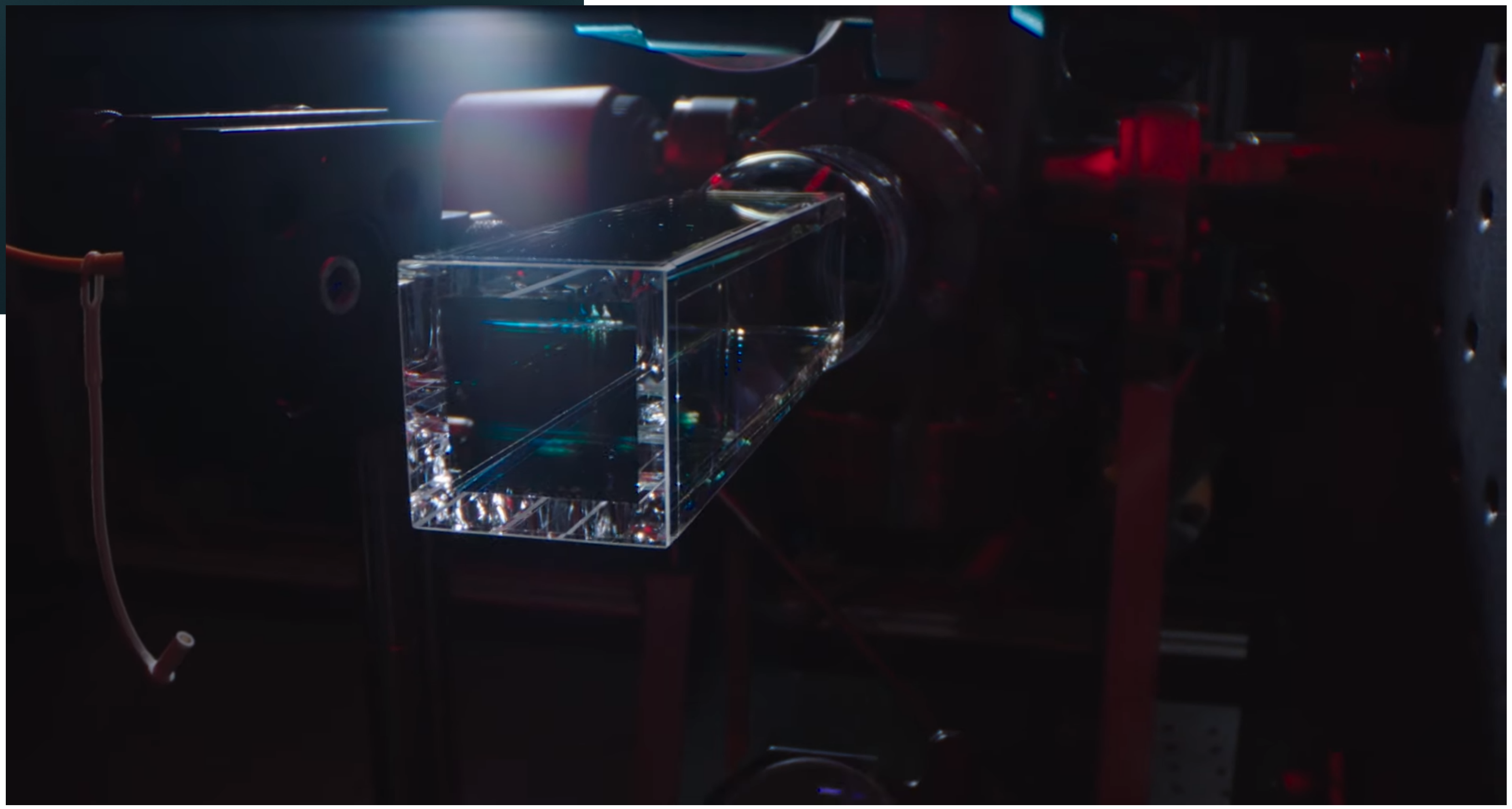
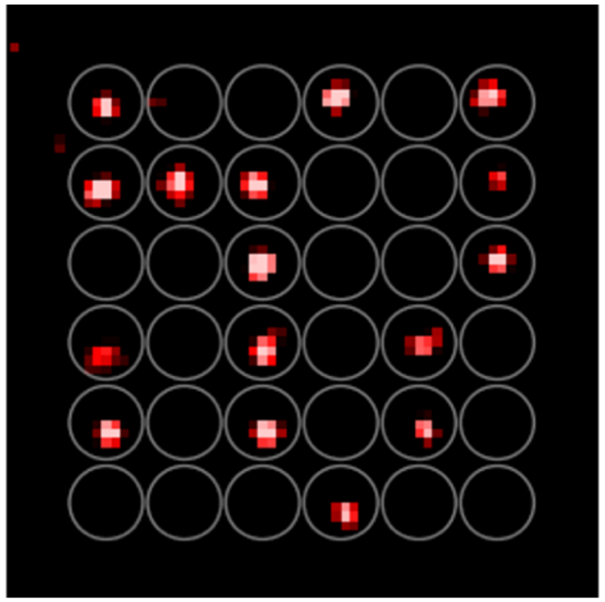
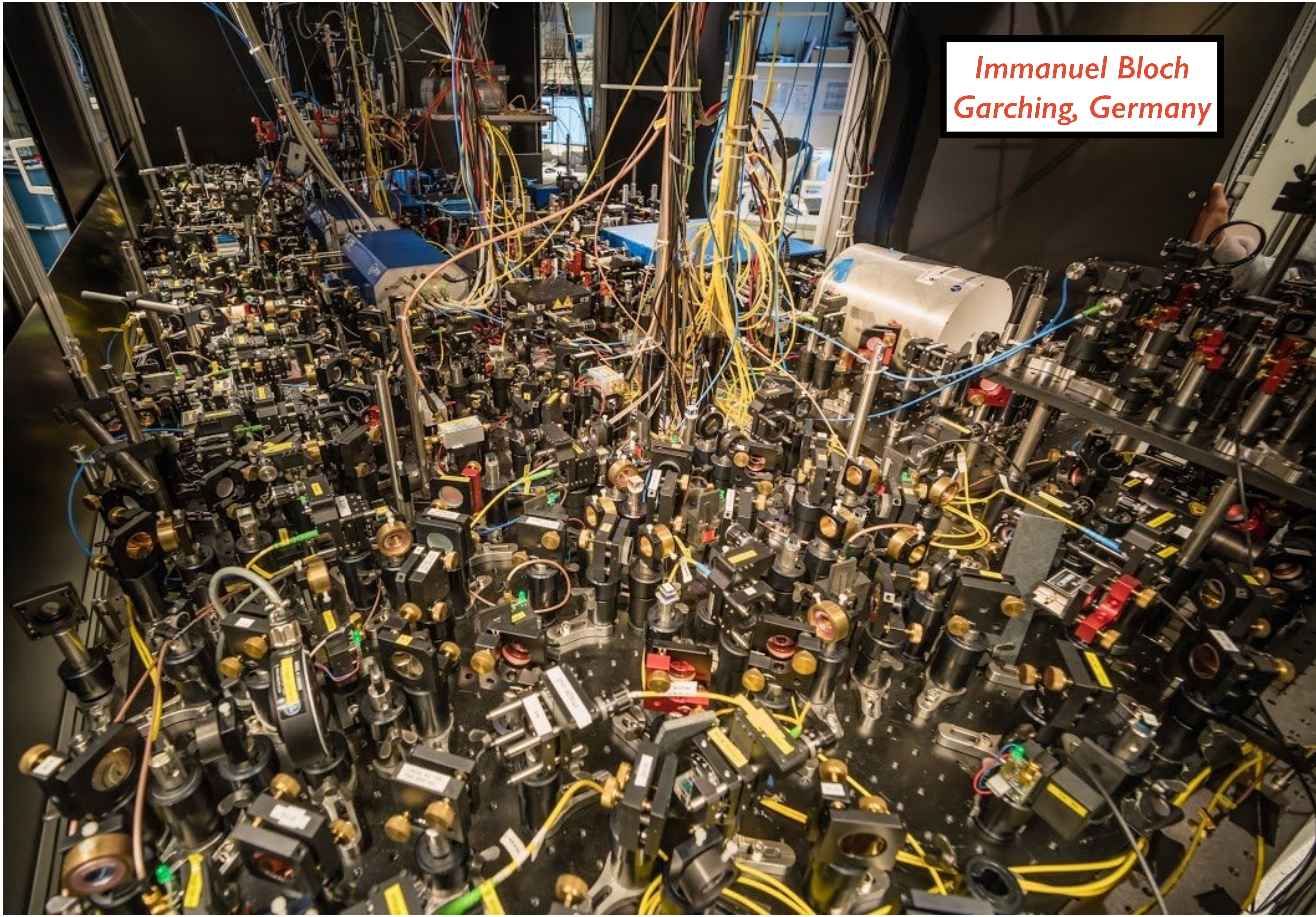


Scalable
Identical
Configurable
Long coherence times






Immanuel Bloch
Garching, Germany

The Task

Quantum Delta NL

In the beginning…

Optimization

Classical (HPC)
Pulse-based
Gate-based

Experiments

Measurements
Implementation
Uncertainties

Quantum Inspire platform

Simulation

Atomic calculations
Hamiltonian generation
Open system evolutions

Pulse Generation

FPGA
Noise characterization
Modularity

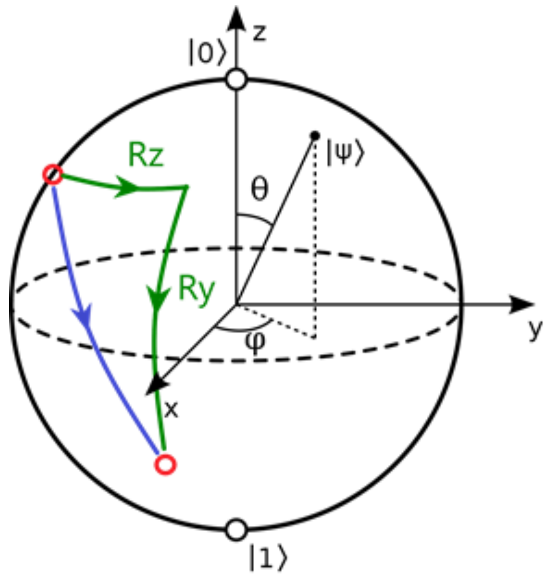
The Qubit

1-Qubit $\quad |\psi\rangle \in \textit{\textbf{Span}}_{\mathbb{C}}\big(\{\,|0\rangle, |1\rangle\}\big) =: \mathscr{H} \cong \mathbb{C}^2$ ***with*** $|\psi| = 1$

Unitary maps $\quad U \in \mathscr{L}(\mathscr{H}) \cong \mathbb{C}^{2\times 2}$ ***unitary***

$$U^\dagger := \overline{U}^\top = U$$

Quantum gates



Bloch sphere

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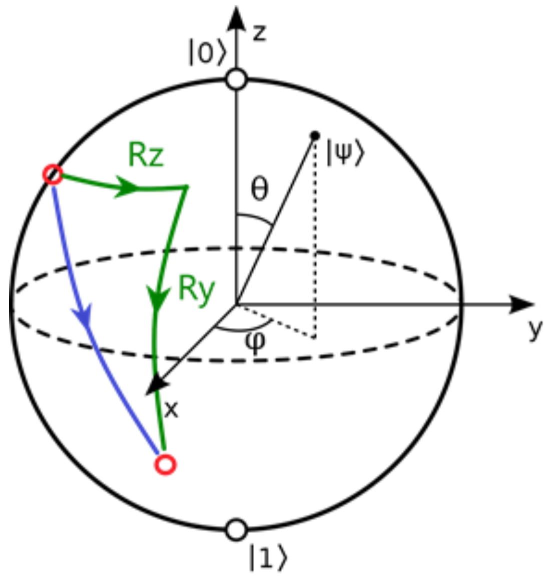
$$U = e^{iH} \qquad H \textbf{ Hermitian}$$



Bloch sphere

Pauli matrices $\sigma_{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_{\mathbf{y}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_{\mathbf{z}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Pauli matrices are *quantum gates*

$$\sigma_{\mathbf{x}}|0\rangle = |1\rangle$$
$$\sigma_{\mathbf{x}}|1\rangle = |0\rangle$$

NOT gate

Pauli rotations

$$R_{\mathbf{j}}(\theta) = \textbf{exp}\left(-i\frac{\theta}{2}\sigma_{\mathbf{j}}\right), \quad \mathbf{j} \in \{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$$

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\langle 0| + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\langle 1|$$

Superposition

The Qubit

2-Qubit $|\psi\rangle \in \mathscr{H}^{\otimes 2} \cong \mathbb{C}^{2^2}$ ***with*** $|\psi| = 1$

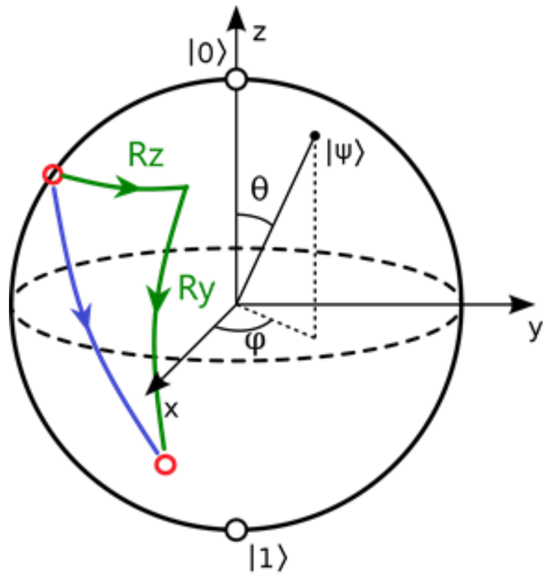
Unitary maps $U \in \mathscr{L}(\mathscr{H}^{\otimes 2})$ ***unitary*** Quantum gates

$U = e^{iH}$ H ***Hermitian***

Product states $|a\rangle \otimes |b\rangle$

$$|\psi\rangle = \frac{1}{2}\big(|00\rangle - |01\rangle + |10\rangle - |11\rangle\big)$$

$$= \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \otimes \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big)$$



Bloch sphere

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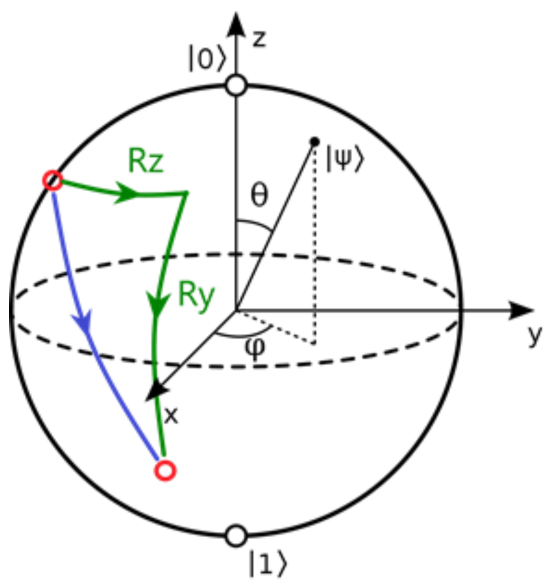

Bloch sphere

Product states $|a\rangle \otimes |b\rangle$

Entangled states $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$

Bell state ***50% probability for*** $|00\rangle$ ***and*** $|11\rangle$
0% for $|01\rangle$ ***and*** $|10\rangle$

Spooky action at a distance

Albert Einstein (1879-1955, German-born theoretical physicist)

The Qubit

2-Qubit $|\psi\rangle \in \mathscr{H}^{\otimes 2} \cong \mathbb{C}^{2^2}$ ***with*** $|\psi| = 1$

Unitary maps $U \in \mathscr{L}(\mathscr{H}^{\otimes 2})$ ***unitary***

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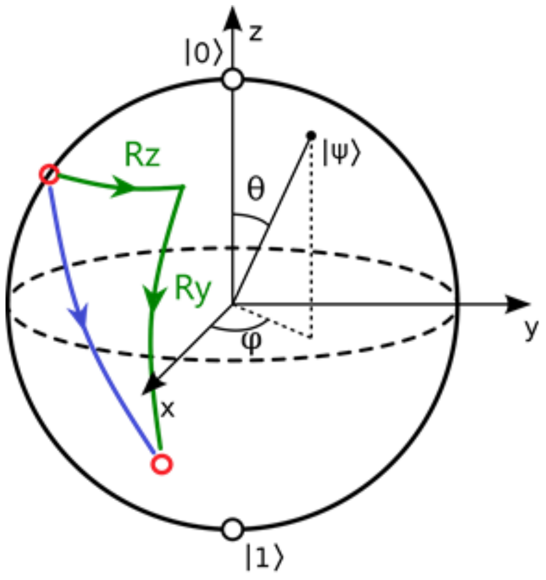


Bloch sphere

Product states $|a\rangle \otimes |b\rangle$

Entangled states $$|\psi\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big)$$

CNOT gate $$U_{CNOT} = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$$

$$|0\rangle|0\rangle \overset{U_H \otimes I}{\longmapsto} \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle \overset{U_{CNOT}}{\longmapsto} \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big)$$

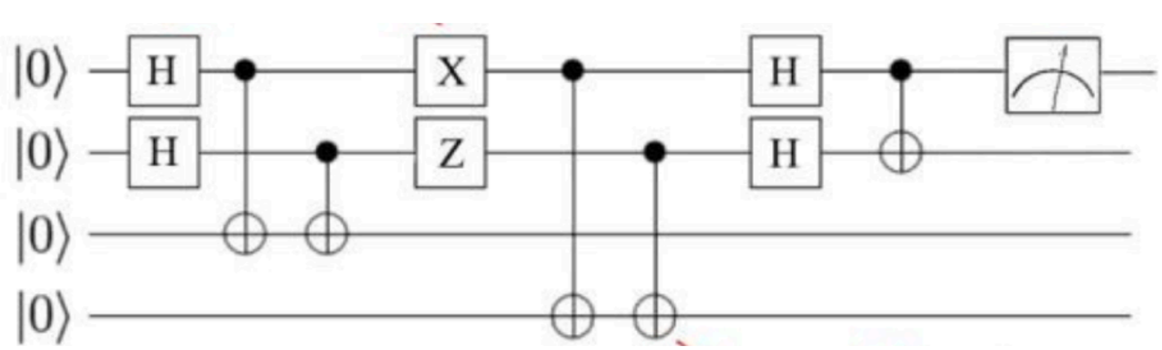
The Quantum Circuit

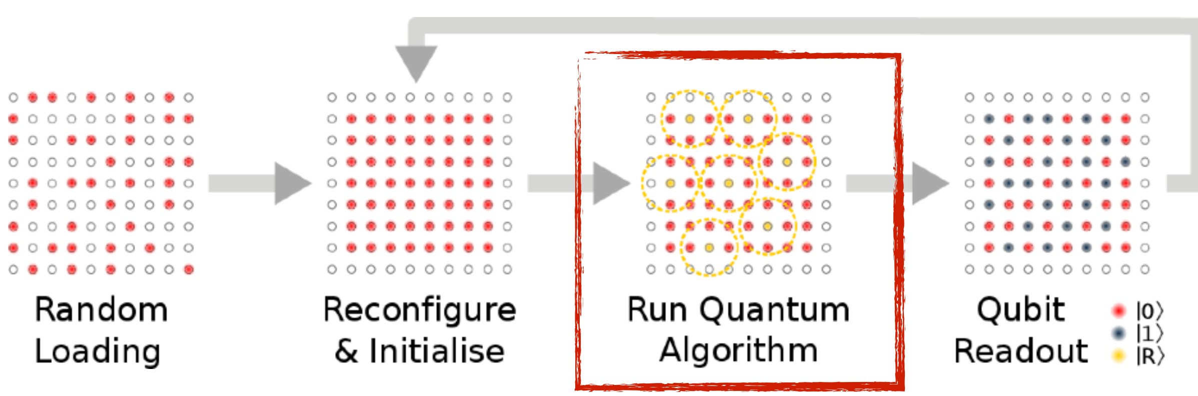
New Scientist @newscientist — *15 July 2023*

In an important benchmark for quantum computing, a record-breaking 51 qubits have been entangled together.

n*-Qubit** $|\psi\rangle \in \mathscr{H}^{\otimes n} \cong \mathbb{C}^{2^n}$ ***with $|\psi| = 1$

Unitary maps $U \in \mathscr{L}(\mathscr{H}^{\otimes n})$ ***unitary***





Operator	Gate(s)	Matrix
Pauli-X (X)	X, ⊕	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
Pauli-Y (Y)	Y	$\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$
Pauli-Z (Z)	Z	$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$
Hadamard (H)	H	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$
Phase (S, P)	S	$\begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$
$\pi/8$ (T)	T	$\begin{bmatrix}1 & 0\\0 & e^{i\pi/4}\end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$
Controlled Z (CZ)	Z	$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}$
SWAP		$\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}$

Realizing Qubits

Any two-level quantum-mechanical system can be used as a qubit

Superconducting (Google, IBM, Intel, China)
Neutral atoms (Atom computing, Coldquanta, QuEra)
Trapped ions (Ionq, Alpine QT)

Optical (China, Xanadu)
Quantum dots (QuTech)
Topological (Microsoft, QuTech)

Issues: *Gate fidelity ~ 90-99%*

Faults from physical implementation (decoherence)

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Noisy Intermediate-Scale Quantum (NISQ) Computers

- *Variational Quantum Algorithms, approximate the lowest energy level of a Hamiltonian.*
- *Quantum Approximate Optimization Algorithm, for CO problems.*
- *Quantum Neural Networks, as quantum analogues of classical neural nets.*
- *The Variational Quantum Linear Solver, for solving linear systems of equations.*
- *Quantum simulator, for simulating low-temperature, many-body physics*

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The Agenda

I. *Pulse-based Quantum Circuits and VQOC*

II. *Learning Quantum Channels*

III. *Understanding Noisy Qubits*

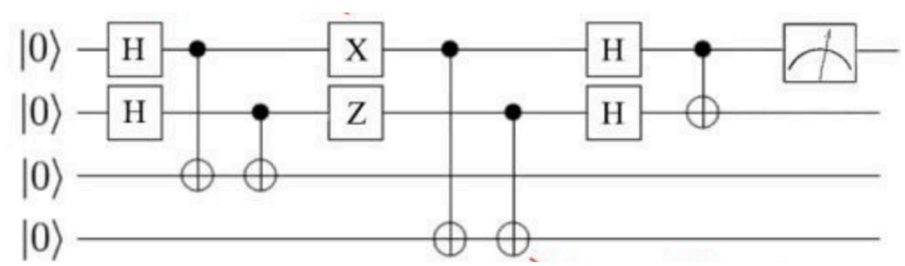
The Agenda

I. Pulse-based Quantum Circuits and VQOC

II. Learning Quantum Channels

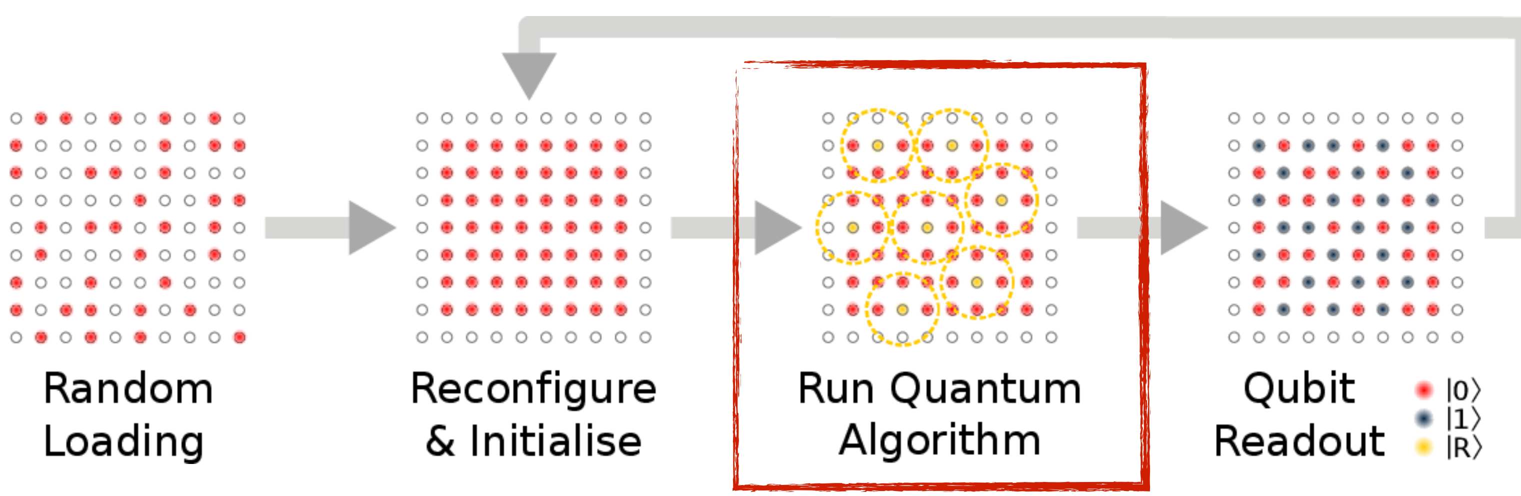
III. Understanding Noisy Qubits

Variational Quantum Optimal Control



Given a molecular Hamiltonian H_{mol}, find $|\psi_g\rangle$:

$$|\psi_g\rangle = \textbf{\textit{argmin}}\ \langle\psi|H_{mol}|\psi\rangle \quad \textbf{\textit{s.t.}} \quad |\psi| = 1$$



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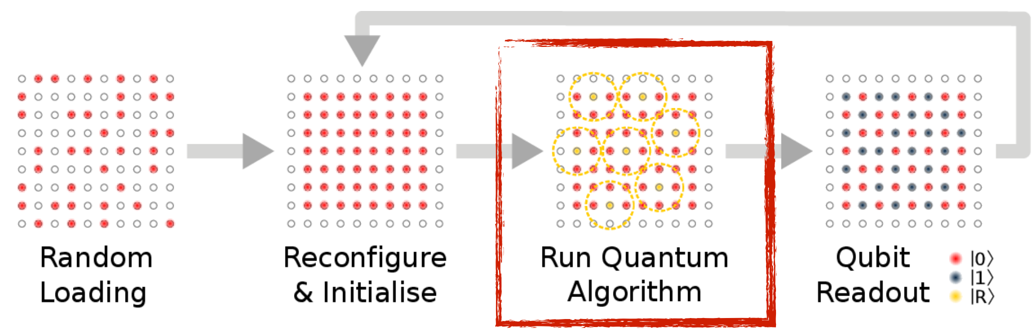
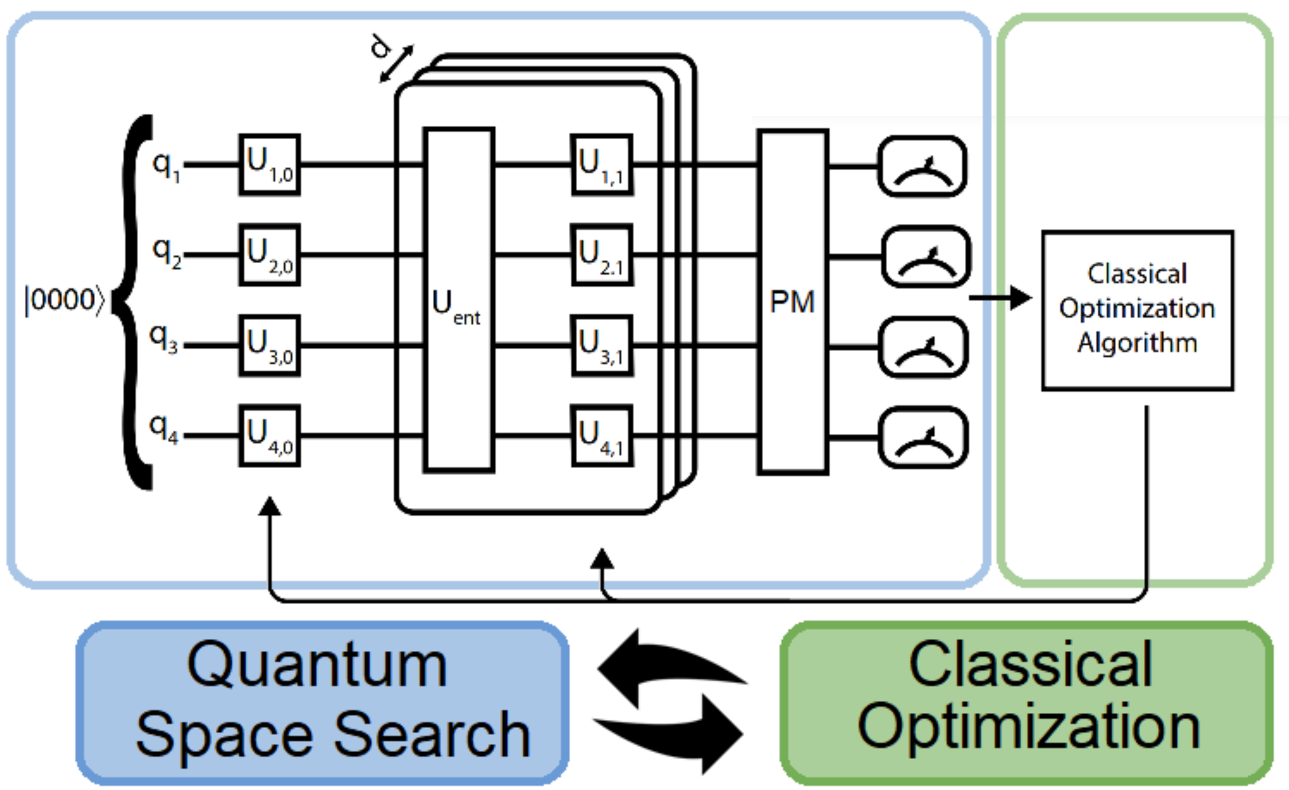
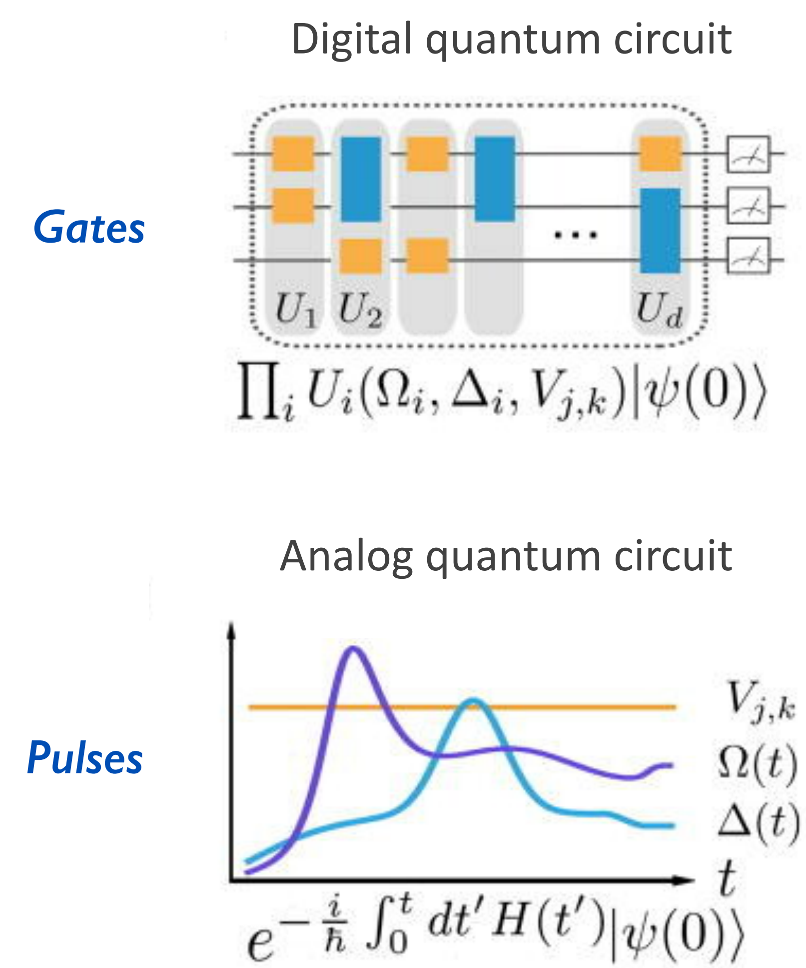






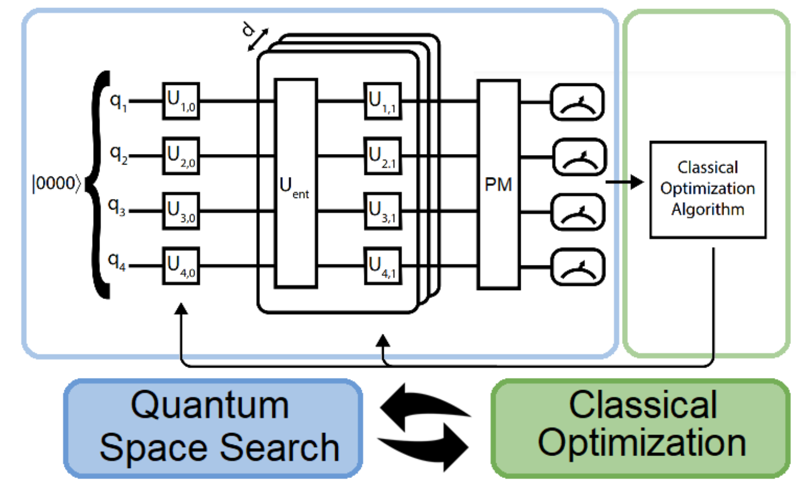
Variational Quantum Optimal Control

Given a molecular Hamiltonian H_{mol}, find $|\psi_g\rangle$:

$$|\psi_g\rangle = \textbf{\textit{argmin}} \ \langle \psi_T | H_{mol} | \psi_T \rangle \quad \textbf{\textit{s.t.}}$$

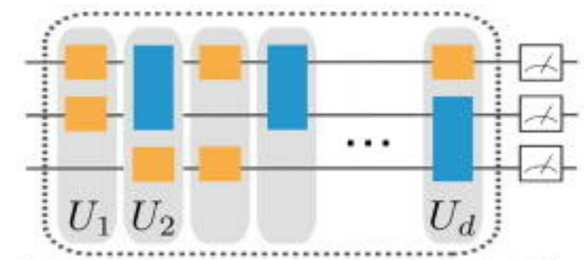
Schrödinger equation: $i\hbar\partial_t |\psi_t\rangle = \underbrace{H_t}_{} |\psi_t\rangle$

Control Hamiltonian



Digital quantum circuit

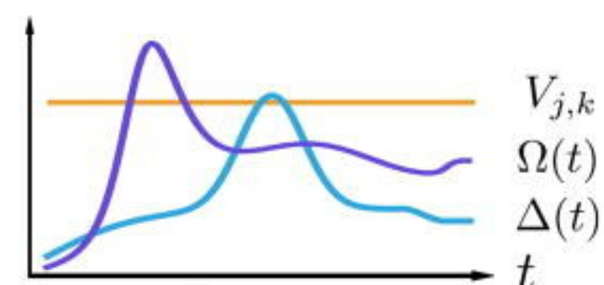
Gates



$\prod_i U_i(\Omega_i, \Delta_i, V_{j,k})|\psi(0)\rangle$

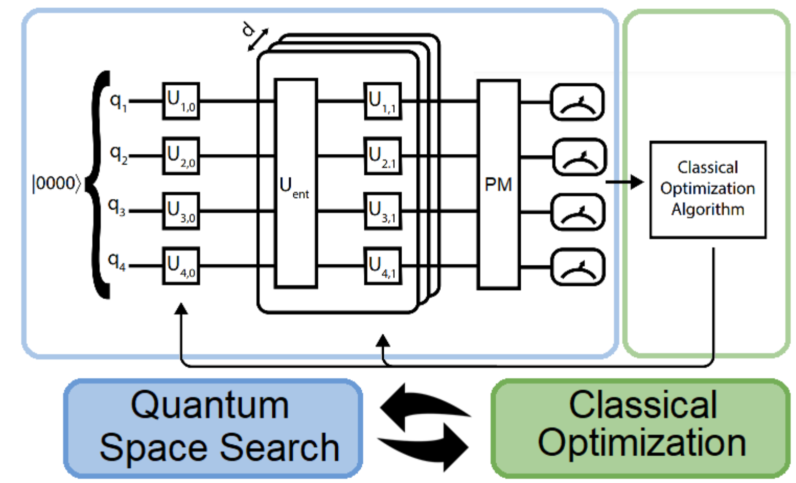
Analog quantum circuit

Pulses



$e^{-\frac{i}{\hbar}\int_0^t dt' H(t')}|\psi(0)\rangle$

Variational Quantum Optimal Control



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$$|\psi_g\rangle = \textbf{argmin } \langle\psi_T|H_{mol}|\psi_T\rangle \quad \textbf{s.t.}$$

Schrödinger equation: $\quad i\hbar\partial_t U_t = H_t U_t$

$U_t \triangleq$ ***Unitary propagator*** $\in \mathscr{L}(\mathscr{H})$

$$|\psi_t\rangle = U_t|\psi_0\rangle$$

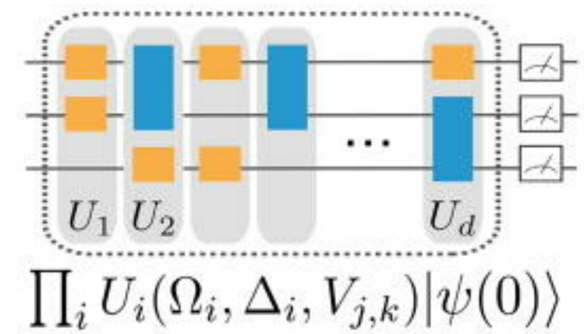
Idea: ***Discretizations of U_t gives rise to quantum gates***

Challenge:

Find a unitary propagator that brings $|\psi_0\rangle$ to $|\psi_g\rangle$ fastest, and with least effort, on a quantum computer

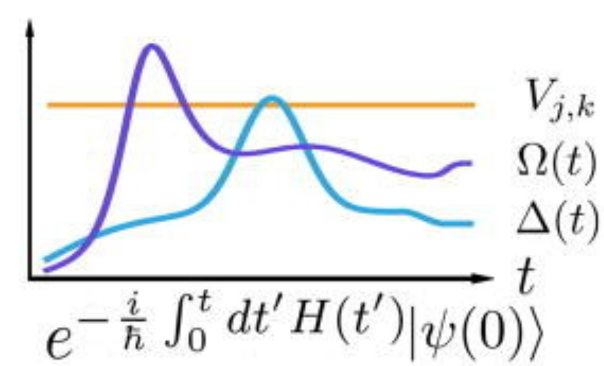
Digital quantum circuit

Gates

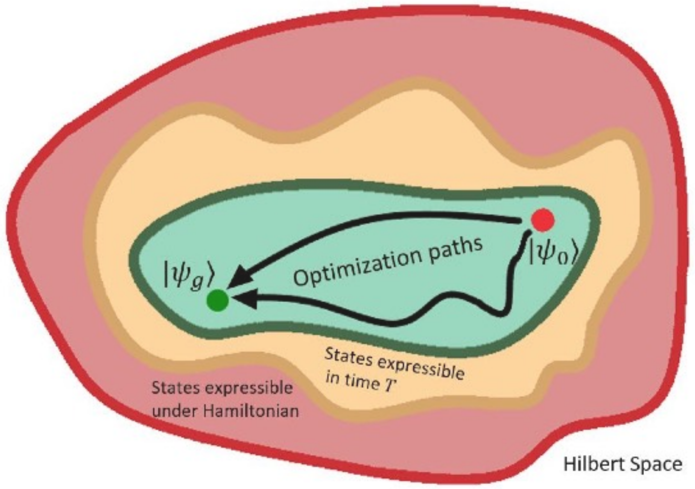


Analog quantum circuit

Pulses



Variational Quantum Optimal Control



Schrödinger equation: $i\hbar\partial_t U_t = H[z_t]U_t$

Physically realizable Hamiltonians

Single qubit Hamiltonian

$$H^{coup} = \sum_{k=1}^{n} H_k^{coup}$$

$$H_k^{coup} = z_k^{coup}\,|0\rangle_k\langle 1|_k + \bar{z}_k^{coup}\,|1\rangle_k\langle 0|_k$$

Entanglement Hamiltonian

$$H^{ent} = \sum_{k=1}^{n}\sum_{l\neq k} H_{kl}^{ent}$$

$$H_{kl}^{ent} = \mathbf{Re}[z^{ent}]\,|11\rangle_{kl}\langle 11|_{kl}$$

Pulse-based Variational Quantum Eigensolver

T ?
Quantum evaluations ?
Topology ?

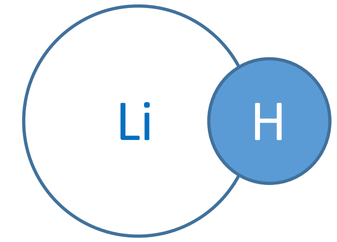
Given a molecular Hamiltonian H_{mol}, find z_g :

$$z_g = \textit{argmin}\ \langle\psi_0|\,U_T^\dagger\,H_{mol}\,U_T|\psi_0\rangle + \lambda\mathscr{R}(z) \quad \textit{s.t.} \quad \textit{Schrödinger equation}$$

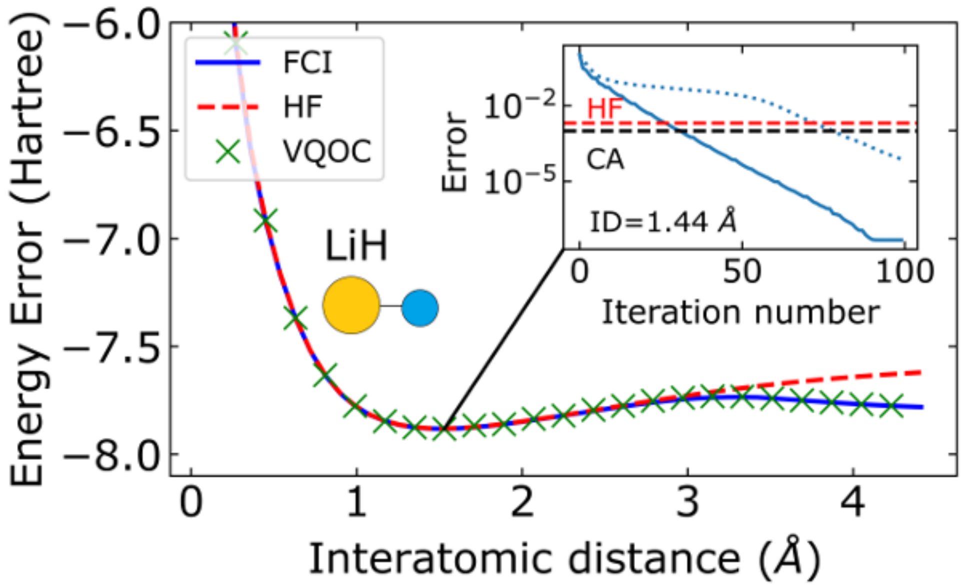
Adjoint-based method

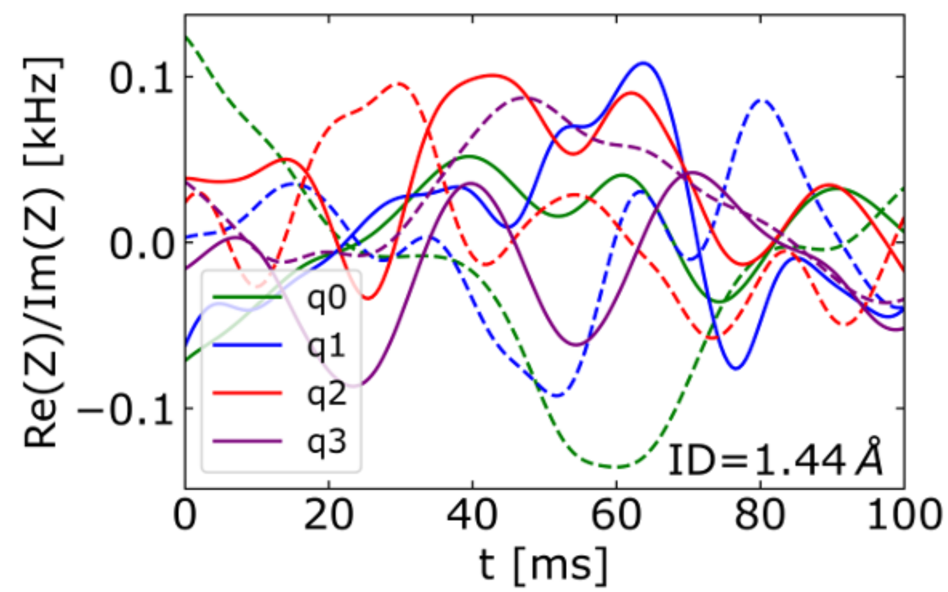
Pulse-based variational quantum optimal control for hybrid quantum computing. Quantum, 2023
Recapture probability for anti-trapped Rydberg states in optical tweezers. APS Physical Review A, 2023

Simulation Results

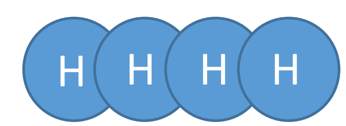


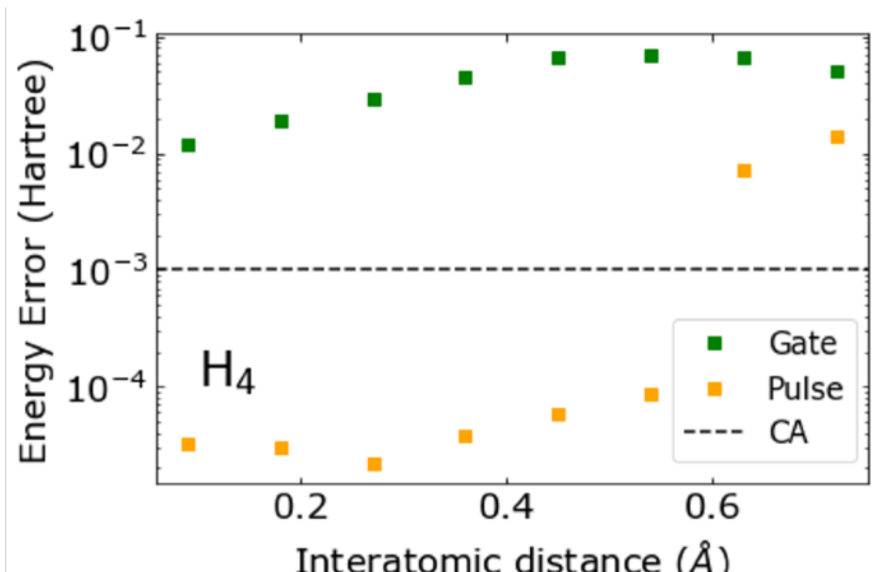
LiH ≙ Lithium hydride *~ 4 qubits, minimally entangled*





H_4 ≙ Hydrogen-4 *~ 6 qubits, highly entangled*





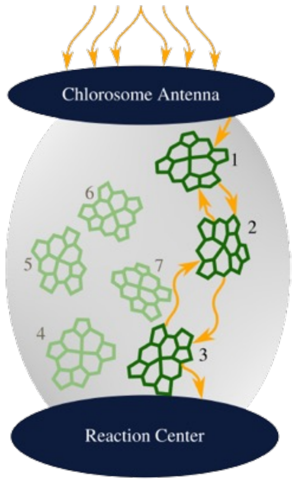
The Agenda

I. Pulse-based Quantum Circuits and VQOC

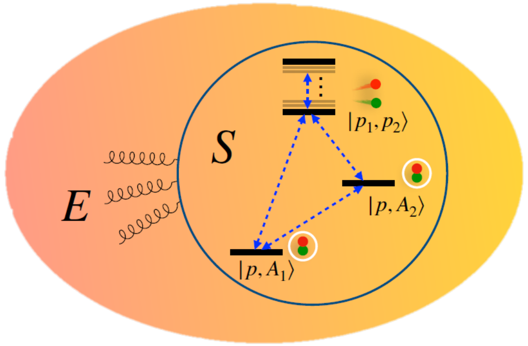
II. Learning Open Quantum Systems

III. Understanding Noisy Qubits

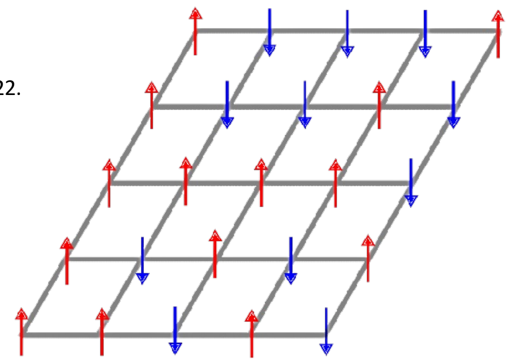
Open Quantum Processes



Z. Hu et al. *A general quantum algorithm for open quantum dynamics demonstrated with the Fenna-Matthews-Olson complex.* Quantum 2022.



De Jong et al. *Quantum Simulation of open quantum systems in heavy-ion collisions.* Phys. Rev. D 2020.



S. Wald. *Thermalisation and Relaxation of Quantum Systems.* 2017

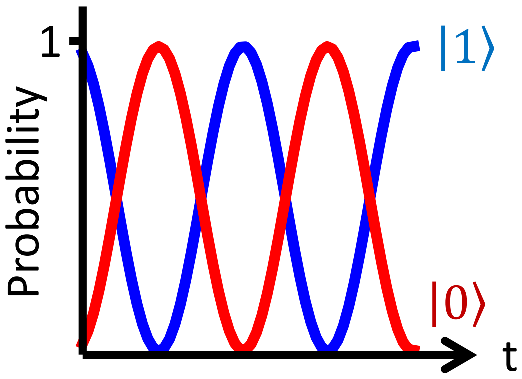
Schrödinger equation:

$$i\hbar\partial_t U_t = H_t U_t$$

Quantum Liouville equation:

$$i\hbar\partial_t \rho_t = [H, \rho_t] := H\rho_t - \rho_t H$$

$$\rho_t = U_t \rho_0 U_t^\dagger$$

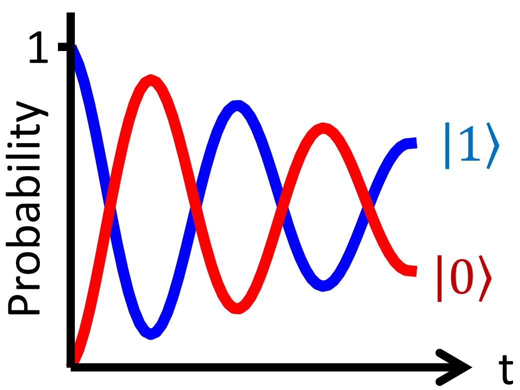


Lindblad equation:

$$i\hbar\partial_t \rho_t = [H, \rho_t] + L\rho_t$$

$$L\rho = \sum \gamma_n \left[A_n \rho A_n^\dagger - \frac{1}{2}\{A_n^\dagger A_n, \rho\}\right]$$

$$\rho_t = \Phi_t(\rho_0)$$ *Quantum channel*



Challenge: ***Learn and construct quantum circuit to simulate quantum channels***

Dilation of Quantum Channels

$i\hbar\partial_t\rho_t = [H, \rho_t] + L\rho_t$

Stinespring Dilation Theorem

For any quantum channel $\Phi : S(\mathscr{H}) \to S(\mathscr{H})$, there exists a Hilbert space $\mathscr{K}$ and a unitary map $U : \mathscr{H} \otimes \mathscr{K} \to \mathscr{H} \otimes \mathscr{K}$ such that

$$\Phi(\rho) = \mathbf{Tr}_{\mathscr{K}}\left[U\rho \otimes |0\rangle\langle 0| U^\dagger\right], \qquad \rho \in S(\mathscr{H})$$

In particular, $\mathscr{K}$ can be chosen such that $\dim \mathscr{K} \leq (\dim \mathscr{H})^2$ $\leq n+1$ *qubits*

General idea:

- *Given a time step τ, learn Φ_τ by learning the unitary map U*
- *Predict ρ at time $n\tau$, $n \geq 1$, by repeated evaluation of Φ_τ, i.e.*

$$\rho_{n\tau} = \Phi_\tau(\rho_{(n-1)\tau}) = \Phi_\tau \circ \Phi_t \circ \cdots \circ \Phi_\tau(\rho_0)$$

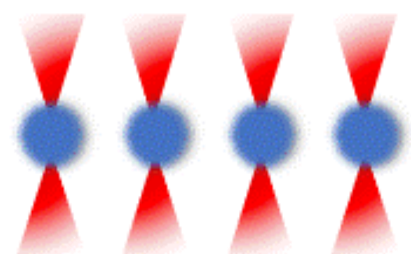
Semigroup property

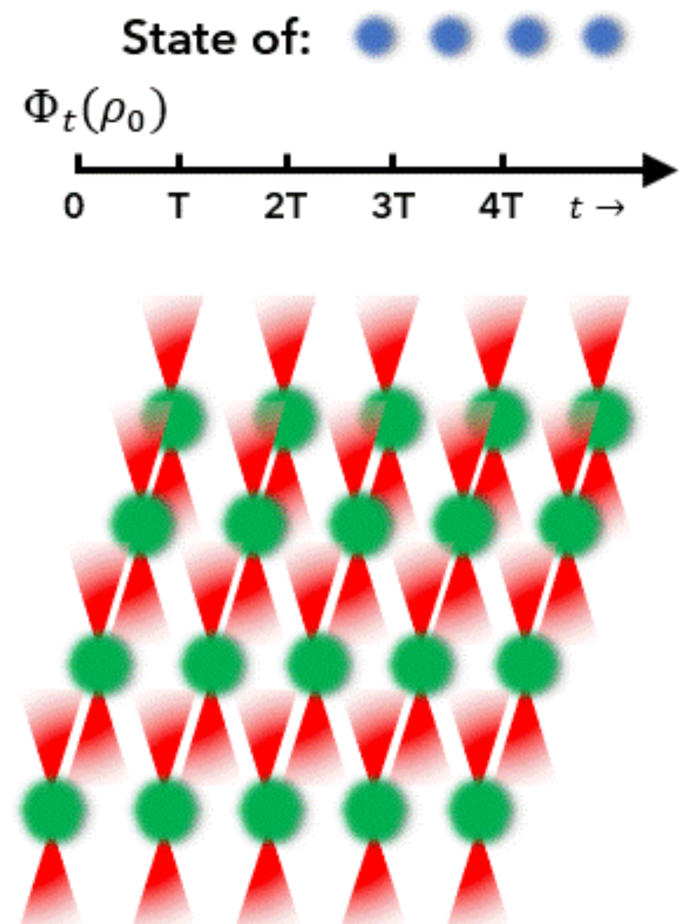
Issue: *Requires repeated initialization of ancilla state on $\mathscr{K}$* *Tweezer magic!*

Tweezer Magic

$$i\hbar\partial_t\rho_t = [H, \rho_t] + L\rho_t$$

1. Insert new ancillas
2. Execute learned $U(\theta)$
3. Deposit ancillas





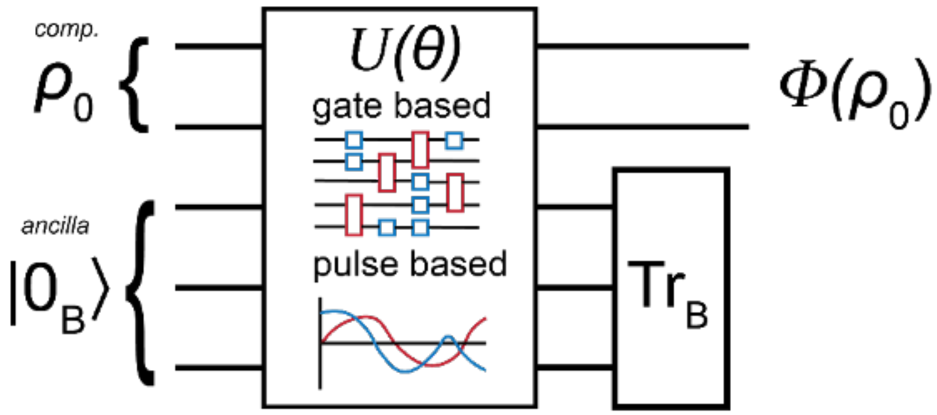
Movable qubits

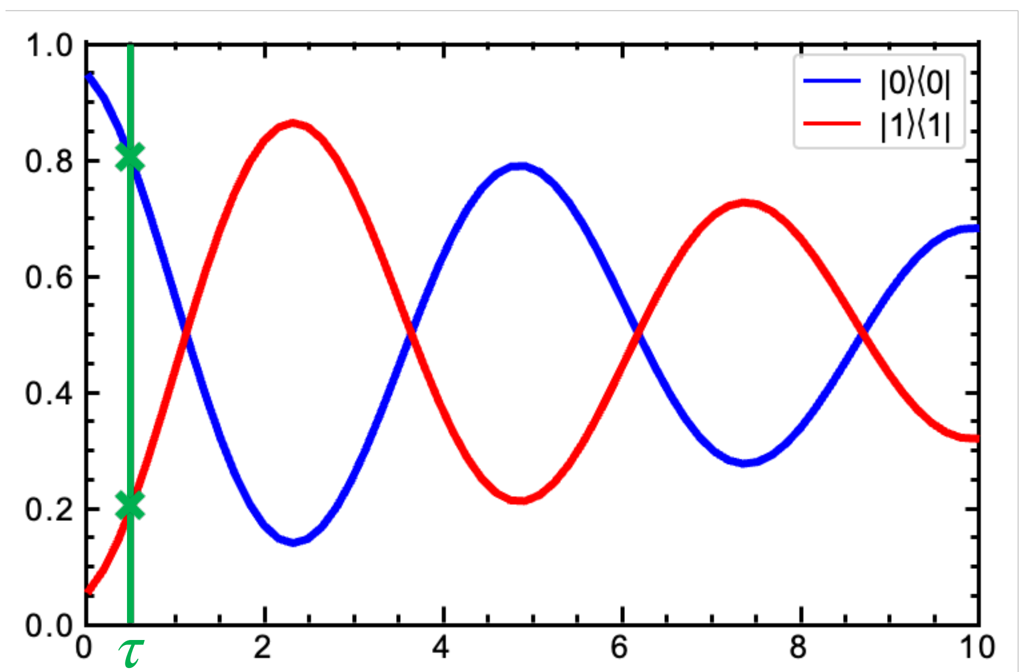
Stable reservoir

Good scalability

Learning Phase

$$i\hbar\partial_t\rho_t = [H, \rho_t] + L\rho_t$$





$$\textbf{Loss}(\theta) = \sum \textbf{Tr}\left[O_\ell\left(\rho_\ell(\theta) - \bar{\rho}_\ell\right)\right]$$

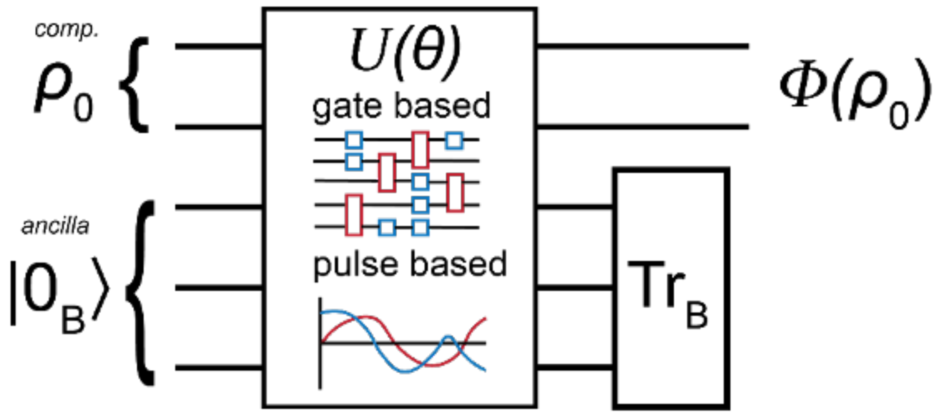
Decaying Rabi oscillations

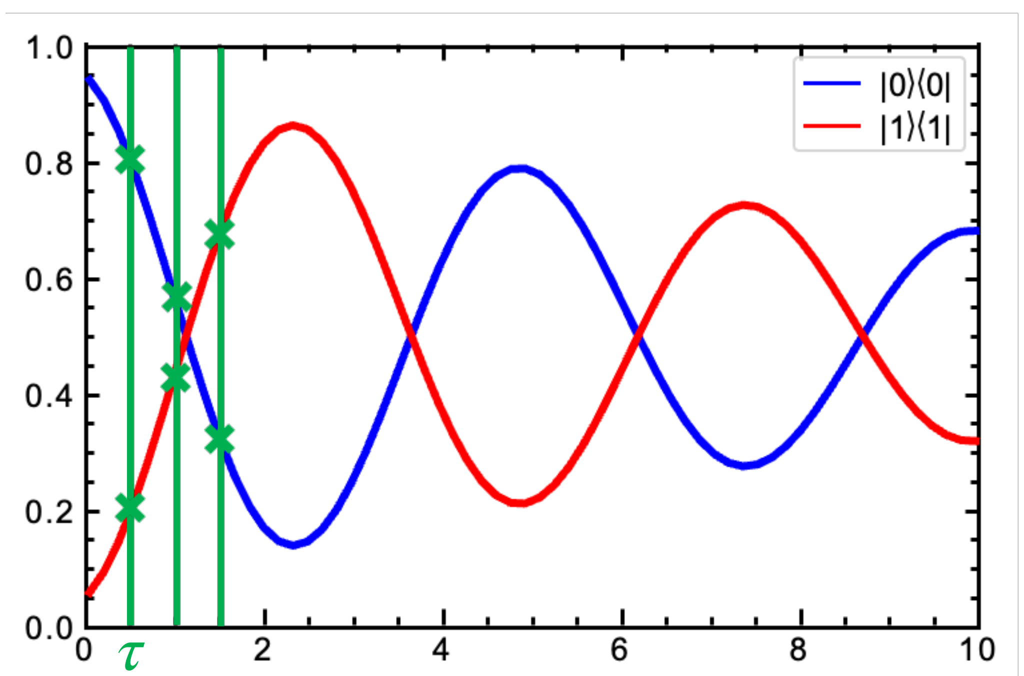
$$H = \omega \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \omega\,\sigma_{\boldsymbol{x}}$$

$$L = \gamma A, \quad A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Learning Phase

$$i\hbar\partial_t\rho_t = [H, \rho_t] + L\rho_t$$





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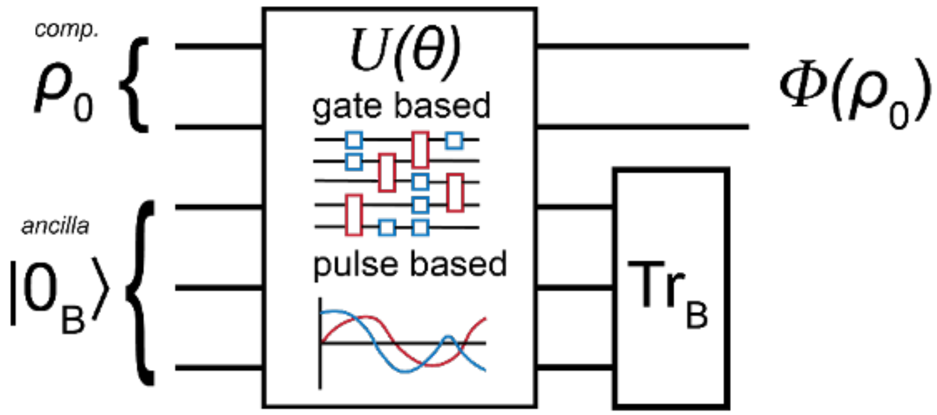

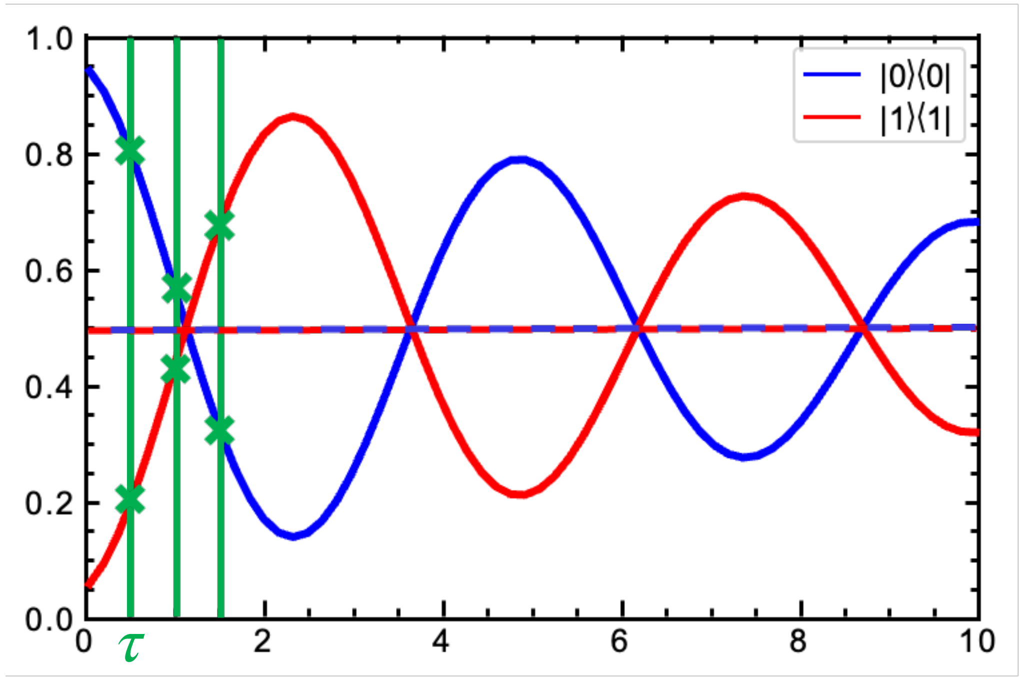

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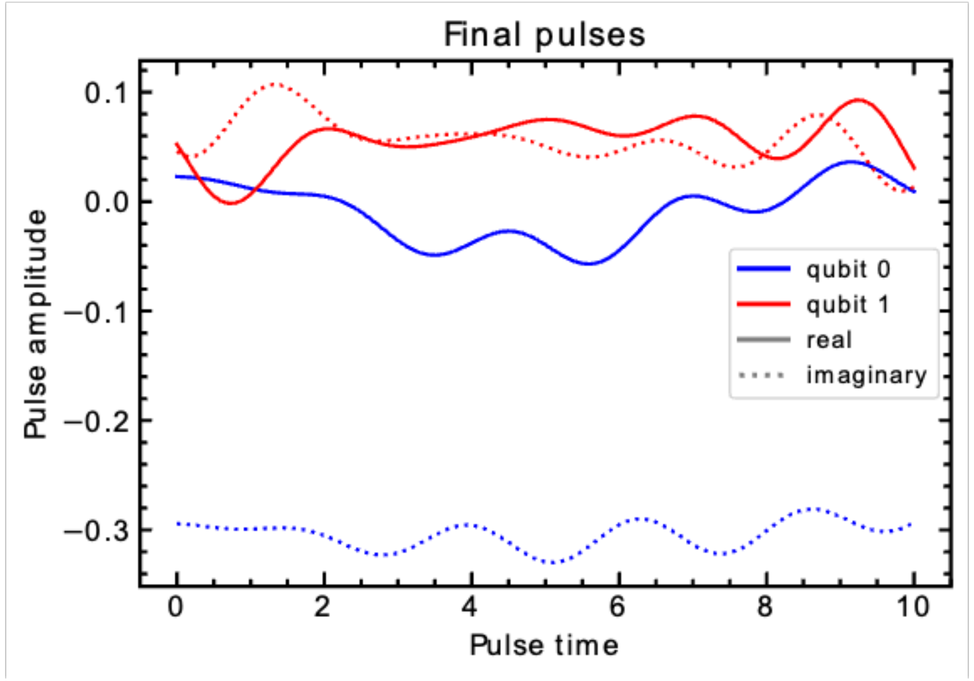
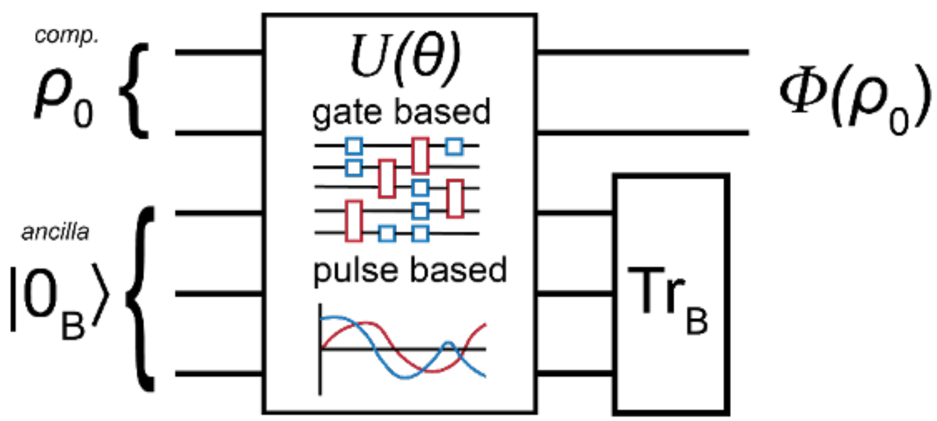
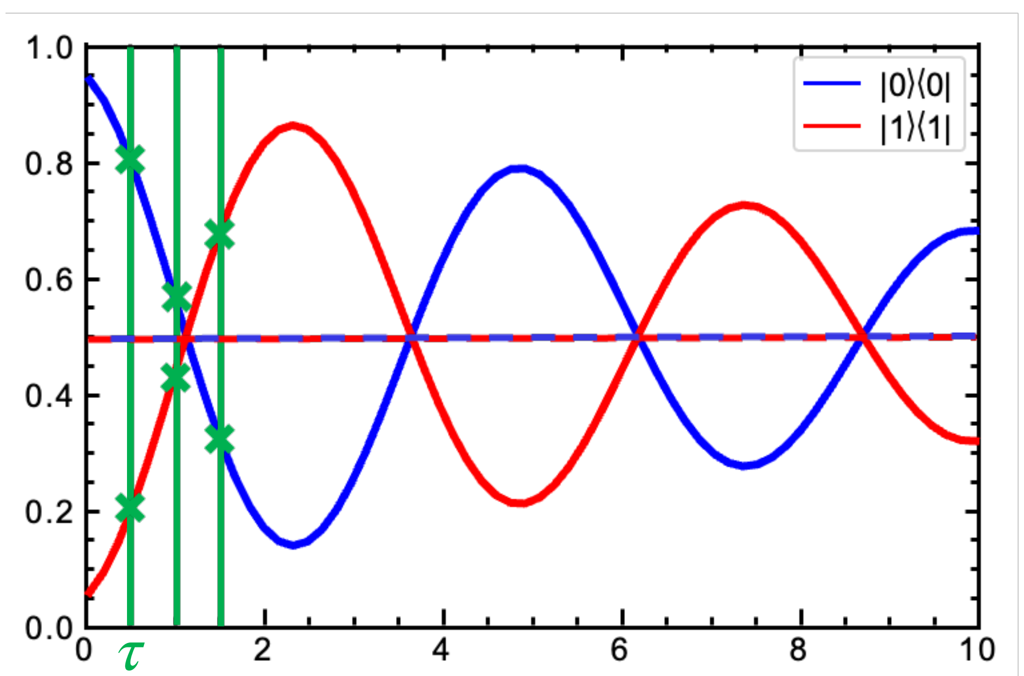


Prediction Phase

$$i\hbar\partial_t\rho_t = [H, \rho_t] + L\rho_t$$





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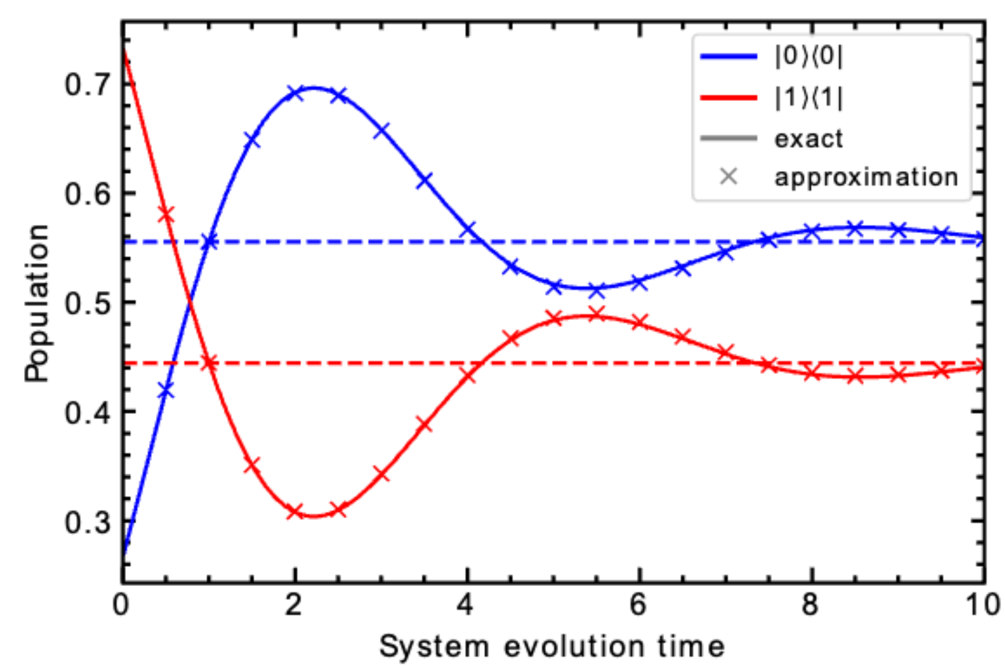
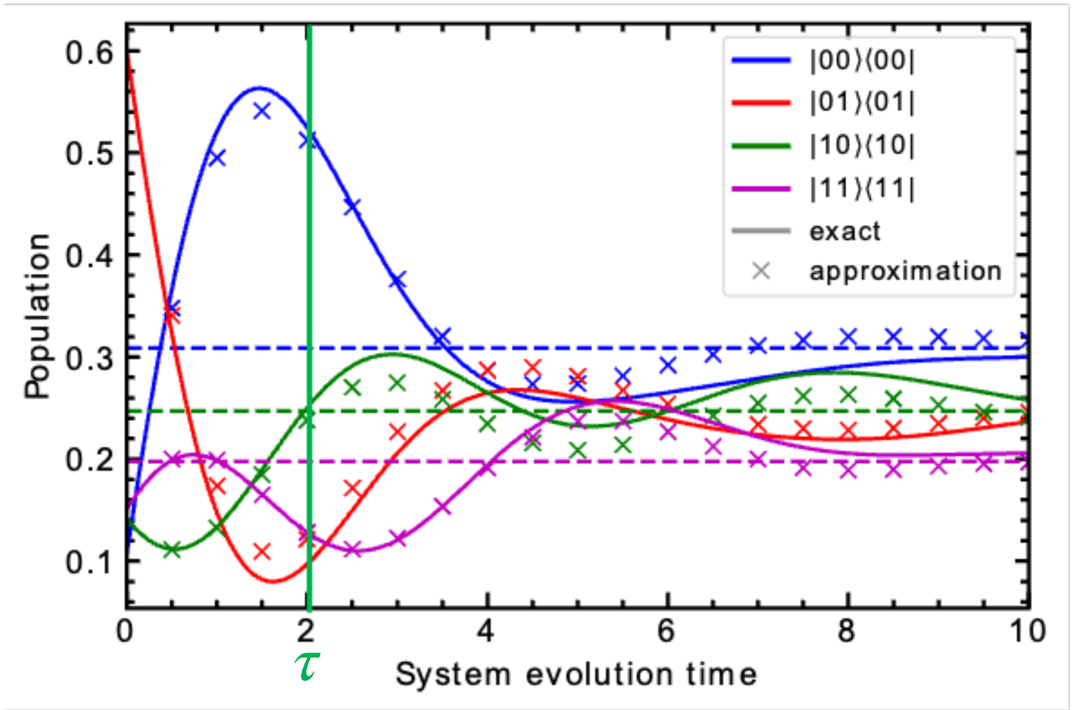


2-Qubit Lindbladians

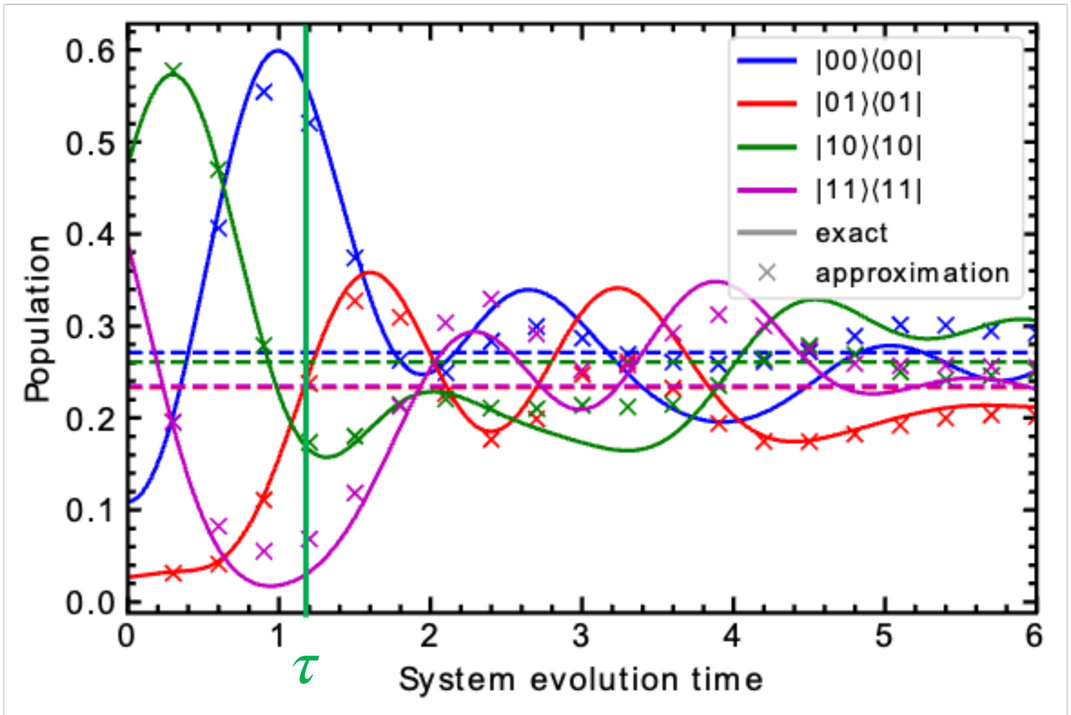
$$i\hbar\partial_t \rho_t = [H, \rho_t] + L\rho_t$$

Decaying Rabi oscillations



$$H = H_{vdW} + \omega\left[\sigma_{\mathbf{x}}^1 + \sigma_{\mathbf{x}}^2\right]$$

Transverse-field Ising model



$$H = J\,\sigma_{\mathbf{z}}^1\,\sigma_{\mathbf{z}}^2 + \omega\left[\sigma_{\mathbf{x}}^1 + \sigma_{\mathbf{x}}^2\right]$$

$$L = \gamma_1 A^1 + \gamma_2 A^2$$

Gate VS Pulse?

Learning quantum channels on a quantum computer. ArXiv

The Agenda

I. Pulse-based Quantum Circuits and VQOC

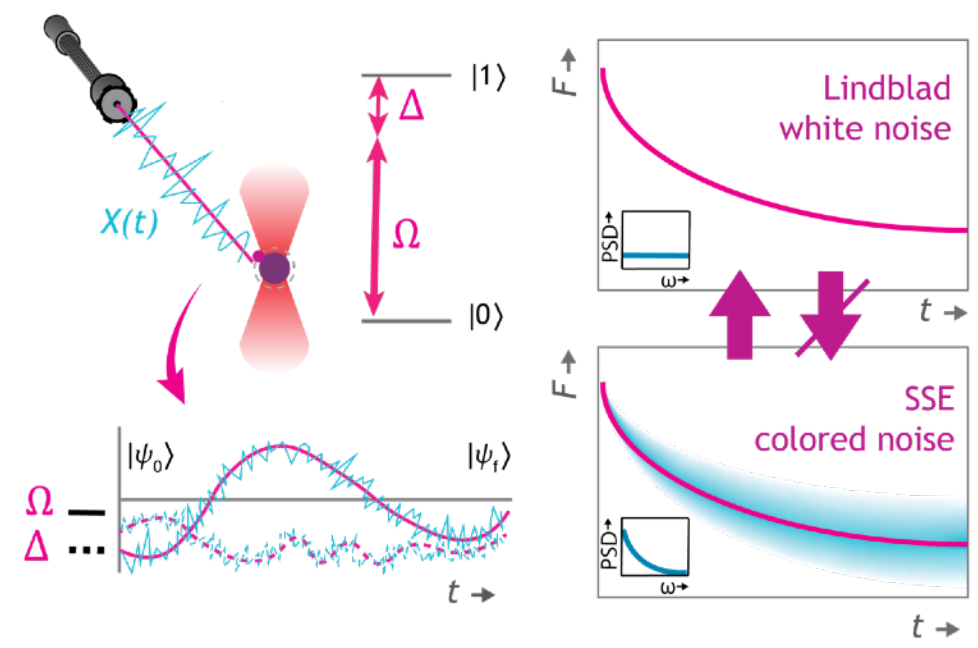
II. Learning Open Quantum Systems

III. Understanding Noisy Qubits

A Model for Noisy Qubits

Stochastic Schrödinger equation

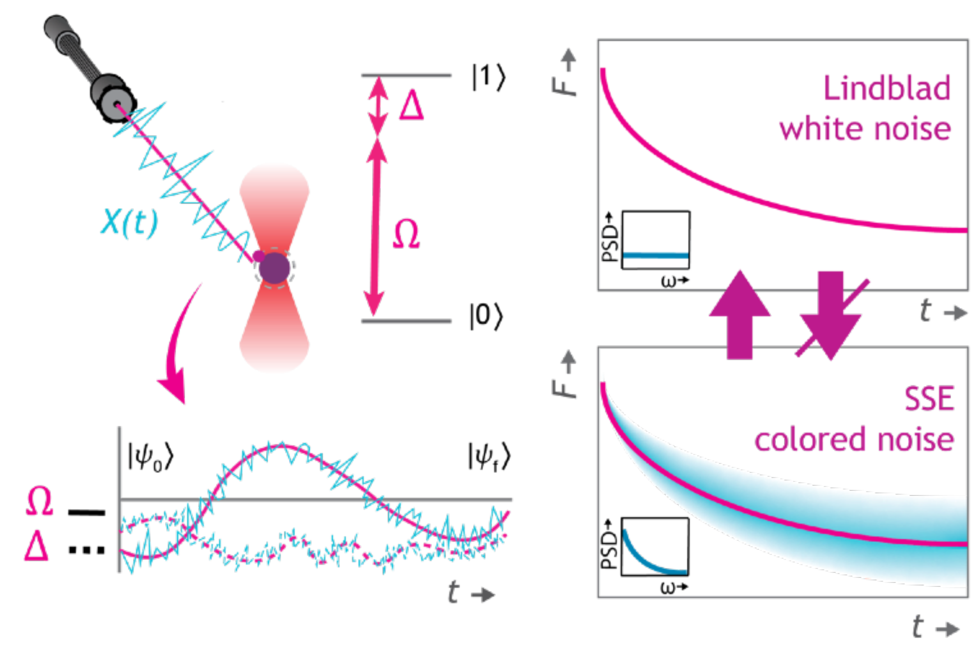
$$i\hbar \, \mathrm{d}\psi_t = H[\xi_t]\psi_t \, \mathrm{d}t$$



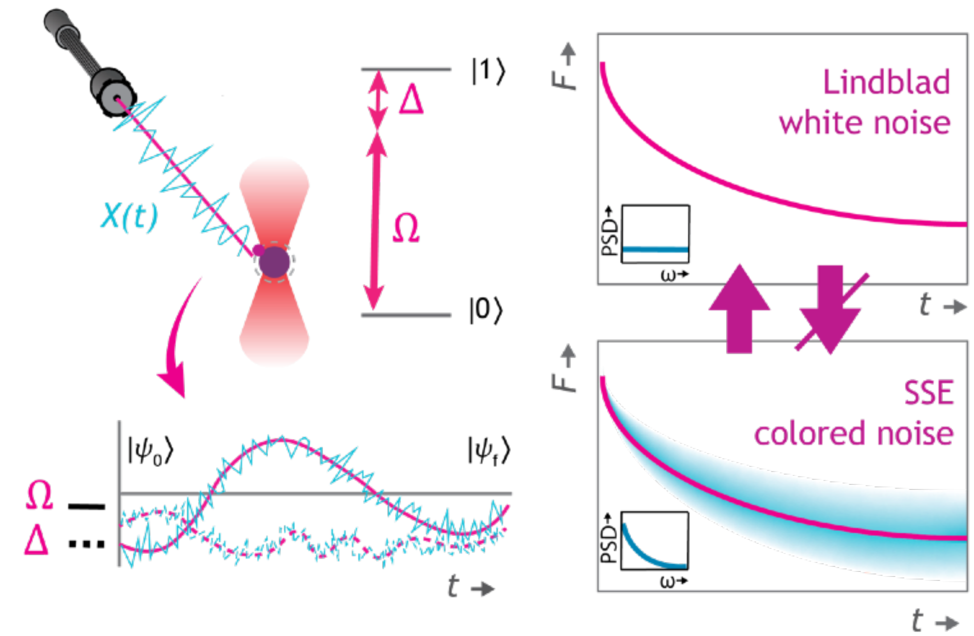
A Model for Noisy Qubits

Stochastic Schrödinger equation

$$i\hbar\, \mathbf{d}\psi_t = H\psi_t\, \mathbf{d}t - i\gamma^2 S^\dagger S\psi_t\, \mathbf{d}t + \gamma S\psi_t\, \mathbf{d}X_t$$



A Model for Noisy Qubits



Stochastic Schrödinger equation

$$i\hbar \, \mathbf{d}\psi_t = H\psi_t \, \mathbf{d}t - i\gamma^2 S^\dagger S\psi_t \, \mathbf{d}t + \gamma S\psi_t \, \mathbf{d}X_t$$

White noise: $\mathbf{d}X_t = \sqrt{2}\,\mathbf{d}B_t$

Ornstein-Uhlenbeck: $\mathbf{d}X_t = -kX_t\,\mathbf{d}t + \sqrt{2}\mathbf{d}B_t$

Conjecture?

Connections with the Lindblad equation: **Setting** $P_t = |\psi_t\rangle\langle\psi_t|$,

$$i\hbar \, \mathbf{d}P_t = \big([H, P_t] + LP_t\big)\mathbf{d}t + \gamma\,[S, P_t]\,\mathbf{d}X_t$$

White noise: $i\hbar \, \mathbf{d}\,\mathbb{E}[P_t] = \big([H, \mathbb{E}[P_t]] + L\mathbb{E}[P_t]\big)\mathbf{d}t$

Fidelity Estimation

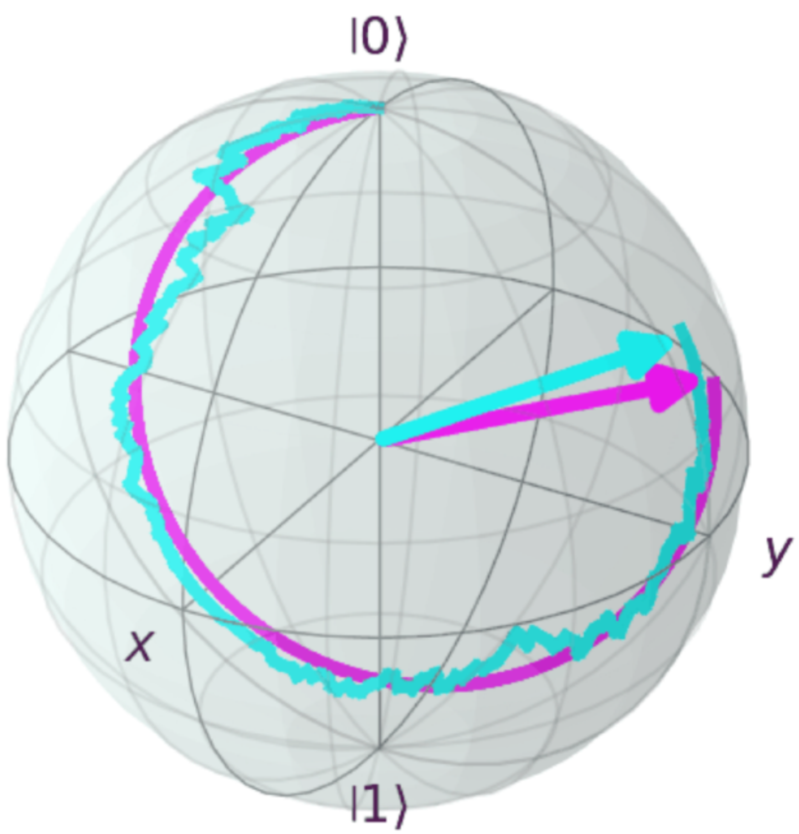
$$i\hbar\, \mathbf{d}\phi_t = H\phi_t\, \mathbf{d}t$$

$$i\hbar\, \mathbf{d}\psi_t = H\psi_t\, \mathbf{d}t - i\gamma^2 S^\dagger S\psi_t\, \mathbf{d}t + \gamma S\psi_t\, \mathbf{d}X_t$$

Goal: *Study statistical properties of the fidelity* $F_t := |\langle\phi_t|\psi_t\rangle|^2$

Laser intensity noise: $S_\sigma = S_\sigma^\dagger,\ S_\sigma^\dagger S_\sigma = I$ *Pauli operators*

Laser detuning noise: $S_p = S_p^\dagger = S_p^\dagger S_p$ *Projection operators*



Simulation Results

$$i\hbar\,\mathbf{d}\phi_t = H\phi_t\,\mathbf{d}t$$

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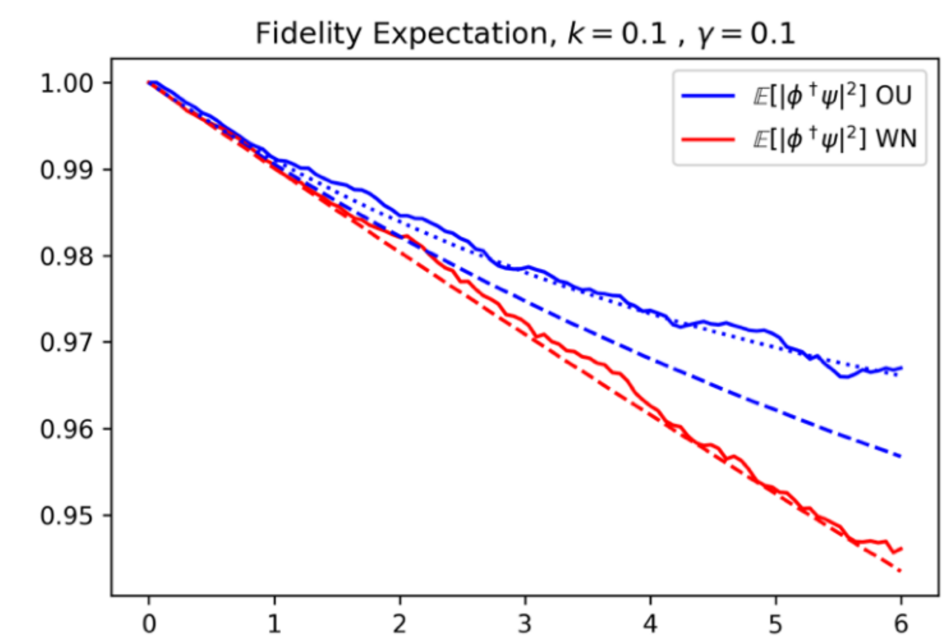

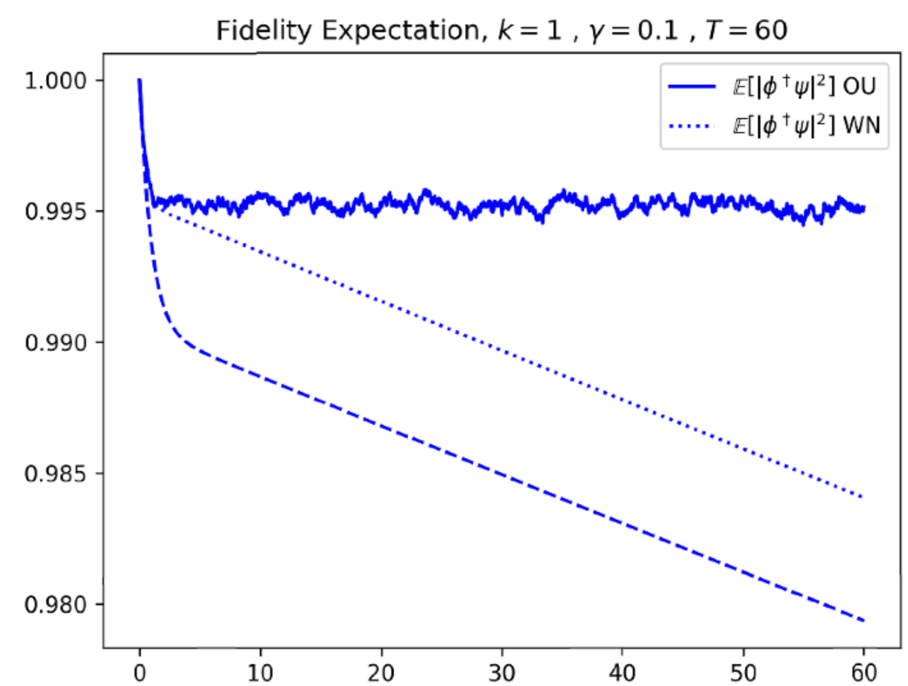

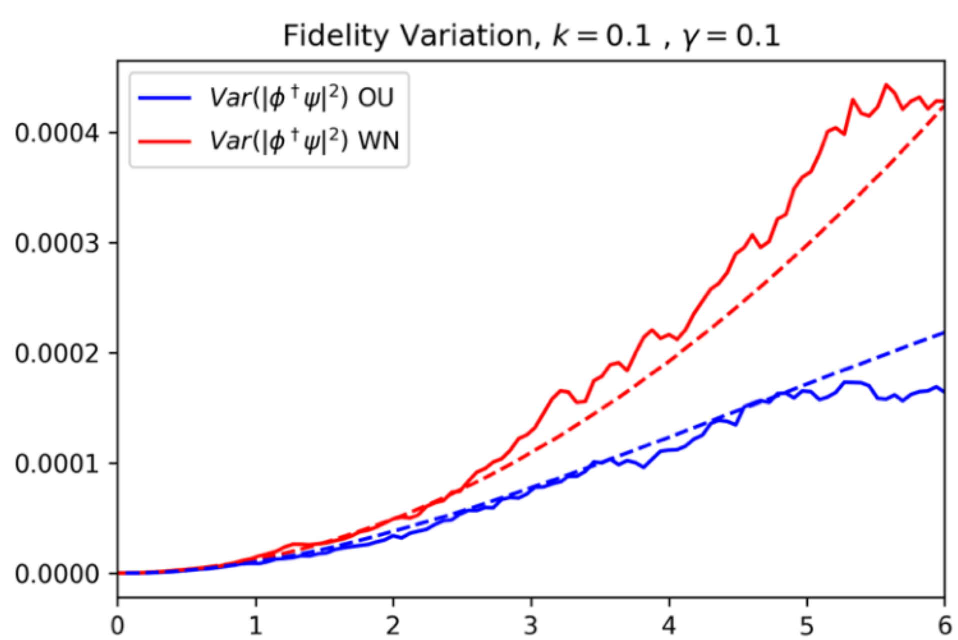

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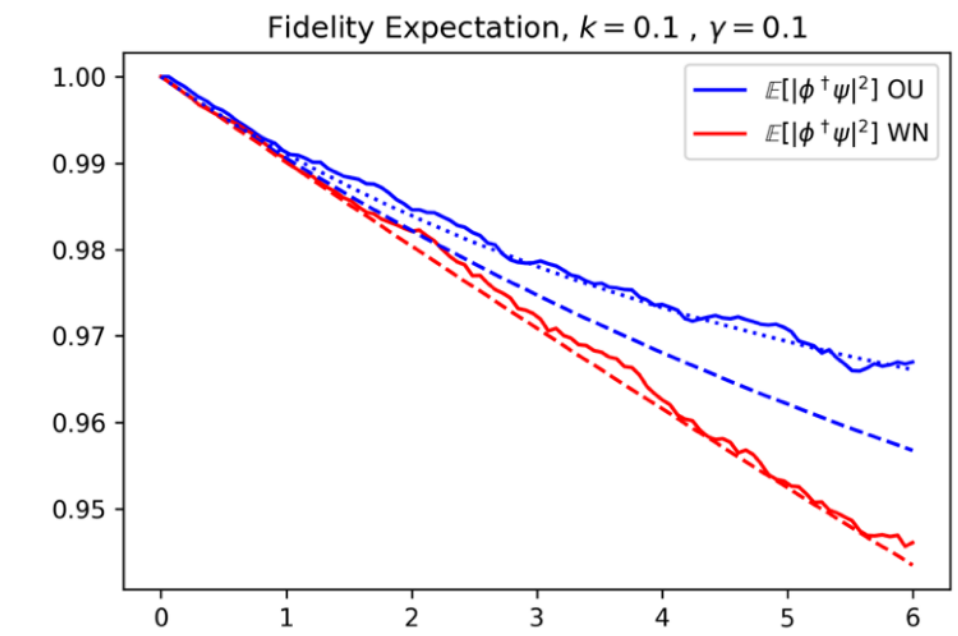

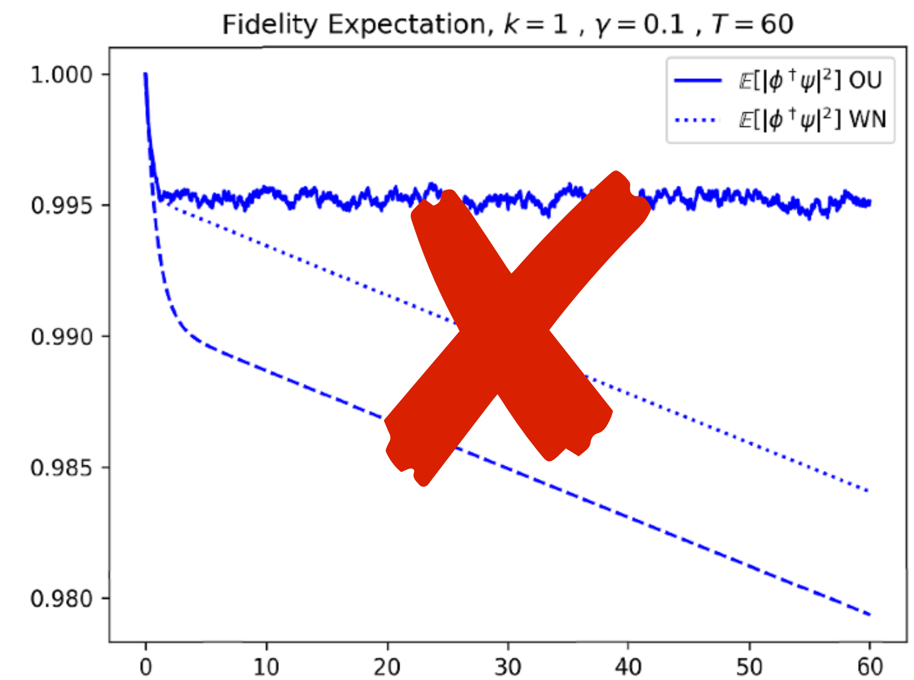

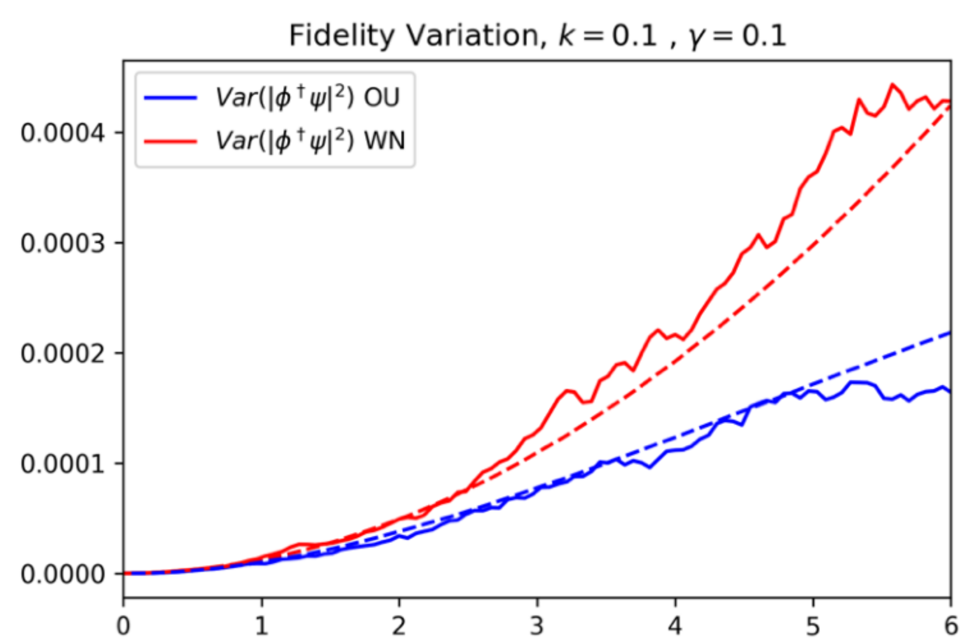

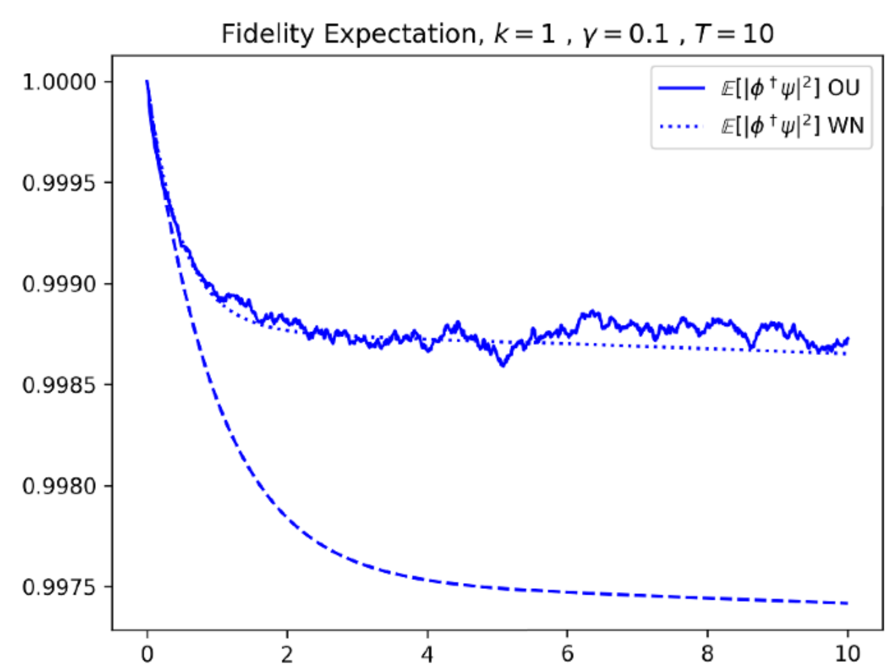

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Results: *Derived exact solutions for Itô processes when* $[H, S] = 0$

$$\mathsf{F}_t^\sigma = \tfrac{1}{2}(1 + \mathsf{F}^{S_\sigma}) + \tfrac{1}{2}(1 - \mathsf{F}^{S_\sigma})\cos\big(2(X_t - X_0)\big), \qquad \mathsf{F}^{S_\sigma} = |\langle\phi_0|S_\sigma|\phi_0\rangle|$$

$$\mathsf{F}_t^p = 1 - 2(1 - \mathsf{F}^{S_p})\,\mathsf{F}^{S_p}\big(1 - \cos(X_t - X_0)\big), \qquad \mathsf{F}^{S_p} = |\langle\phi_0|S_p|\phi_0\rangle|$$

Derived hierarchical approximations when $[H, S] \neq 0$

Fidelity Estimation

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Goal:

Laser i

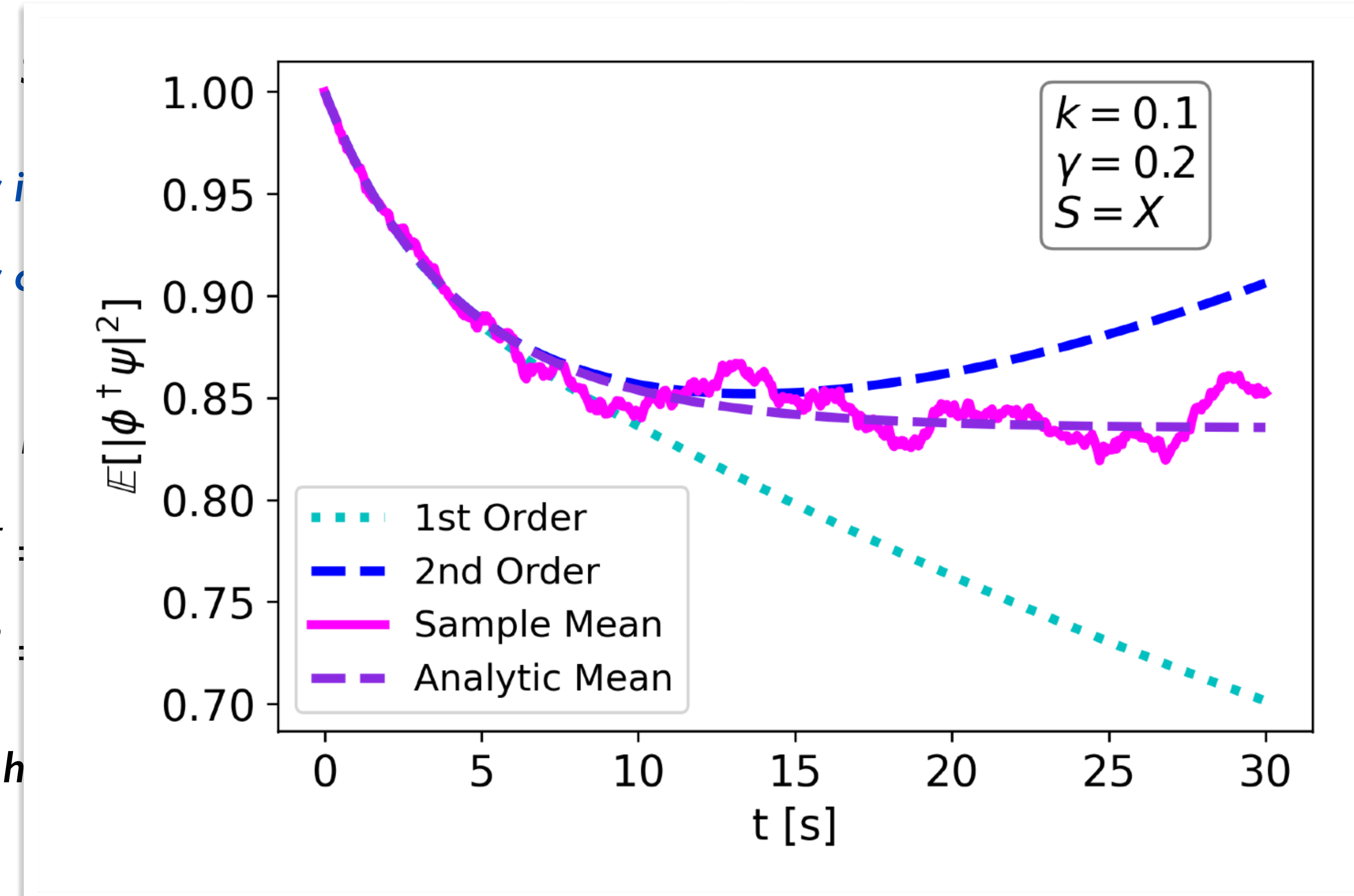
Laser c

Results:

$F_t^\sigma =$ … $_0\rangle\,|$

$F_t^p =$ … $_0\rangle\,|$

Derived h



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Derived hierarchical approximations when $[H, S] \neq 0$

Extensions: ***2-qubit, multiple noise frequencies*** ⤳ ***Power spectral density***

Qubit fidelity under stochastic Schrödinger equations driven by colored noise. In preparation

Towards controlled gates

$$i\hbar\, \mathbf{d}\phi_t = H\phi_t\, \mathbf{d}t$$

$$i\hbar\, \mathbf{d}\psi_t = H\psi_t\, \mathbf{d}t - i\gamma^2 S^\dagger S\psi_t\, \mathbf{d}t + \gamma S\psi_t\, \mathbf{d}X_t$$

Example: ***Feedback process***

$$Y_t = \begin{cases} X_t & \textbf{\textit{for}}\ t < \tau \\ X_t - X_{t-\tau} & \textbf{\textit{for}}\ t \geq \tau \end{cases}$$

Towards controlled gates

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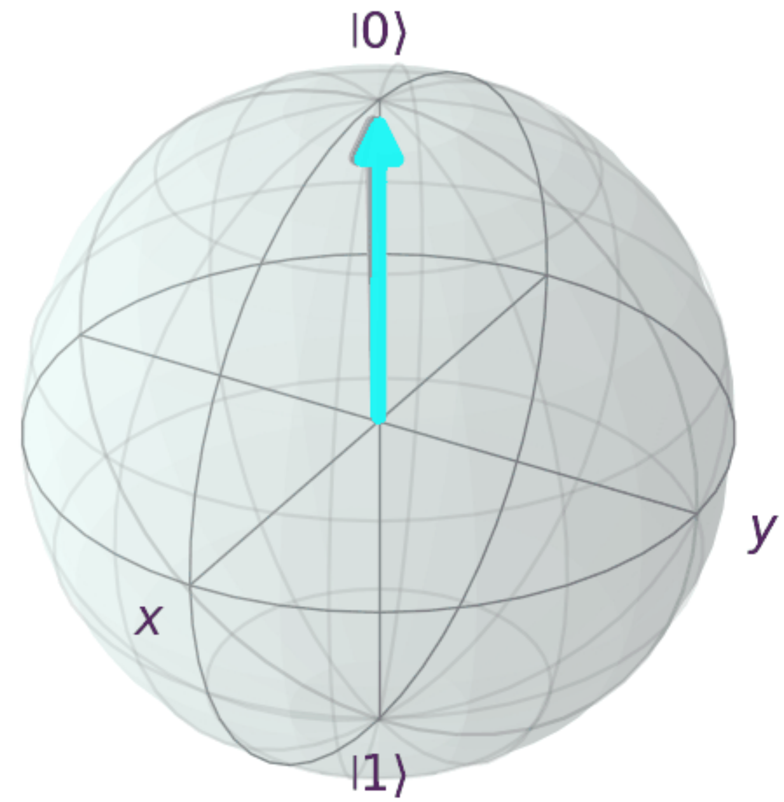

Example: *Feedback process*

$$Y_t = \begin{cases} X_t & \text{for } t < \tau \\ X_t - X_{t-\tau} & \text{for } t \geq \tau \end{cases}$$

Example: *Stochastic control*

Towards controlled gates

$$i\hbar\, \mathbf{d}\phi_t = H\phi_t\, \mathbf{d}t$$

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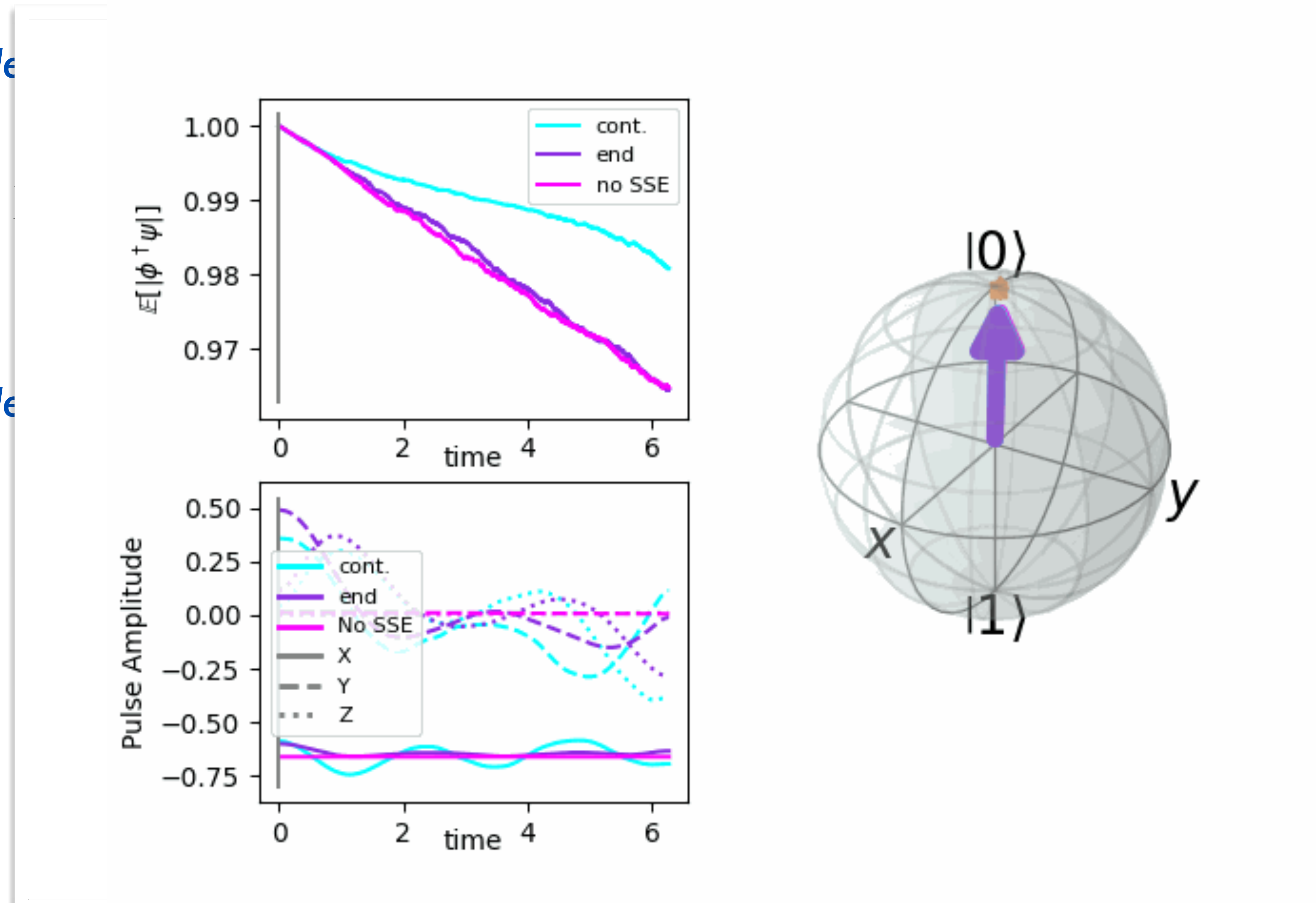
Example

Example

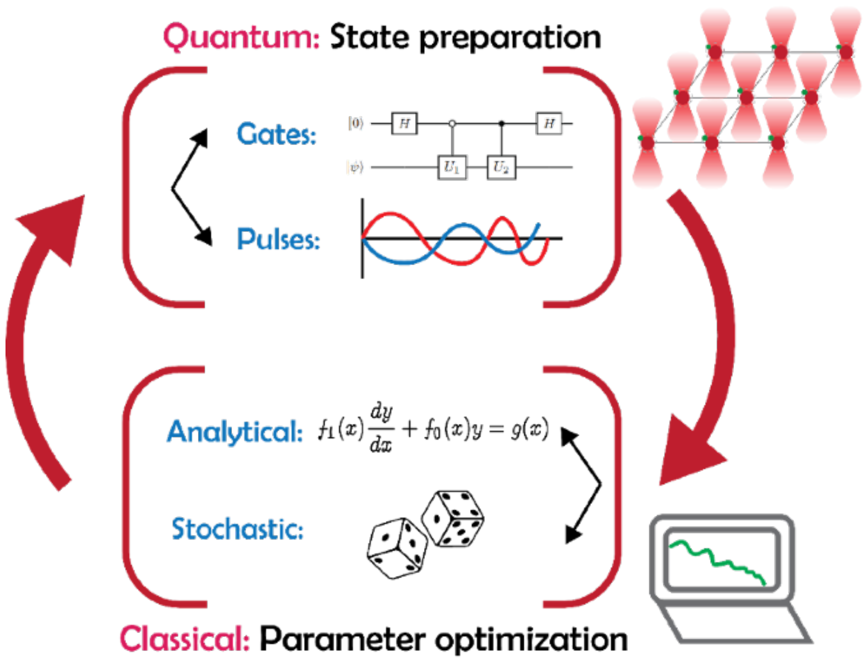


Summary

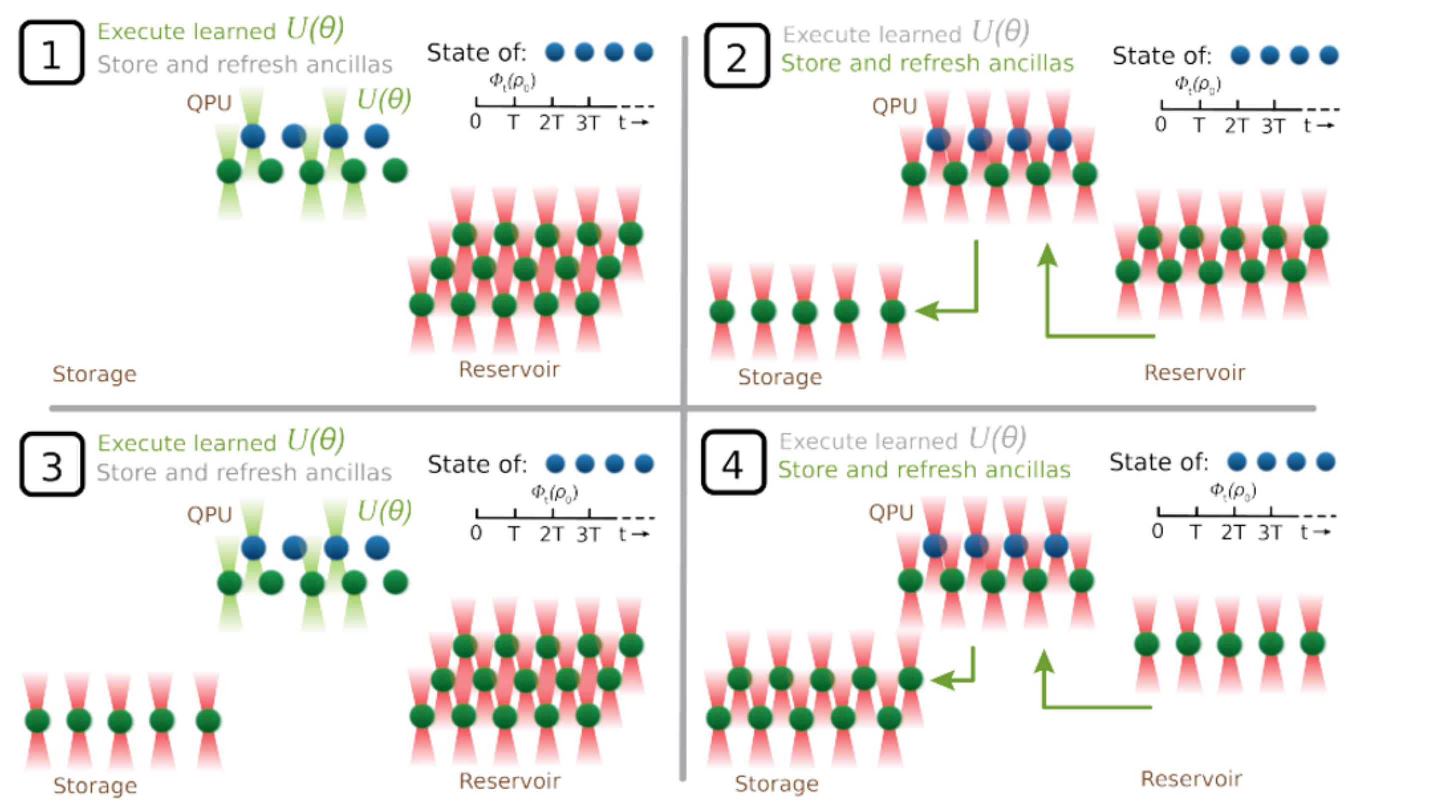
What's next?

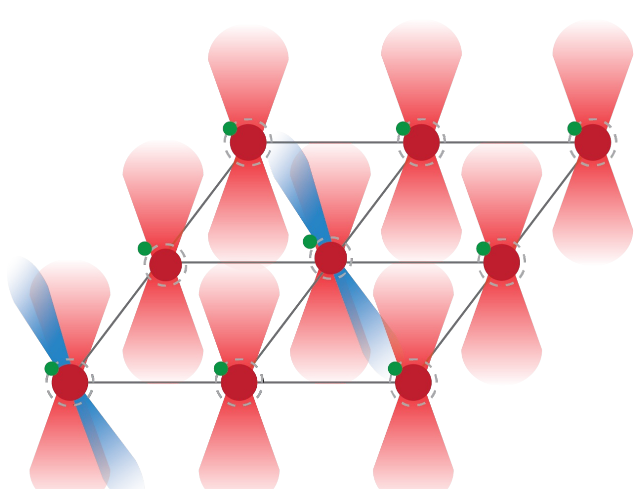
- *VQOC with noisy qubits*
- *Quantum optimal transport*
- *Integrating HPC*

Pulse-based Quantum Circuits and VQOC



Learning Open Quantum Systems





Understanding Noisy Qubits

Towards controlled qubits

Summary

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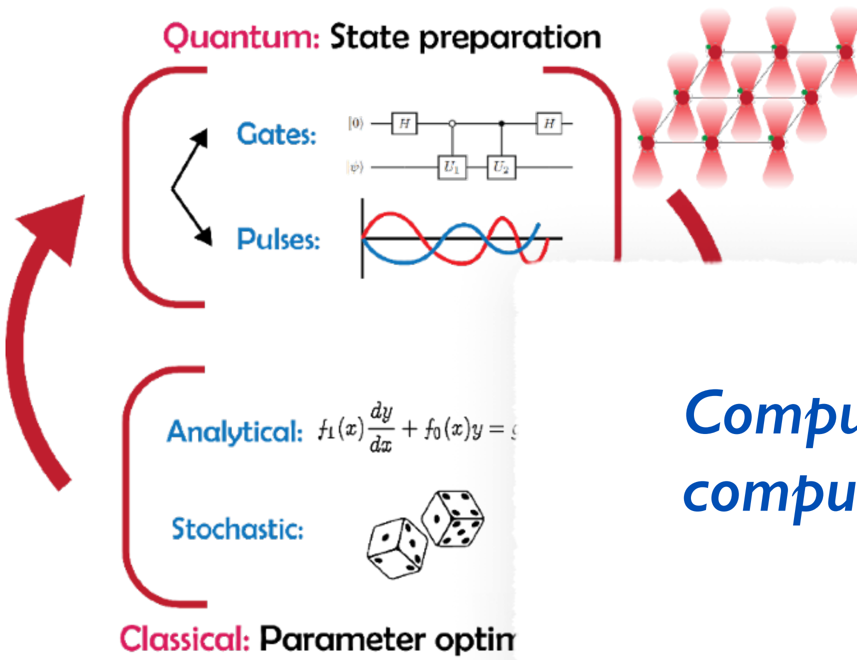
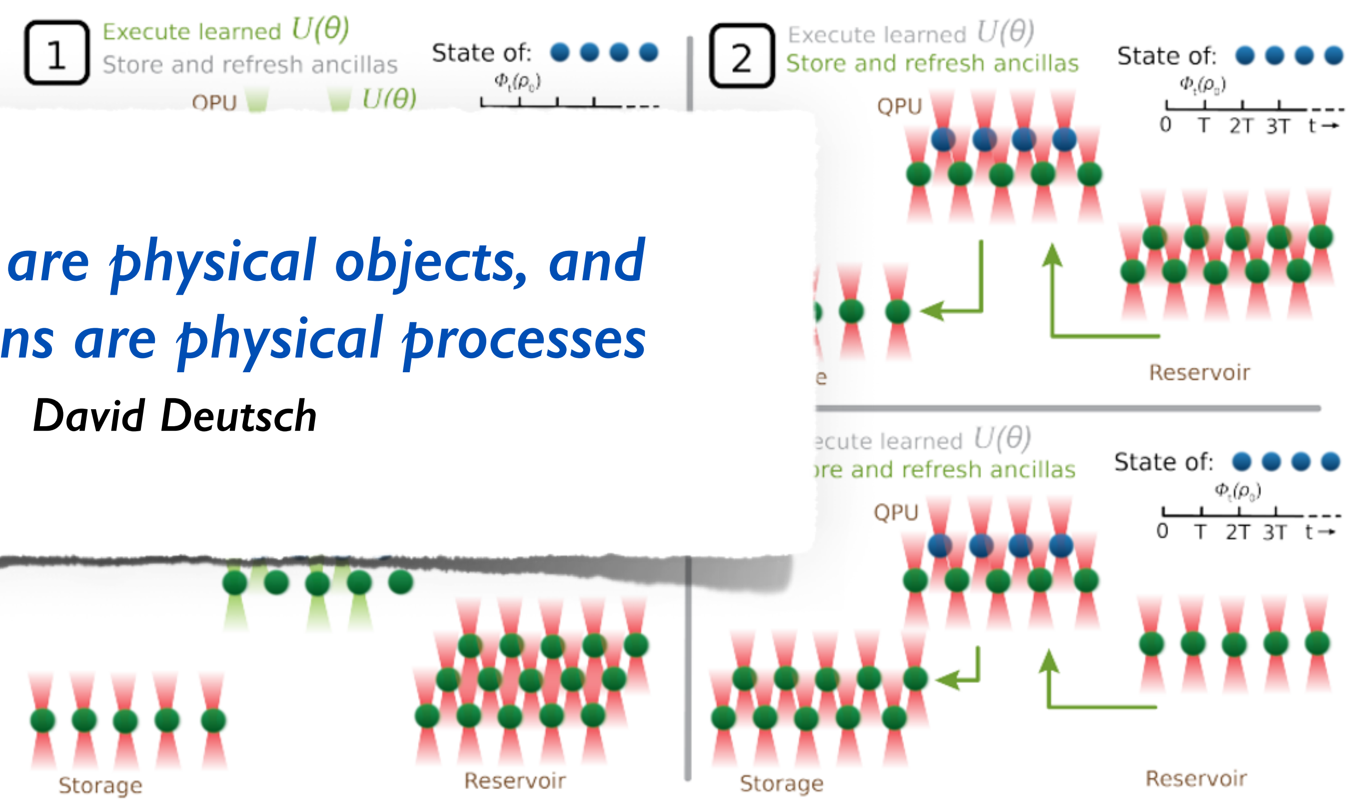
Pulse-based Quantum Circuits and VQOC

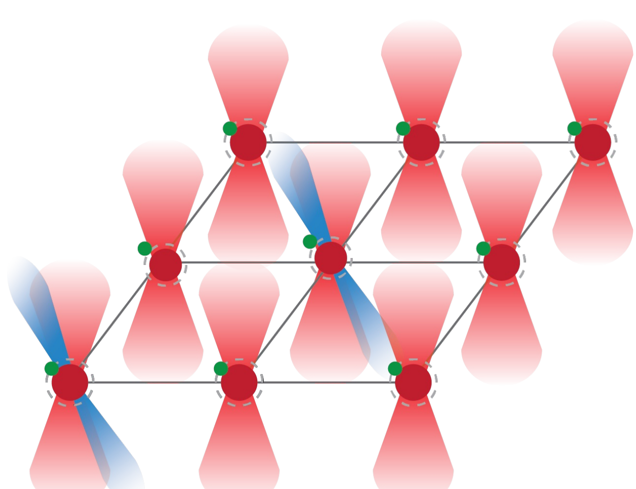


Learning Open Quantum Systems



Computers are physical objects, and computations are physical processes

David Deutsch



Understanding Noisy Qubits

Towards controlled qubits

Thank you!