Postdoc/ Graduate Student Introductions
Wednesday, January 31, 2024

John Carter, Rensselaer Polytechnic Institute
Casey Cavanaugh, Louisiana State University
Tristan Goodwill, University of Chicago
Sijing Liu, Brown University
Marissa Masden, ICERM
Henry von Wahl, Friedrich Schiller University Jena
Christopher Wang, Cornell University
Yukun Yue, University of Wisconsin, Madison
Introduction: Casey Cavanaugh

Postdoctoral Researcher
Louisiana State University
Center for Computation & Technology and Dept. of Mathematics
Supervisor: Susanne Brenner

PhD: Tufts University, 2022
Advisors: James Adler and Xiaozhe Hu
Thesis: *Structure-preserving discretizations for PDEs*

Research interests: finite element methods, finite difference methods, linear solvers, multigrid methods
Thesis: Structure-preserving discretizations for PDEs

**de Rham exact sequence**

\[
H(\text{grad}) \xrightarrow{\text{grad}} H(\text{curl}) \xrightarrow{\text{curl}} H(\text{div}) \xrightarrow{\text{div}} L^2
\]

- discrete exterior calculus (DEC)
- finite element exterior calculus (FEEC)

**Idea:** Draw connections to use FE theory for DEC scheme.

- **Maxwell’s equations:** enforce \( \text{div} \ B = 0 \)
  

- **Convection-dominated diffusion equations:** stability
  
Current: FEM and multigrid for 3D quad-curl equation

Joint work with S. C. Brenner and L. -Y. Sung (LSU)

Find \( u \in E \) such that for all \( v \in E \),

\[
\langle \text{curl} \text{ curl} \, u, \text{curl} \text{ curl} \, v \rangle + \beta \langle \text{curl} \, u, \text{curl} \, v \rangle + \gamma \langle u, v \rangle = \langle f, v \rangle.
\]

Hodge decomposition

A divergence-free vector field \( u \) has decomposition,

\[
u = \text{curl} \, \phi + \sum_{i=1}^{n} c_i \text{grad} \lambda_i.
\]

Idea: Reduce quad-curl to standard saddle point problems.
• High order methods for PDEs
• Particularly fast and accurate integral equation methods
• Fast hierarchical linear algebra (FMM, etc…)
• PDEs on surfaces
• PDEs with discontinuous coefficients
Laplace-Beltrami equation

We want to solve

\[ \Delta_{\Gamma} u = f \text{ on } \Gamma \]

The 2D Green’s function \( K(x, x') = \log \|x - x'\| \) satisfies

\[ \Delta_{\Gamma} K(x, x') = \delta(x - x') + R(x, x'). \]

Thus if \( u(x) = \int_{\Gamma} K(x, x') \sigma(x') da(x') \),
then \( \sigma \) must solve

\[ \sigma(x) + \int_{\Gamma} R(x, x') \sigma(x') da(x') = f(x). \]
Discretized system

Discretize the integrals and enforce at nodes

\[ \sigma_i + \sum_{j=1}^{N} R(x_i, x_j) \sigma_j \sqrt{(\det g)(x_j)} \ w_{i,j} = f(x_i) \]

Use recursive skeletonization to build an \( O(N) \) direct solver.

384 8th order patches
Other elliptic equations

General form \( \nabla_\Gamma \cdot a(x) \nabla_\Gamma u + b(x) \cdot \nabla_\Gamma u + c(x)u = f \)

\[ K(x, x') = \frac{i}{4a(x')} H_0^{(1)} \left( \sqrt{\frac{c(x')}{a(x')}} \| x - x' \| \right) \]
Time dependent problems

Solve $\partial_t u = \Delta_{\Gamma} u$ with Crank-Nicolson

$t = 0$ \hspace{1cm} $t = 0.1$ \hspace{1cm} $t = 5$ \hspace{1cm} $t = 20$

Solve $\partial_t^2 u + \gamma \partial_t u = \Delta_{\Gamma} u + f(x)g(t)$ in frequency space

$u(x,0)$ \hspace{1cm} $u(x,2)$ \hspace{1cm} $u(x,4)$ \hspace{1cm} $u(x,6)$
ICERM Introduction

Sijing Liu

ICERM

Brown University

January 31, 2024
Background

- Louisiana State University, PhD, 2020.
- University of Connecticut, Assistant Research Professor, 2020-2023.
- ICERM, Institute Postdoc, 2023-2024.
  - Numerical PDEs: Analysis, Algorithms, and Data Challenges, Spring 2024.

Research Interests:
- (Stabilized) Finite Element Methods, Discontinuous Galerkin Methods.
- Multigrid Methods.
- PDE-constrained Optimizations, Optimal Control Problems.
- Fluid-structure Interaction.
We consider a region $\Omega \subset \mathbb{R}^2$ or $\mathbb{R}^3$ to be heated or cooled.

The linear-quadratic elliptic optimal control problem is to find

$$\min_{(y,u)} \left[ \frac{1}{2} \| y - y_d \|_{L^2(\Omega)}^2 + \frac{\beta}{2} \| u \|_{L^2(\Omega)}^2 \right],$$

subject to

$$-\Delta y = u \quad \text{in} \quad \Omega,$$

$$y = 0 \quad \text{on} \quad \partial \Omega.$$
We consider

\[-\varepsilon \Delta u + \zeta \cdot \nabla u + \gamma u = f \quad \text{in} \quad \Omega,\]

\[u = g \quad \text{on} \quad \partial \Omega,\]

where \( \varepsilon \ll \| \zeta \|_\infty \).

Stabilization methods:
- SUPG, Local Projection, EAFE, DG, DWDG, etc.
Fluid-Structure Interaction (FSI) is the multiphysics interaction of a fluid flow with a solid structure.
Marissa Masden

- B.S. Math and Chemistry 2015
  *Walla Walla University*

- High school math teacher

- Ph.D. Mathematics, 2023
  *University of Oregon*

- Postdoctoral Fellow, 2023-2024
  *ICERM*

https://mmasden.github.io
Research Interests

- (Combinatorial, algebraic) approaches to topology of neural network functions
- Relationships between shape of training data and shape of neural networks
- Geometry and topology in scientific data, machine learning
Establish conditions when face poset is determined by combinatorial information. (Differential Topology)

Understand algebraic properties and establish its duality to a cubical complex. (Oriented Matroid Theory)

Tracking Betti numbers of ReLU networks during training. (Algebraic Topology)
Obtaining topological properties of the true decision boundary of a neural network

Extend PL Morse theory concepts to non-compact, non-general position ReLU functions (With Grigsby, Lindsey)

Current: Using face poset knowledge to develop algorithms for PL and Discrete Morse theory on ReLU networks (With R. Brooks)
Other Projects (Near Future):

• Probability distributions of geometric properties (e.g. discrete curvature of substructures) under random initialization assumptions

• Developing theory describing combinatorial changes on paths in parameter space.

• Expanding tools about ReLU networks to other architectures (non-fully-connected, convolutional)

• Using tools from this work to computationally analyze dynamical systems

Thank you!
Personal Introduction
ICERM Semester Program “Numerical PDEs: Analysis, Algorithms, and Data Challenges”

Henry von Wahl

Institute for Mathematics, Friedrich-Schiller-University Jena, Ernst-Abbe-Platz 2, Jena, Germany

28th January 2024
Unfitted Finite Element Methods

- Unfitted Finite Elements
  - Geometry representation implicit through level sets.
  - Stability through ghost-penalties.
  - Error estimates including higher-order geometry approximation.

- Moving domain problems.
  - Time-stepping with fixed background mesh.
  - Applications: Fluid-structure interaction.
    - Artificial neural network to predict forces governing rigid-body motion.

- Structure preserving discretisations $\rightarrow H(\text{div})$-conforming.
- Trefftz Methods $\rightarrow$ Reduced basis to gain efficiency.
- ngsfem: Add-on package to NGSolve for unfitted FEM.

Video: Unfitted FEM simulation of an experiment with a ball falling in a fluid.
Other Recent Work

- Discontinuous Galerkin for moist air
  - Hyperbolic conservation law with multiple densities.
  - Mass exchange between densities contain algebraic constraints.
  - Highly-parallelizable scheme to use HPC.
- Crack-propagation
  - Phase-field approach to simulate pressurized crack.
  - Geometry reconstruction to couple with FSI with ALE discretization.

Video: Stabilized DG discretization for a rising thermal in an under-saturated atmosphere leading to rain.
Christopher Wang

- My name is Christopher Wang (I go by Chris)
- 2nd year PhD
- Cornell University
- Adviser: Alex Townsend
Research Interests: Operator Learning

- data-driven algorithms for learning solution operators of PDEs

\[ \{(u_j, f_j)\}_{j=1}^N, \quad \mathcal{L}u_j = f_j \]

\[ u_j(x) = \int_{\Omega} K(x, y)f_j(y)\,dy \]

- learning characteristics of hyperbolic PDEs
Research Interests: Complexity & RandNLA

- operator learning for nonlinear PDEs
- curvature flows
- Solvability Complexity Index (SCI) hierarchy
  - classify complexity of computational problems, like computing the spectrum of a differential operator
- randomized numerical linear algebra (RandNLA), e.g., rSVD
Other Interests

- hiking, skiing, rock climbing, swimming
- singing, piano
Introduction

Yukun Yue

University of Wisconsin-Madison

January 29, 2024
I received my Ph.D. in Mathematics from Carnegie Mellon University in 2023, under the guidance of Prof. Franziska Weber and Prof. Noel Walkington.

Following my graduation, I joined the University of Wisconsin-Madison as a Van Vleck Visiting Assistant Professor in Fall 2023, collaborating with Prof. Qin Li.
Research Interest

Classical Numerical Analysis

- Studying convergence and stability analysis of different numerical schemes.
- Constructing more efficient numerical schemes to solve PDE-related problems.
- Both computational and theoretical performance are emphasized.

PDE-Constrained Optimization

- Focus on finding optimal control in a PDE system.
- Applying data-related method to solve inverse problem.
Current Research Problems

Gradient-Flow Related Physics Problems
- Liquid crystals
- Phase-field problems.
- Application of Invariant Energy Quadratization (IEQ), Scalar Auxiliary Variable (SAV) and Convex-Splitting method

Analysis of HDG Method
- Application onto various problems
- Focus on stability analysis.

Plasma-Related Problems
- Control of plasma instability based on Vlasov equation.
- Simulation of plasma behavior.