On a Generalization of Jones Polynomial and its Categorification for Legendrian Knots

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Based on a joint work with Dr. Dheeraj Kulkarni
A Legendrian knot in the contact manifold \((\mathbb{R}^3, \xi_{st} = \ker(dz - ydx))\) is a smooth embedding of \(S^1\), which is always tangent to \(\xi_{st}\).

A front projection of a Legendrian knot is its projection onto the \(xz\)-plane.
Legendrian Jones Polynomial

**Legendrian Bracket polynomial** of a front projection of a Legendrian link is defined by the following rules:

1. \[ \langle \begin{array}{c} \bullet \bullet \\ \end{array} \rangle = -A^2 r^{-1} - A^{-2} r \]

2. \[ \langle K_F \sqcup \begin{array}{c} \bullet \bullet \\ \end{array} \rangle = (-A^2 r^{-1} - A^{-2} r) \langle K_F \rangle \]

3. \[ \langle \begin{array}{c} \bullet \bullet \bullet \\ \end{array} \rangle = A \langle \begin{array}{c} \bullet \bullet \\ \end{array} \rangle + A^{-1} r \langle \begin{array}{c} \bullet \bullet \bullet \\ \end{array} \rangle \]

The polynomial defined as

\[ P_{K_F}(A, r) = (-A)^{-3 \omega(K_F)} r_2^{\frac{c}{2}} - l(K_F) \langle K_F \rangle \]

is an invariant of Legendrian knots up to Legendrian isotopy\(^1\).

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