On a Generalization of Jones Polynomial and its Categorification for Legendrian Knots

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Tangled in Knot Theory

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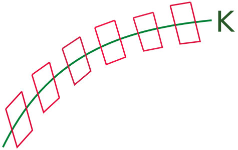
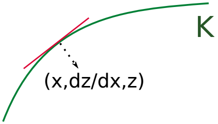
Based on a joint work with Dr. Dheeraj Kulkarni

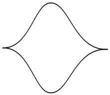
Legendrian Knots and Their Front Projections

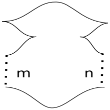
- A **Legendrian** knot in the contact manifold $(\mathbb{R}^3, \xi_{st} = ker(dz - ydx))$ is a smooth embedding of $\mathbb{S}^1$, which is always tangent to ξ_{st}.



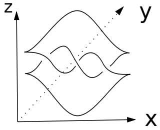


- A **front projection** of a Legendrian knot is its projection onto the *xz*-plane.


unknot



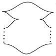
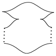
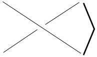
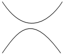
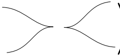
unknot



right handed trefoil

Legendrian Jones Polynomial

Legendrian Bracket polynomial of a front projection of a Legendrian link is defined by the following rules :

1. $\langle$  $\rangle = -A^2 r^{-1} - A^{-2} r$
2. $\langle K_F \sqcup$  $\rangle = (-A^2 r^{-1} - A^{-2} r)\langle K_F \rangle$
3. $\langle$  $\rangle = A \langle$  $\rangle + A^{-1} r \langle$  $\rangle$

The polynomial defined as

$$P_{K_F}(A, r) = (-A)^{-3\omega(K_F)} r^{\frac{c}{2} - l(K_F)} \langle K_F \rangle$$

is an invariant of Legendrian knots upto Legendrian isotopy[1].

[1] Dheeraj Kulkarni and Monika Yadav, On a generalization of Jones polynomial and its categorification for Legendrian knots, Bull. Sci. Math. 182 (2023), https://doi.org/10.1016/j.bulsci.2022.103212