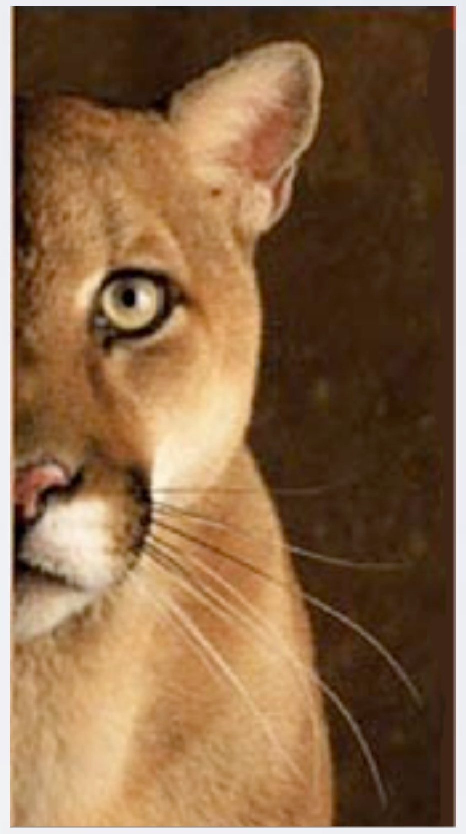
Plasticity and Balance in Neuronal Networks

Krešimir Josić   University of Houston



Plasticity and Balance

University of Houston
Alan Akil

University of Notre Dame
Robert Rosenbaum

UT, Houston
Ariana Andrei
Valentin Dragoi

Fly brain community detection

University of Houston
Kevin Bassler
Jiahao Guo

Baylor College of Medicine
Xaq Pitkow
Alex Kunin

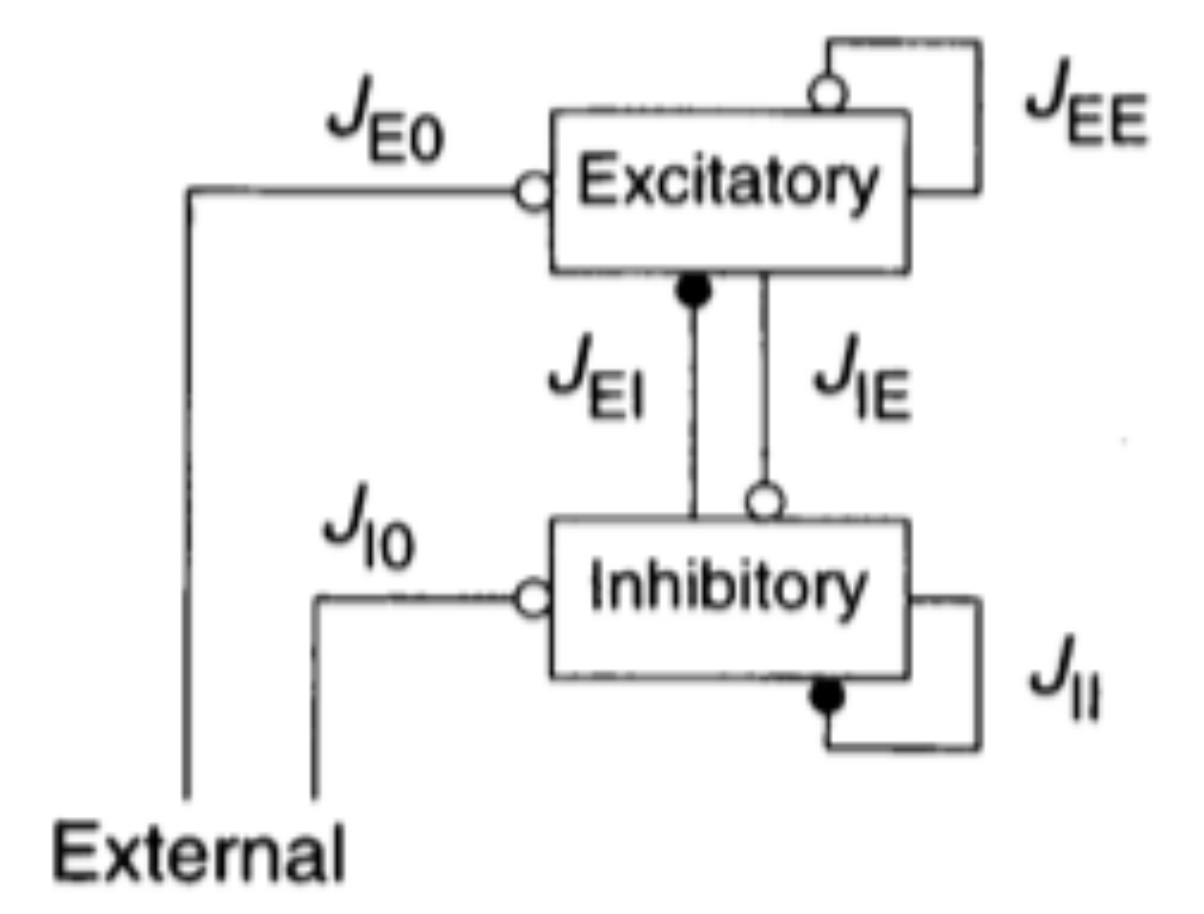
Janelia
Brad Hulse
Romain Franconville

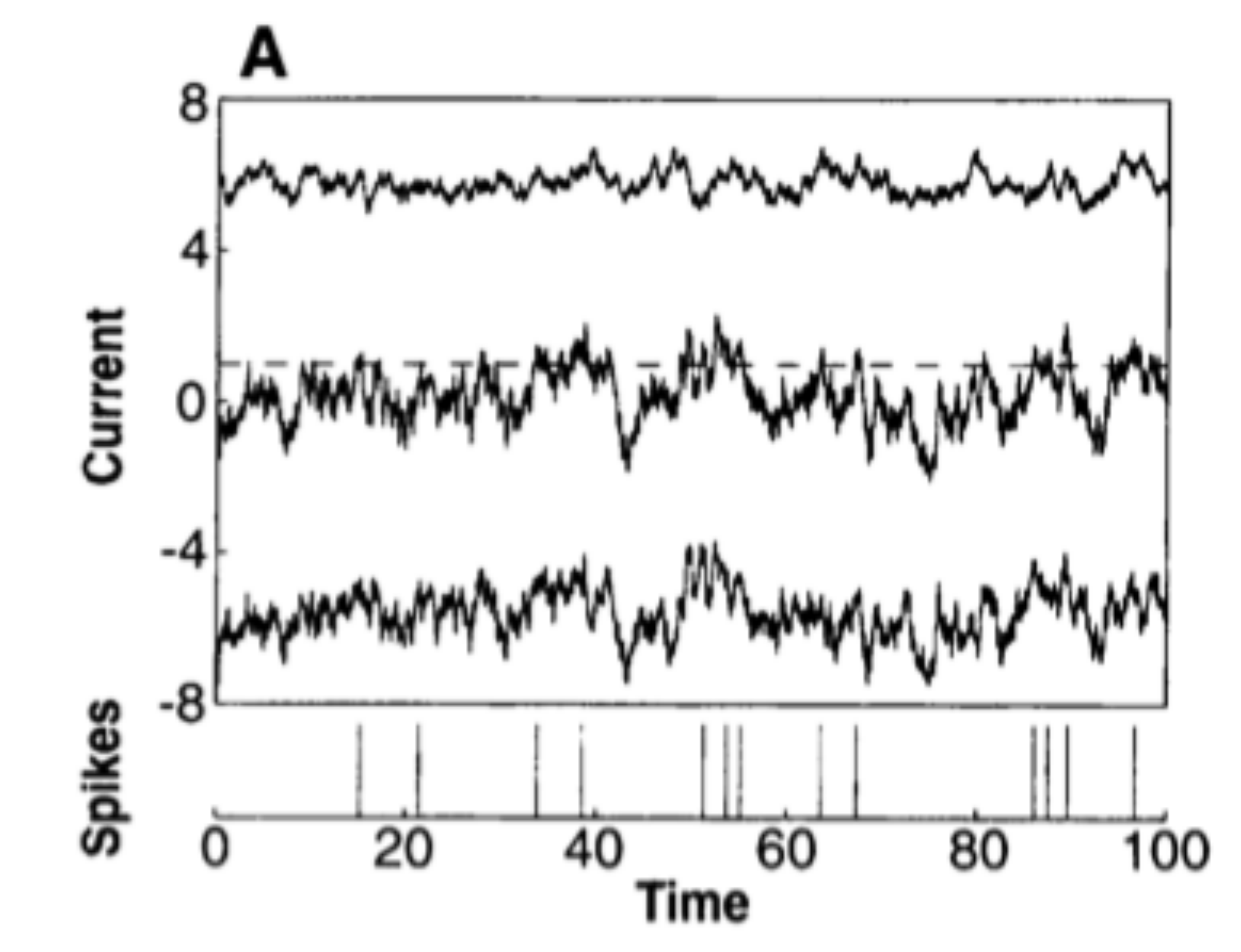
Funding: NSF NeuroNex, NIH

Outline

- Synaptic plasticity in balanced networks
- Homeostatic control of correlations in primate visual cortex
- Automated discovery of large scale organization in neuronal networks

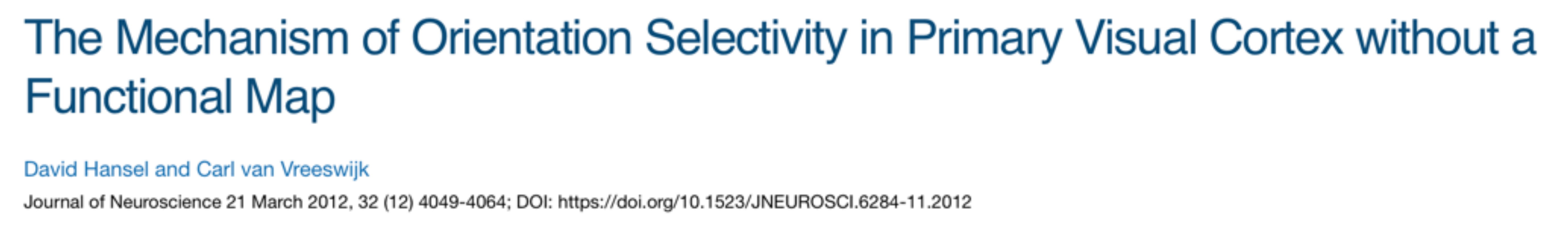
Balanced Networks





van Vreeswijk and Sompolinsky, 1996

Balanced Networks - theory and experiments



The Mechanism of Orientation Selectivity in Primary Visual Cortex without a Functional Map

David Hansel and Carl van Vreeswijk

Journal of Neuroscience 21 March 2012, 32 (12) 4049-4064; DOI: https://doi.org/10.1523/JNEUROSCI.6284-11.2012

Published: 31 October 2016

The spatial structure of correlated neuronal variability

Robert Rosenbaum, Matthew A Smith, Adam Kohn, Jonathan E Rubin & Brent Doiron

Nature Neuroscience **20**, 107–114 (2017) | Cite this article

Published: 23 February 2016

Efficient codes and balanced networks

Sophie Denève & Christian K Machens

Nature Neuroscience **19**, 375–382 (2016) | Cite this article

Published: 30 March 2008

Instantaneous correlation of excitation and inhibition during ongoing and sensory-evoked activities

Michael Okun & Ilan Lampl

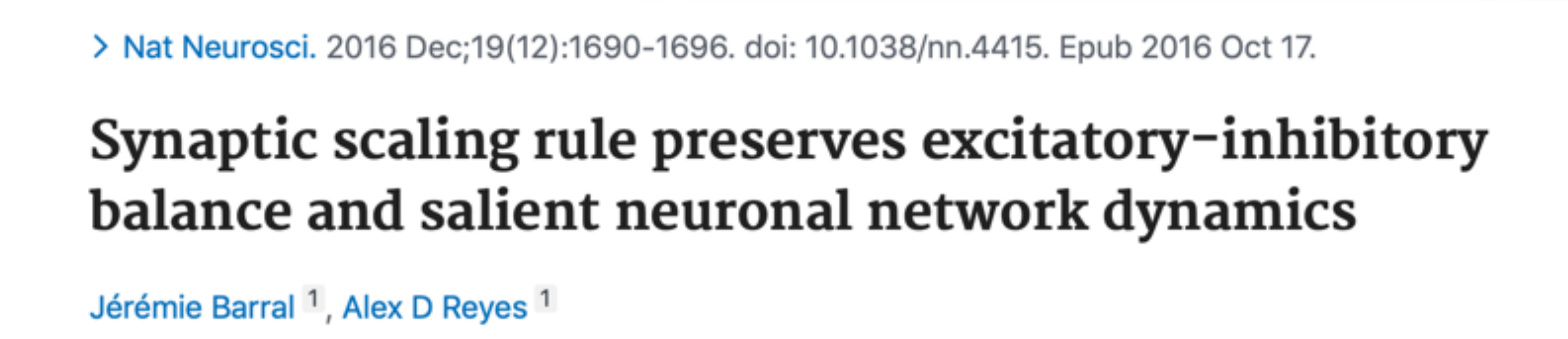
Nature Neuroscience **11**, 535–537 (2008) | Cite this article

PLOS COMPUTATIONAL BIOLOGY

Nonlinear stimulus representations in neural circuits with approximate excitatory-inhibitory balance

Cody Baker, Vicky Zhu, Robert Rosenbaum

Version 2 Published: September 18, 2020 • https://doi.org/10.1371/journal.pcbi.1008192

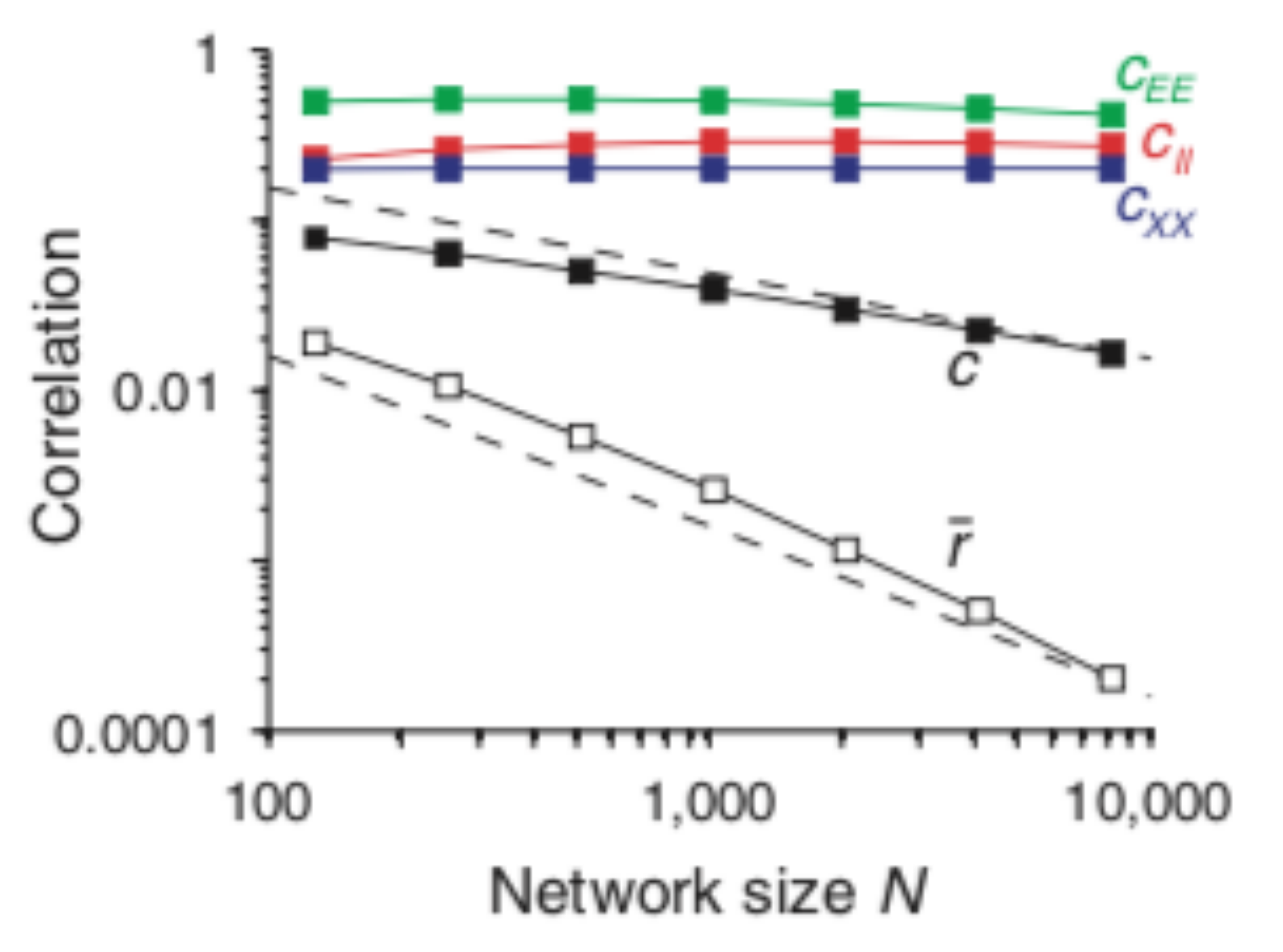


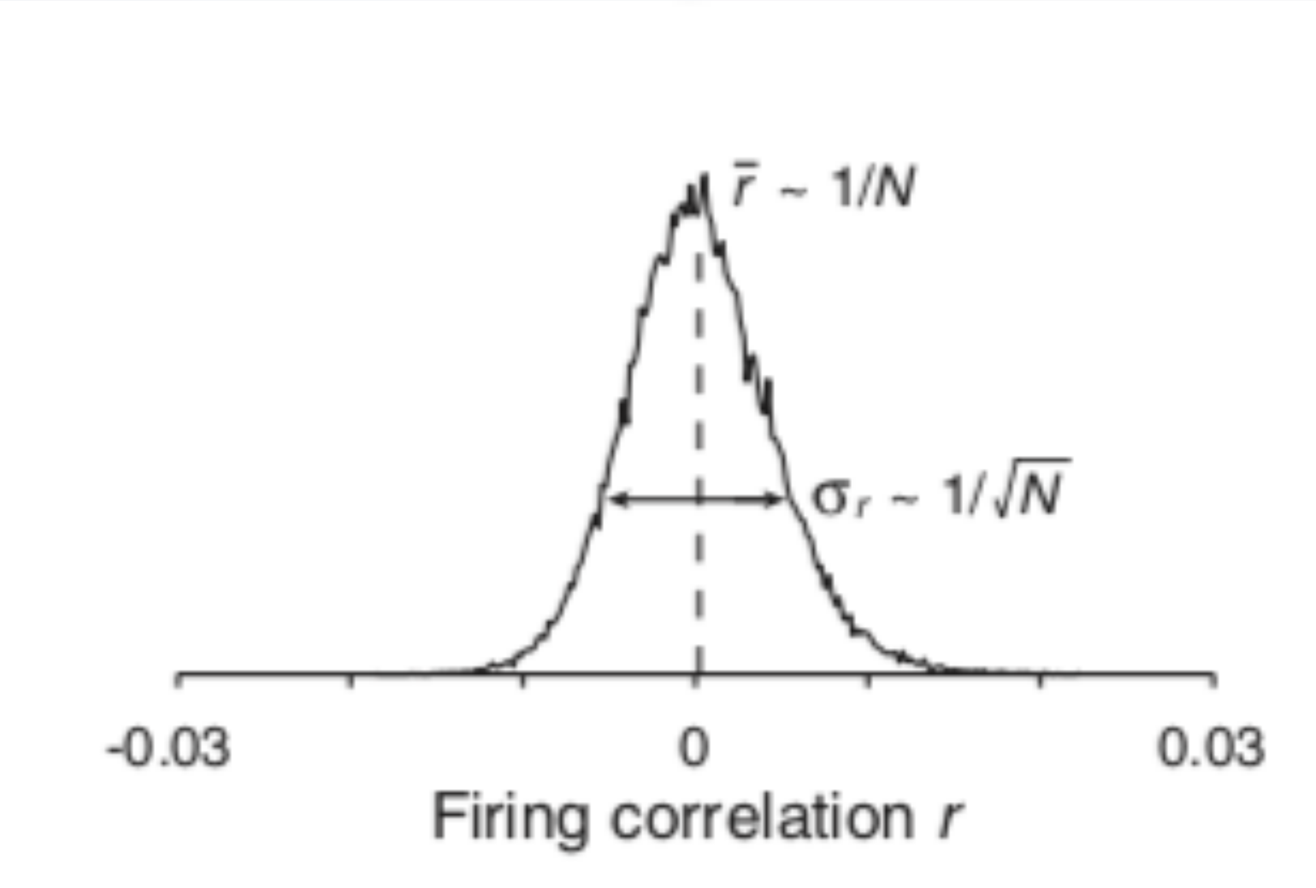
> Nat Neurosci. 2016 Dec;19(12):1690-1696. doi: 10.1038/nn.4415. Epub 2016 Oct 17.

Synaptic scaling rule preserves excitatory-inhibitory balance and salient neuronal network dynamics

Jérémie Barral [1], Alex D Reyes [1]

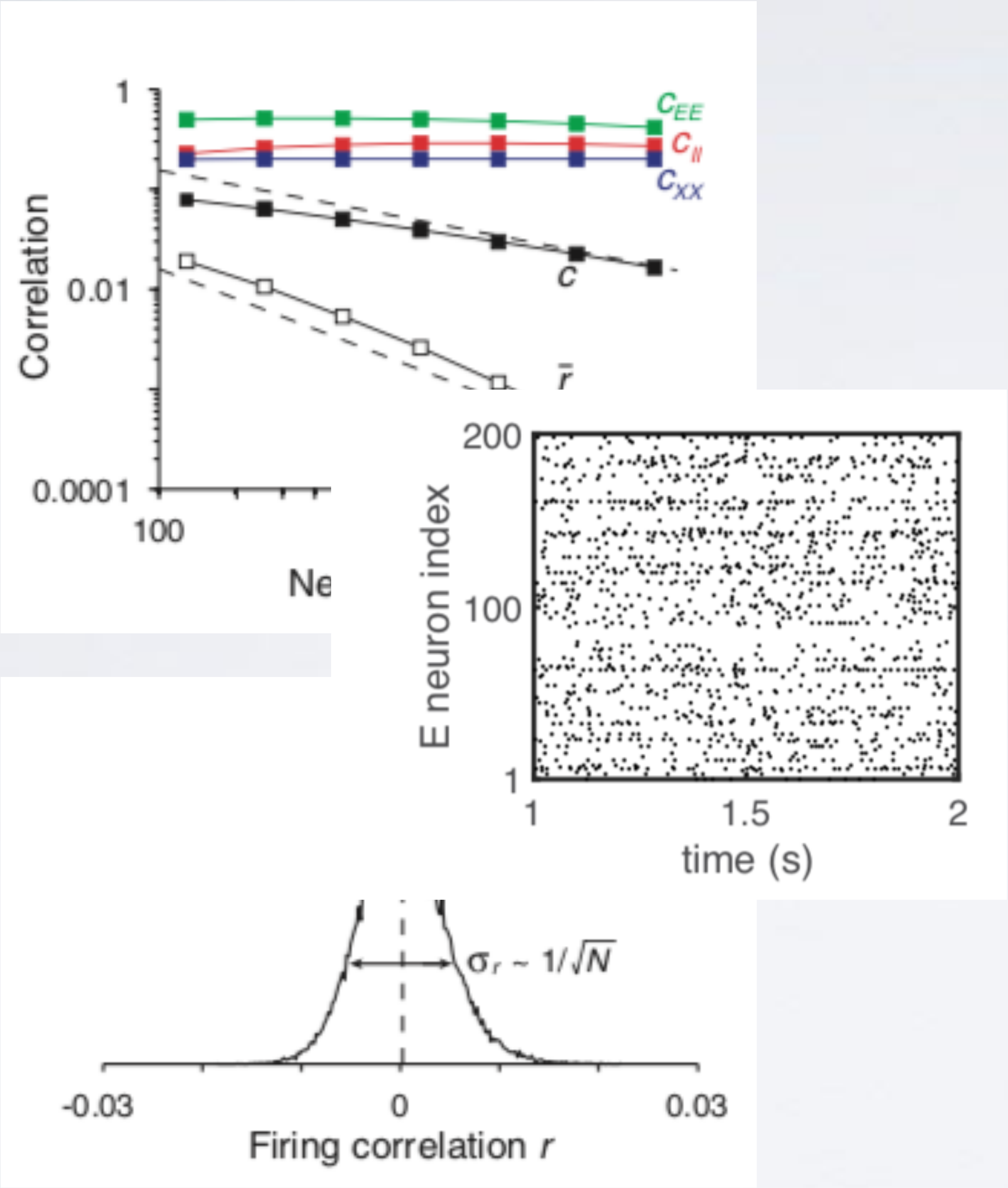
Uncorrelated and Correlated Balanced Networks





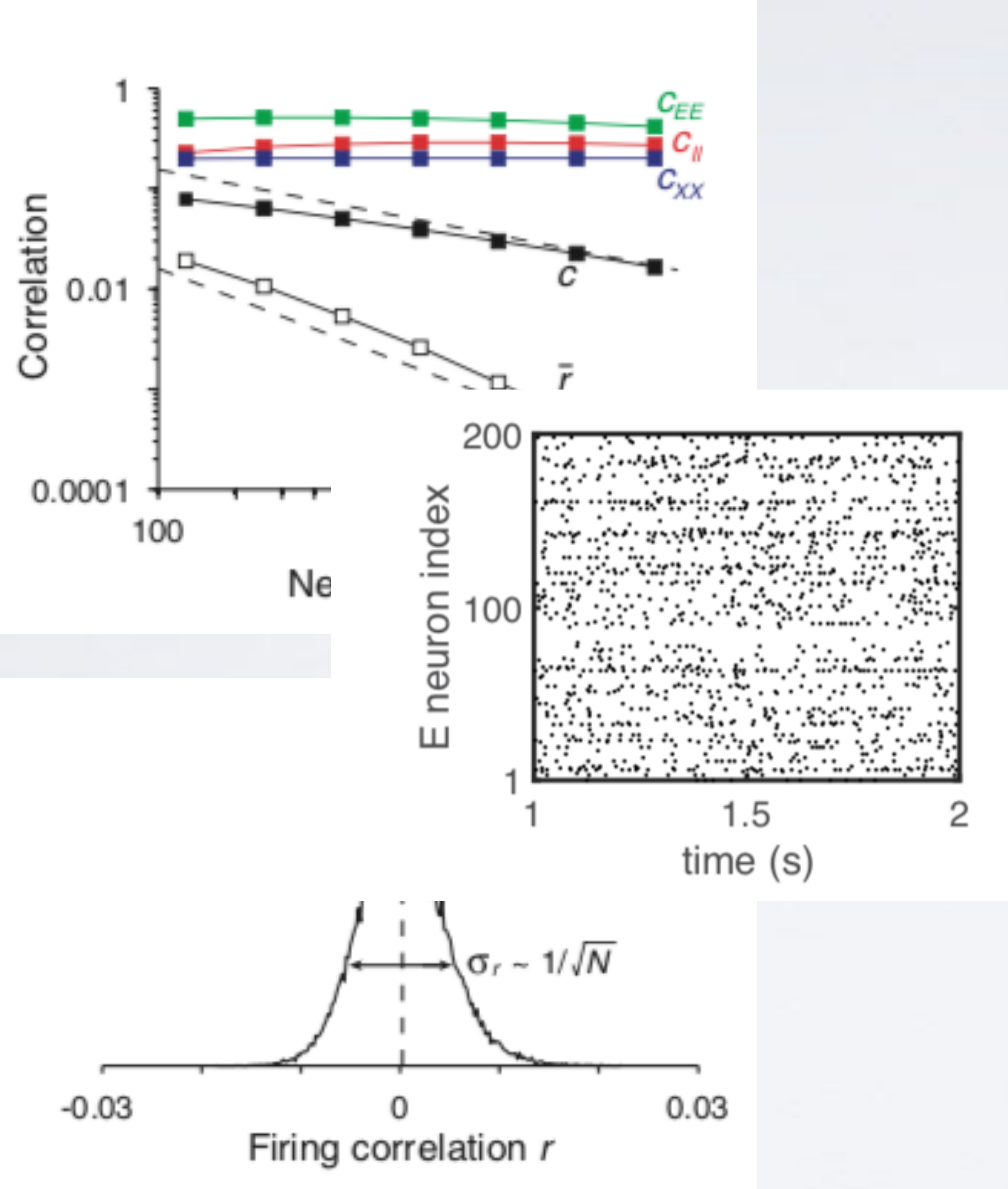
Renart, de la Rocha, et al 2010

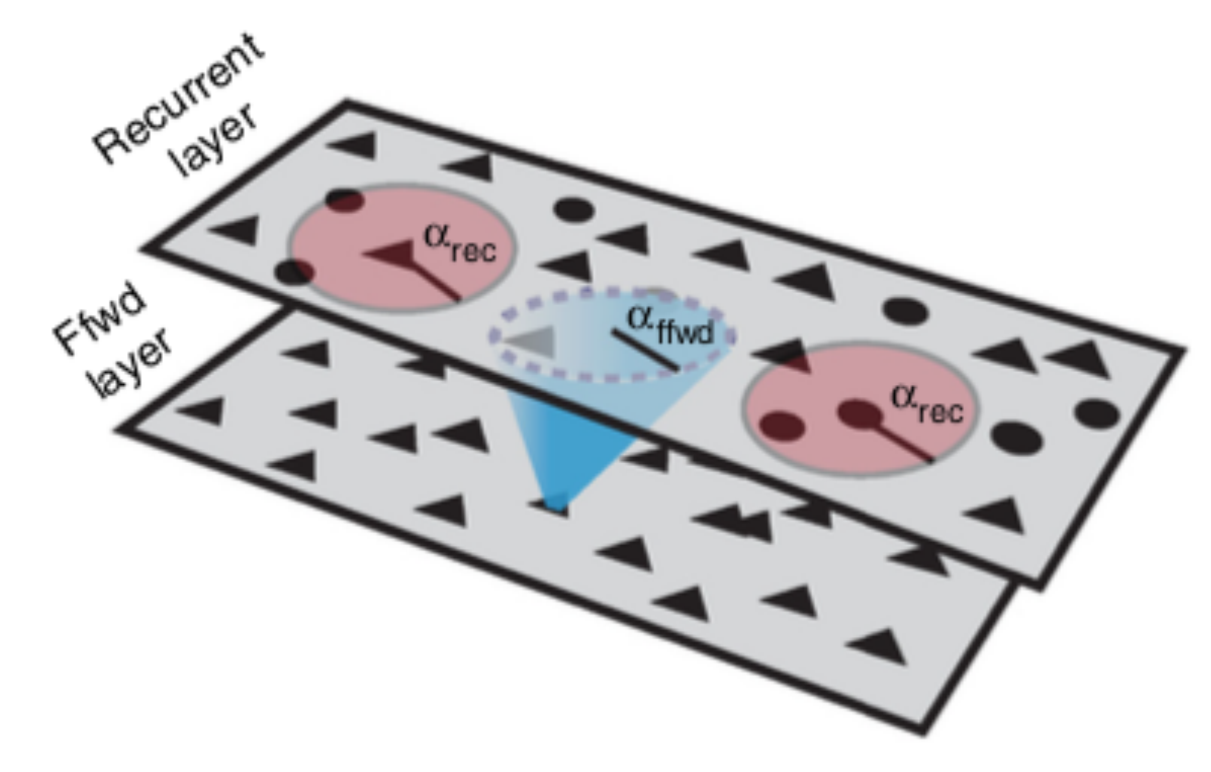
Uncorrelated and Correlated Balanced Networks



Renart, de la Rocha, et al 2010

Uncorrelated and Correlated Balanced Networks

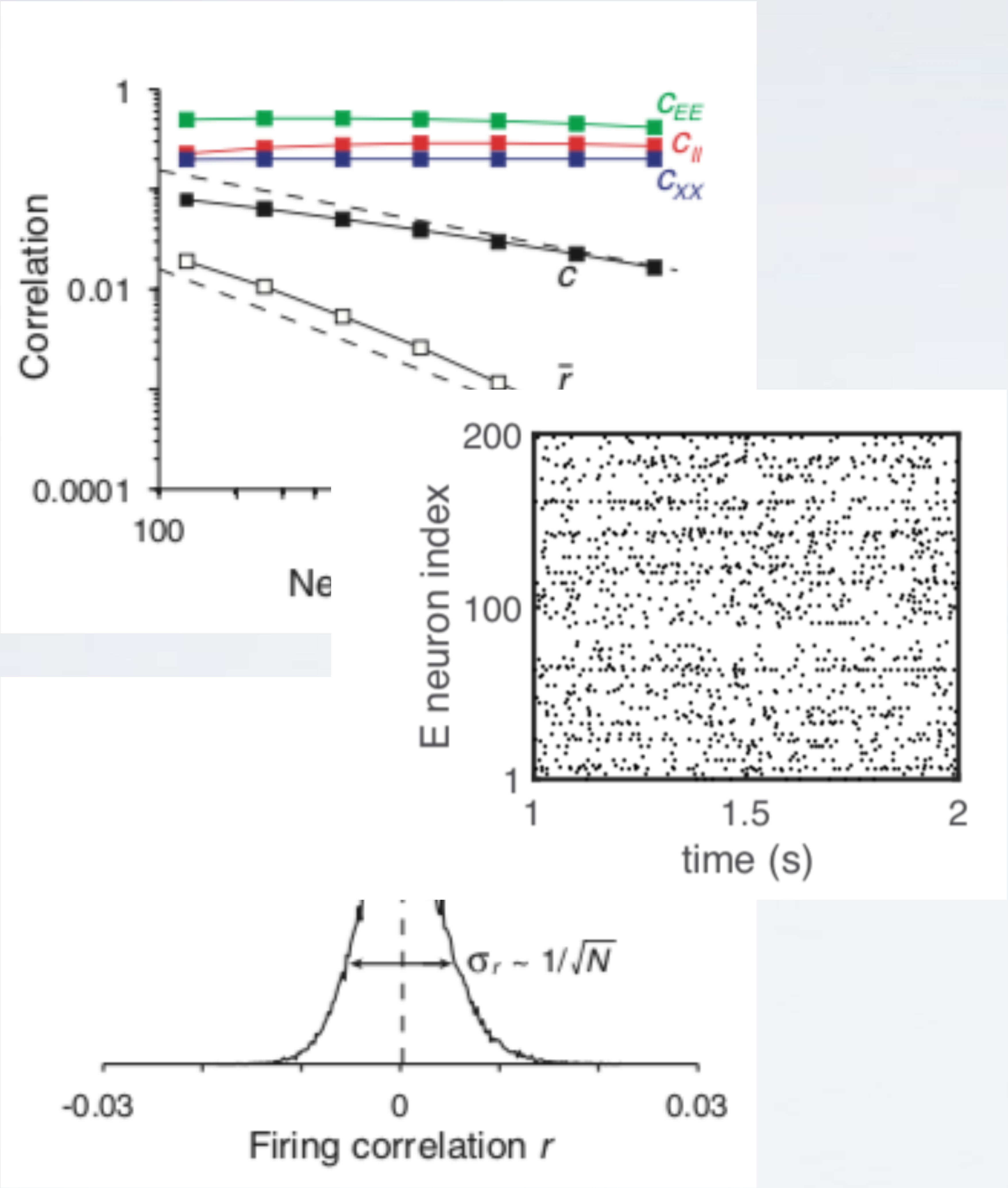




Rosenbaum, et al. 2017

Renart, de la Rocha, et al 2010

Uncorrelated and Correlated Balanced Networks



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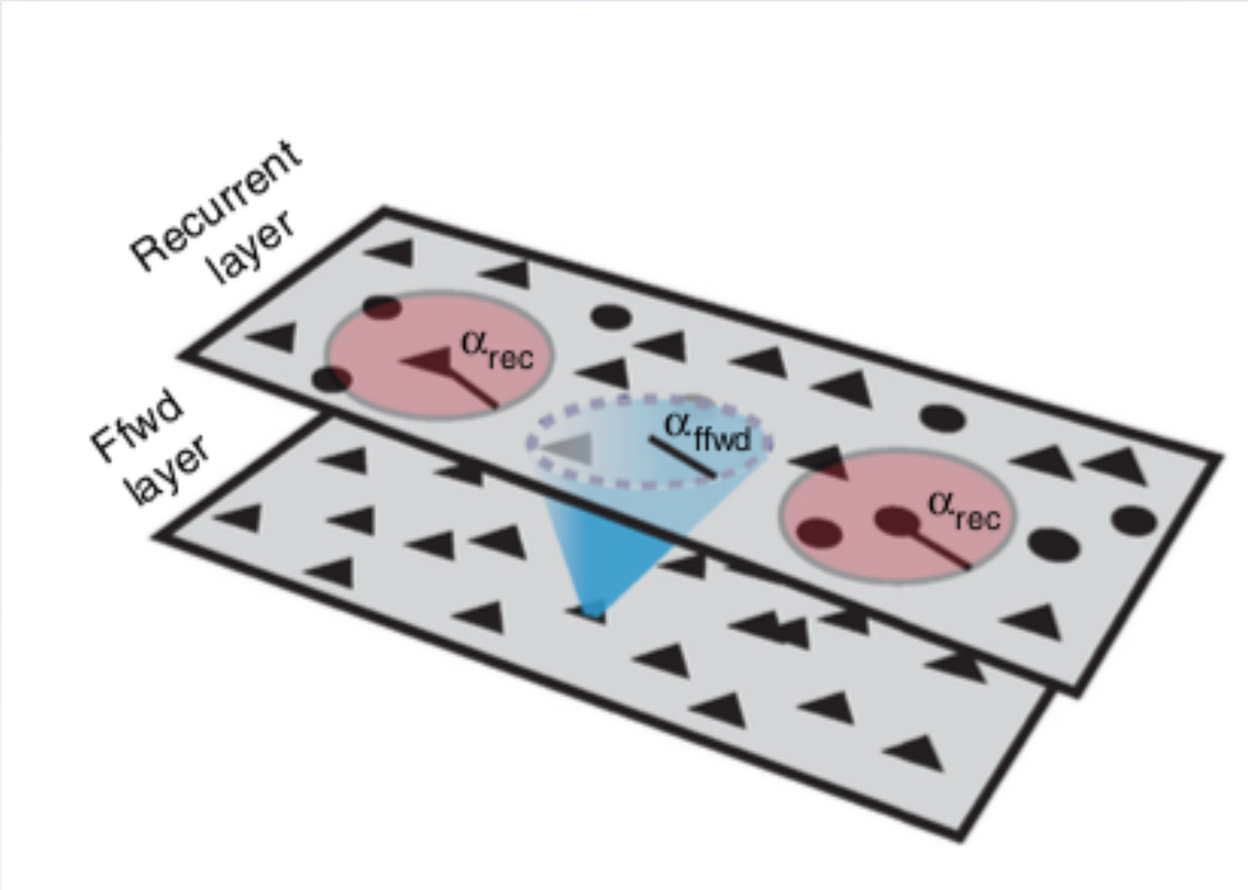
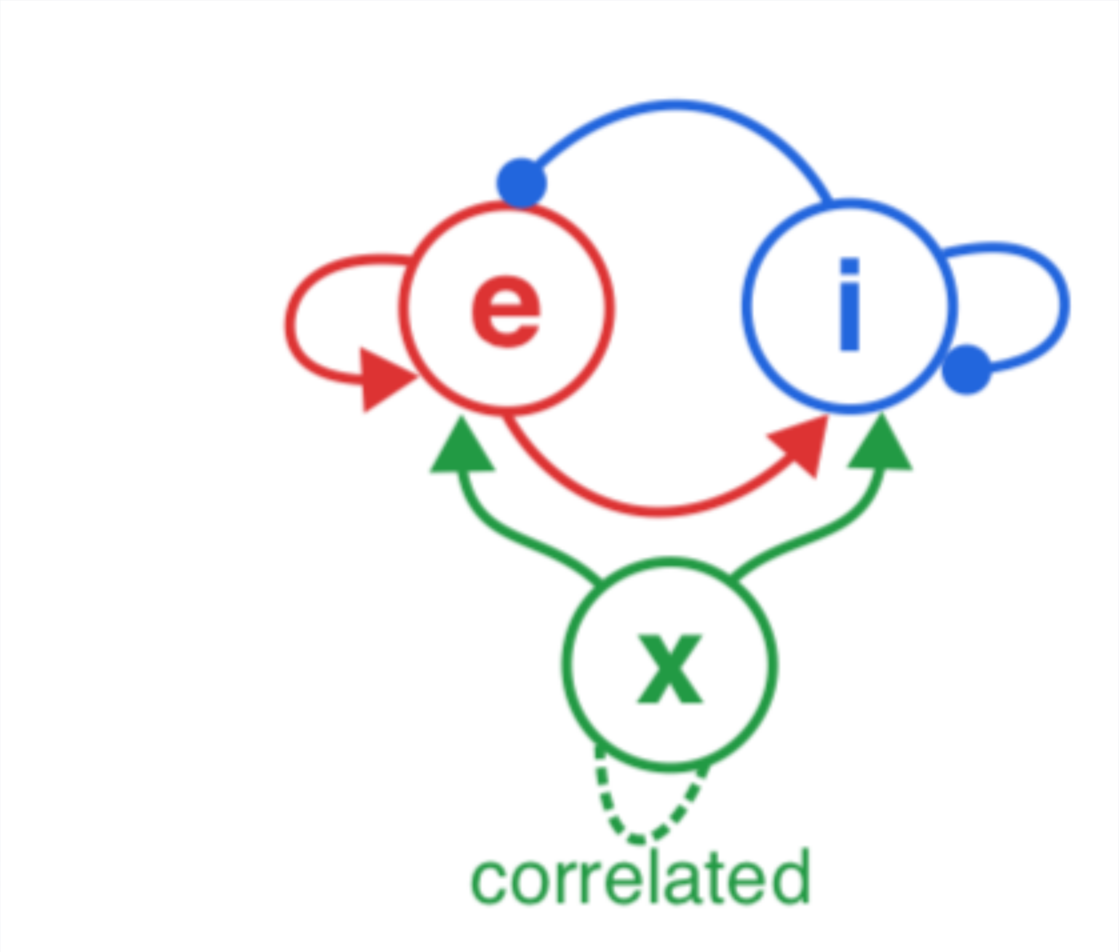
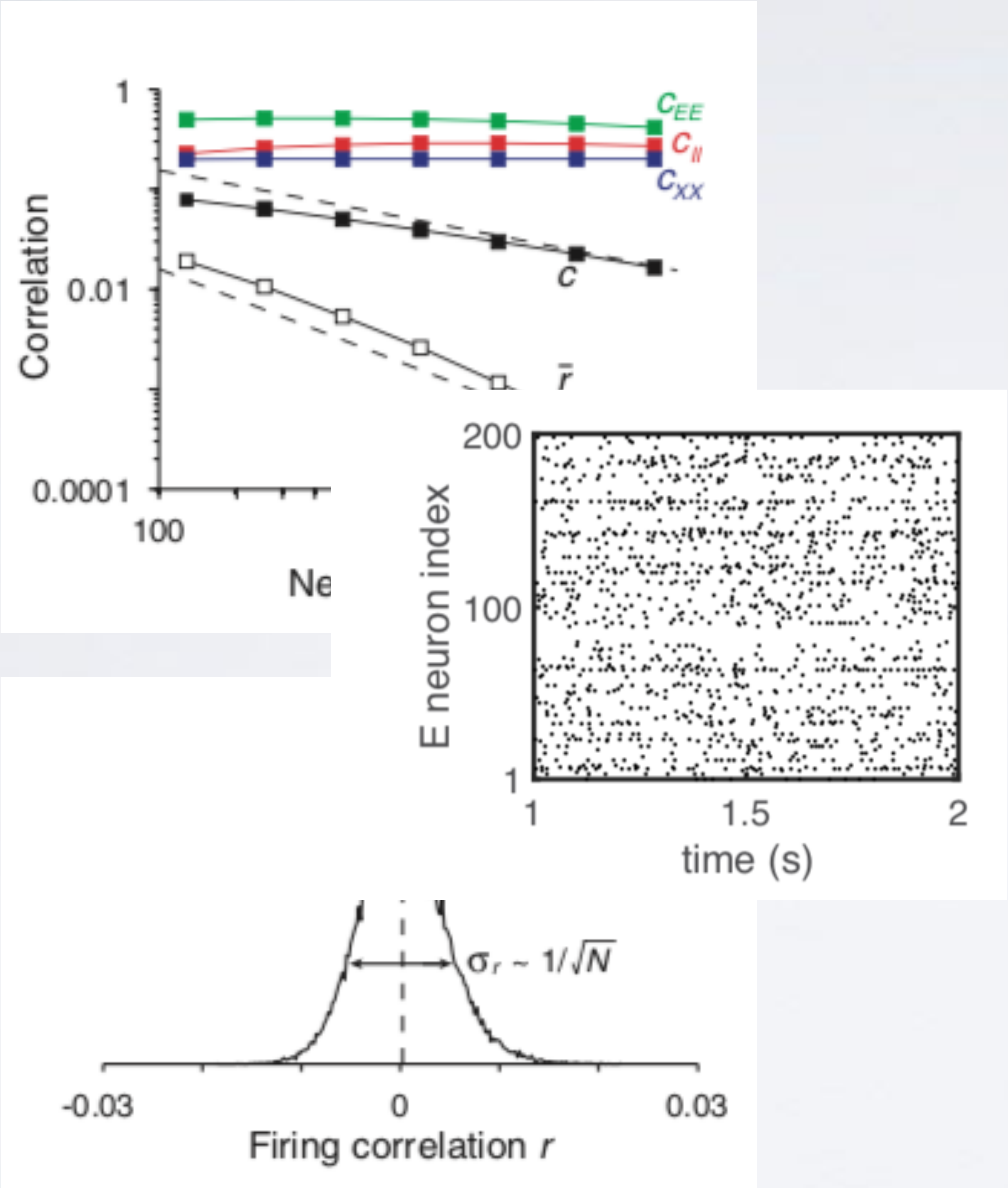


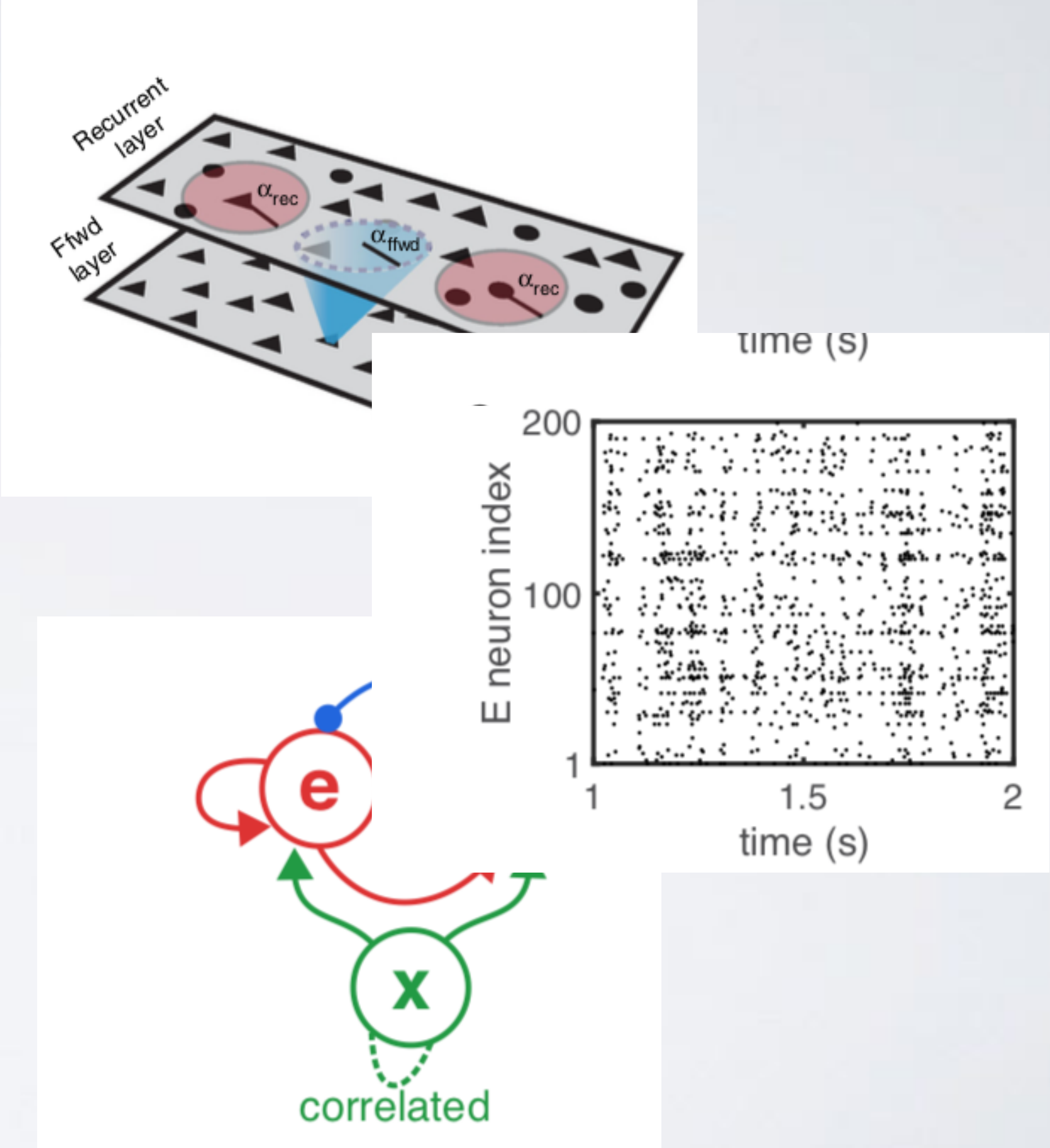
Rosenbaum, et al. 2017



Baker, et al. 2019

Uncorrelated and Correlated Balanced Networks

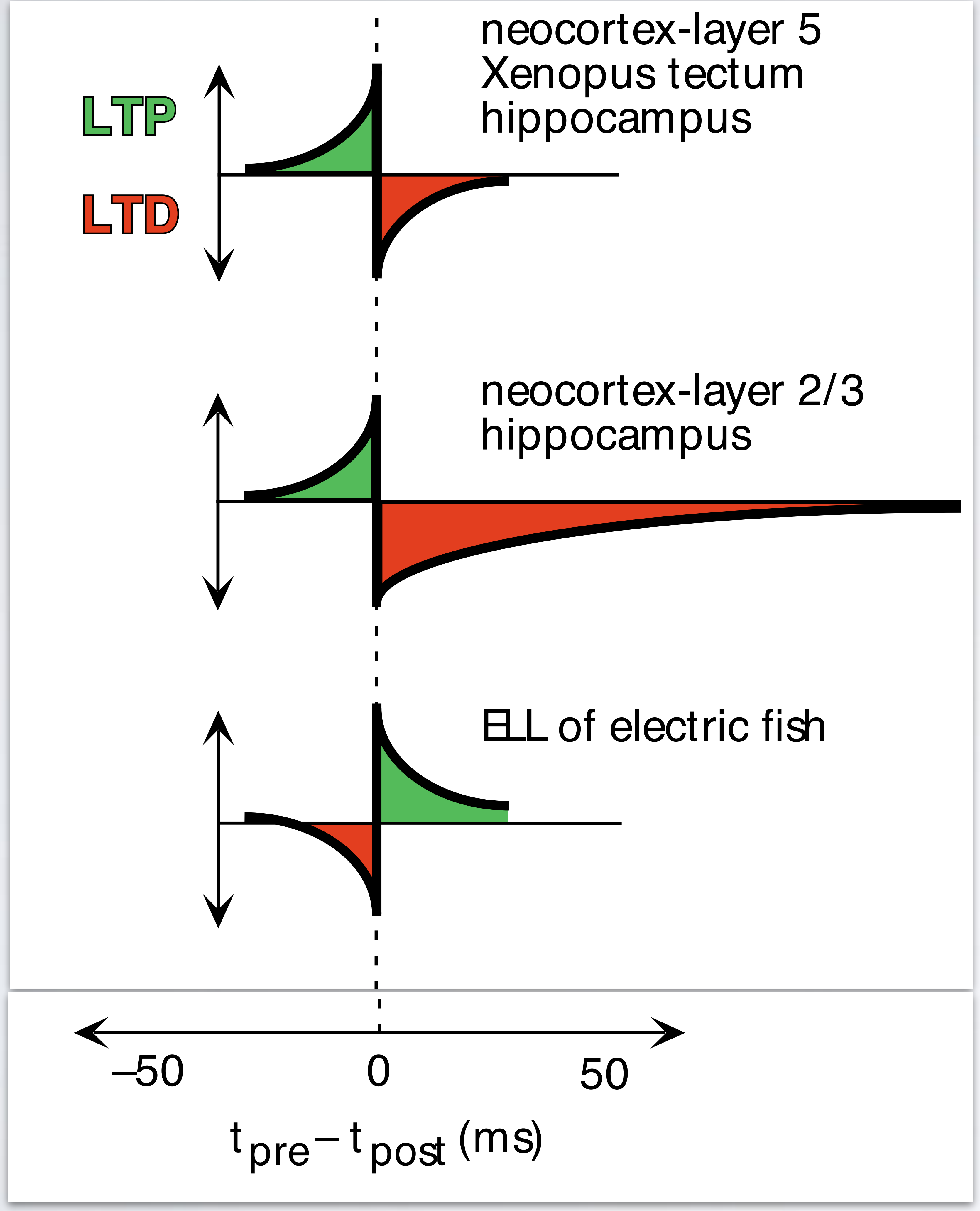




Renart, de la Rocha, et al 2010

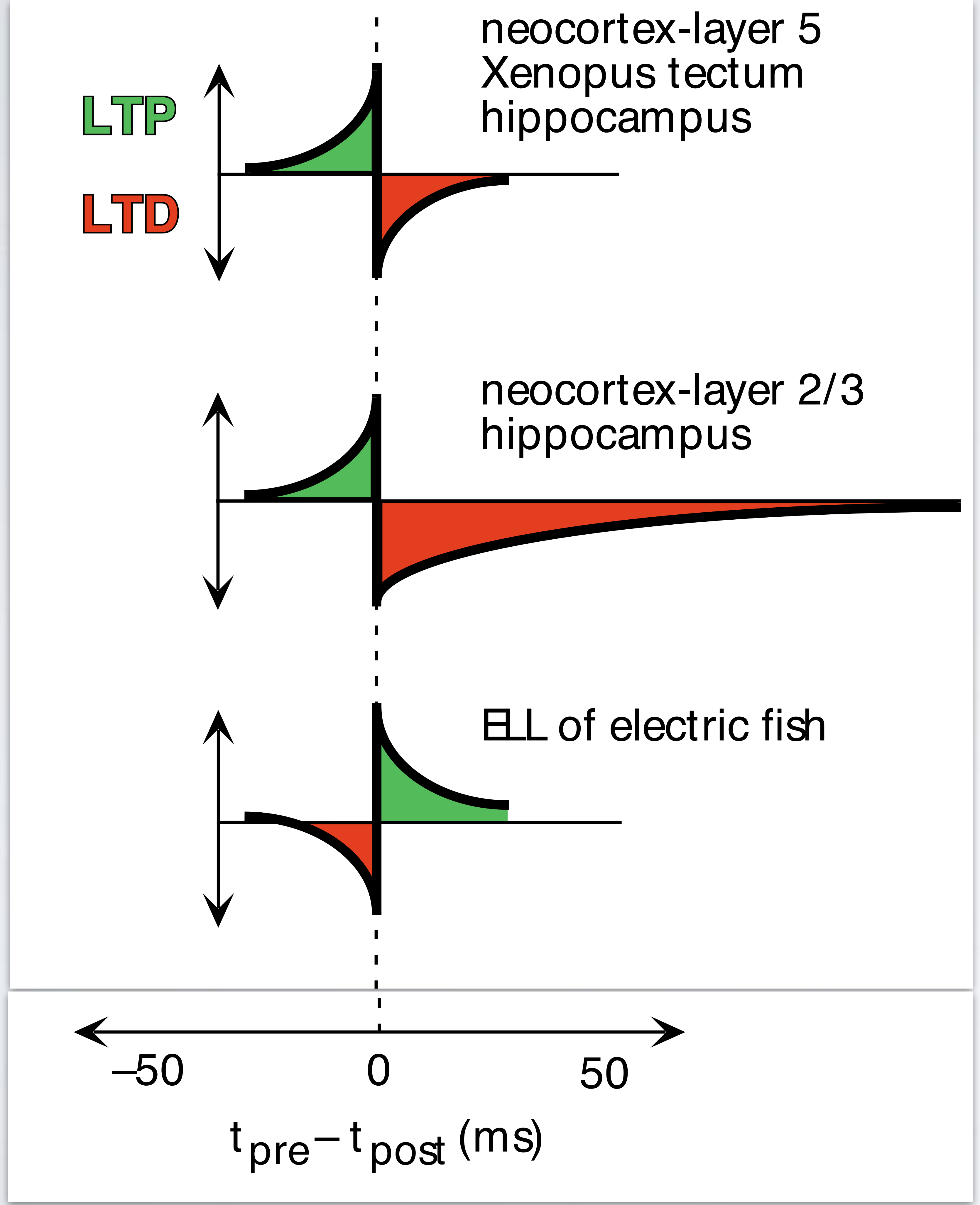
Baker, et al. 2019

Spike Time Dependent Plasticity

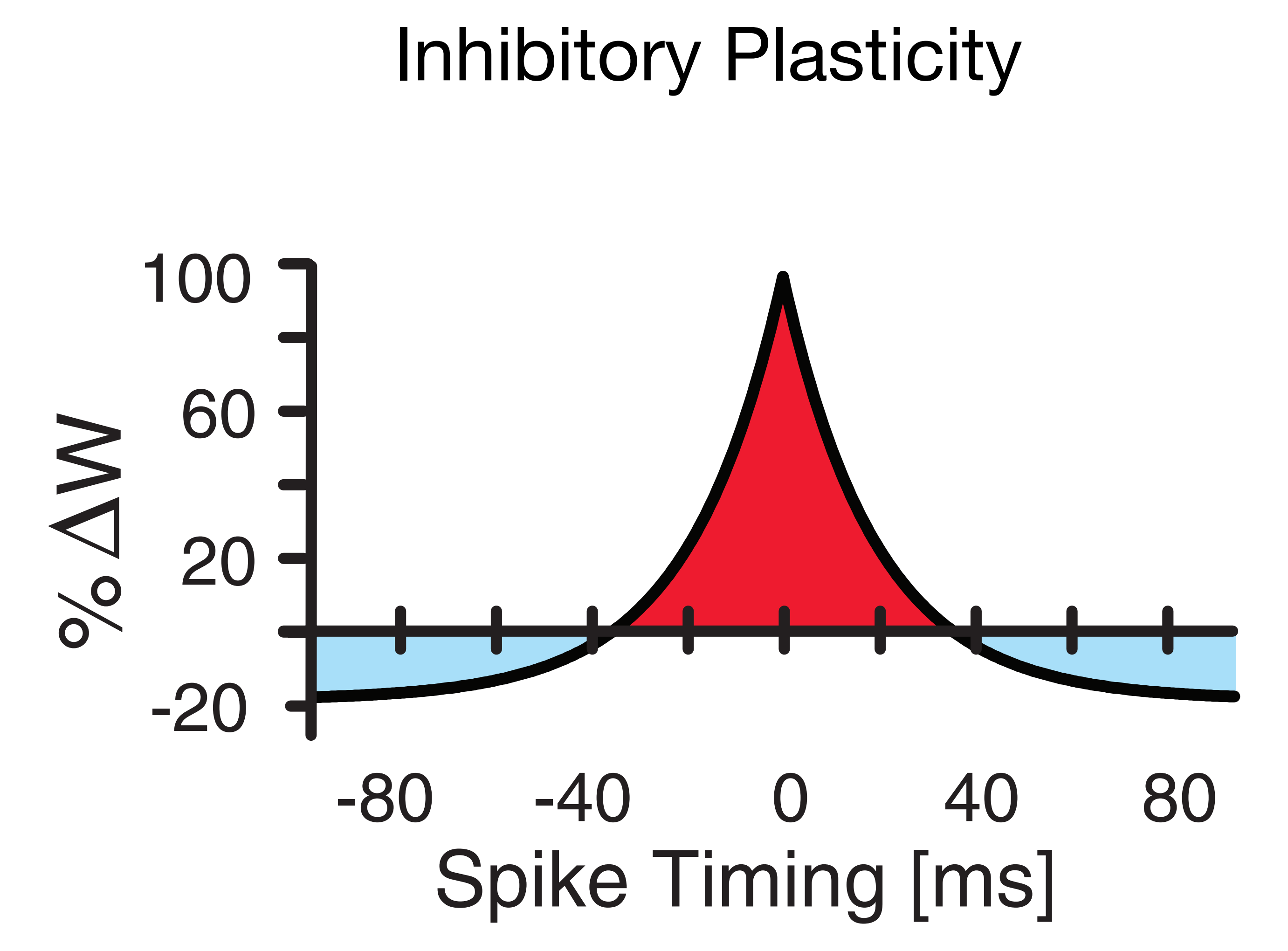


Abbott and Nelson, 2000

Spike Time Dependent Plasticity



Abbott and Nelson, 2000

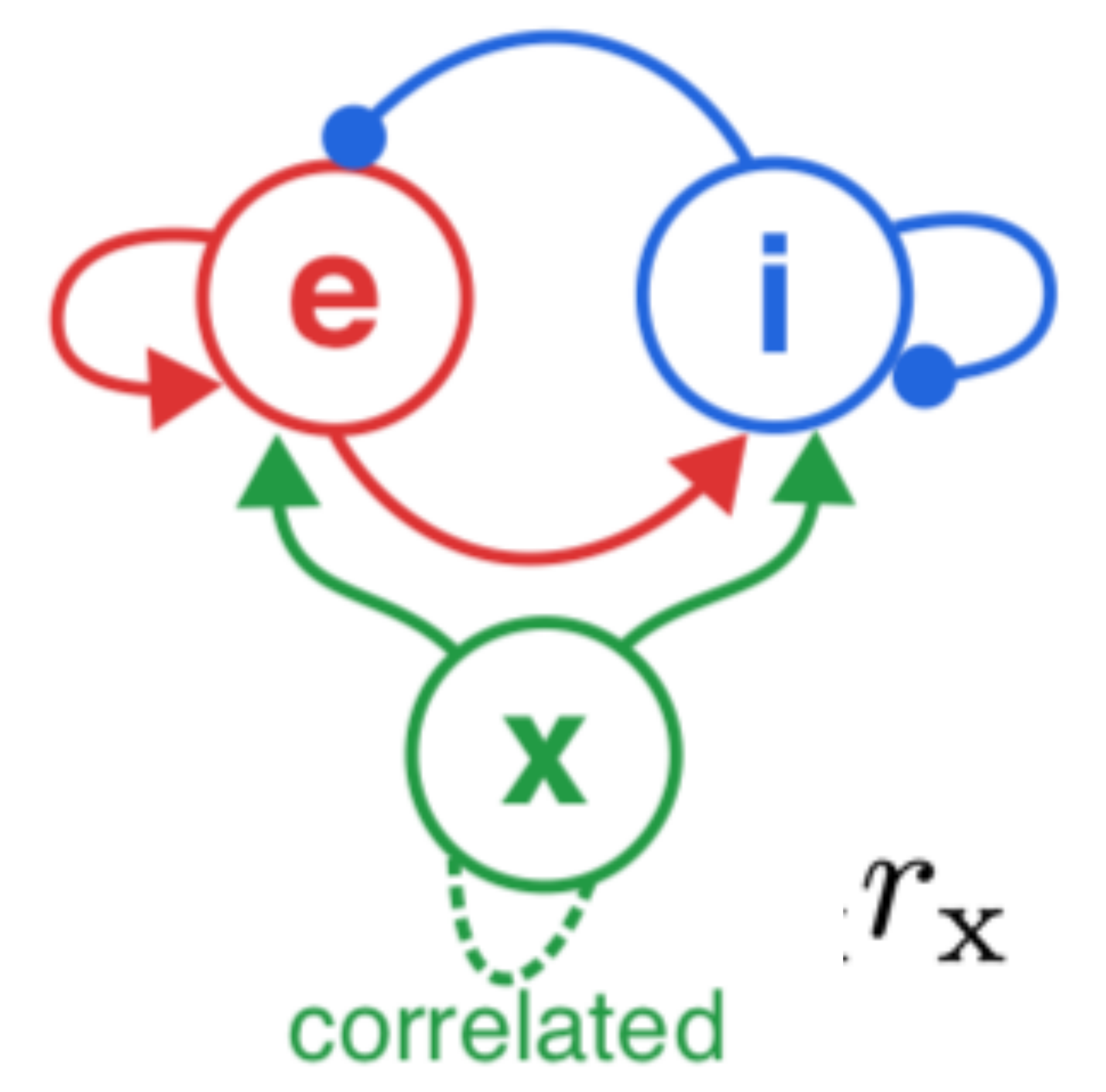


Vogels, et al 2011
Hennequin, et al 2017

Can we extend the mean-field theory of balanced networks to include plasticity?

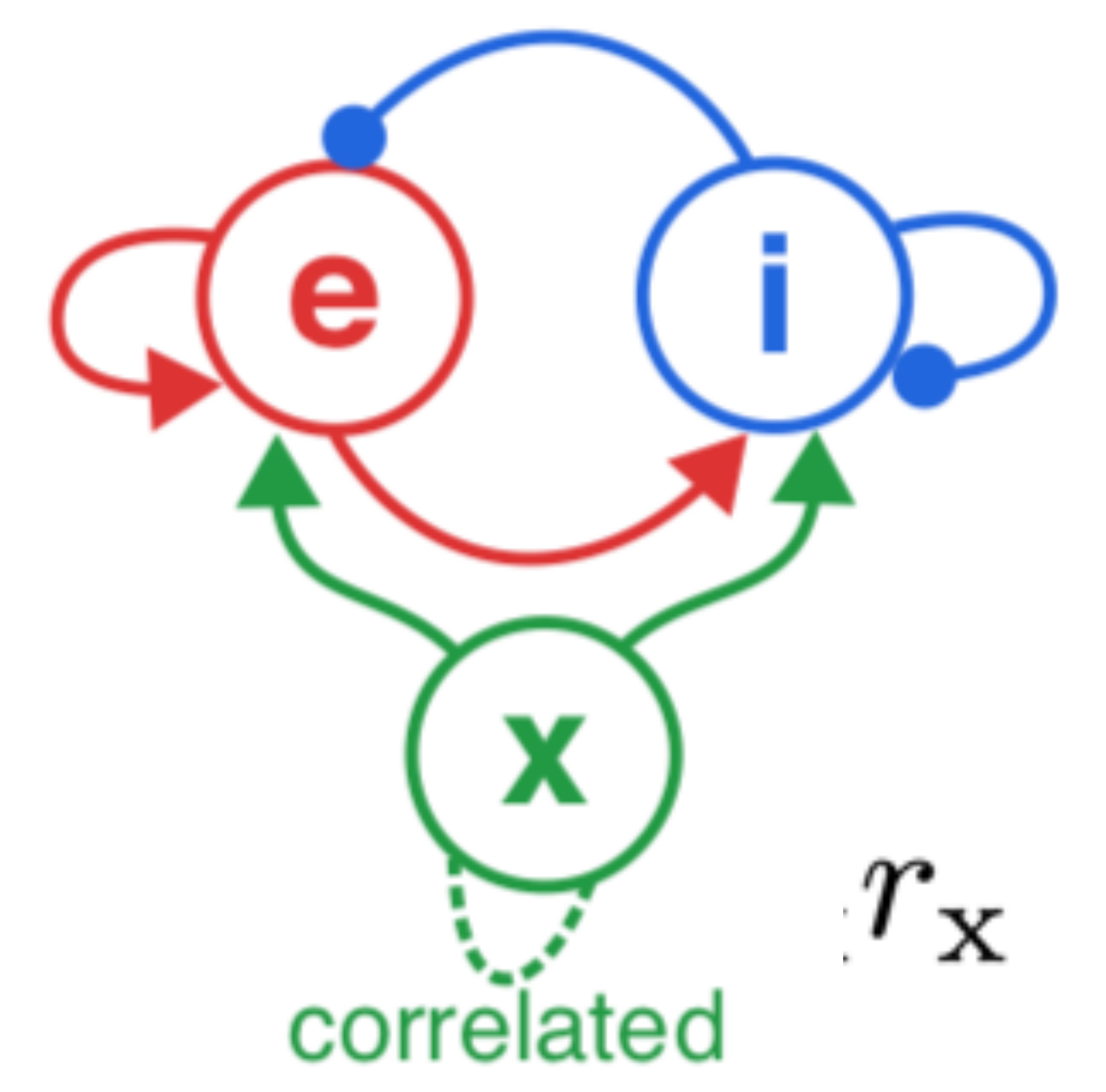
Can the theory predict when a balanced state is preserved?

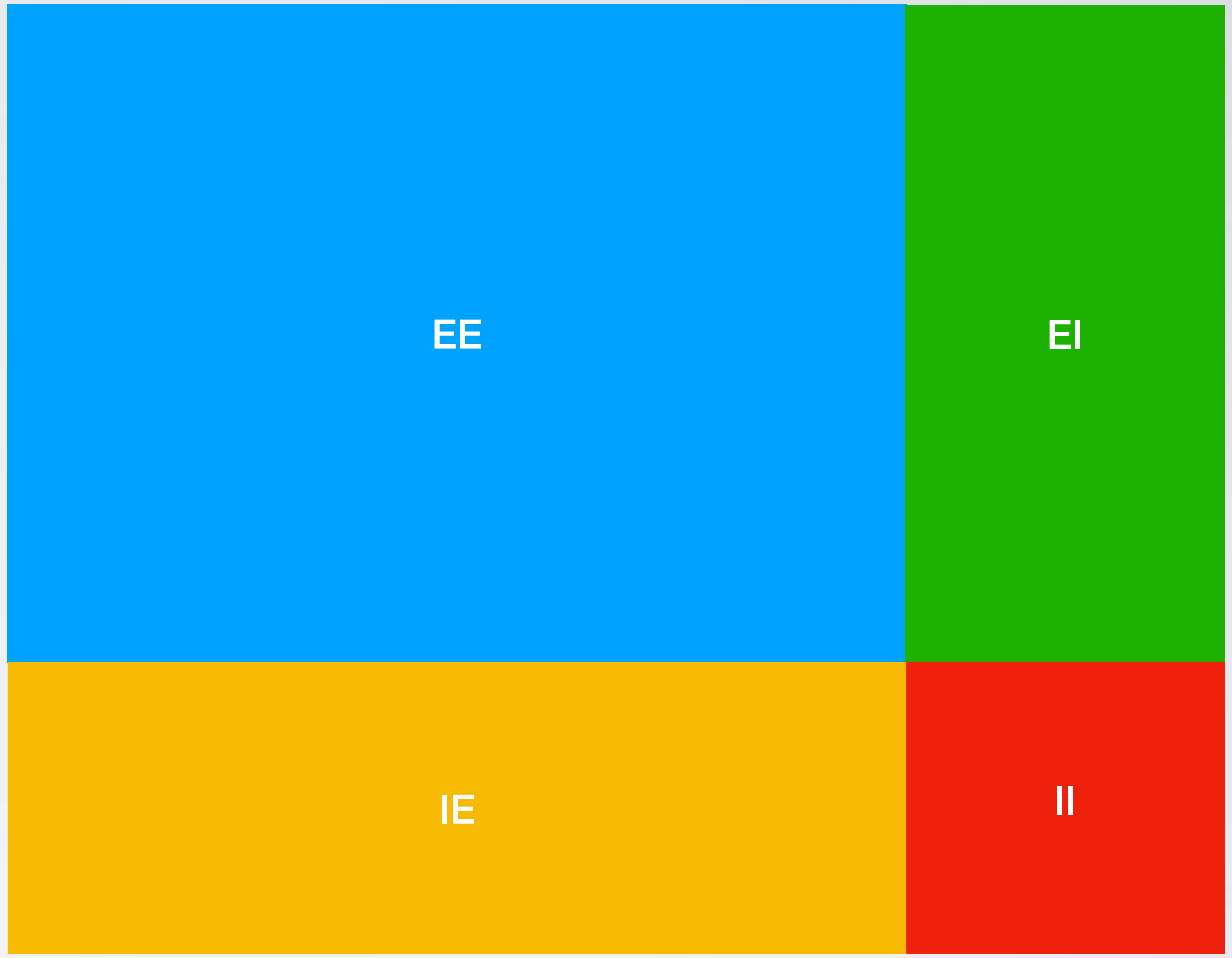
Network Model



Akil, Rosenbaum and Josić, PLoS Comp Bio 2021

Network Model

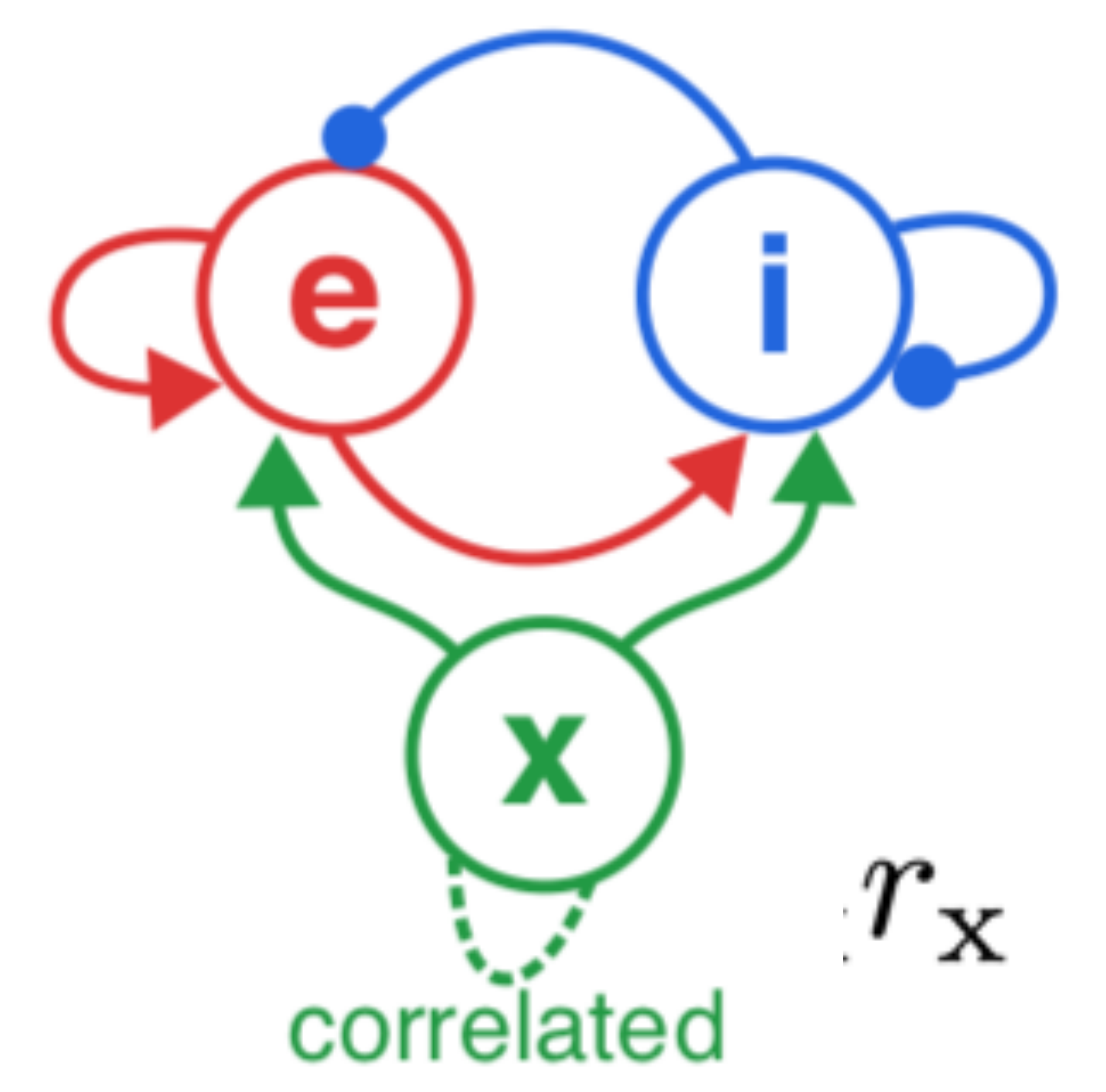




$$J_{jk}^{ab} = \frac{1}{\sqrt{N}} \begin{cases} j_{ab} & \text{with probability } p_{ab}, \\ 0 & \text{otherwise.} \end{cases}$$

Akil, Rosenbaum and Josić, PLoS Comp Bio 2021

Network Model



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Akil, Rosenbaum and Josić, PLoS Comp Bio 2021

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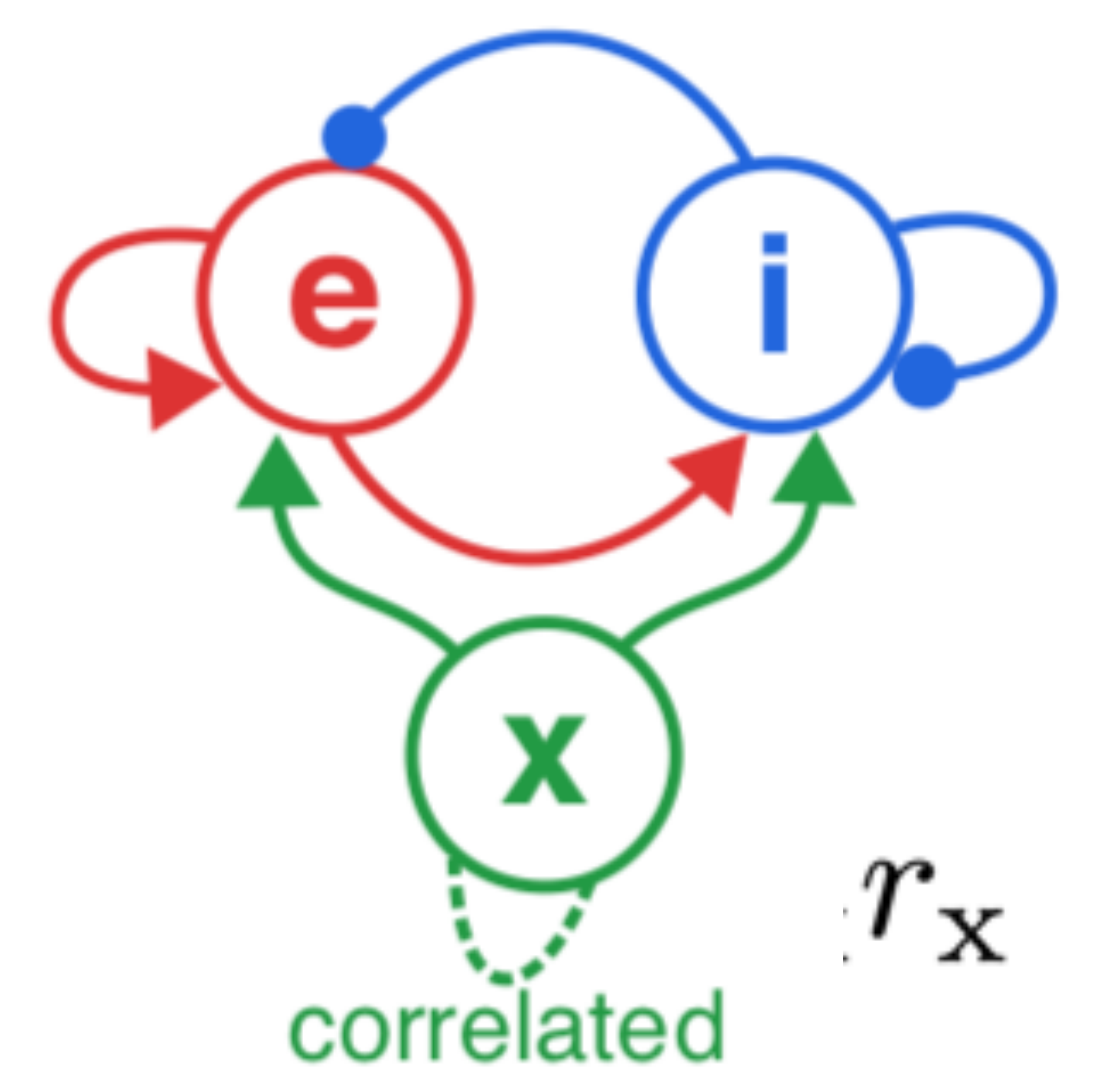


$$J_{jk}^{ab} = \frac{1}{\sqrt{N}} \begin{cases} j_{ab} & \text{with probability } p_{ab}, \\ 0 & \text{otherwise.} \end{cases}$$

$$\overline{W} = \begin{bmatrix} \overline{w}_{\rm ee} & \overline{w}_{\rm ei} \\ \overline{w}_{\rm ie} & \overline{w}_{\rm ii} \end{bmatrix}$$

Mean Recurrent Weights

$$\overline{W}_x = \begin{bmatrix} \overline{w}_{\rm ex} \\ \overline{w}_{\rm ix} \end{bmatrix}$$

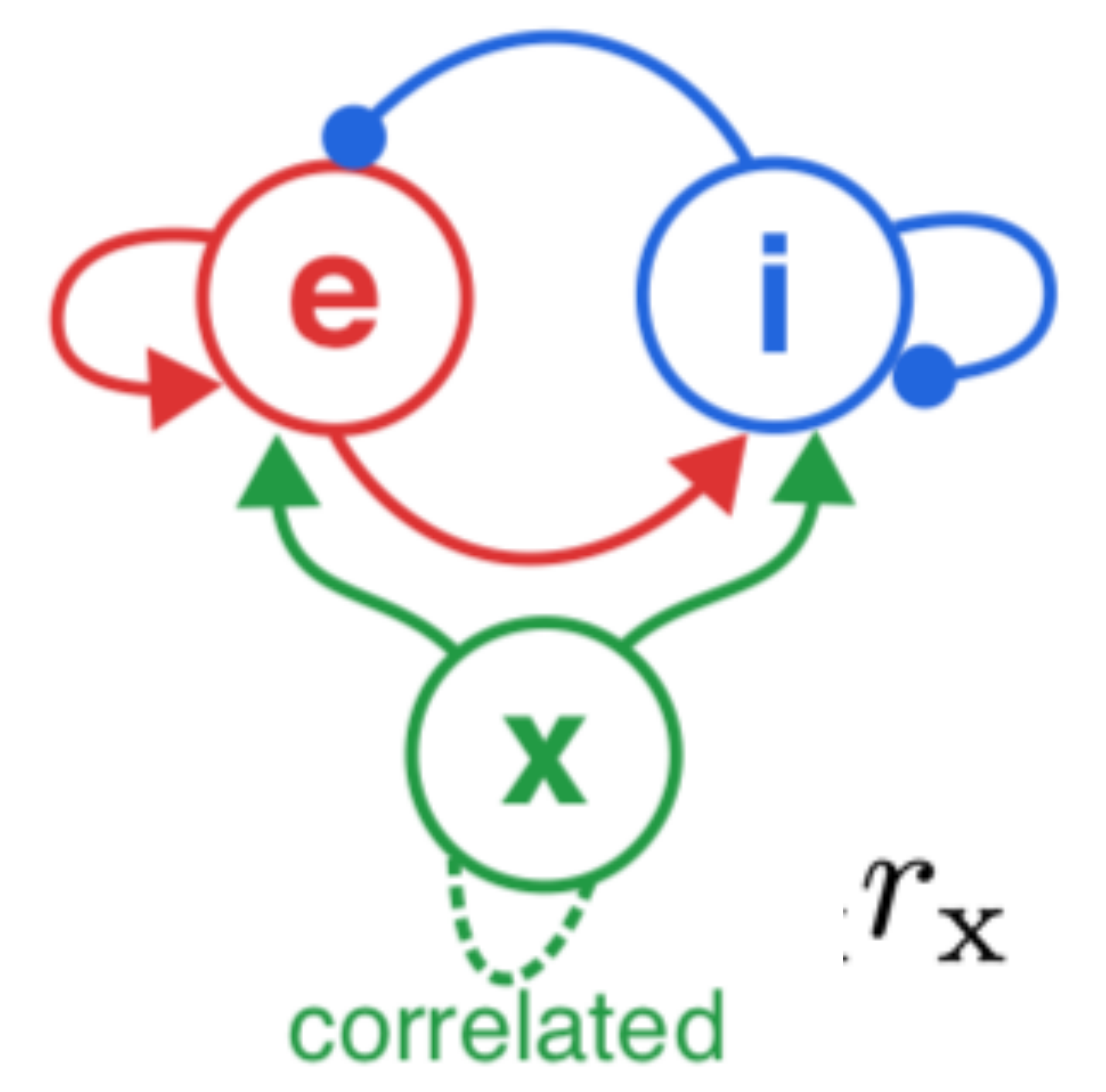
Mean Feedforward Weights

$$\mathbf{r} = \begin{bmatrix} \overline{r}_{\rm e} \\ \overline{r}_{\rm i} \end{bmatrix}$$

Mean Firing Rates

Akil, Rosenbaum and Josić, PLoS Comp Bio 2021

Network Model



$$J^{ab}_{jk} = \frac{1}{\sqrt{N}} \begin{cases} j_{ab} & \text{with probability } p_{ab}, \\ 0 & \text{otherwise.} \end{cases}$$

Akil, Rosenbaum and Josić, PLoS Comp Bio 2021

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Mean Feedforward Weights

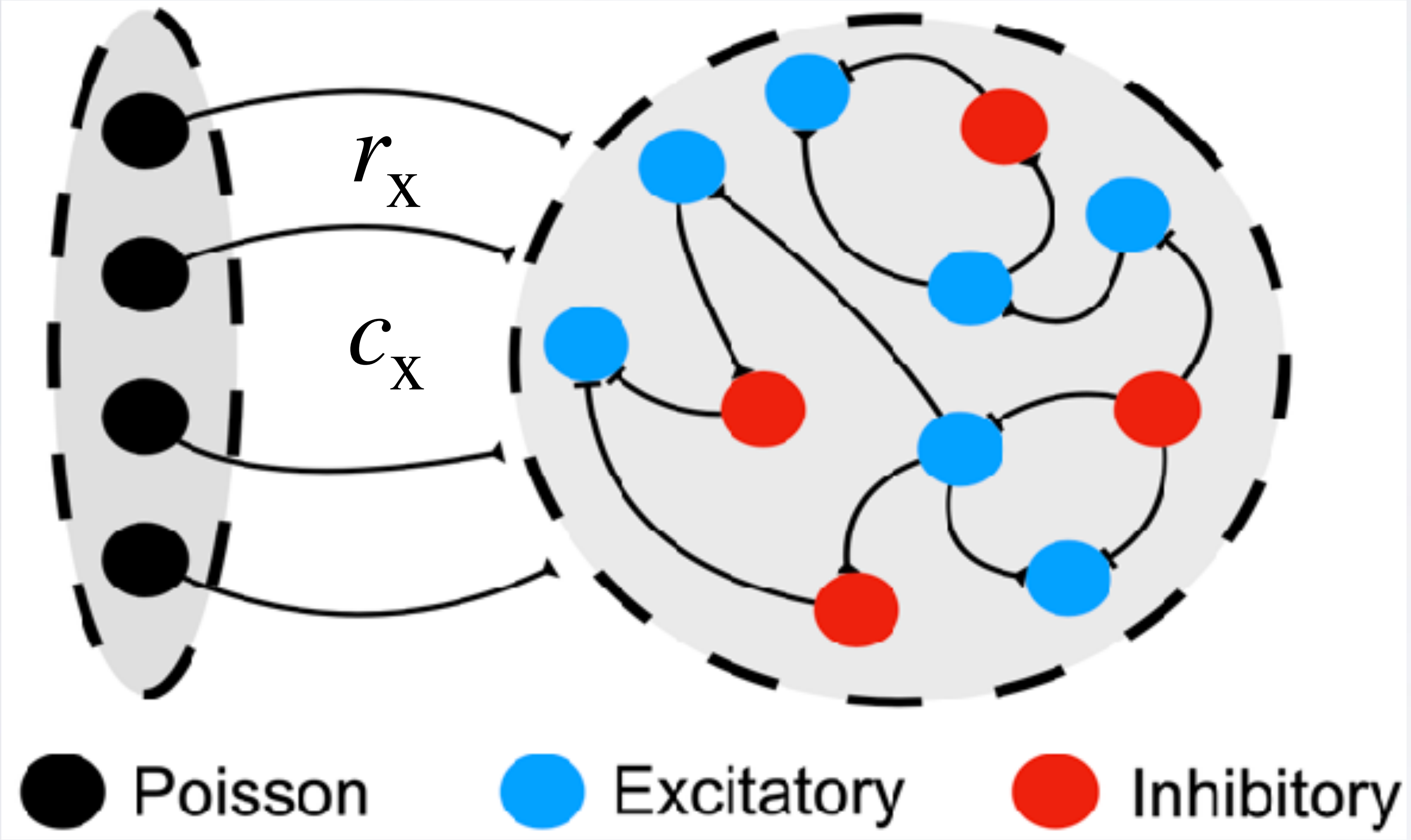
$$\mathbf{r} = \begin{bmatrix} \overline{r}_{\mathrm{e}} \\ \overline{r}_{\mathrm{i}} \end{bmatrix}$$

Mean Firing Rates

In balance the mean firing rates are:

$$\lim_{N\to\infty} \boldsymbol{r} = -\overline{W}^{-1}\overline{W}_{\mathrm{x}} r_{\mathrm{x}}$$

Correlated Balanced State

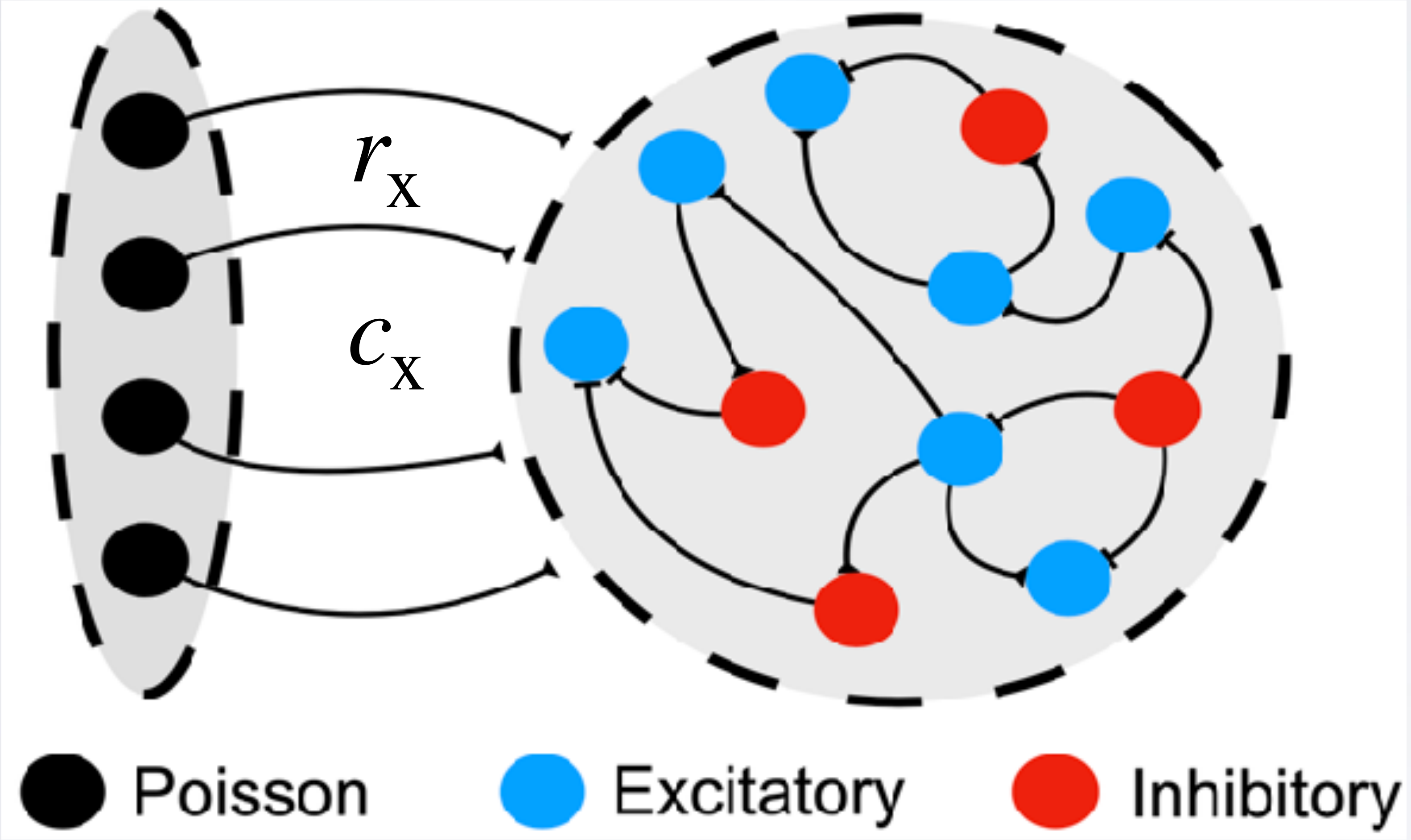


Spike count covariance matrix in window T_{win}

$$C = \begin{bmatrix} C_{\text{ee}} & C_{\text{ei}} \\ C_{\text{ie}} & C_{\text{ii}} \end{bmatrix}$$

Baker, et al. 2019

Correlated Balanced State



Spike count covariance matrix in window T_{win}

$$C = \begin{bmatrix} C_{ee} & C_{ei} \\ C_{ie} & C_{ii} \end{bmatrix}$$

If $c_x \neq 0$, then *C* remains *O(1),* and the network is in a correlated state.

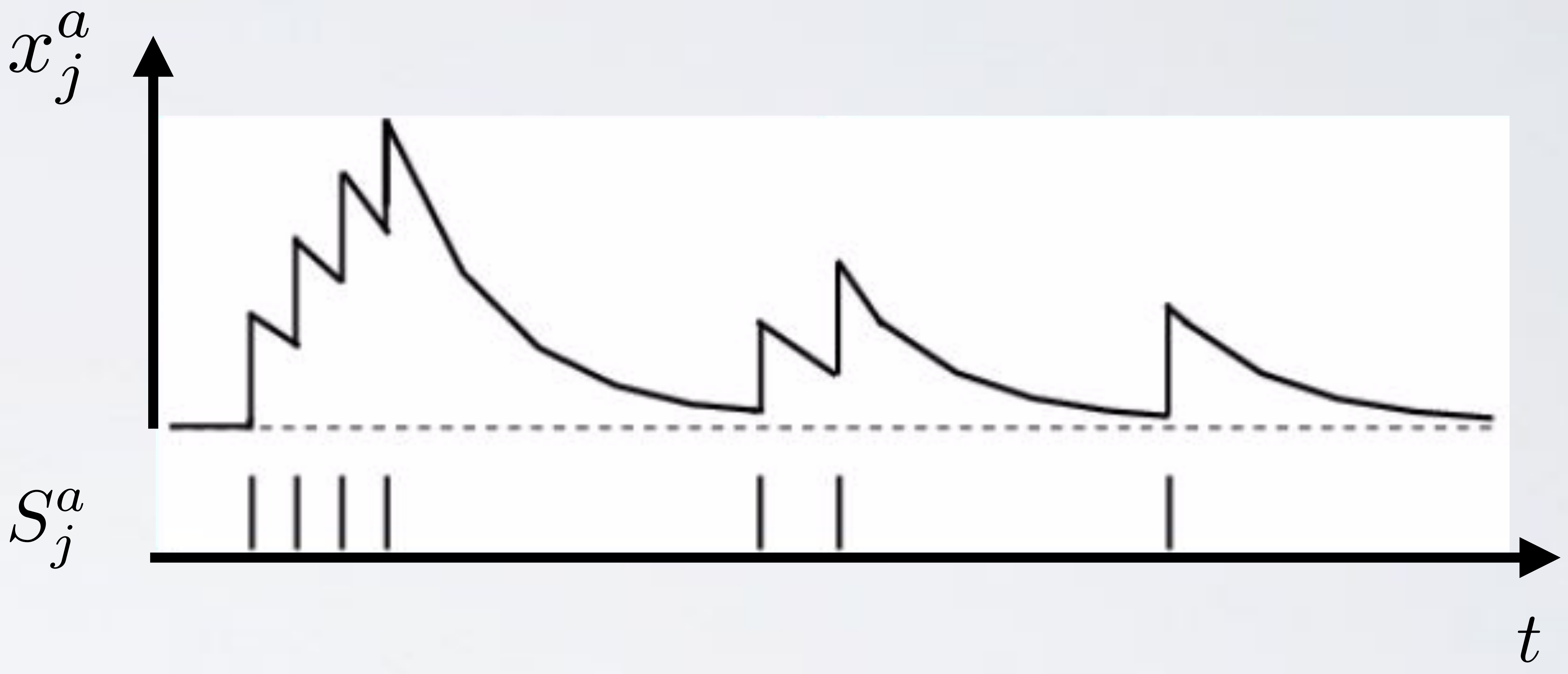
$$C \approx T_{win} c_x r_x \overline{W}^{-1} \overline{W}_x \overline{W}_x^T \overline{W}^{-T}$$

Baker, et al. 2019

Plasticity and Eligibility Traces

$$\tau_{\text{STDP}} \frac{dx_j^a(t)}{dt} = -x_j^a(t) + S_j^a(t)$$

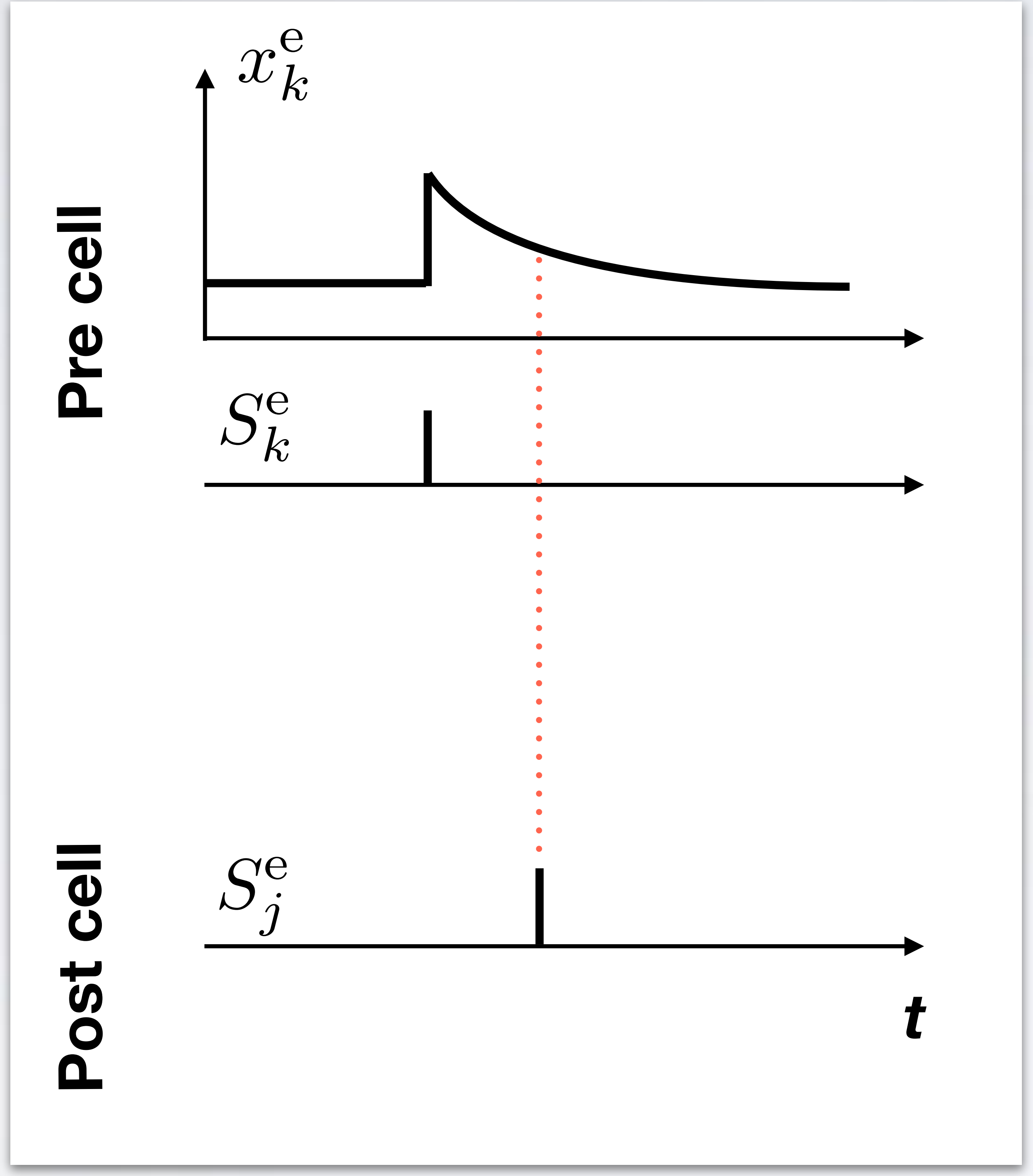
$$S_j^a(t) = \sum_n \delta(t - t_n^{a,j})$$



Example: Classical Hebbian plasticity

Klopf 1982, many others

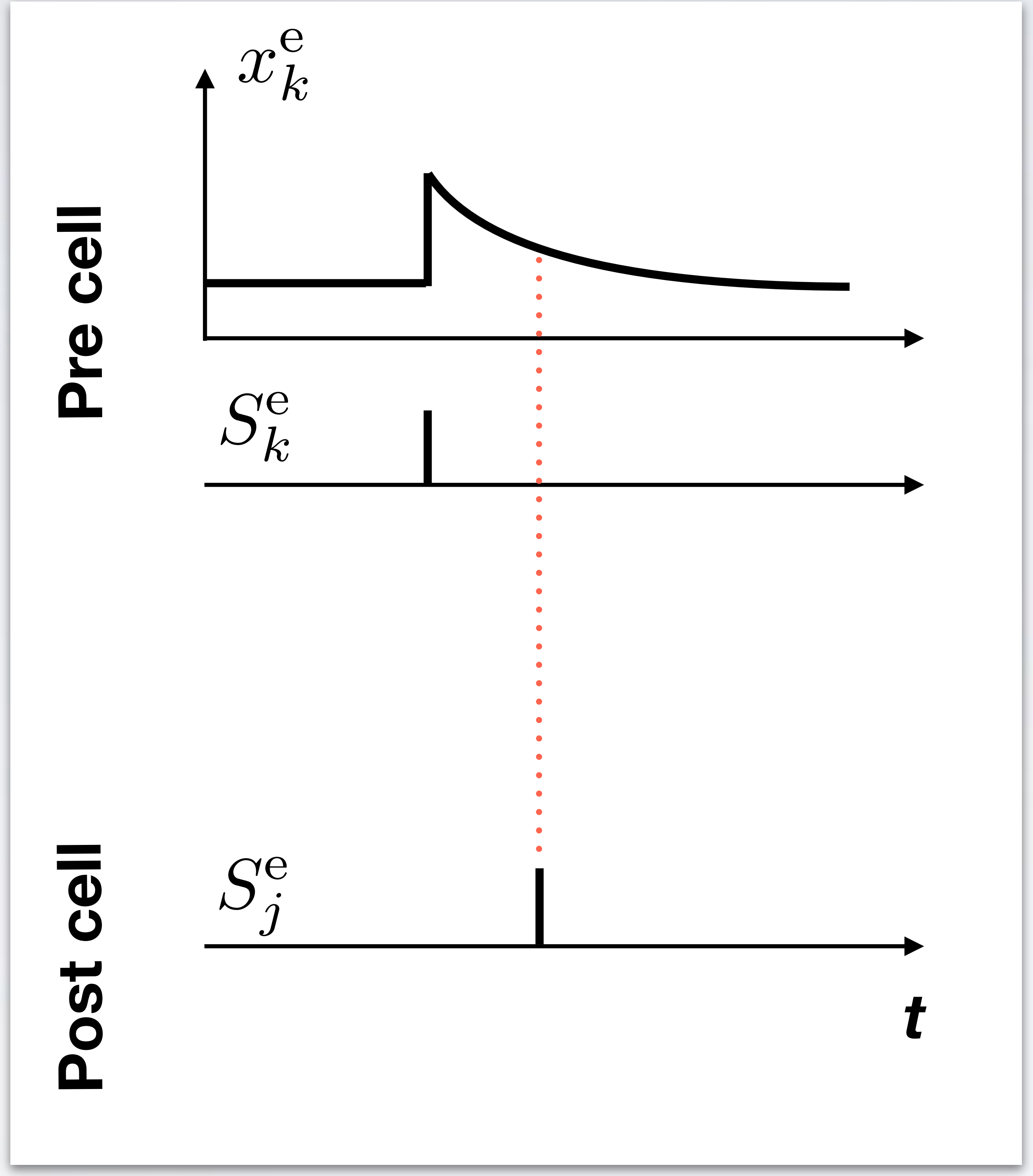
Example: Classical Hebbian plasticity



$$\frac{dJ_{jk}^{\mathrm{ee}}}{dt} =$$

Klopf 1982, many others

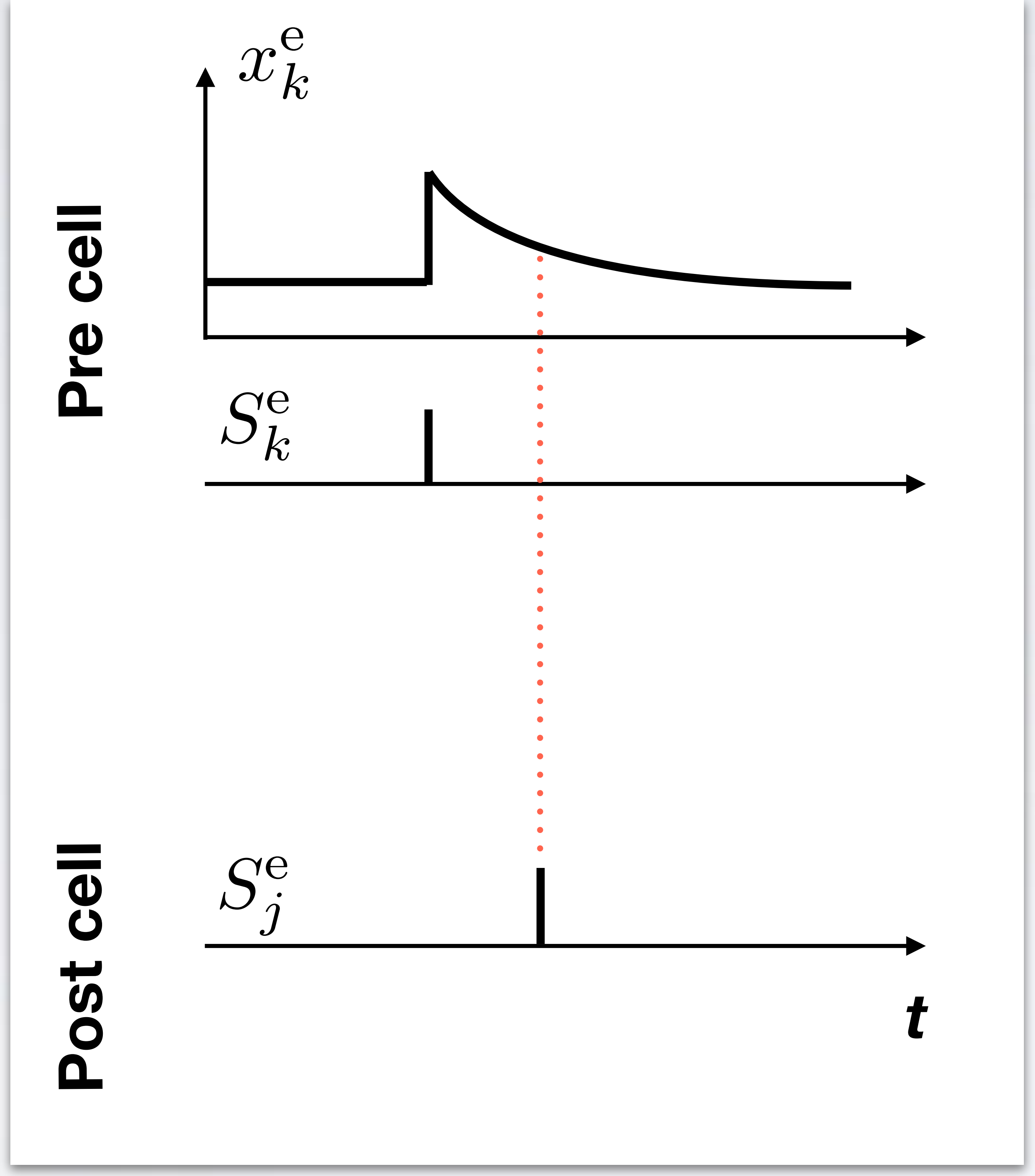
Example: Classical Hebbian plasticity

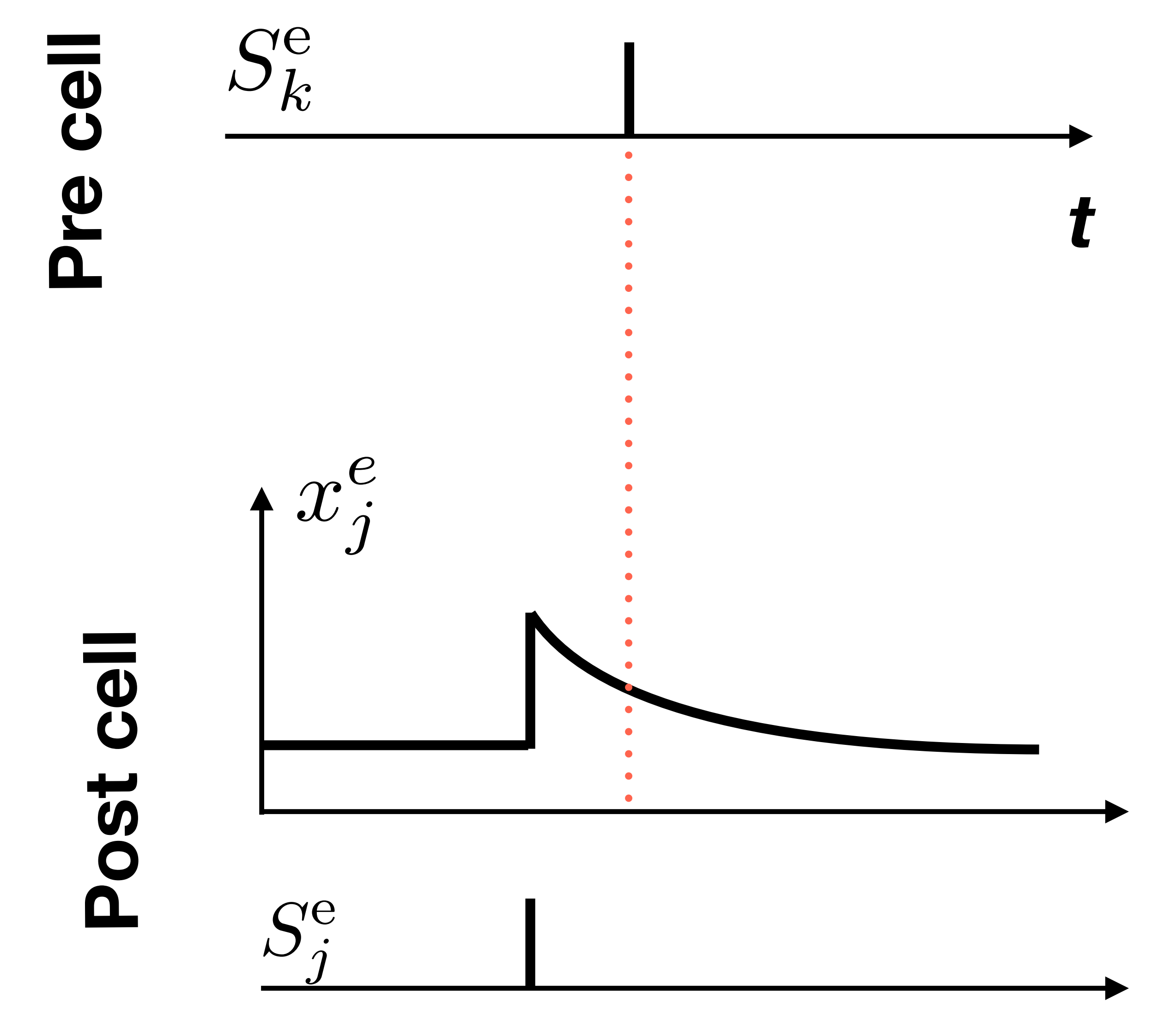


$$\frac{dJ_{jk}^{\mathrm{ee}}}{dt} = \qquad C_{\mathrm{pre-post}} x_k^{\mathrm{e}} S_j^{\mathrm{e}}$$

Klopf 1982, many others

Example: Classical Hebbian plasticity





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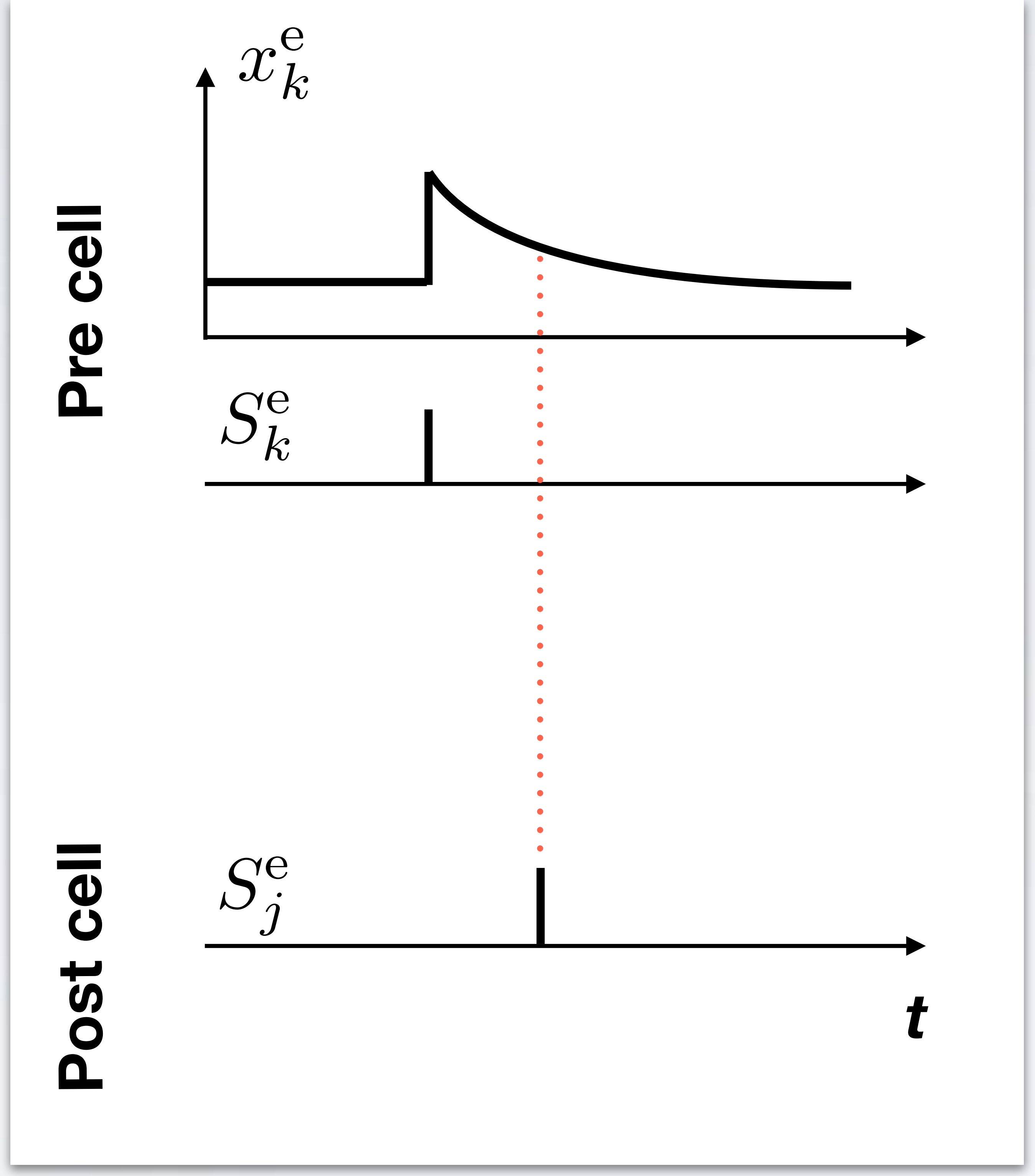
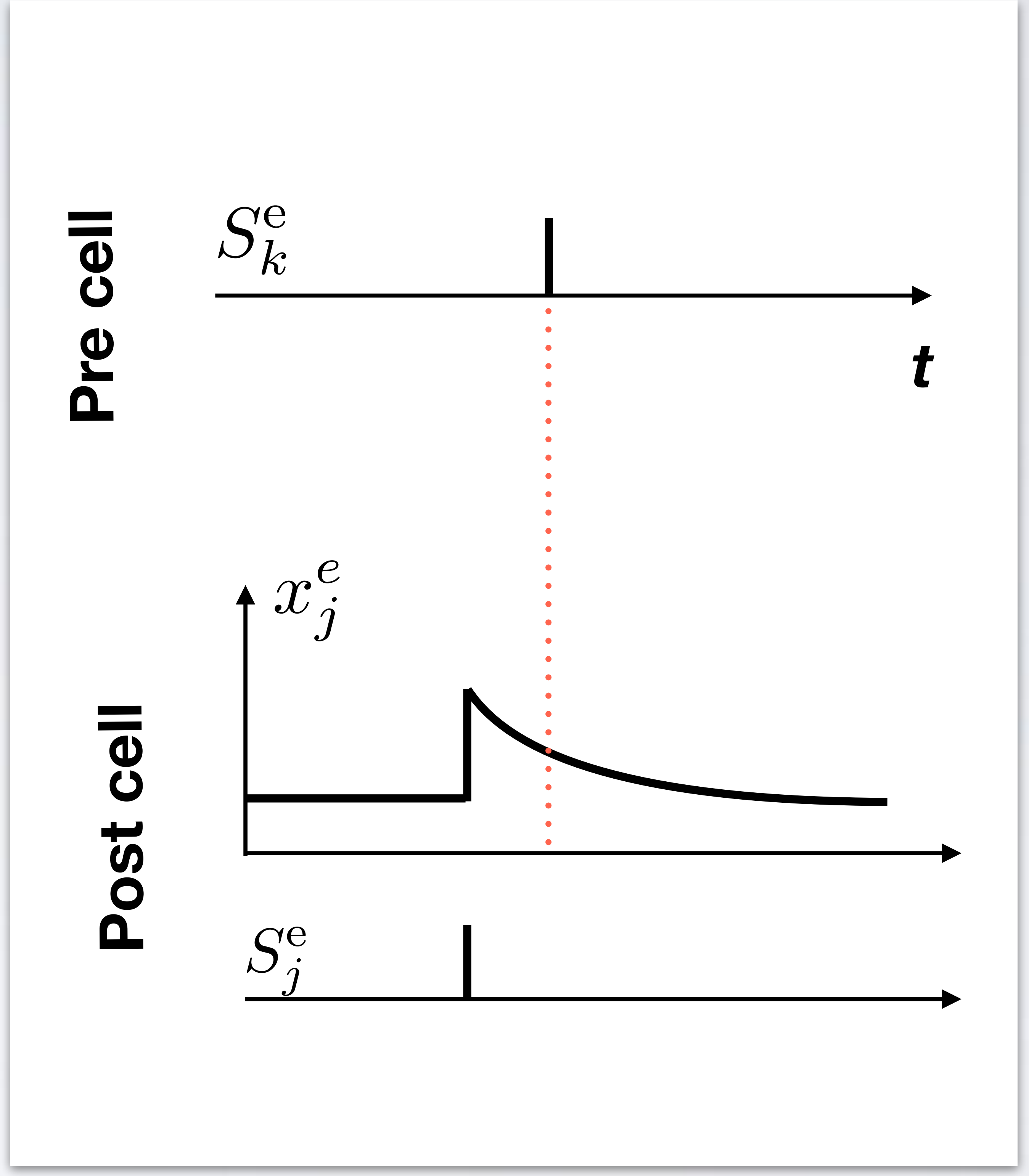
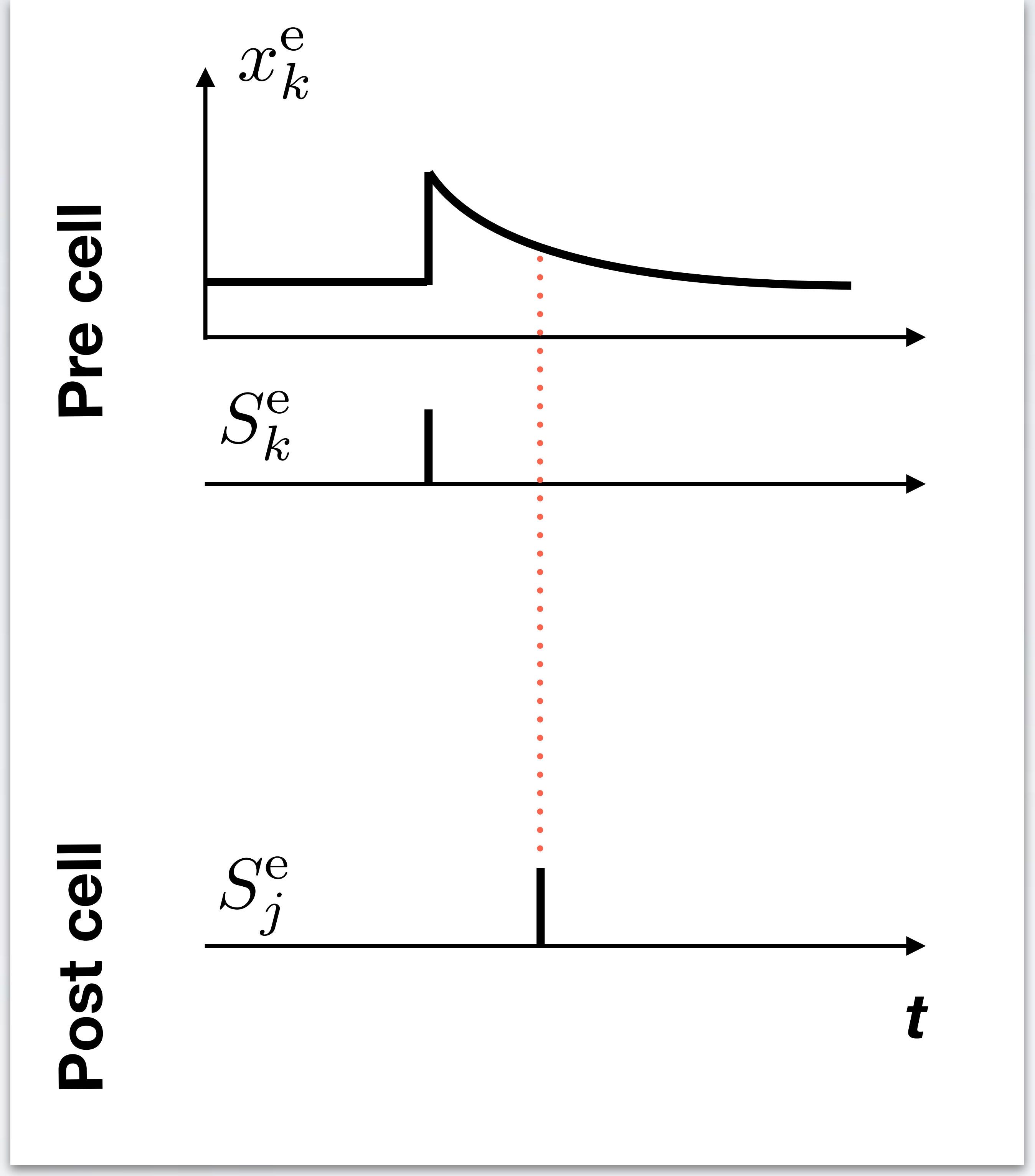


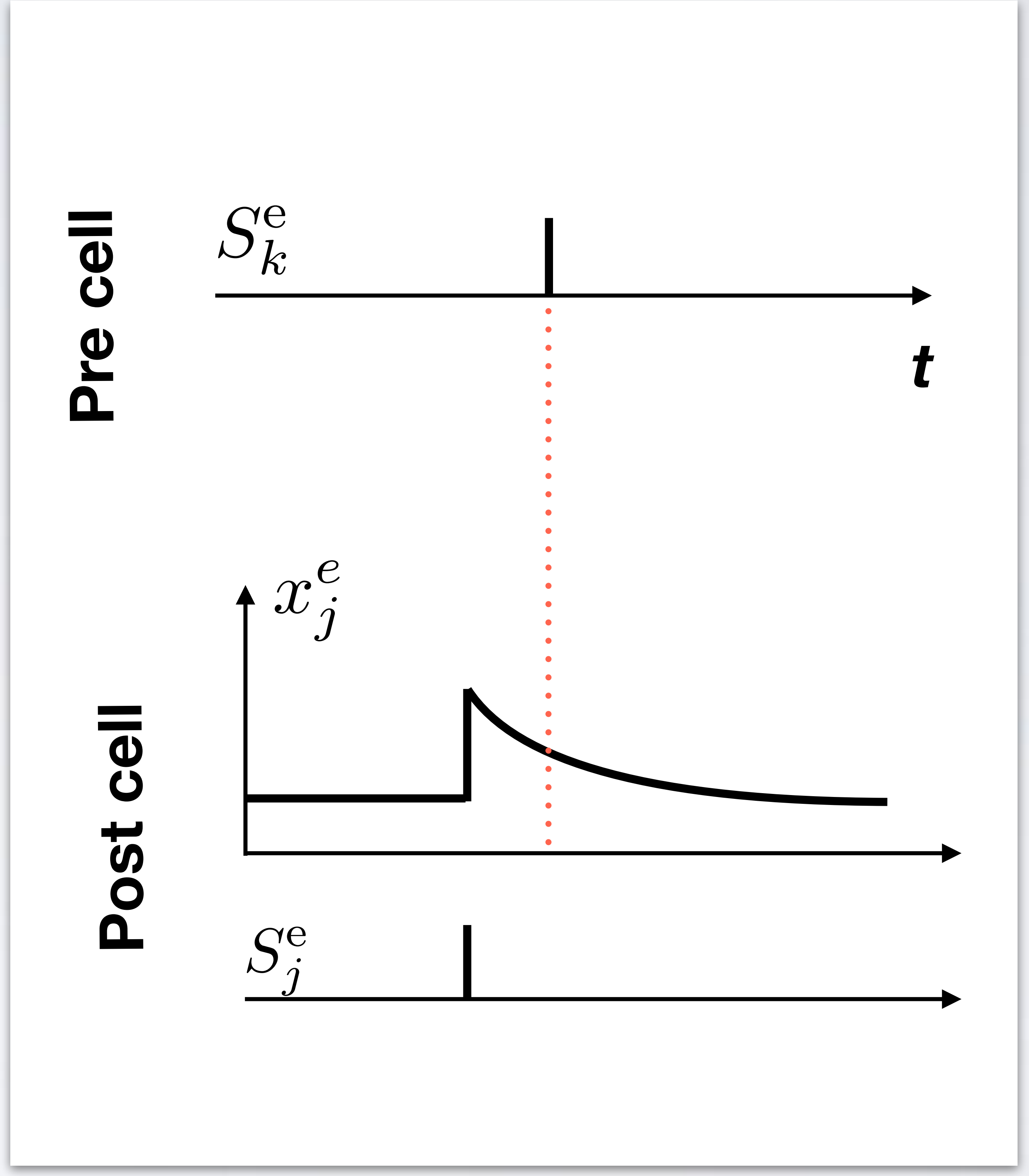


$$\frac{dJ_{jk}^{\mathrm{ee}}}{dt} = C_{\mathrm{pre-post}} x_k^\mathrm{e} S_j^\mathrm{e} - C_{\mathrm{post-pre}} x_j^\mathrm{e} S_k^\mathrm{e}$$

Klopf 1982, many others

Example: Classical Hebbian plasticity

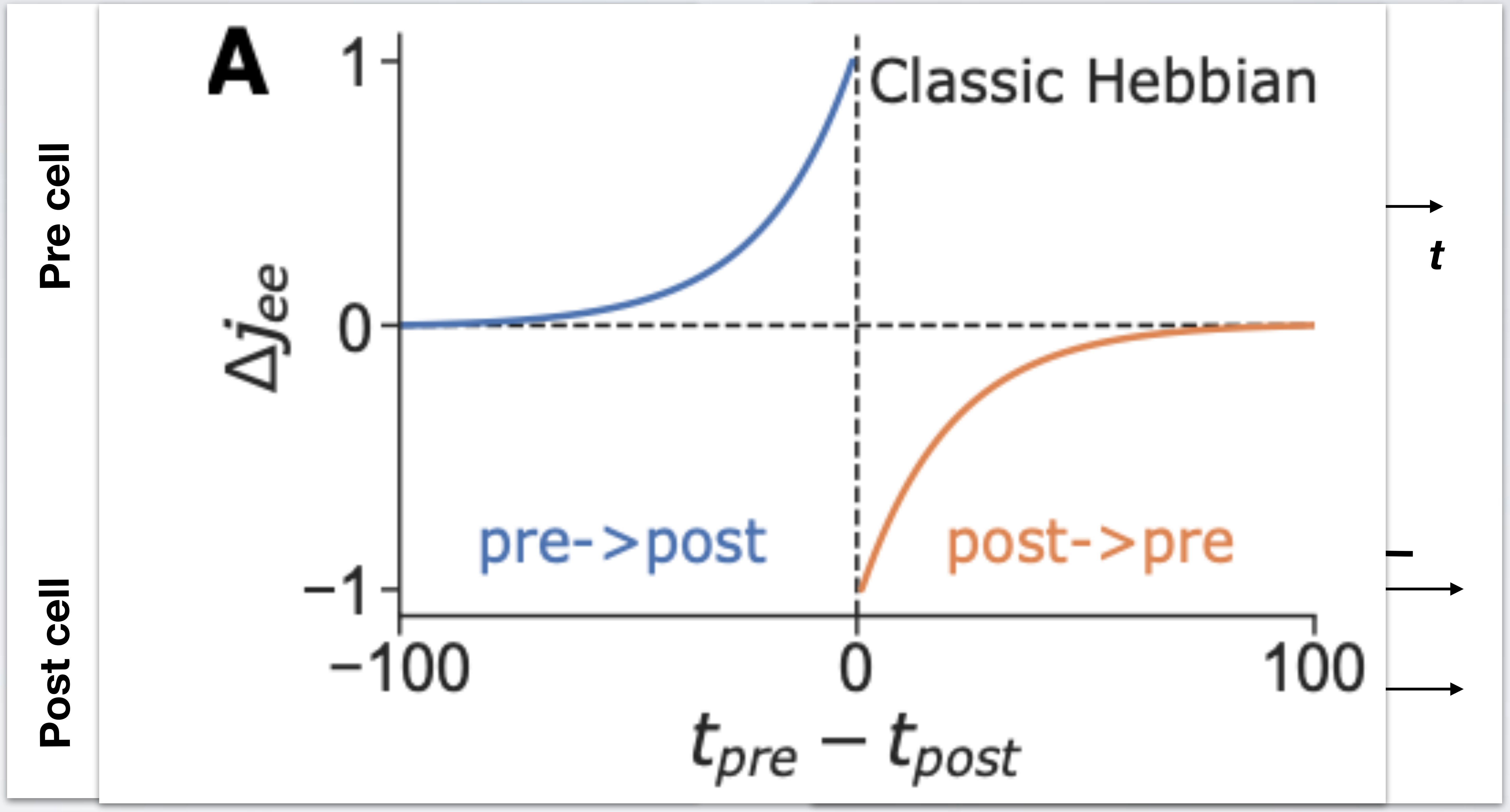




$$\frac{dJ_{jk}^{\mathrm{ee}}}{dt} = \eta_{\mathrm{ee}}\left(C_{\mathrm{pre-post}} x_k^{\mathrm{e}} S_j^{\mathrm{e}} - C_{\mathrm{post-pre}} x_j^{\mathrm{e}} S_k^{\mathrm{e}} \right)$$

Klopf 1982, many others

Example: Classical Hebbian plasticity



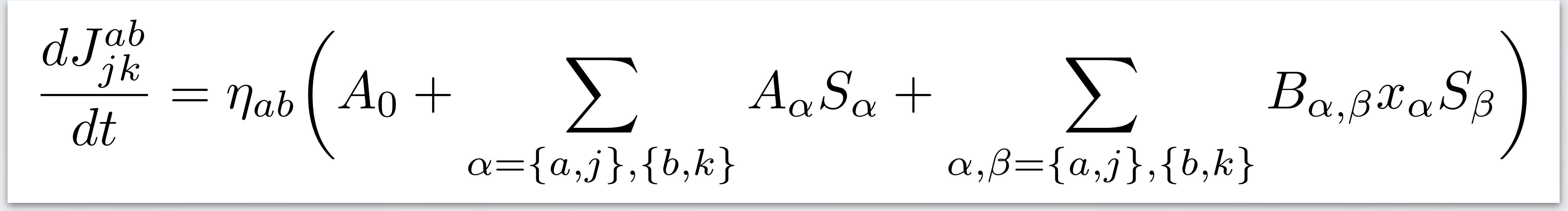
$$\frac{dJ^{\mathrm{ee}}_{jk}}{dt} = \eta_{\mathrm{ee}}\left(C_{\mathrm{pre-post}} x^{\mathrm{e}}_k S^{\mathrm{e}}_j \quad - \quad C_{\mathrm{post-pre}} x^{\mathrm{e}}_j S^{\mathrm{e}}_k \right)$$

Klopf 1982, many others

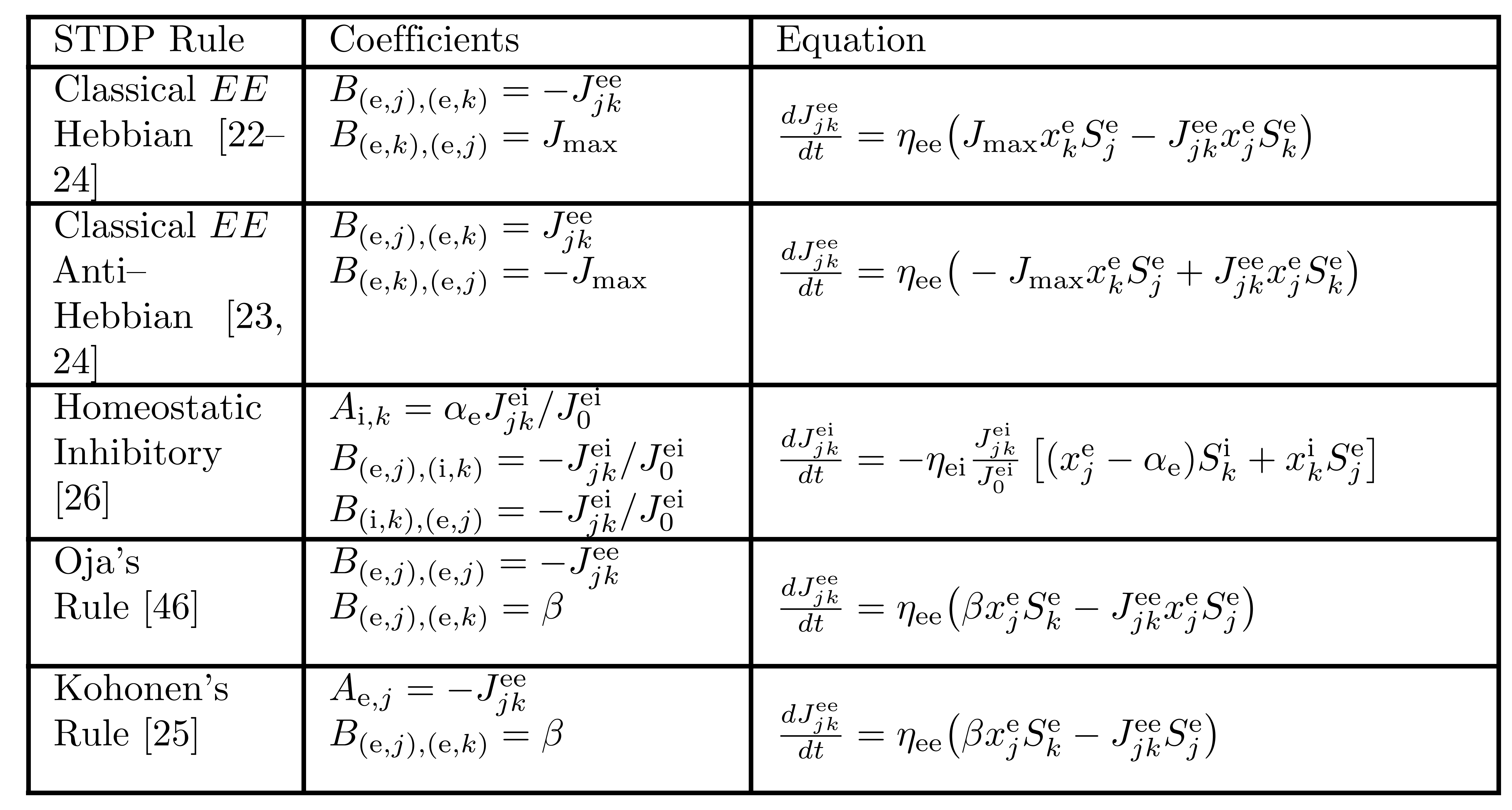
Plasticity Rules with up to Second Order Interactions

$$\frac{dJ_{jk}^{ab}}{dt} = \eta_{ab}\left(A_0 + \sum_{\alpha=\{a,j\},\{b,k\}} A_\alpha S_\alpha + \sum_{\alpha,\beta=\{a,j\},\{b,k\}} B_{\alpha,\beta} x_\alpha S_\beta\right)$$

Plasticity Rules with up to Second Order Interactions



$$\frac{dJ^{ab}_{jk}}{dt} = \eta_{ab}\Bigg(A_0 + \sum_{\alpha=\{a,j\},\{b,k\}} A_\alpha S_\alpha + \sum_{\alpha,\beta=\{a,j\},\{b,k\}} B_{\alpha,\beta} x_\alpha S_\beta\Bigg)$$



STDP Rule	Coefficients	Equation
Classical EE Hebbian [22–24]	$B_{(\mathrm{e},j),(\mathrm{e},k)} = -J^{\mathrm{ee}}_{jk}$ $B_{(\mathrm{e},k),(\mathrm{e},j)} = J_{\max}$	$\frac{dJ^{\mathrm{ee}}_{jk}}{dt} = \eta_{\mathrm{ee}}\big(J_{\max} x^{\mathrm{e}}_k S^{\mathrm{e}}_j - J^{\mathrm{ee}}_{jk} x^{\mathrm{e}}_j S^{\mathrm{e}}_k\big)$
Classical EE Anti–Hebbian [23, 24]	$B_{(\mathrm{e},j),(\mathrm{e},k)} = J^{\mathrm{ee}}_{jk}$ $B_{(\mathrm{e},k),(\mathrm{e},j)} = -J_{\max}$	$\frac{dJ^{\mathrm{ee}}_{jk}}{dt} = \eta_{\mathrm{ee}}\big(- J_{\max} x^{\mathrm{e}}_k S^{\mathrm{e}}_j + J^{\mathrm{ee}}_{jk} x^{\mathrm{e}}_j S^{\mathrm{e}}_k\big)$
Homeostatic Inhibitory [26]	$A_{\mathrm{i},k} = \alpha_{\mathrm{e}} J^{\mathrm{ei}}_{jk}/J^{\mathrm{ei}}_0$ $B_{(\mathrm{e},j),(\mathrm{i},k)} = -J^{\mathrm{ei}}_{jk}/J^{\mathrm{ei}}_0$ $B_{(\mathrm{i},k),(\mathrm{e},j)} = -J^{\mathrm{ei}}_{jk}/J^{\mathrm{ei}}_0$	$\frac{dJ^{\mathrm{ei}}_{jk}}{dt} = -\eta_{\mathrm{ei}} \frac{J^{\mathrm{ei}}_{jk}}{J^{\mathrm{ei}}_0} \left[(x^{\mathrm{e}}_j - \alpha_{\mathrm{e}}) S^{\mathrm{i}}_k + x^{\mathrm{i}}_k S^{\mathrm{e}}_j\right]$
Oja's Rule [46]	$B_{(\mathrm{e},j),(\mathrm{e},j)} = -J^{\mathrm{ee}}_{jk}$ $B_{(\mathrm{e},j),(\mathrm{e},k)} = \beta$	$\frac{dJ^{\mathrm{ee}}_{jk}}{dt} = \eta_{\mathrm{ee}}\big(\beta x^{\mathrm{e}}_j S^{\mathrm{e}}_k - J^{\mathrm{ee}}_{jk} x^{\mathrm{e}}_j S^{\mathrm{e}}_j\big)$
Kohonen's Rule [25]	$A_{\mathrm{e},j} = -J^{\mathrm{ee}}_{jk}$ $B_{(\mathrm{e},j),(\mathrm{e},k)} = \beta$	$\frac{dJ^{\mathrm{ee}}_{jk}}{dt} = \eta_{\mathrm{ee}}\big(\beta x^{\mathrm{e}}_j S^{\mathrm{e}}_k - J^{\mathrm{ee}}_{jk} S^{\mathrm{e}}_j\big)$

Averaging synaptic weight dynamics

$$\frac{dJ^{ab}_{jk}}{dt} = \eta_{ab}\left(A_0 + \sum_{\alpha=\{a,j\},\{b,k\}} A_\alpha S_\alpha + \sum_{\alpha,\beta=\{a,j\},\{b,k\}} B_{\alpha,\beta} x_\alpha S_\beta\right)$$

↓

Average in time, and over different network realizations

↓

$$\frac{dJ_{ab}}{dt} = \eta_{ab}\left(A_0 + A_a r_a + \textbf{Rate}_{a,b} + \textbf{Cov}_{a,b}\right)$$

Closing Equations using Quasi Steady State Assumption

Firing rates:

$$\boxed{\lim_{N\to\infty} \boldsymbol{r} = -\overline{W}^{-1}\overline{W}_{\mathrm{x}} r_{\mathrm{x}}}$$

Spike count covariances:

$$\boxed{C \approx \frac{1}{N} T_{\mathrm{win}} \overline{W}^{-1} \Gamma \overline{W}^{-T}}$$

Mean synaptic weights:

$$\boxed{\tau_{J_{ab}} \frac{dJ_{ab}}{dt} = A_0 + \sum_{\alpha,\beta=\{a,b\}} \mathrm{Rate}_{\alpha,\beta} + \mathrm{Cov}_{\alpha,\beta}}$$

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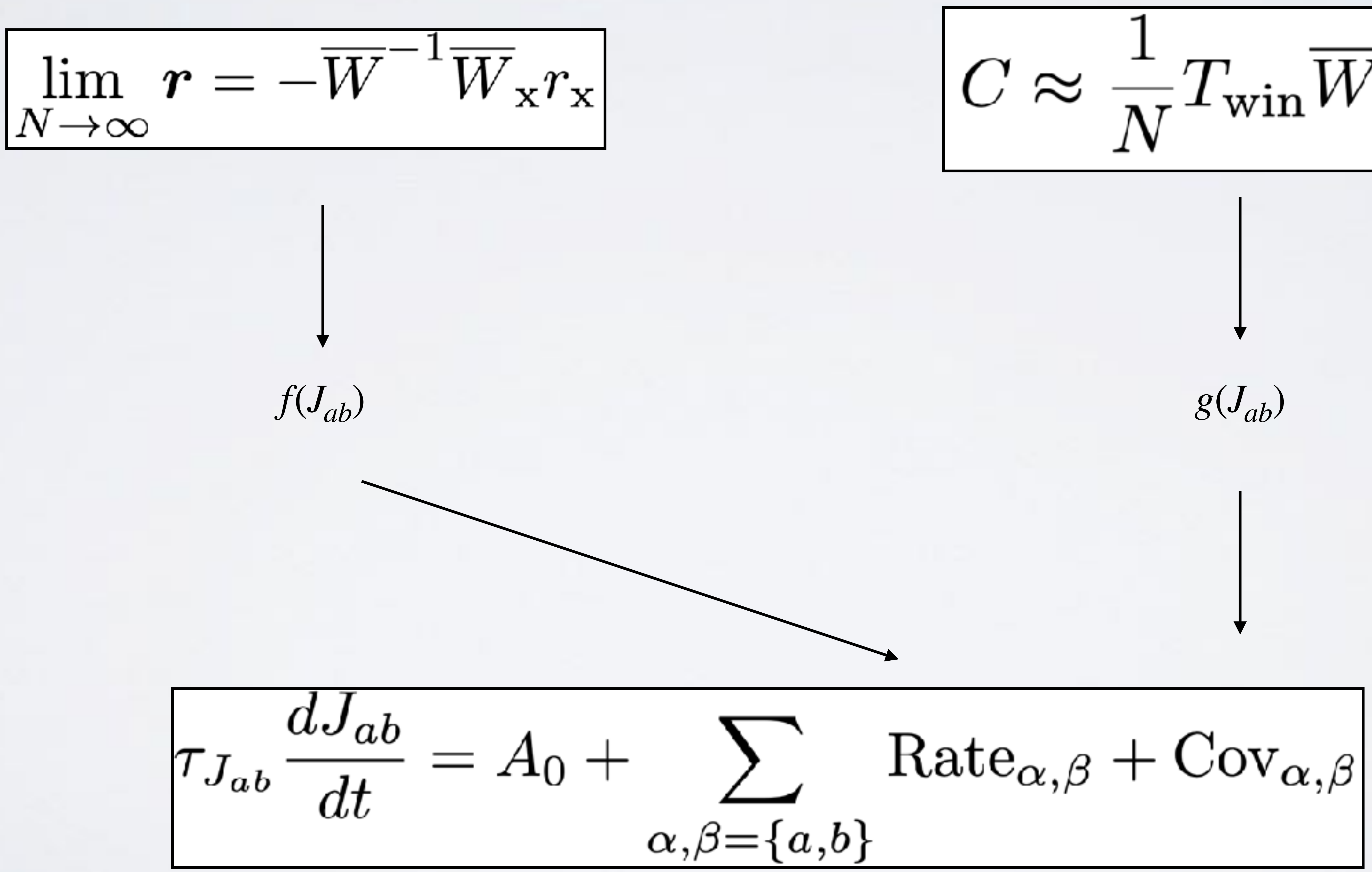


$$\boxed{\lim_{N\to\infty} \boldsymbol{r} = -\overline{W}^{-1}\overline{W}_{\mathrm{x}} r_{\mathrm{x}}}$$

$$\boxed{C \approx \frac{1}{N} T_{\mathrm{win}} \overline{W}^{-1}\Gamma\overline{W}^{-T}}$$

$f(J_{ab})$

$g(J_{ab})$

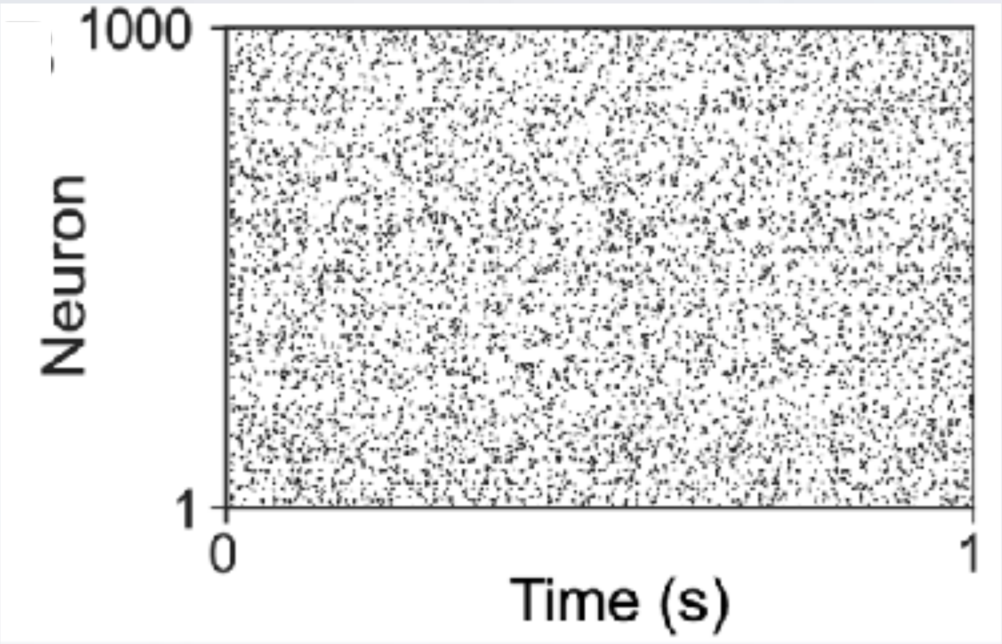
$$\boxed{\tau_{J_{ab}}\frac{dJ_{ab}}{dt} = A_0 + \sum_{\alpha,\beta=\{a,b\}} \mathrm{Rate}_{\alpha,\beta} + \mathrm{Cov}_{\alpha,\beta}}$$

Closing Equations using Quasi Steady State Assumption

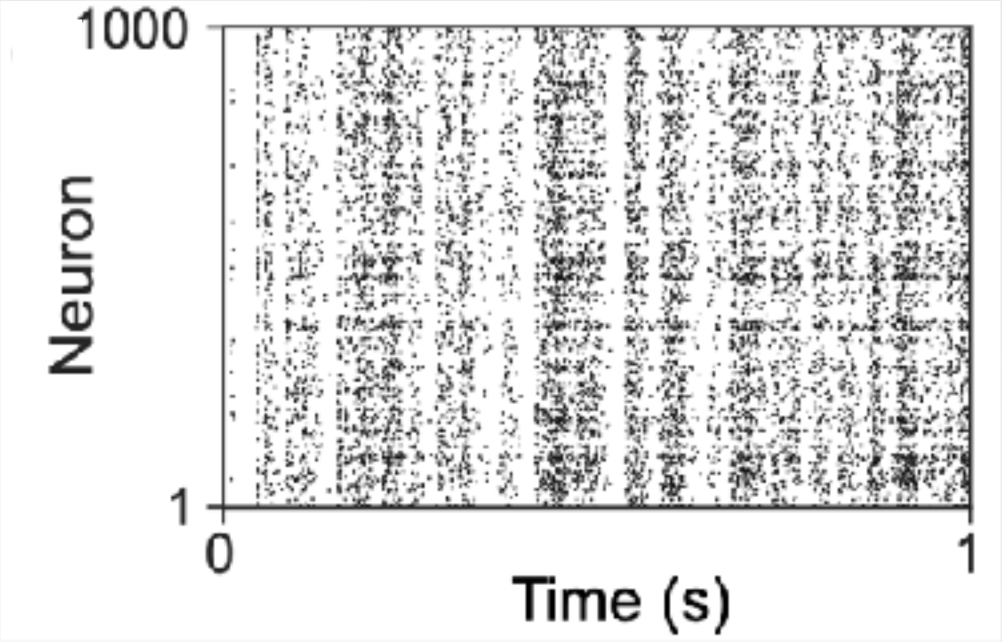
$$\begin{tikzcd}
\boxed{\lim_{N\to\infty} \boldsymbol{r} = -\overline{W}^{-1}\overline{W}_{\mathrm{x}} r_{\mathrm{x}}} \arrow[d] & & \boxed{C \approx \frac{1}{N} T_{\mathrm{win}} \overline{W}^{-1} \Gamma \overline{W}^{-T}} \arrow[d] \\
f(J_{ab}) \arrow[dr] & & g(J_{ab}) \arrow[d] \\
& \boxed{\tau_{J_{ab}} \frac{dJ_{ab}}{dt} = A_0 + \sum_{\alpha,\beta=\{a,b\}} \mathrm{Rate}_{\alpha,\beta} + \mathrm{Cov}_{\alpha,\beta}} & {}
\end{tikzcd}$$

Theory predicts stable asynchronous and correlated states

Asynchronous



Correlated



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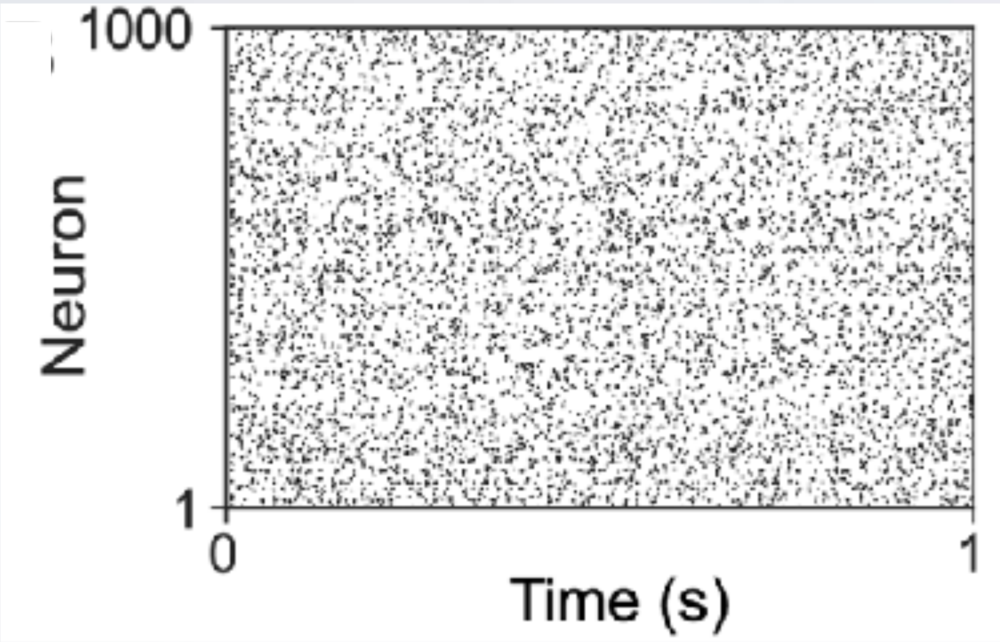

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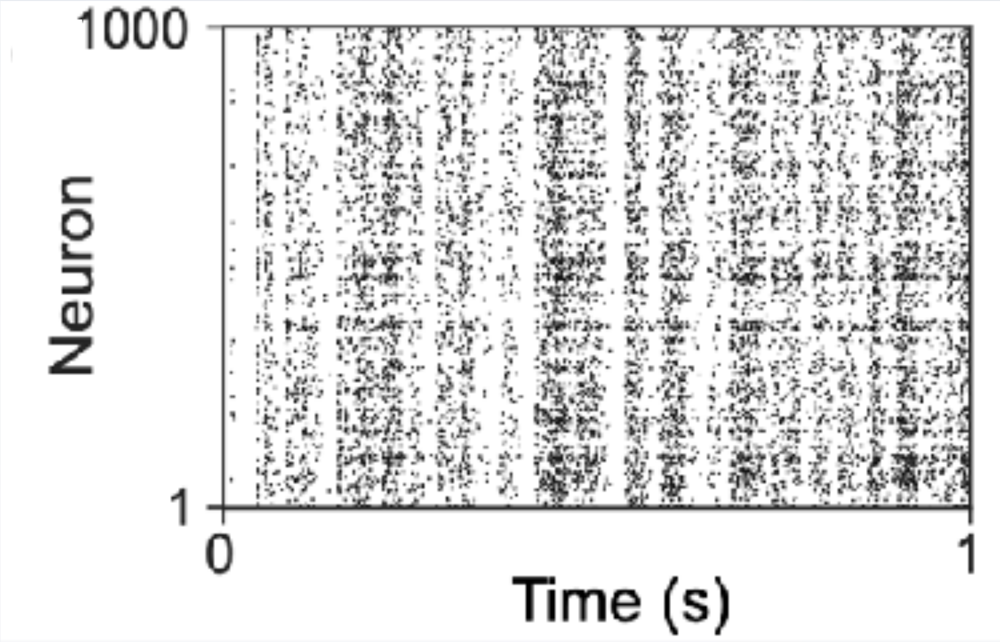
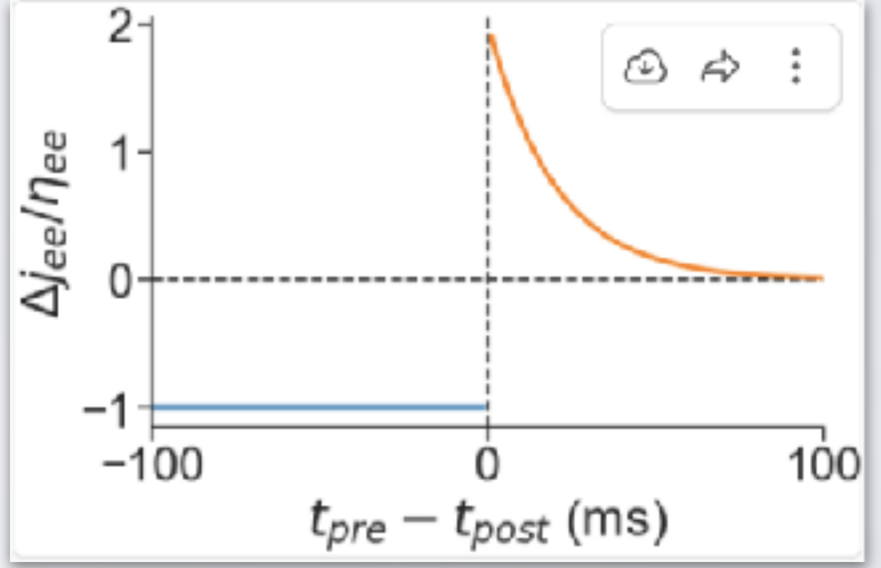


Kohonen's Rule

$$\frac{dJ^{\mathrm{ee}}_{jk}}{dt} = \eta_{\mathrm{ee}}\big(\beta x^{\mathrm{e}}_j S^{\mathrm{e}}_k - J^{\mathrm{ee}}_{jk} S^{\mathrm{e}}_j\big)$$



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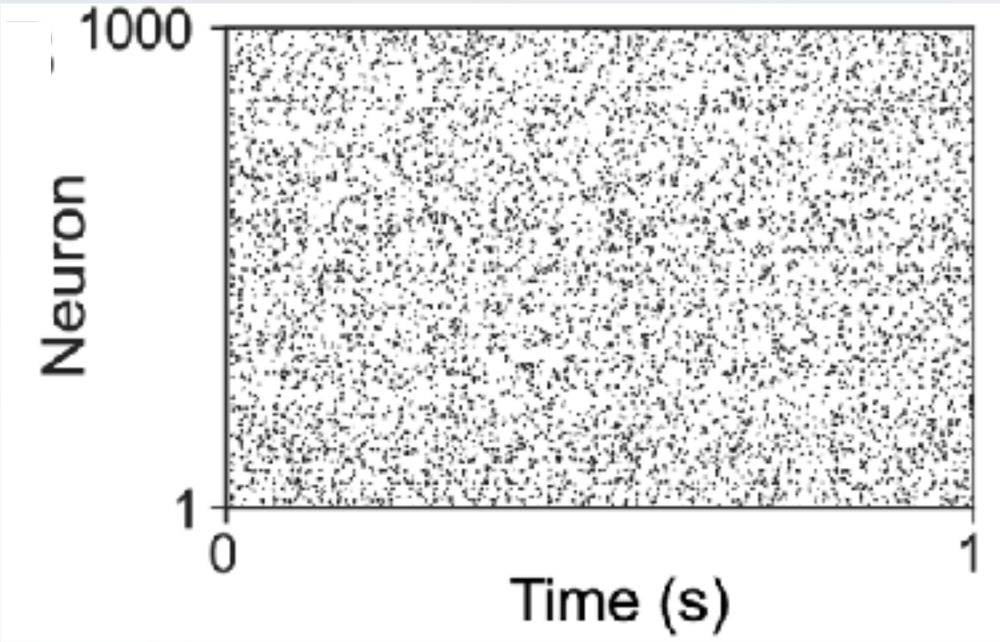

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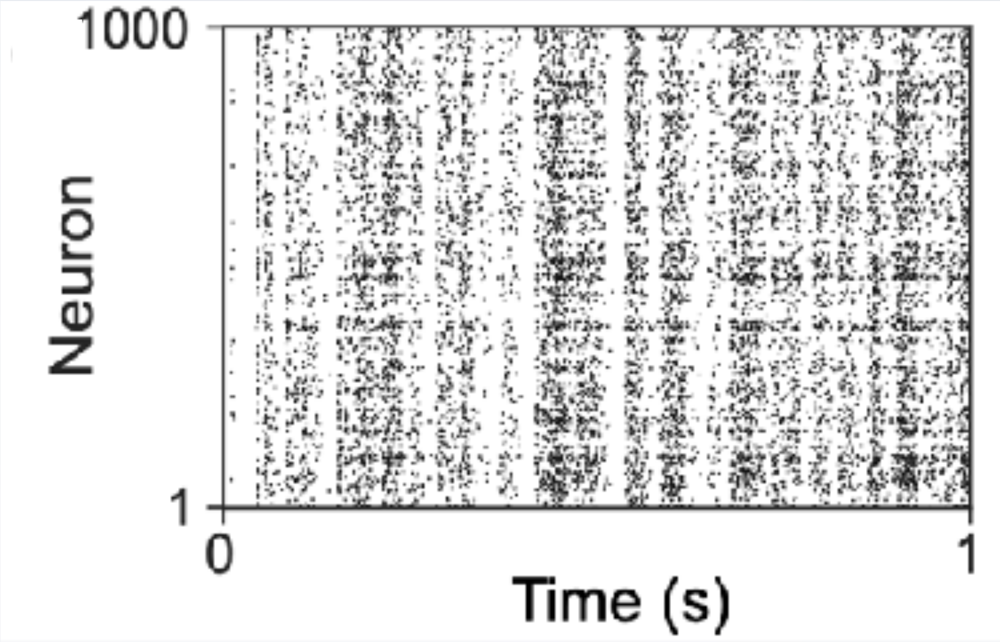

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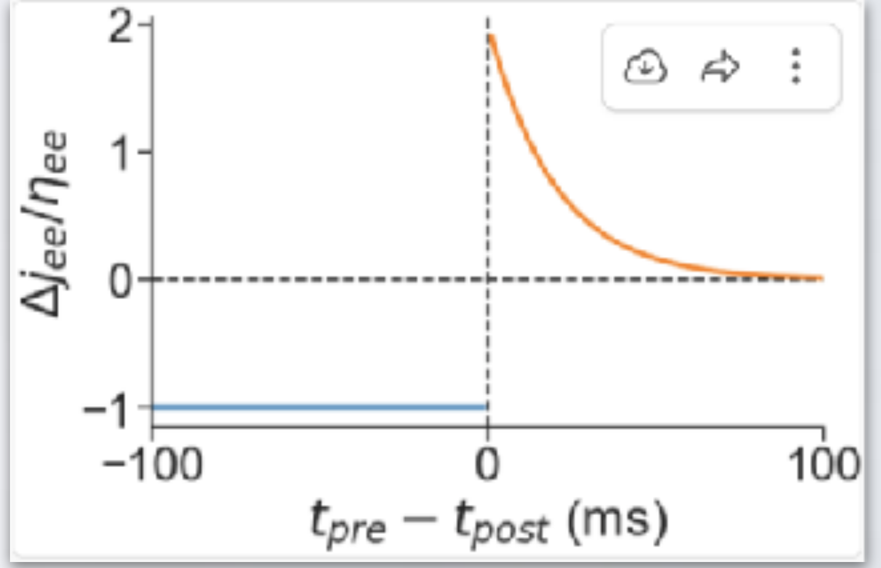

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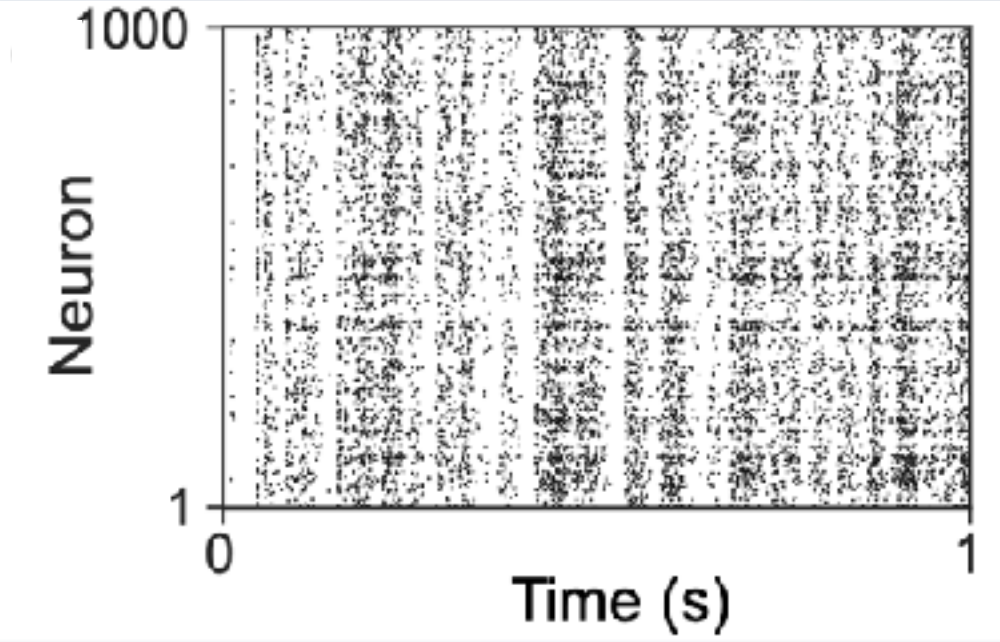

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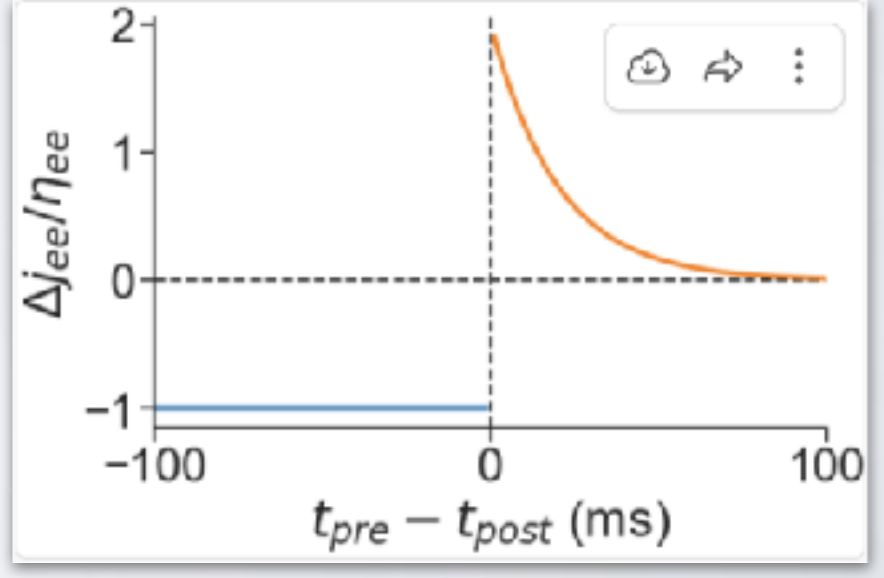

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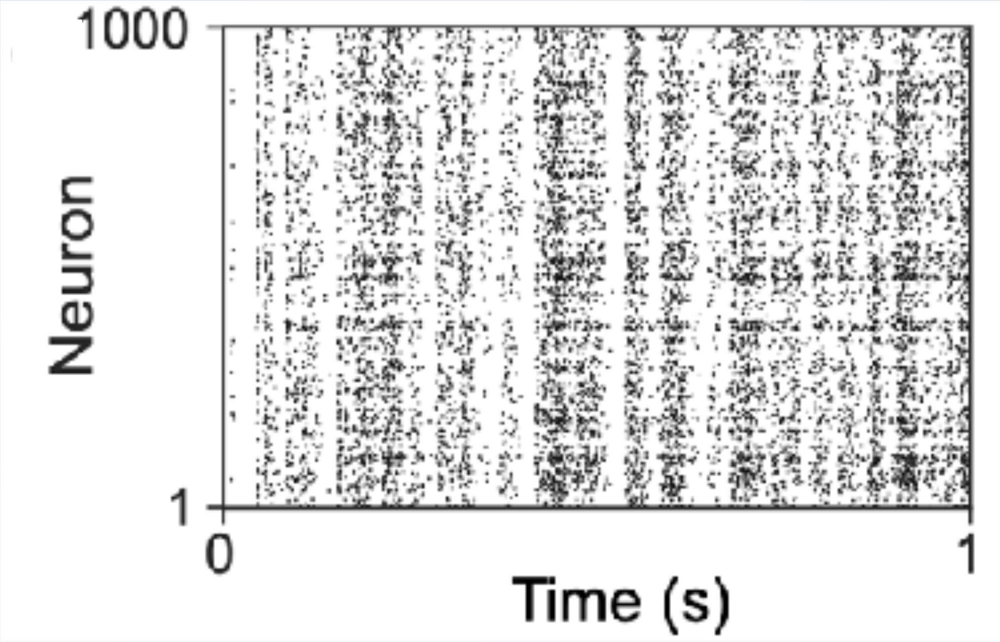

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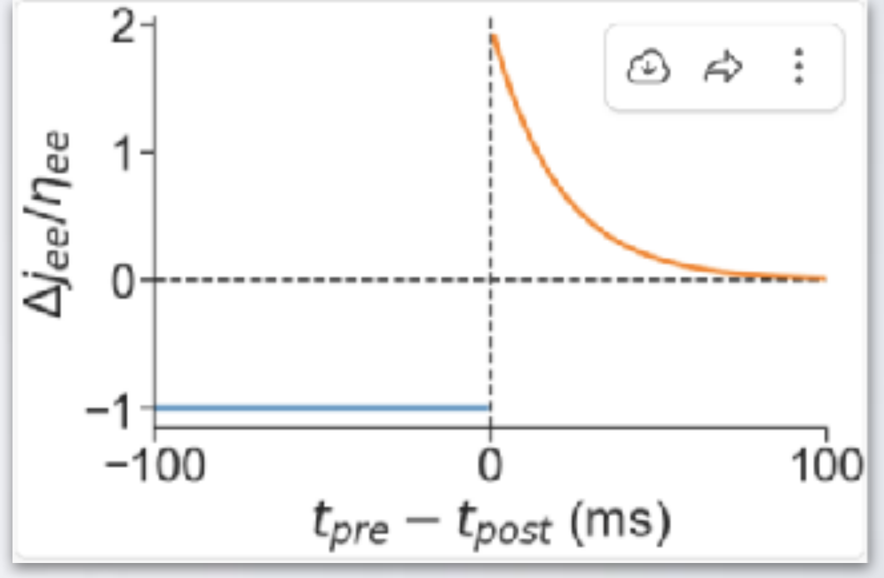

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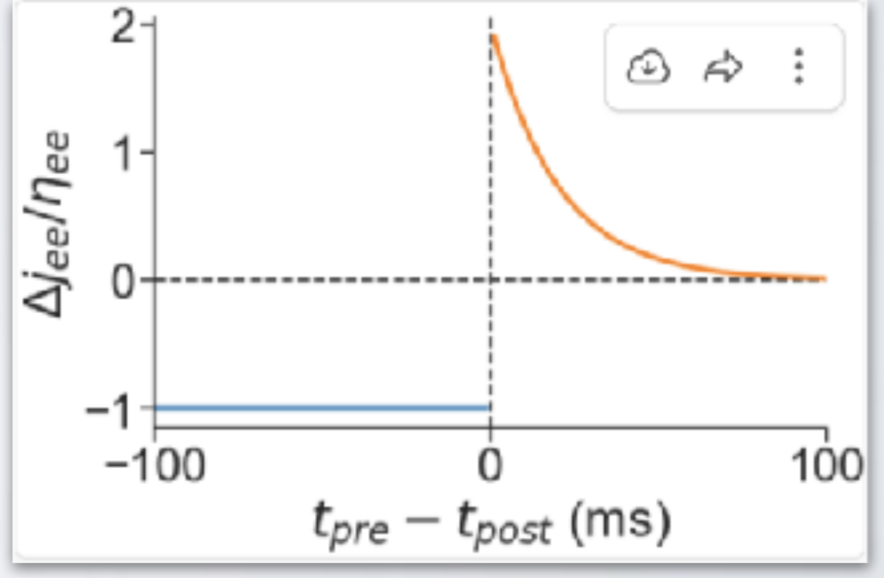

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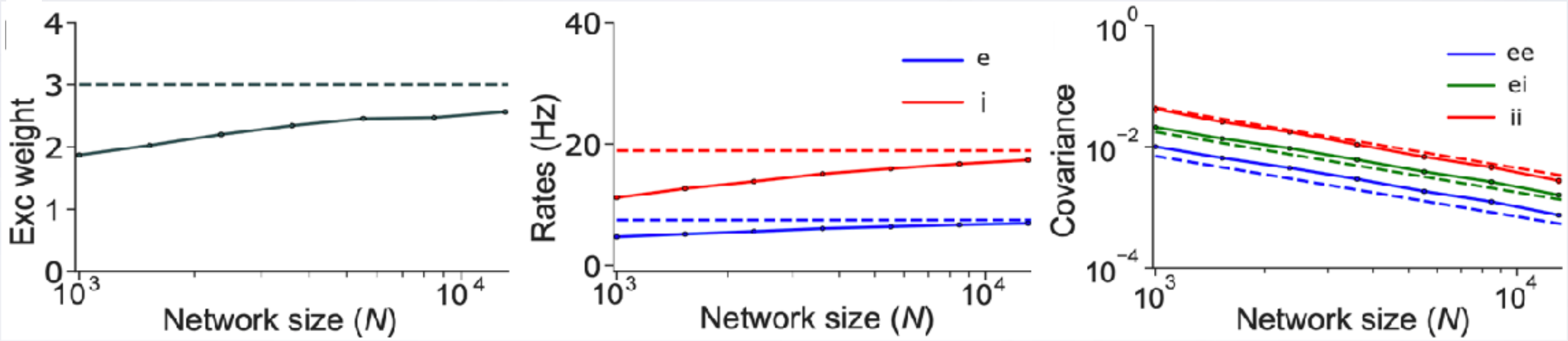

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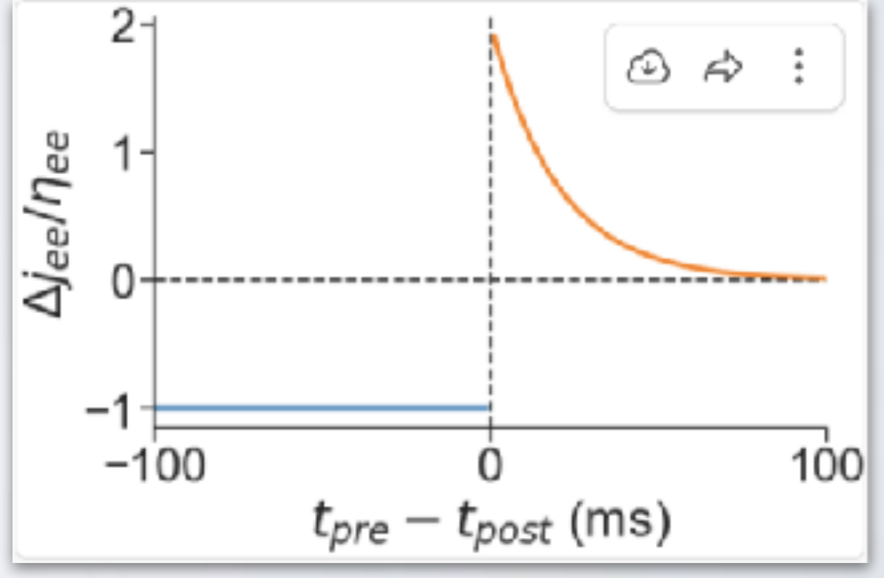

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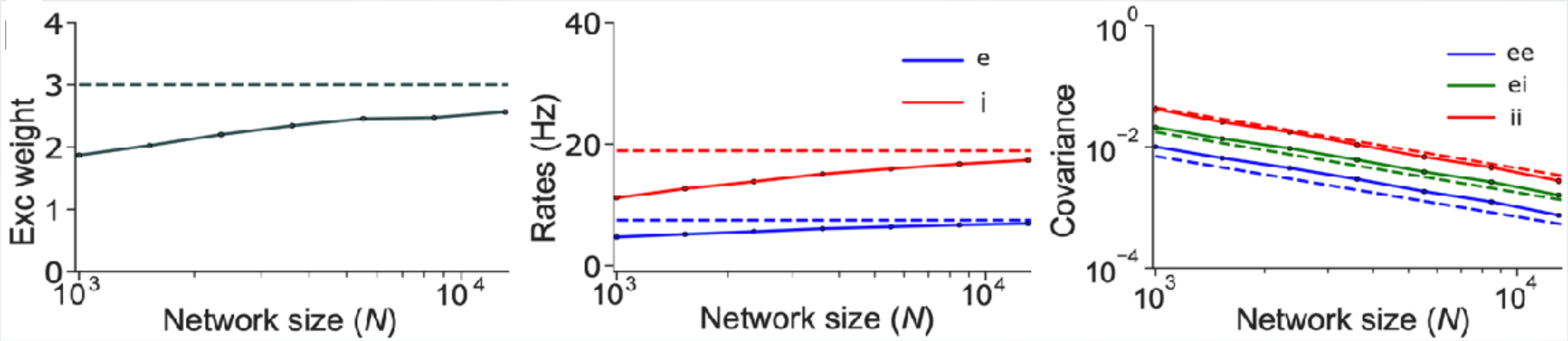

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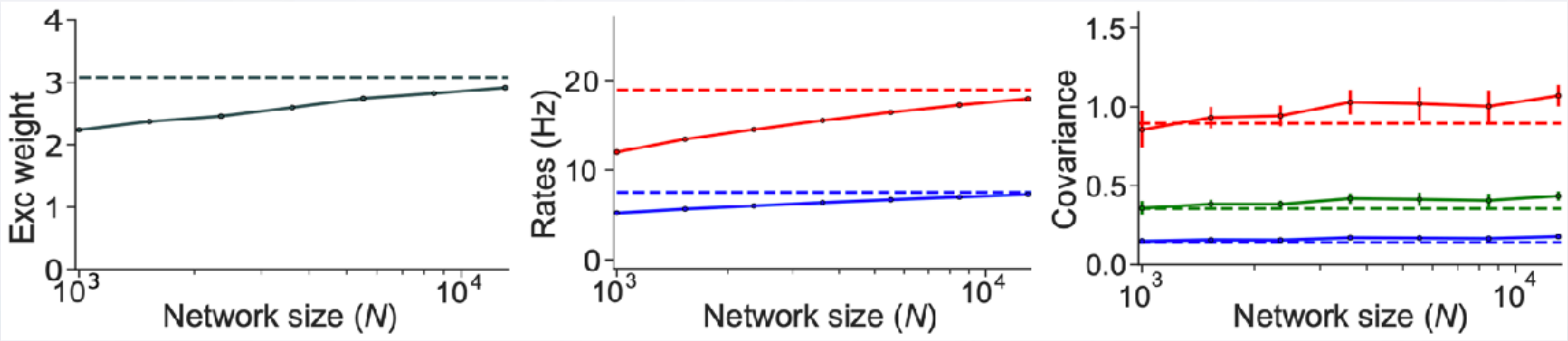

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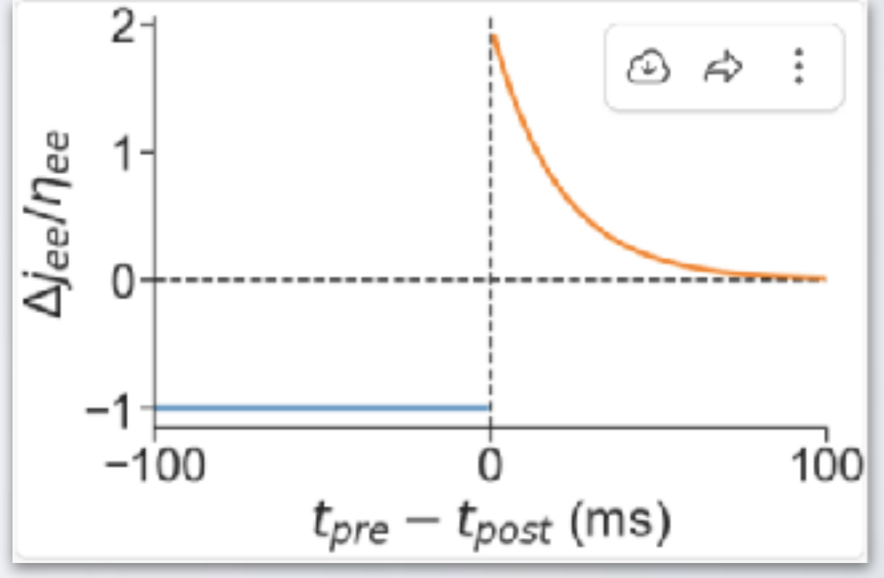

Loss of Stability with Kohonen's Rule

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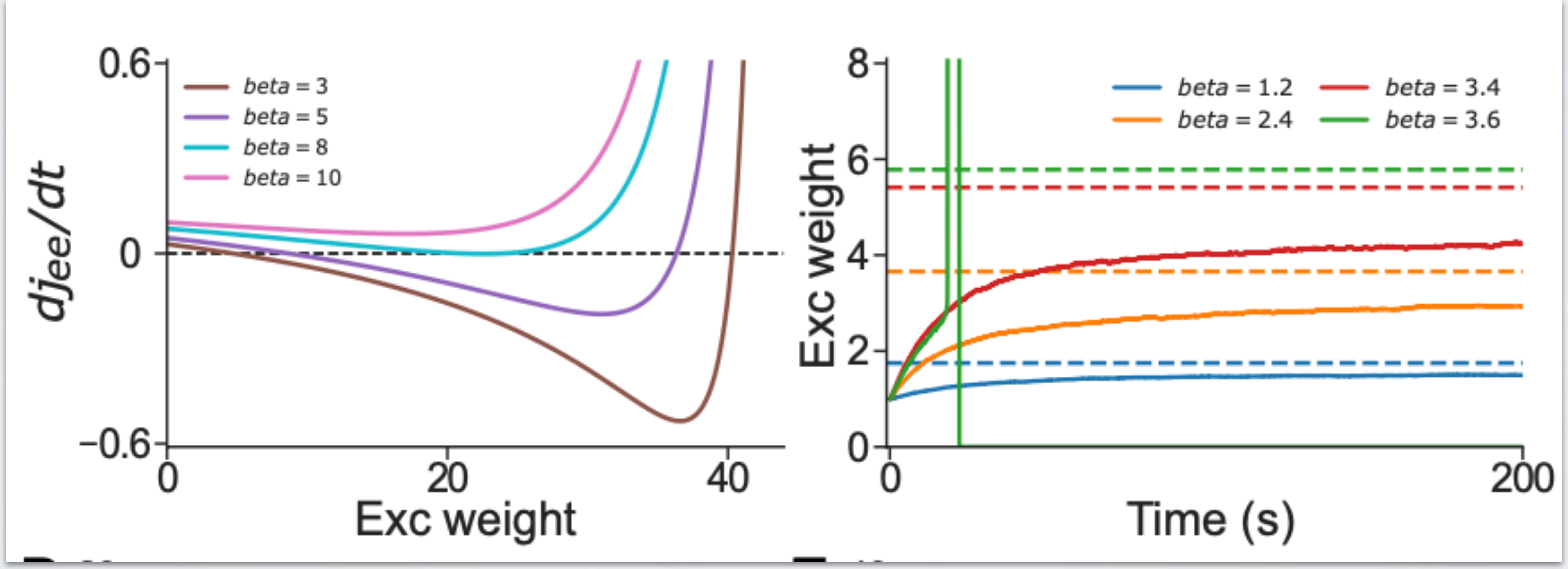
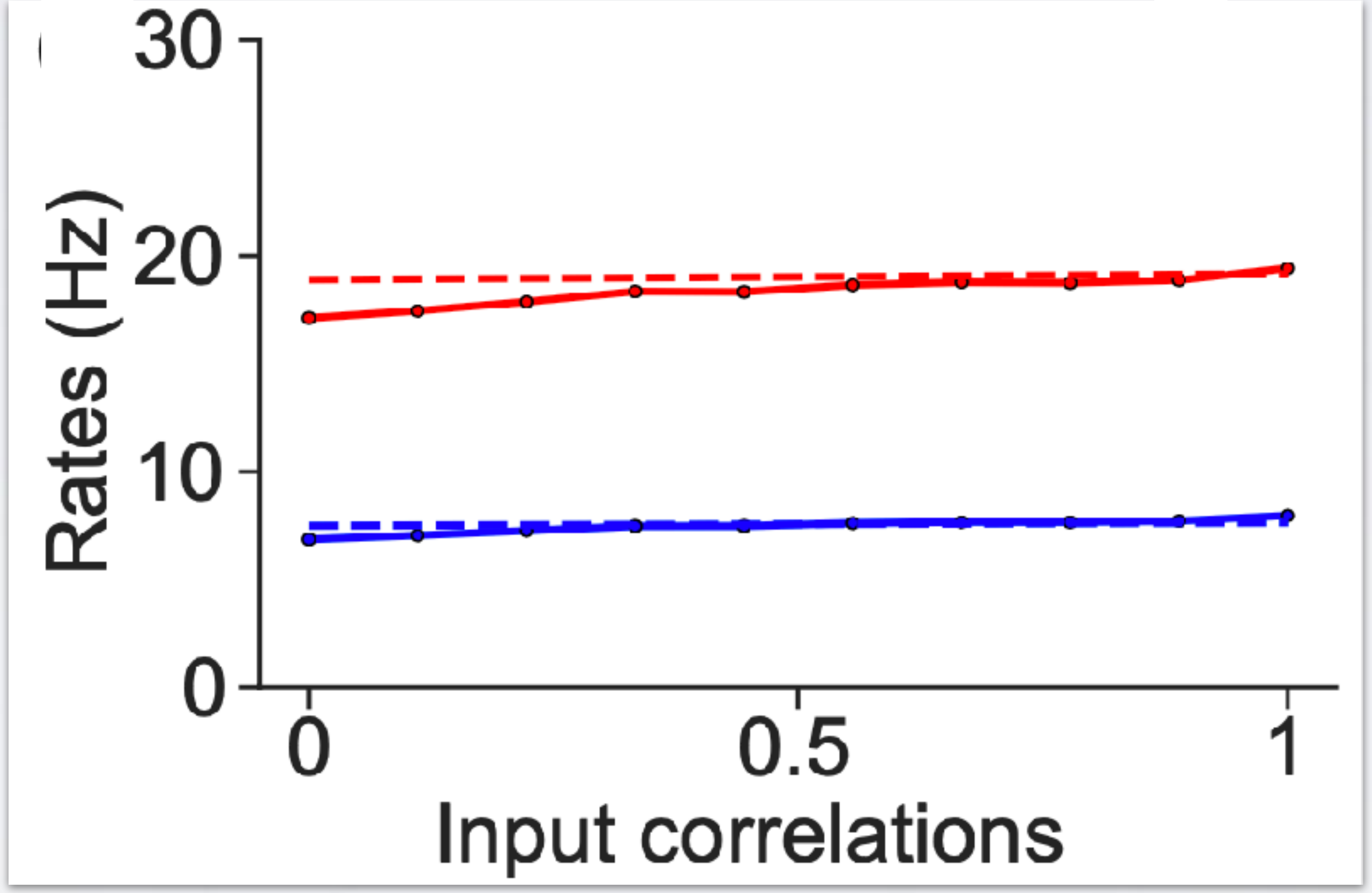


Correlations have a small impact

$$\frac{dJ_{ab}}{dt} = \eta_{ab}\left(A_0 + A_a r_a + \textbf{Rate}_{a,b} + \textbf{Cov}_{a,b}\right)$$



See also Graupner, *et al*, 2016, Ocker and Doiron, 2015

Can we extend the mean-field theory of balanced networks to include plasticity?

Can the theory predict when the balanced state is preserved?

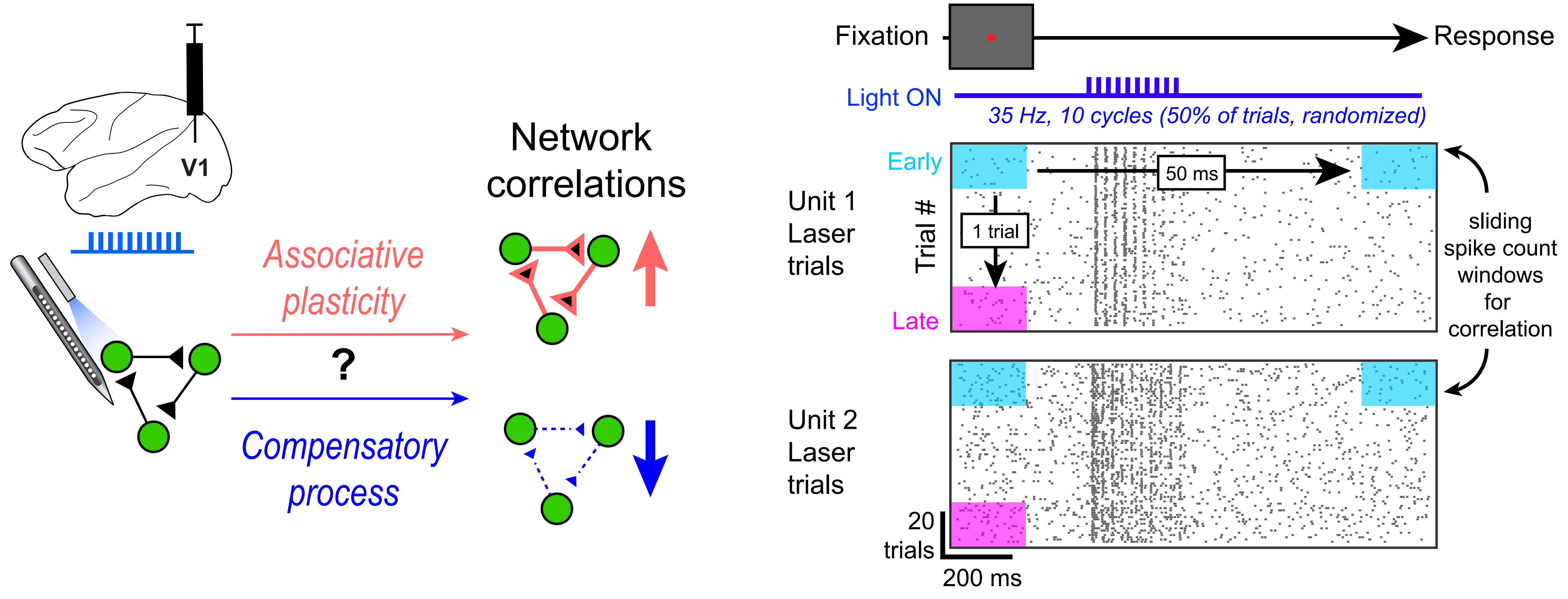
Can we extend the mean-field theory of balanced networks to include plasticity?

Can the theory predict when the balanced state is preserved?

Yes - We can get a closed set of equations that describe the evolving dynamics of the network

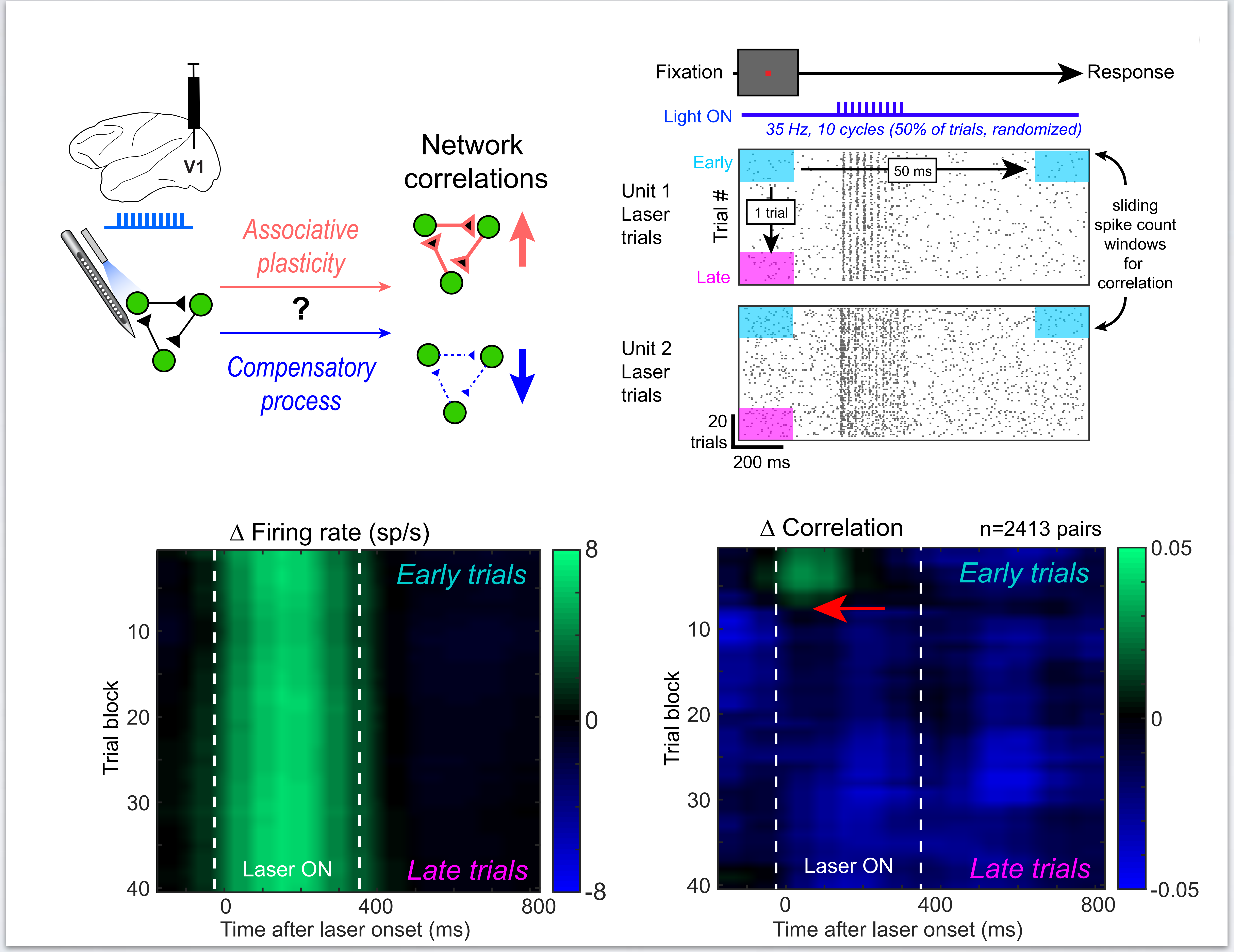
Multiple forms of plasticity acting together

Homeostatic Plasticity and Optogenetic Stimulation?



Andrei, et al. Nature Neuroscience, 2023

Homeostatic Plasticity and Optogenetic Stimulation?



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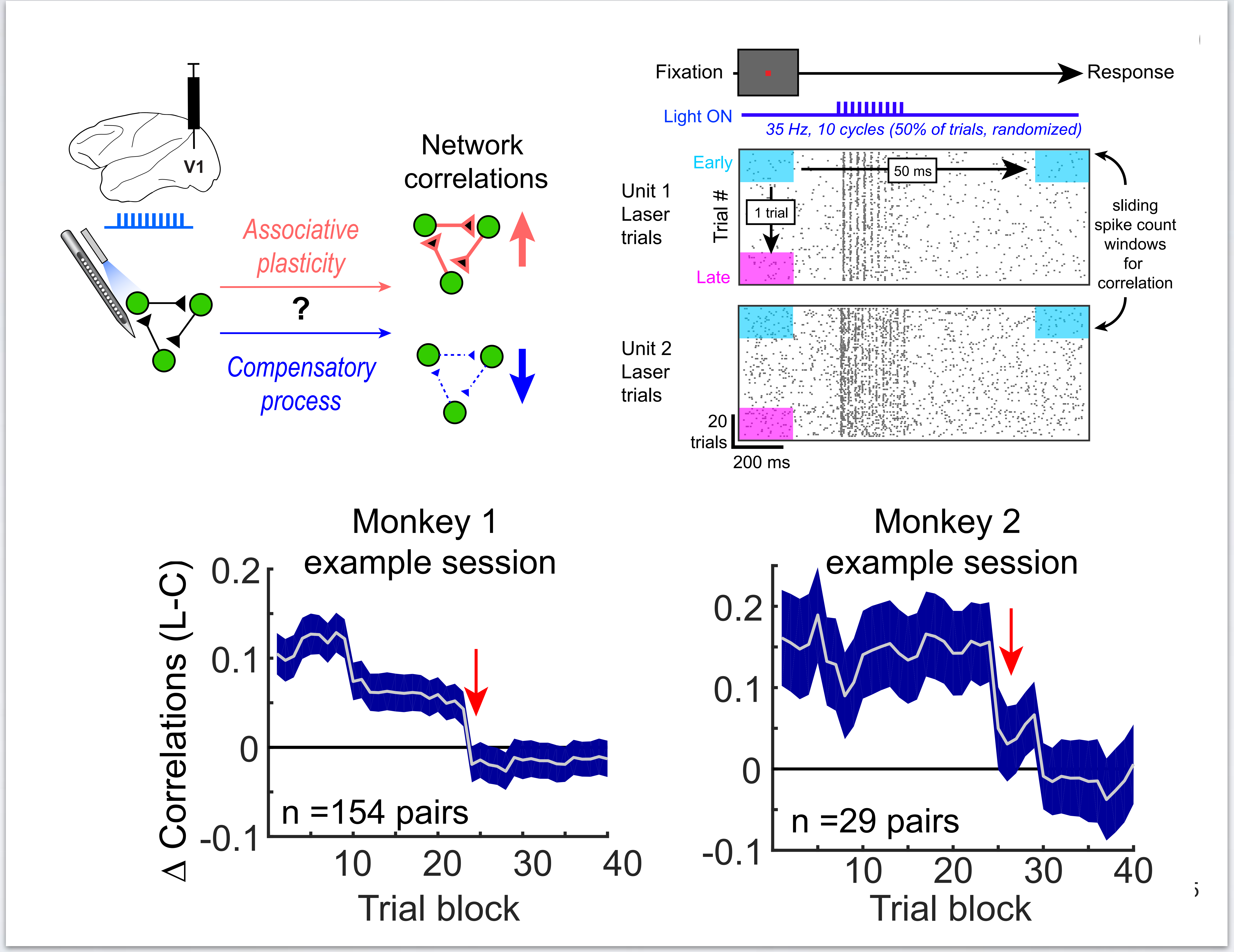
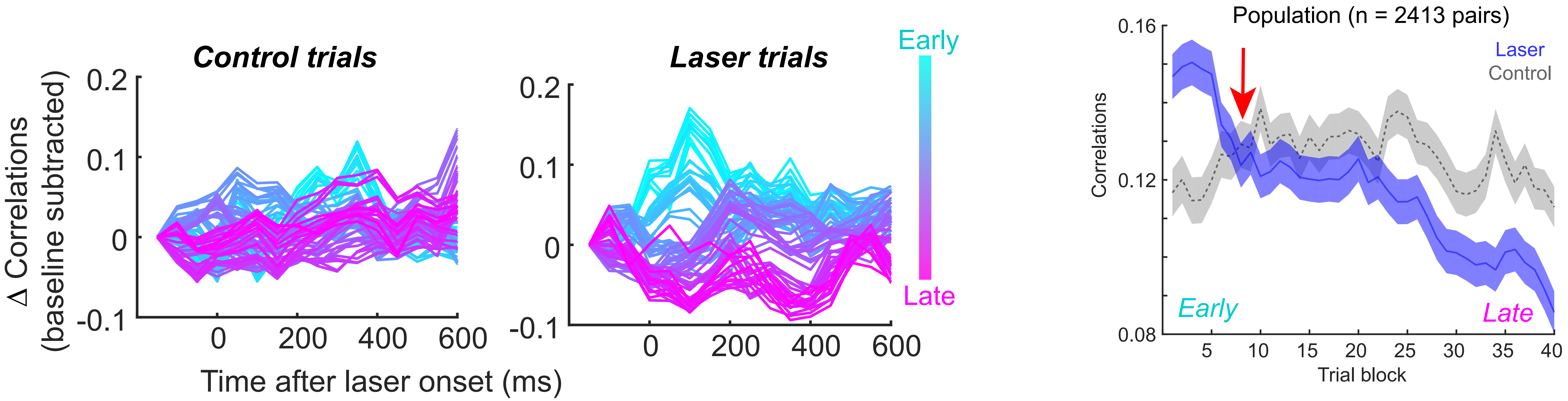
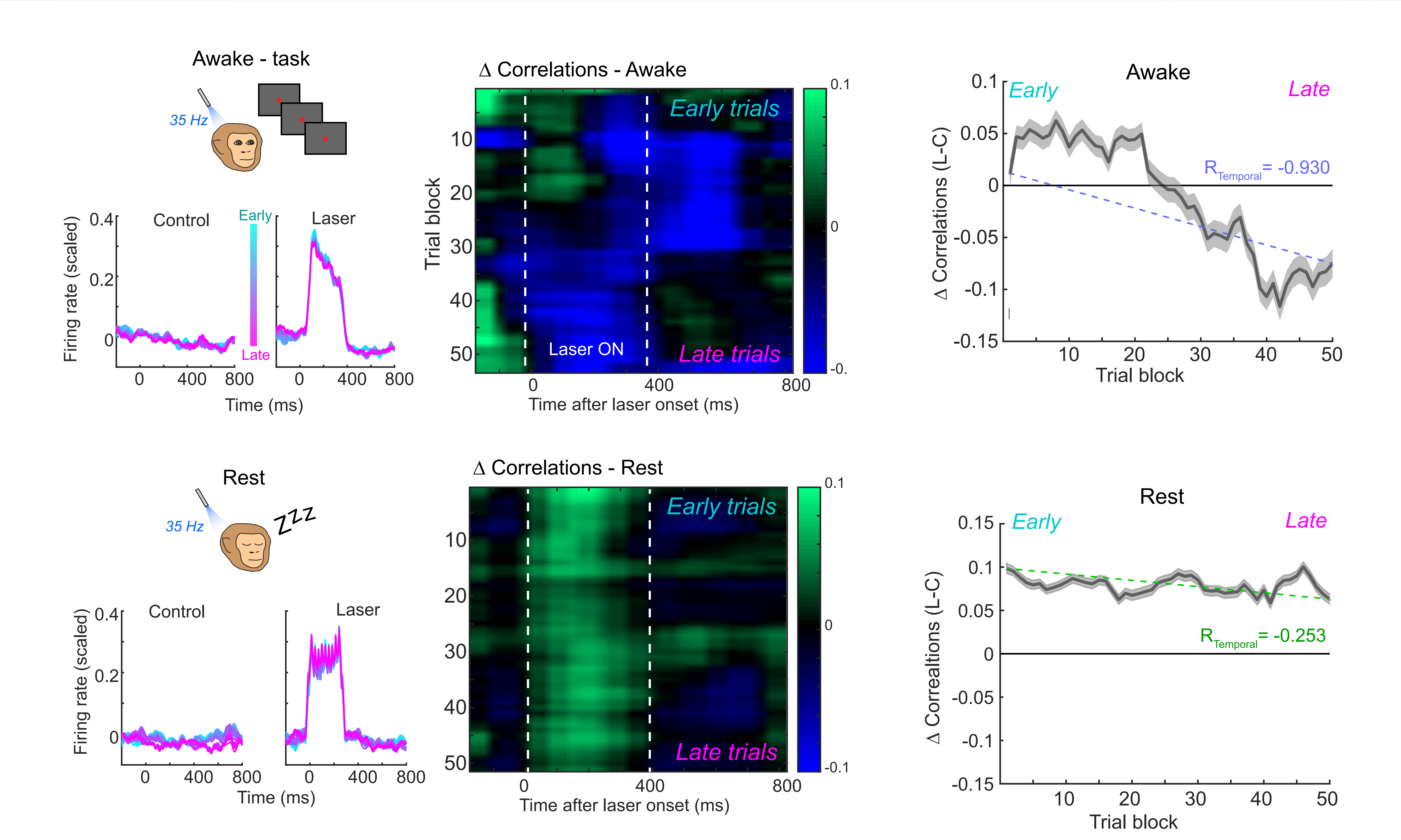


Andrei, et al. Nature Neuroscience, 2023

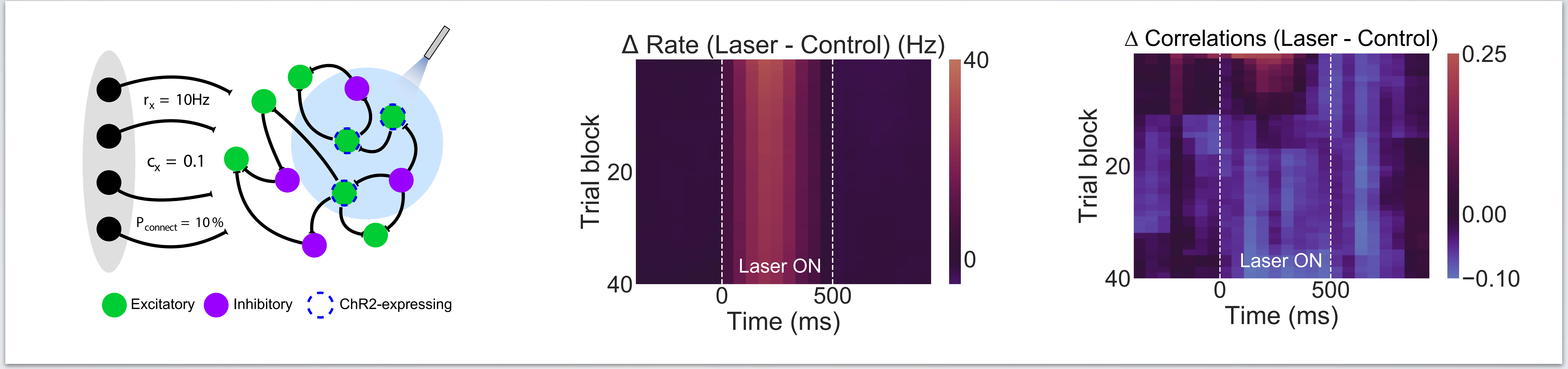
Correlation changes are heterogeneous



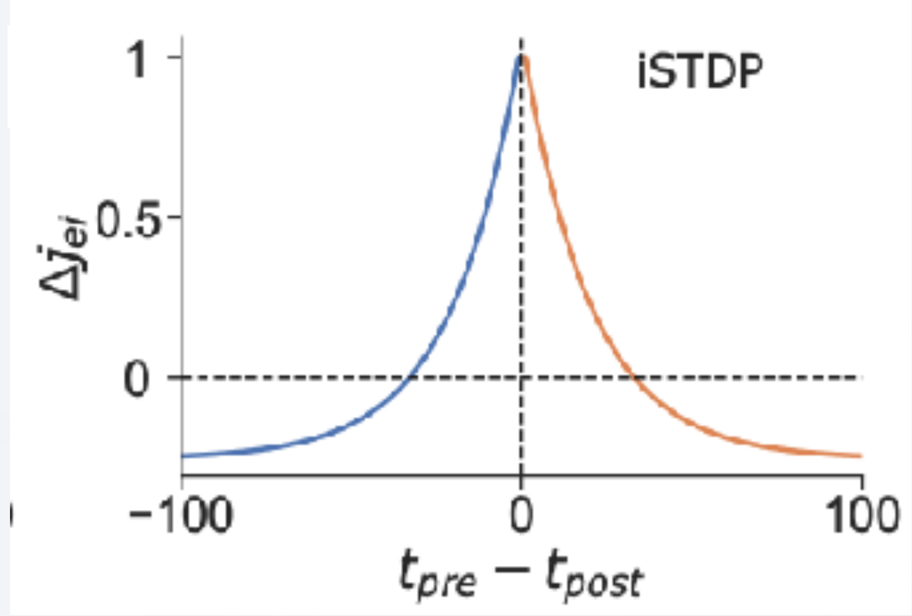
No changes during rest



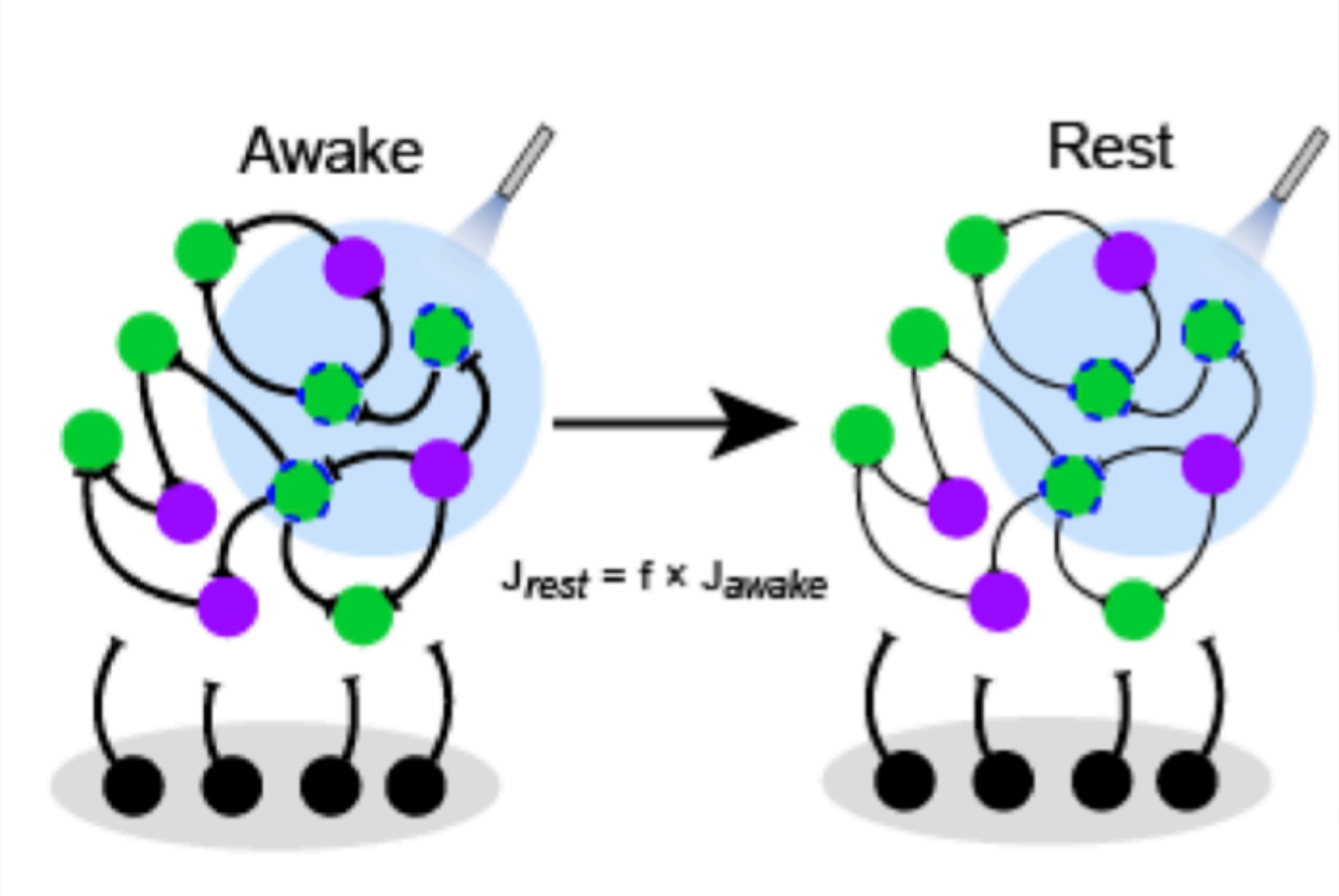
Model with Homeostatic Inhibitory Plasticity

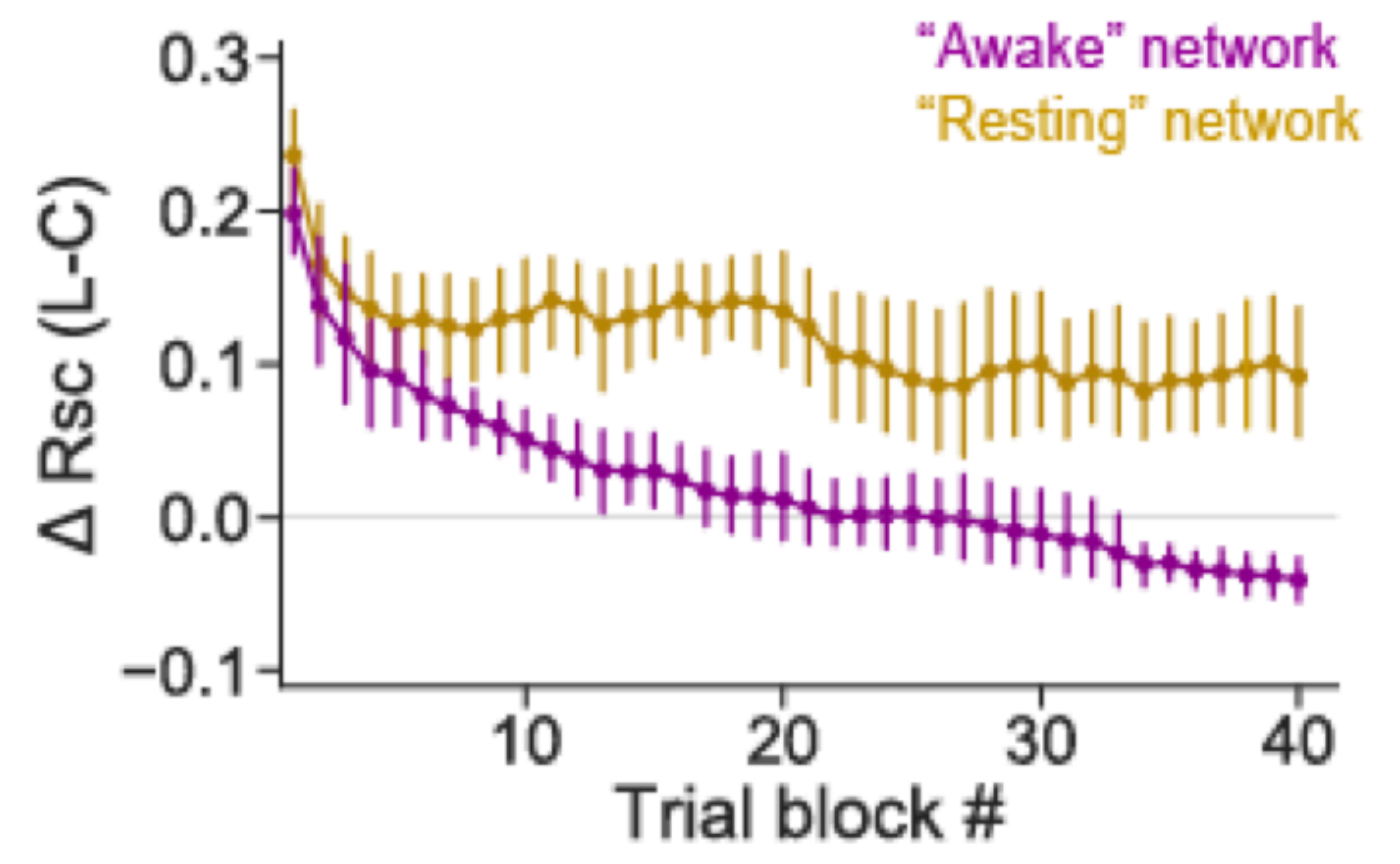


$$\tau_{J_{\mathrm{ei}}} \frac{dJ^{\mathrm{ei}}_{jk}}{dt} = -\frac{J^{\mathrm{ei}}_{jk}}{J^{0}_{\mathrm{ei}}} \left[(x^{\mathrm{e}}_j - \alpha_{\mathrm{e}})S^{\mathrm{i}}_k + x^{\mathrm{i}}_k S^{\mathrm{e}}_j\right]$$



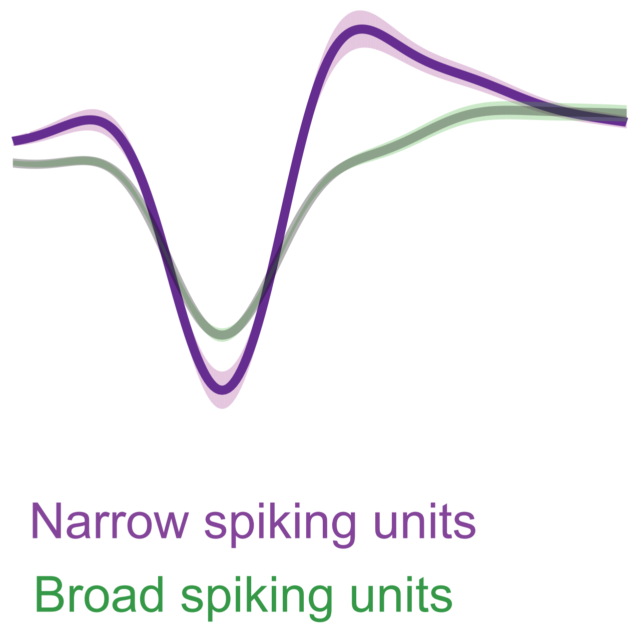
Model Shows Small Effect During Rest



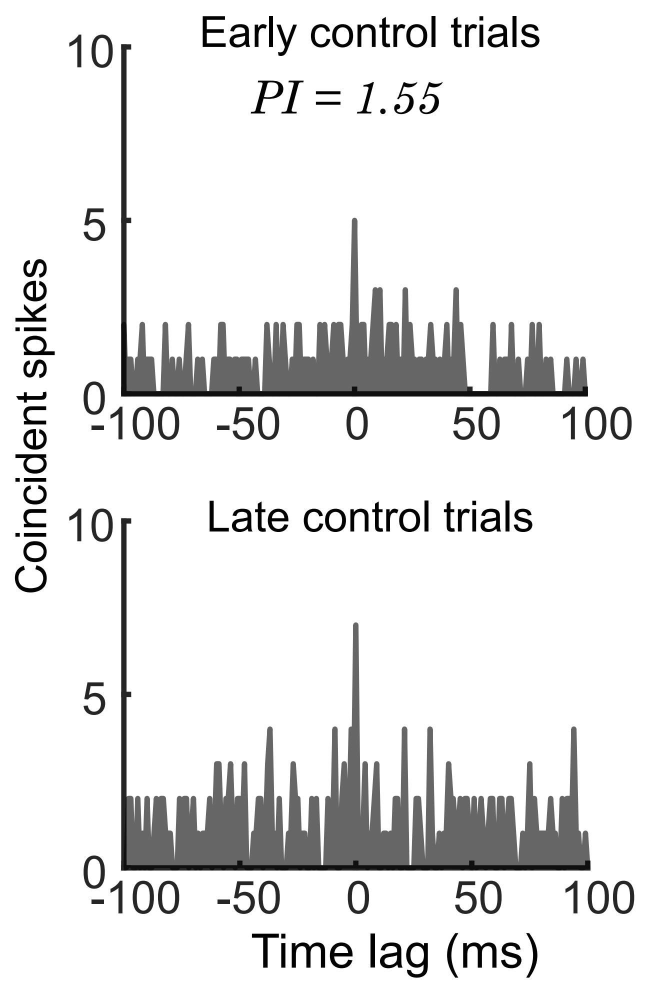


Vyazovkiy, et al 2008,
Johansen, et al 2010

Putative IE Synapses are Potentiated



$$\textit{Potentiation index ("PI")} = \frac{\textit{AUC late trials}}{\textit{AUC early trials}}$$



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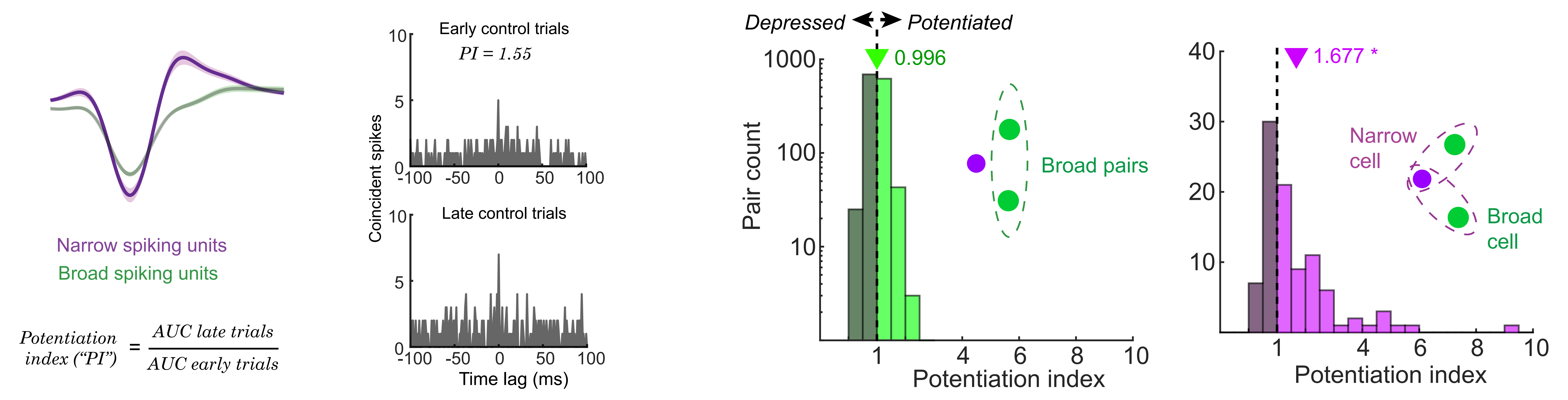

Enormous Efforts to Reconstruct the Brain

Synapse-level dense reconstructions

C. Elegans ~300 neurons (1986)

Larval MB ~300 neurons (2020)

Drosophila Hemibrain, ~22,000 neurons (2020)

Platinum Mouse ~80,000 neurons (2022)

Human brain by Lichtman’s lab (2021…)

Drosophila Brain, ~130,00 neurons (2023)

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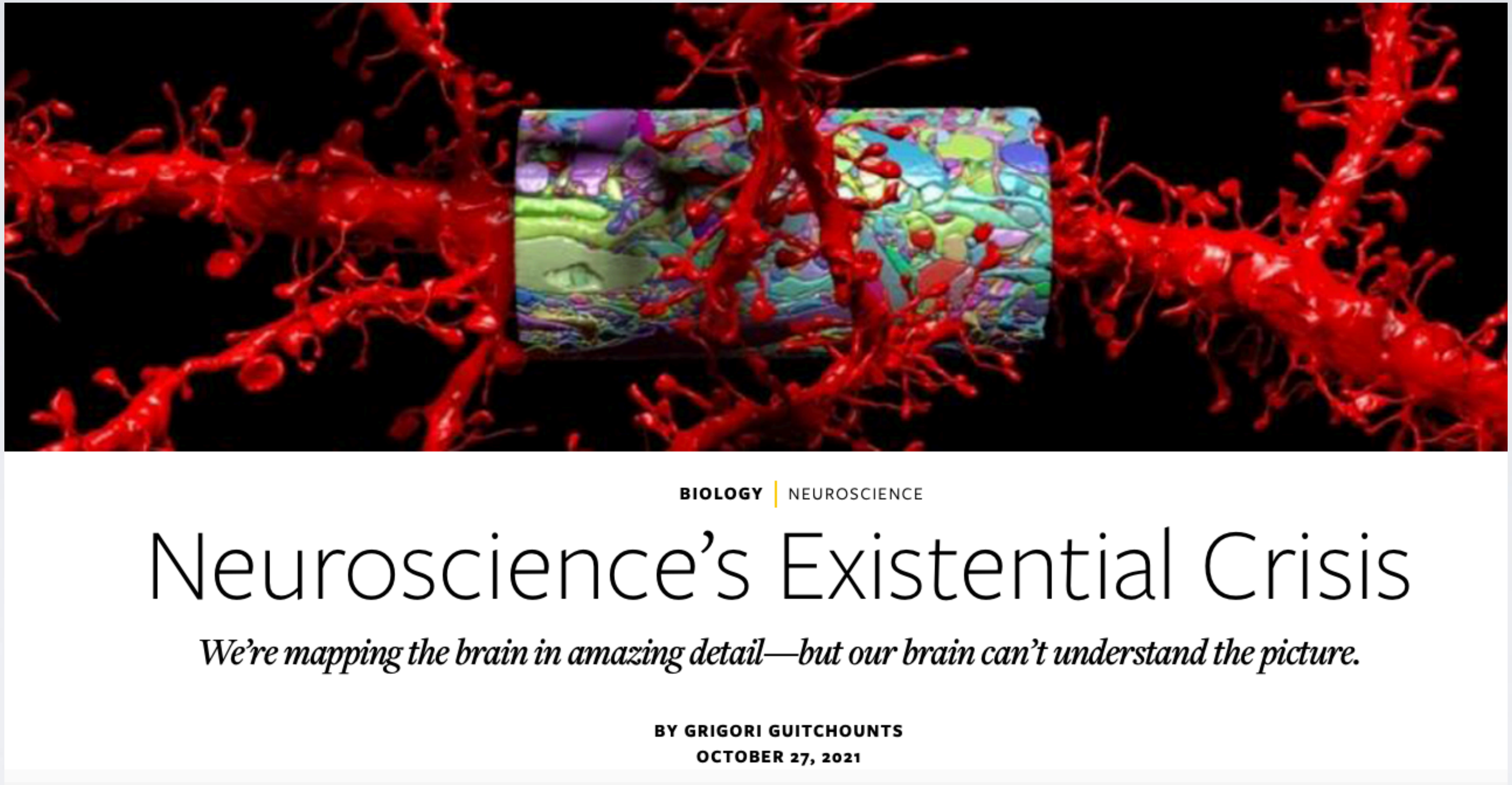
Analysis requires new tools and methods



BIOLOGY | NEUROSCIENCE

Neuroscience’s Existential Crisis

We’re mapping the brain in amazing detail—but our brain can’t understand the picture.

BY GRIGORI GUITCHOUNTS
OCTOBER 27, 2021

The Fly Hemibrain

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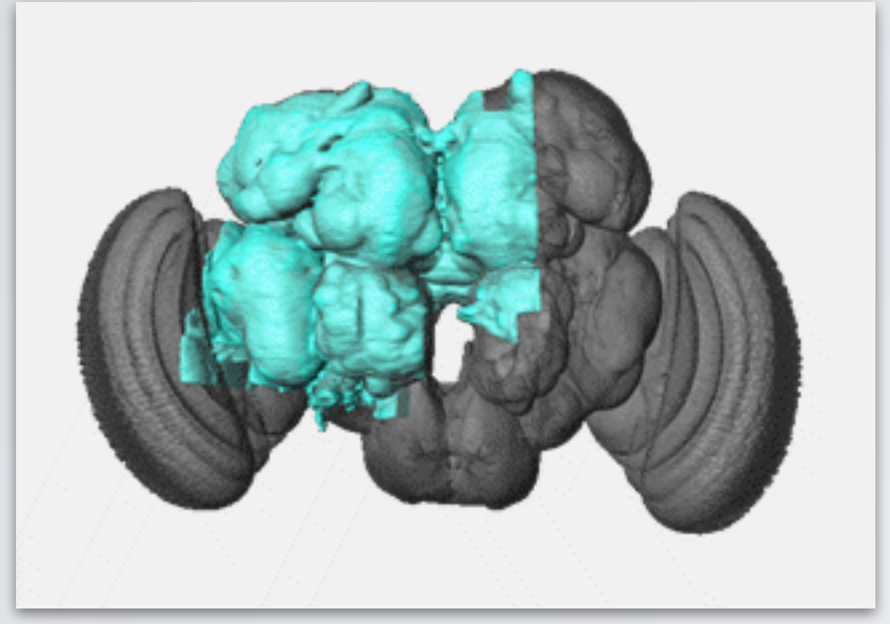

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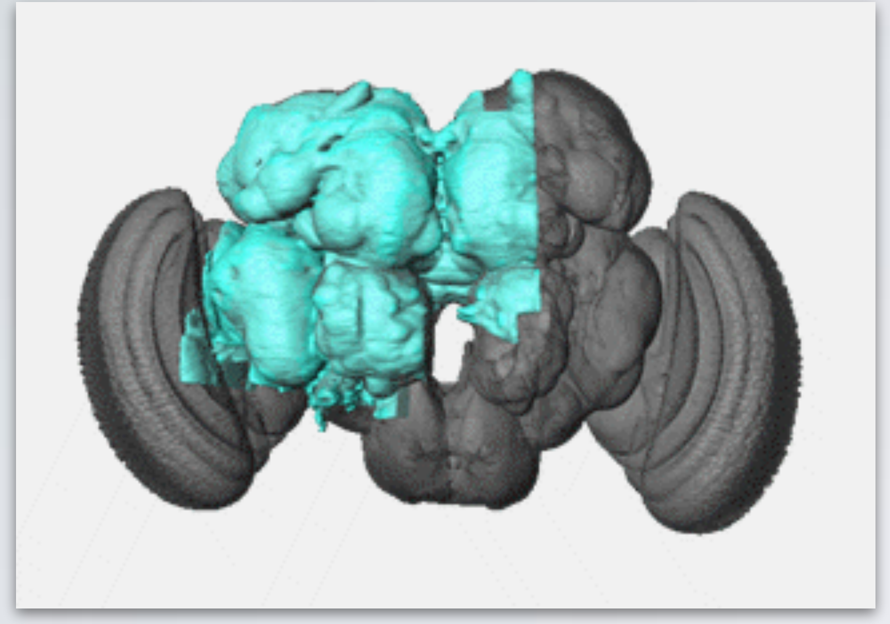

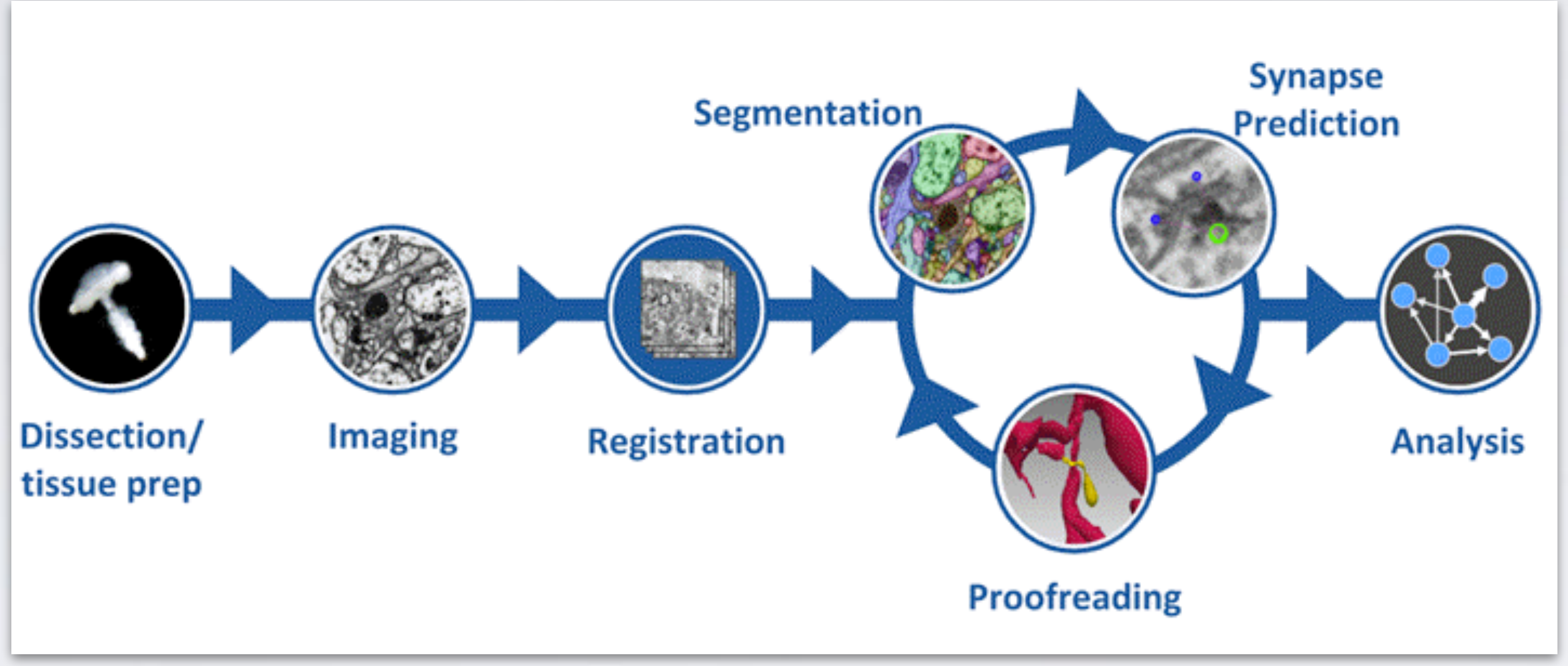

janelia.org/project-team/flyem/hemibrain

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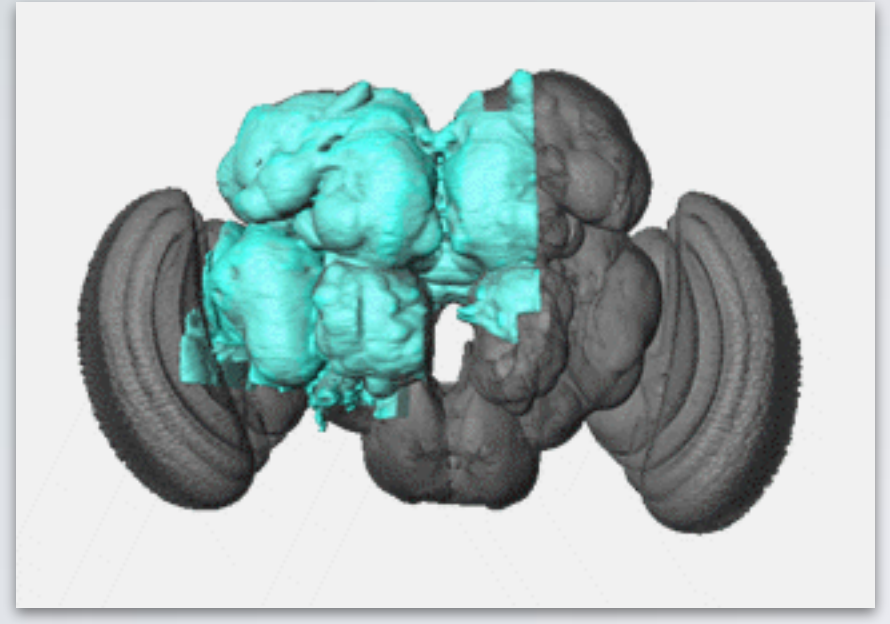

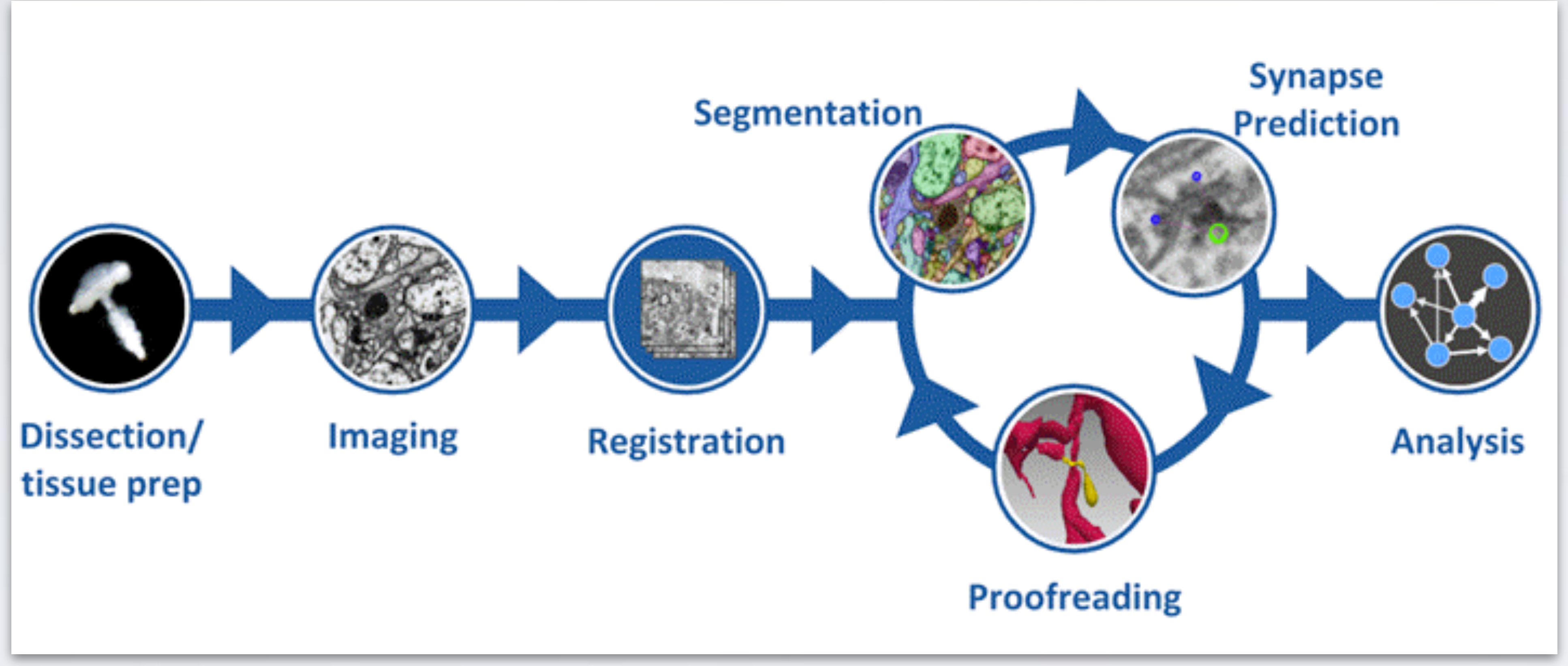

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Gives us a graph of 21,733 nodes (neurons)

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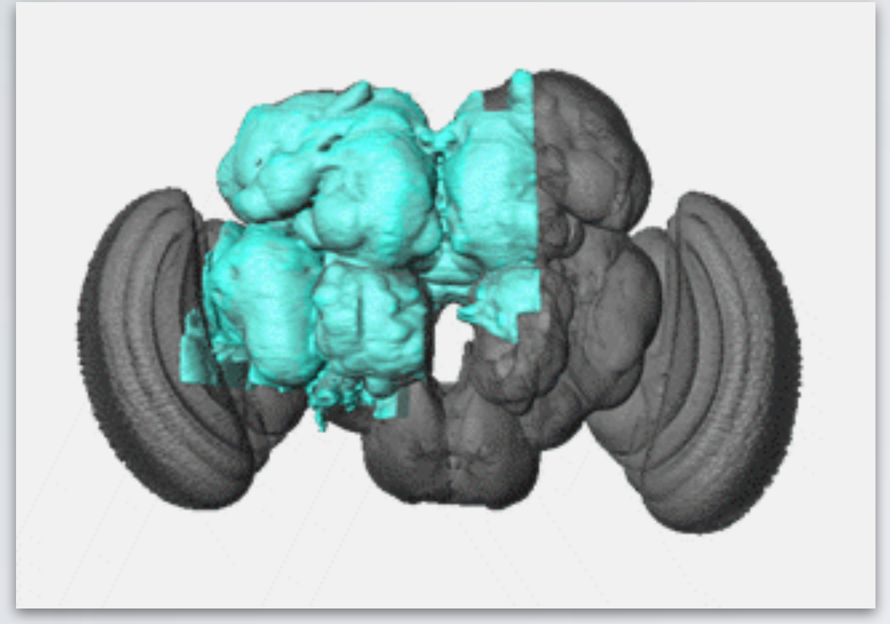

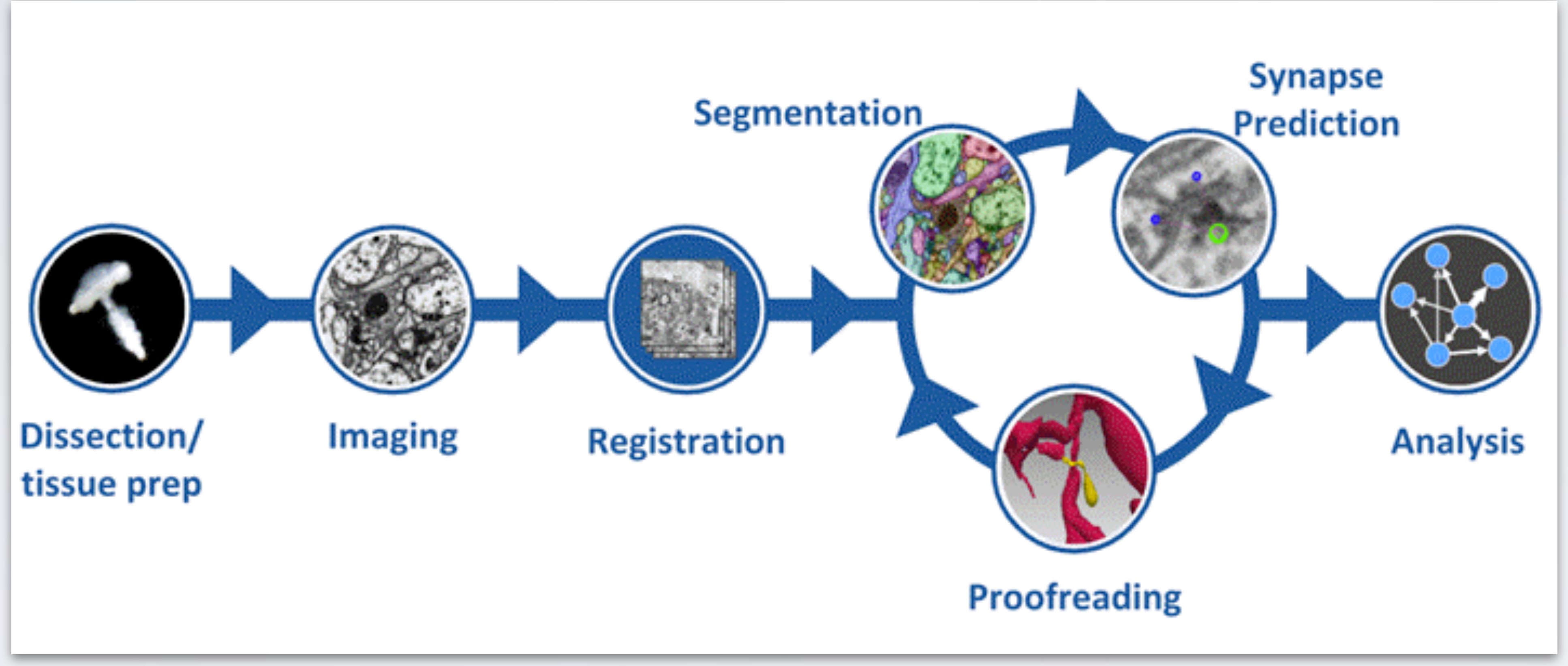

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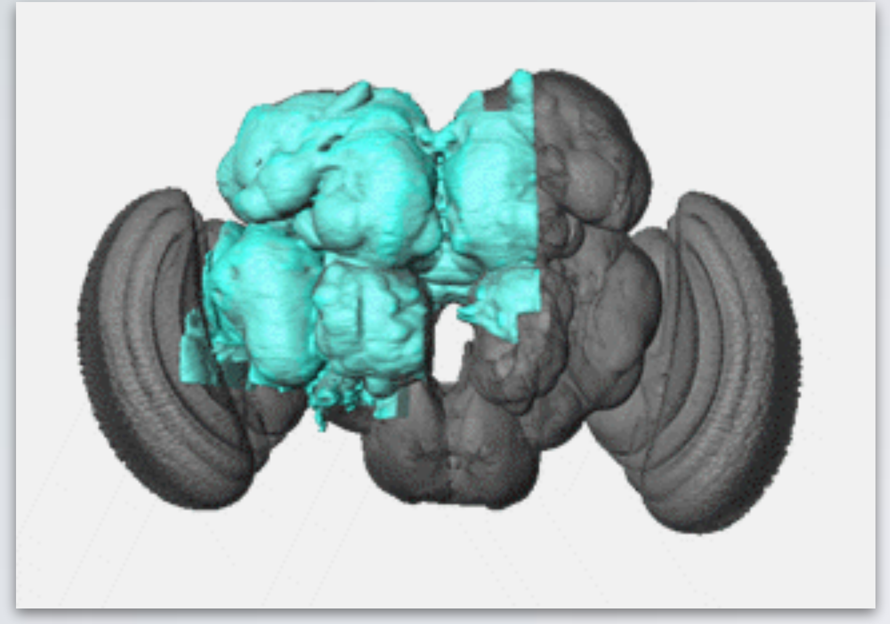

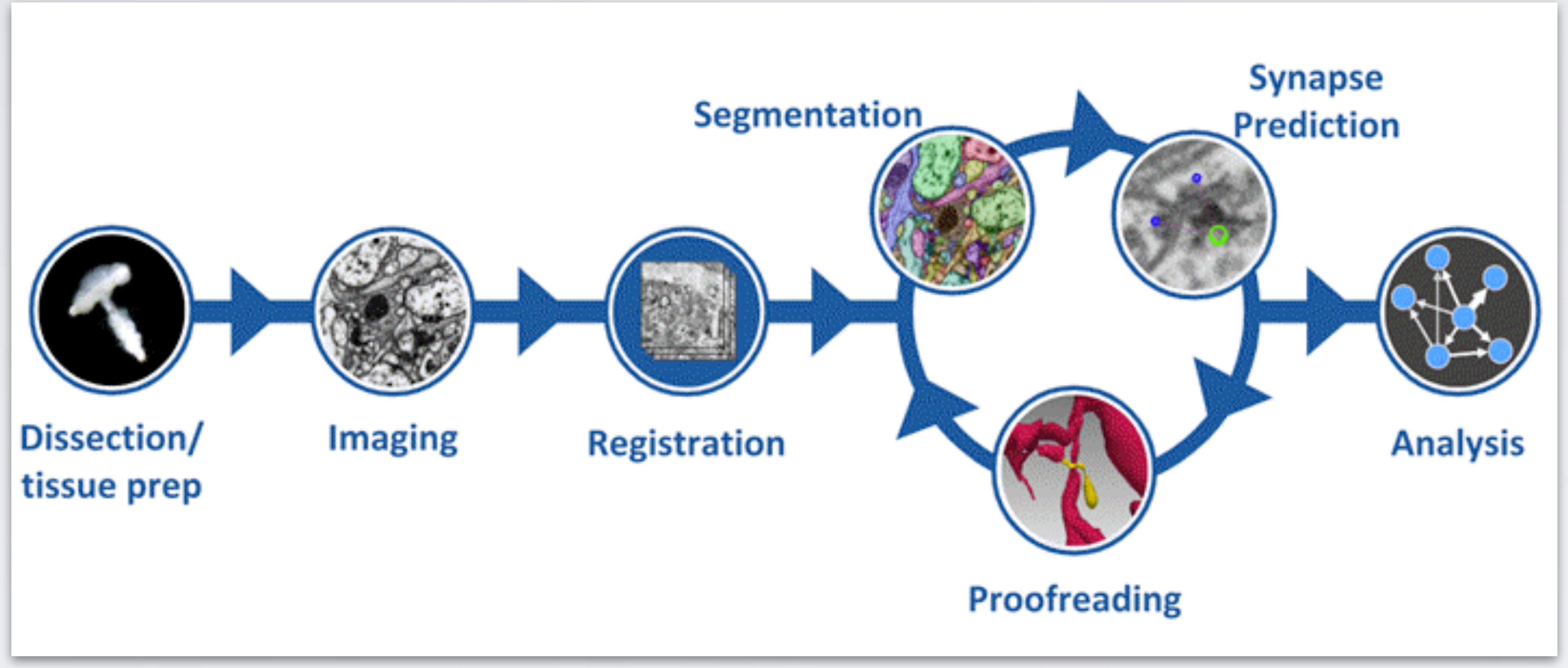

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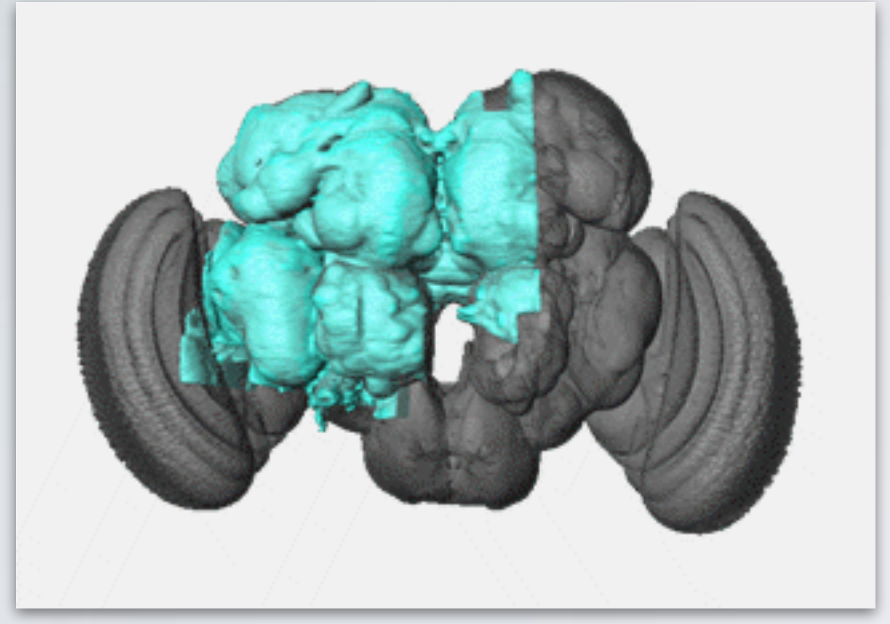

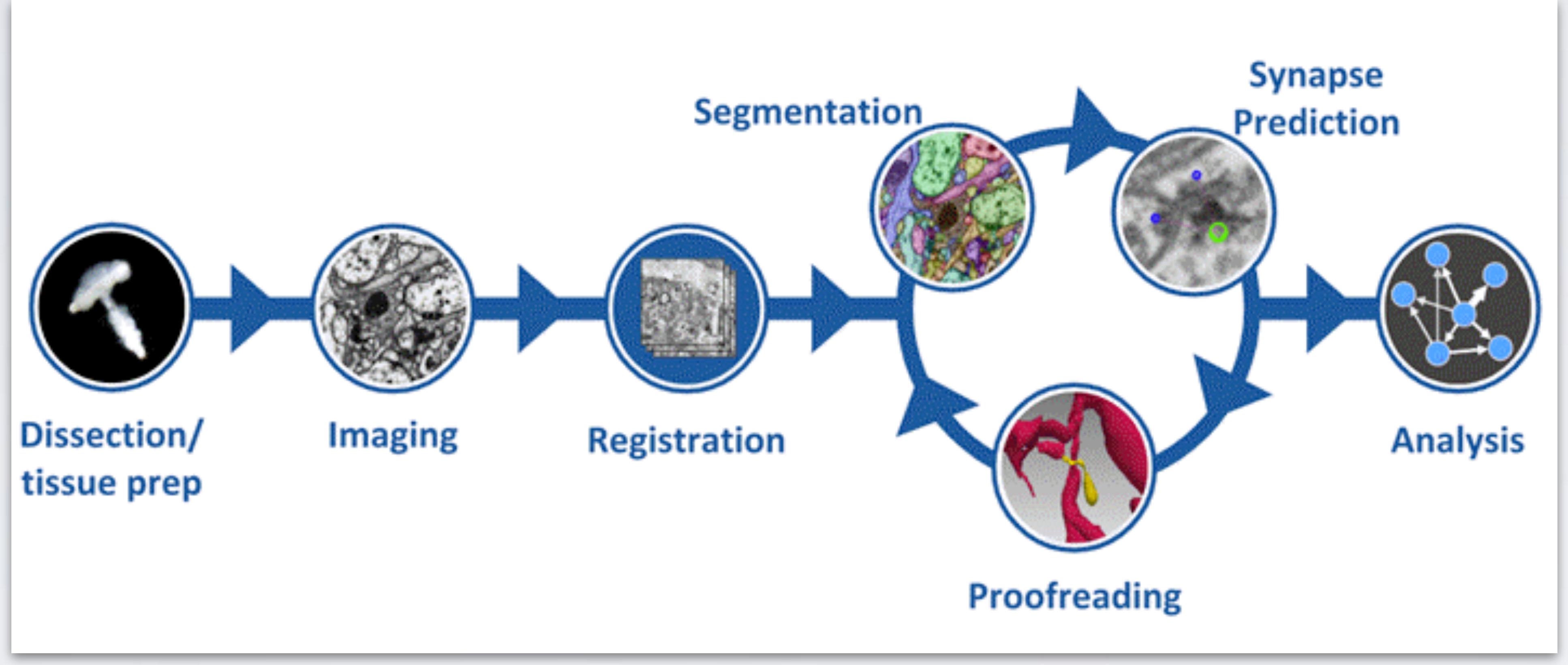

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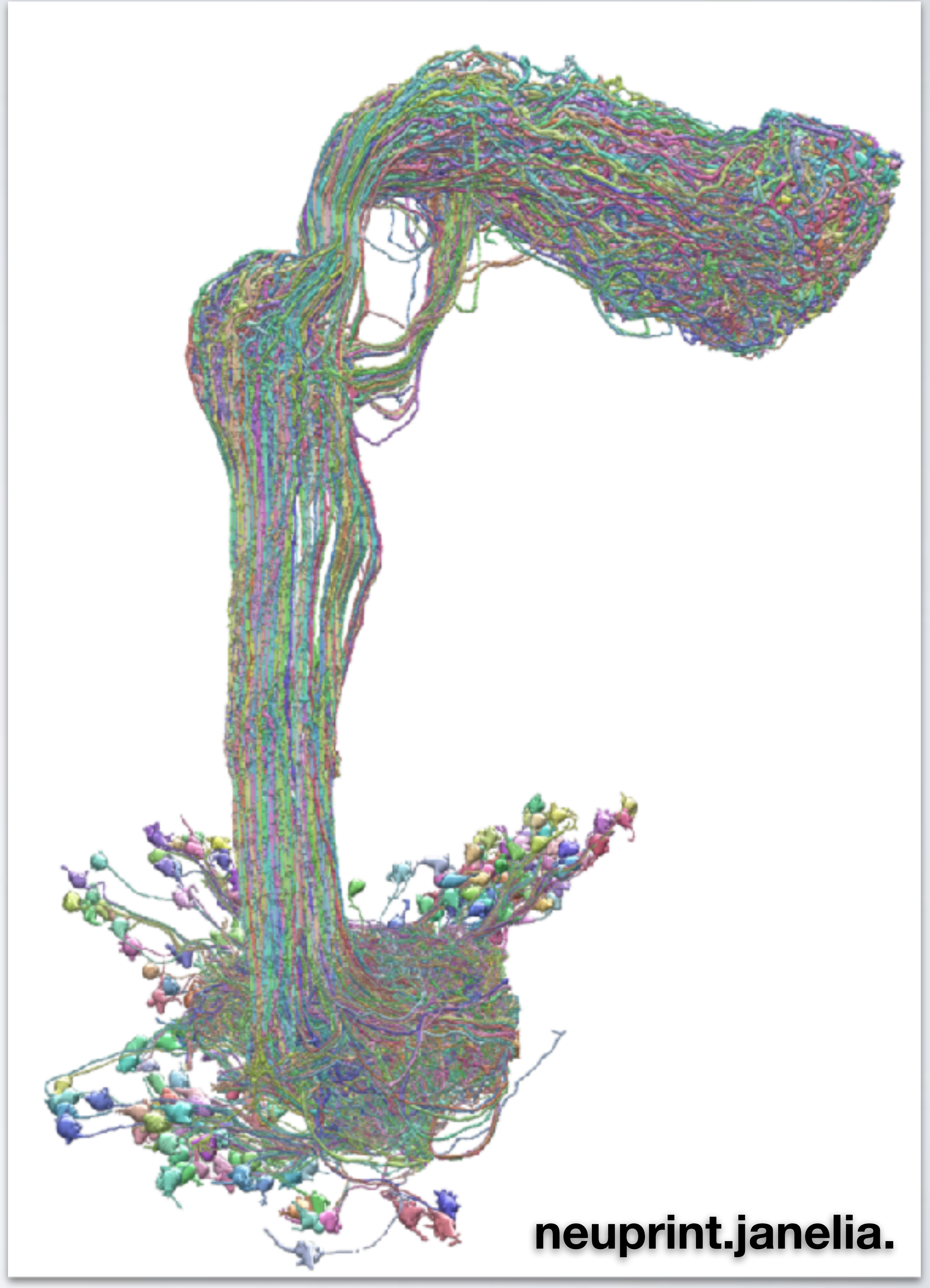

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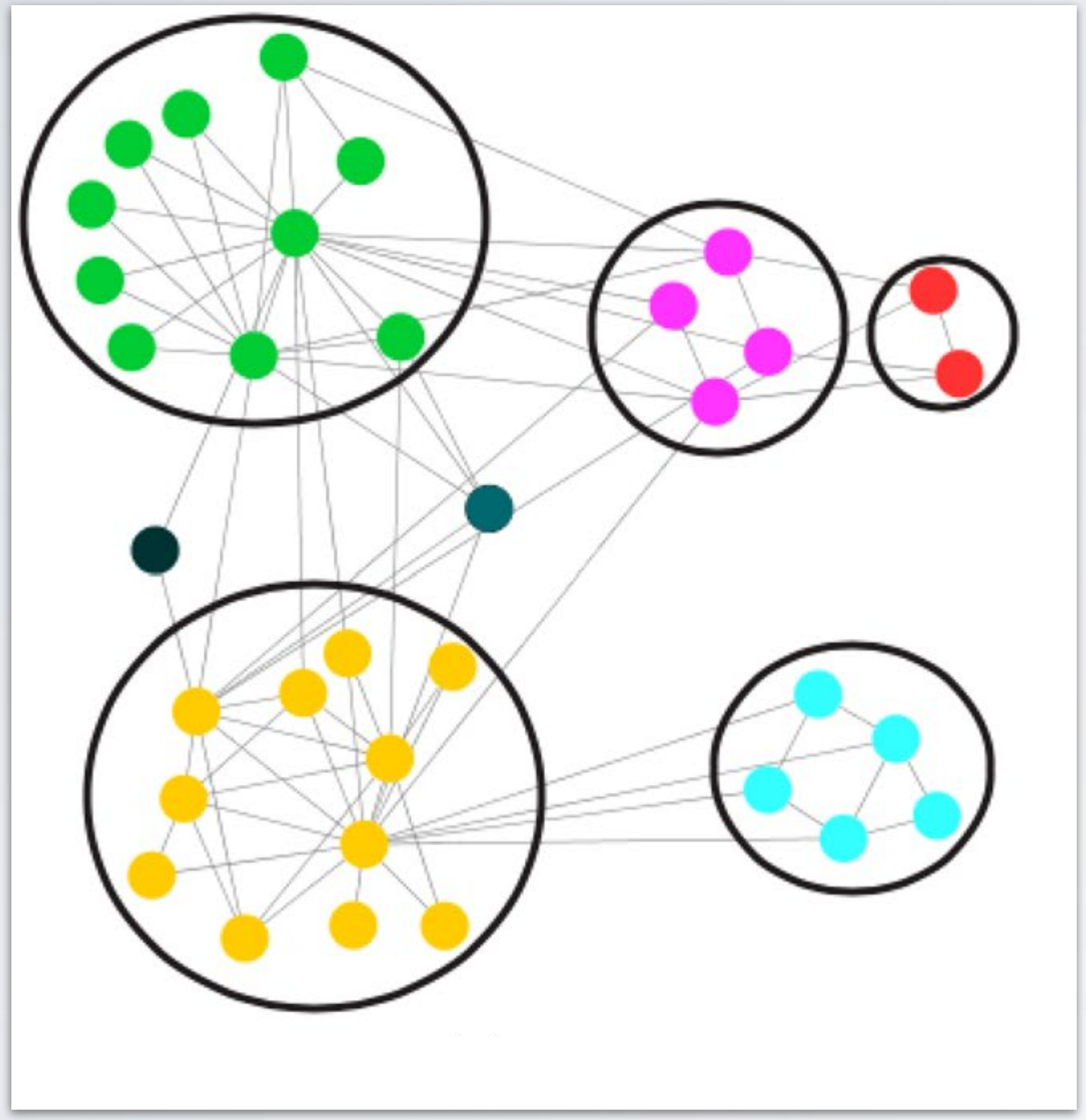
Community Detection



K_c is total degree of nodes in c

m, m_c are number of edges in graph or in c respectively

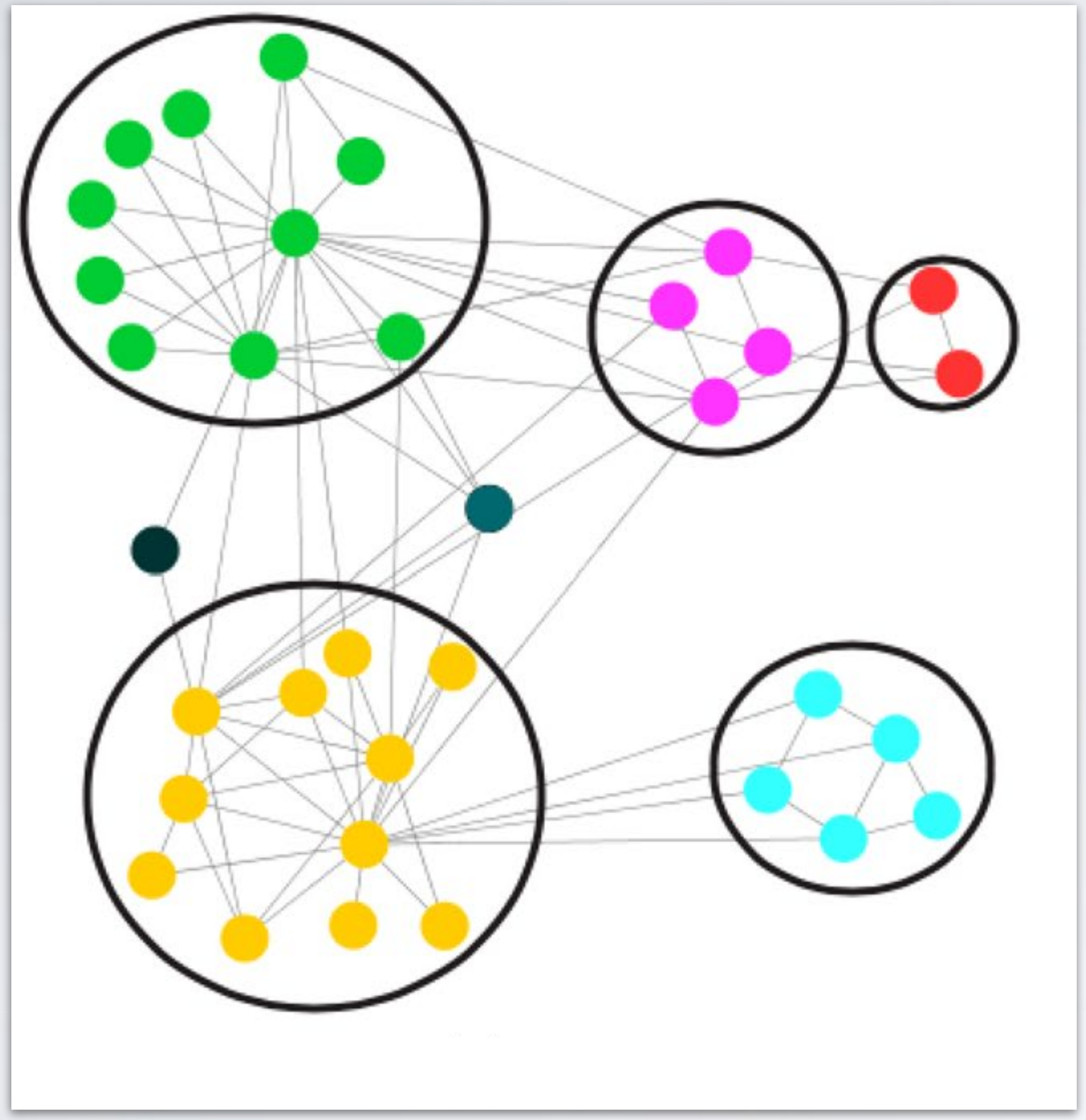
$$Q_g = \frac{1}{2m} \sum_c \left(2m_c - \frac{K_c^2}{2m} \right)$$

Number of edges in c

Expected number of edges in c if randomly assigned.

Guo et al, 2019

Community Detection



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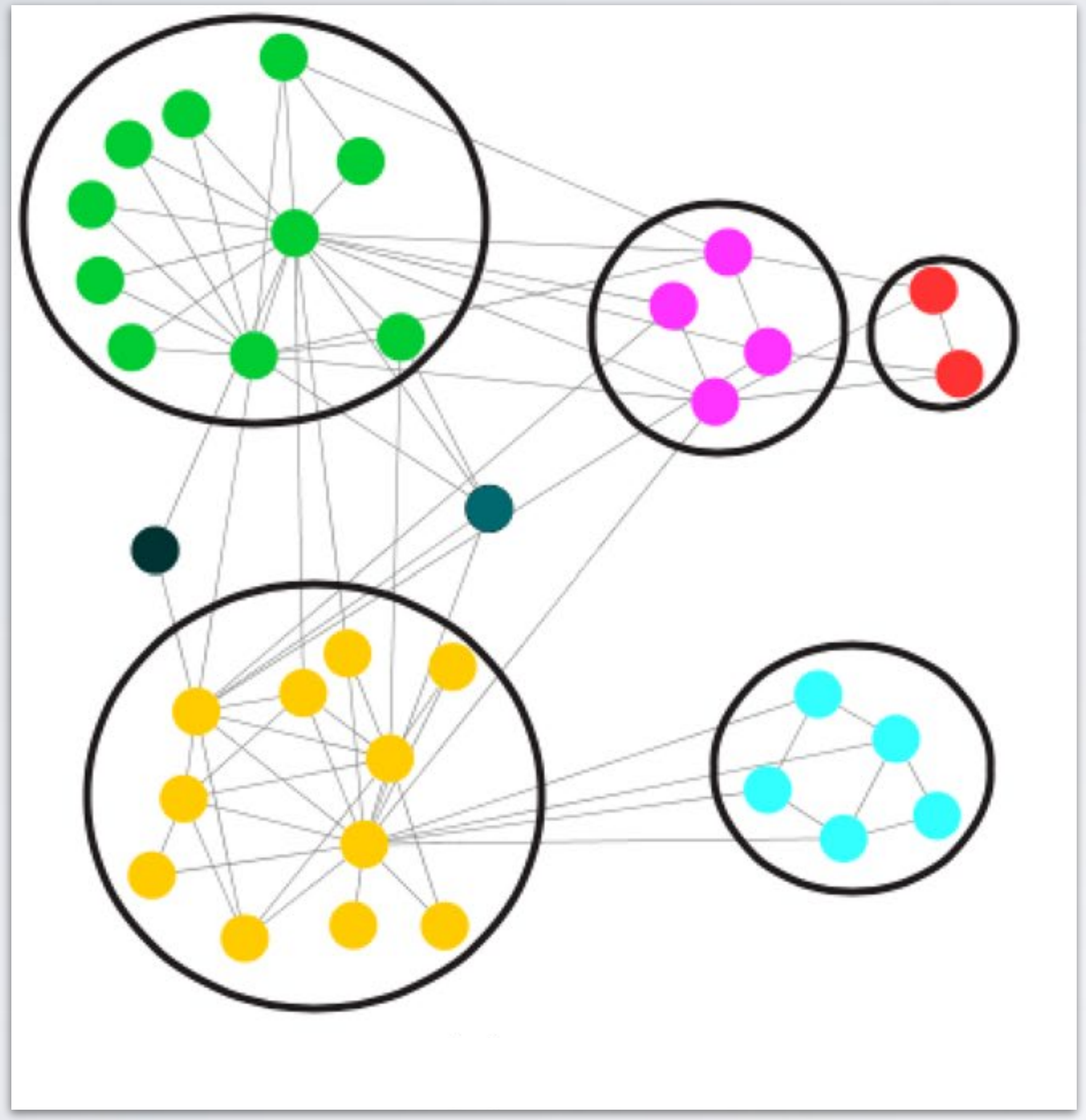

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$$Q_g = \frac{1}{2m} \sum_c \left(2m_c - \frac{K_c^2}{2m} \right) \rho_c^\chi$$

$$\rho_c = \frac{2m_c}{n_c(n_c - 1)}$$

- Link density in community c

Guo et al, 2019

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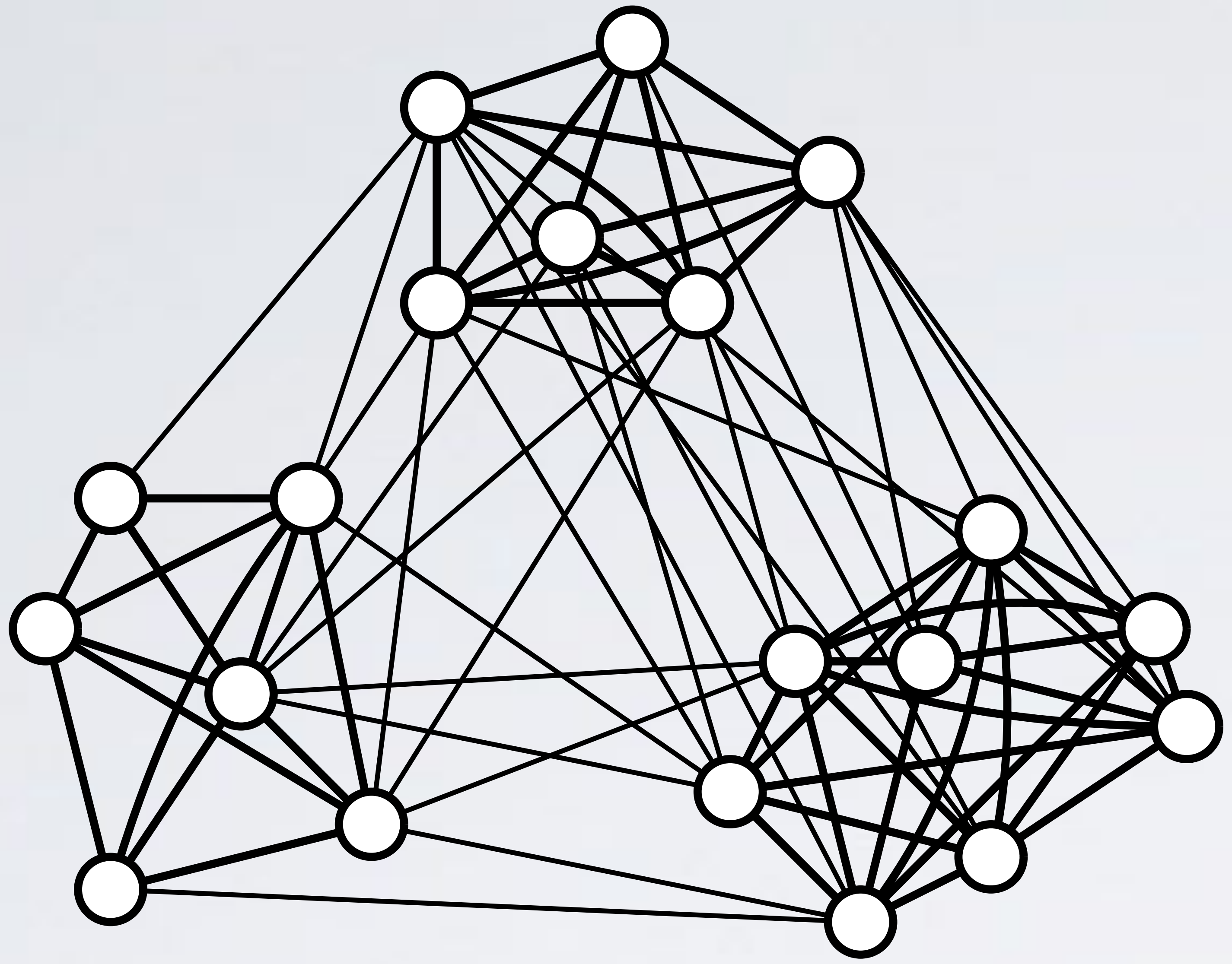

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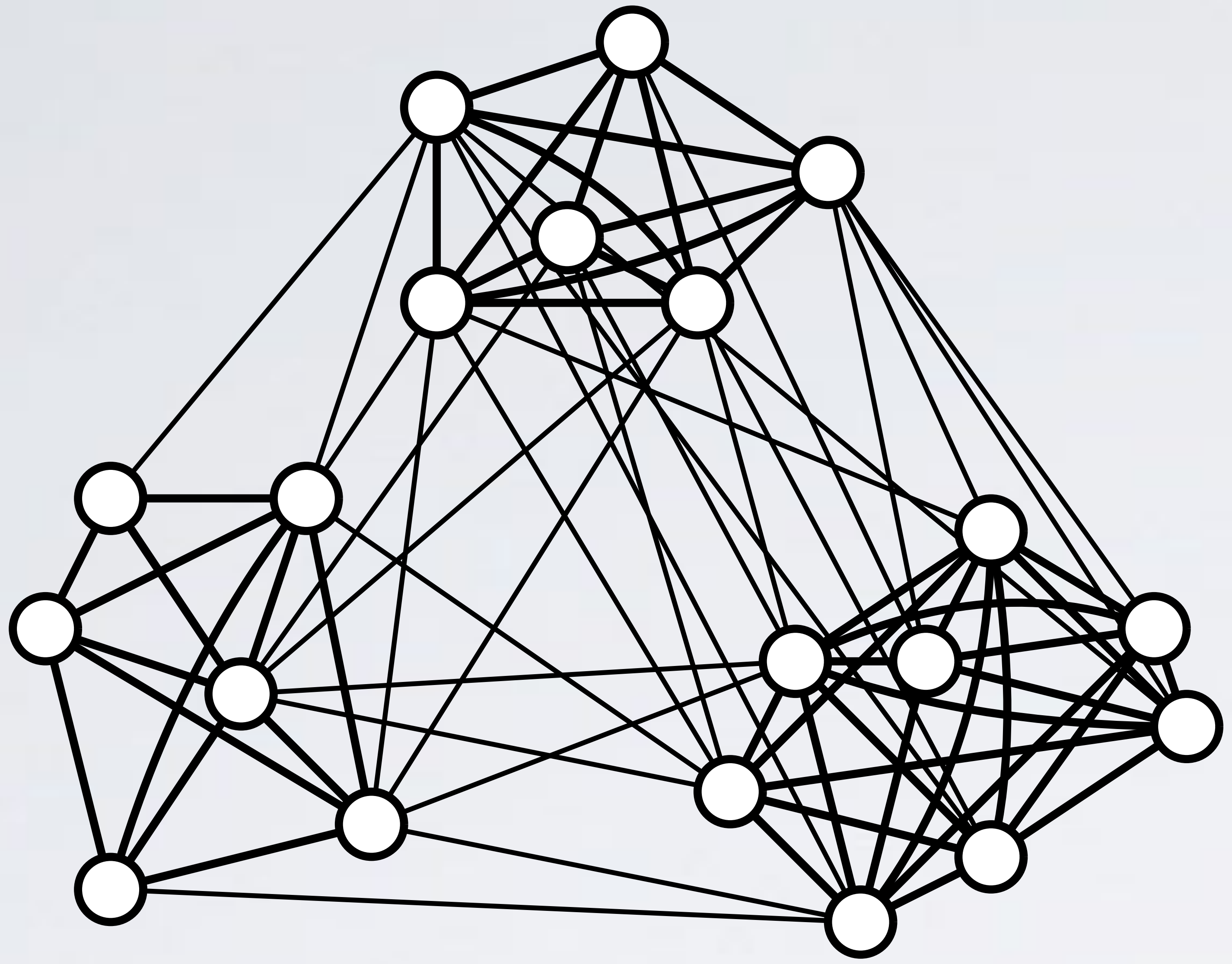

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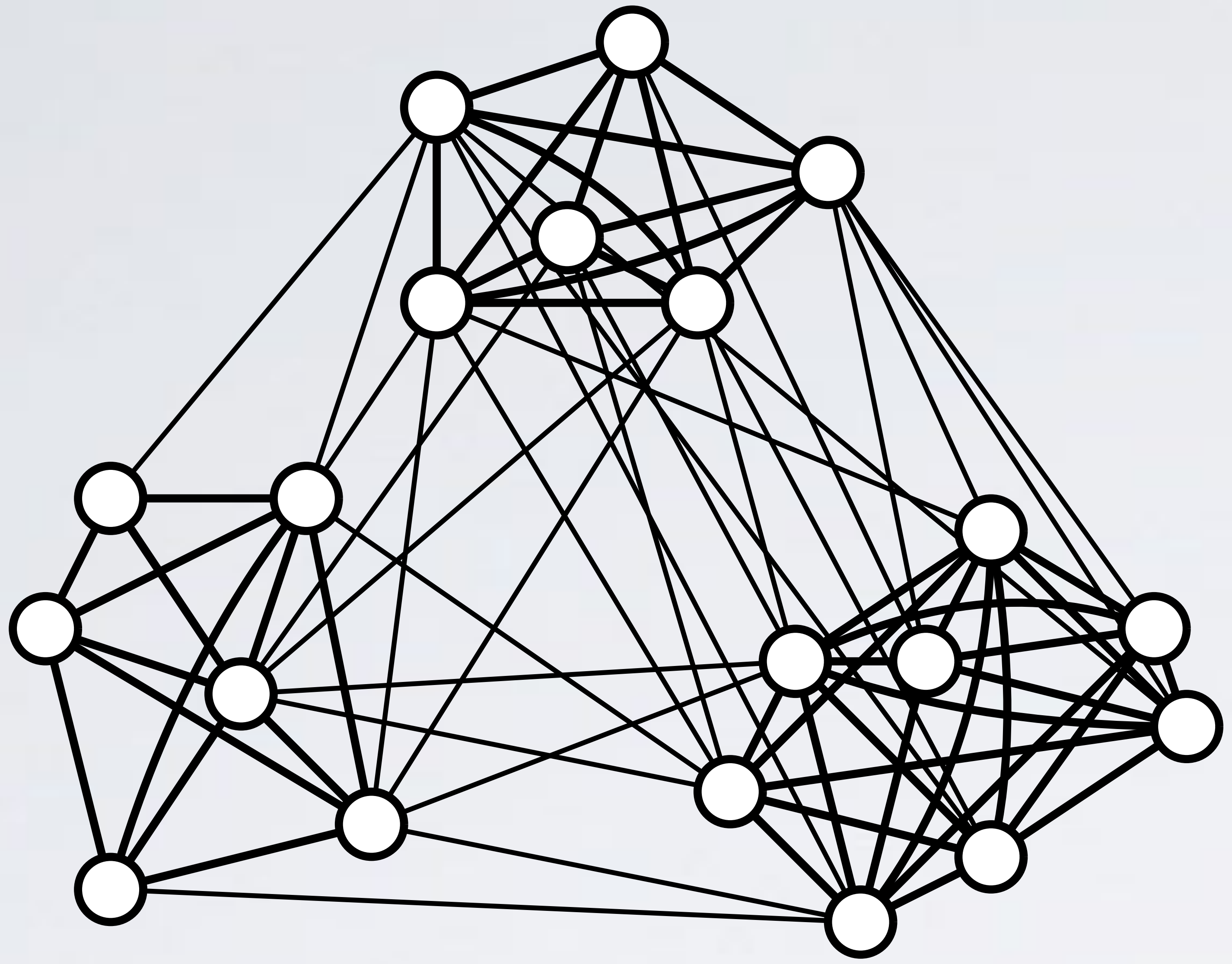

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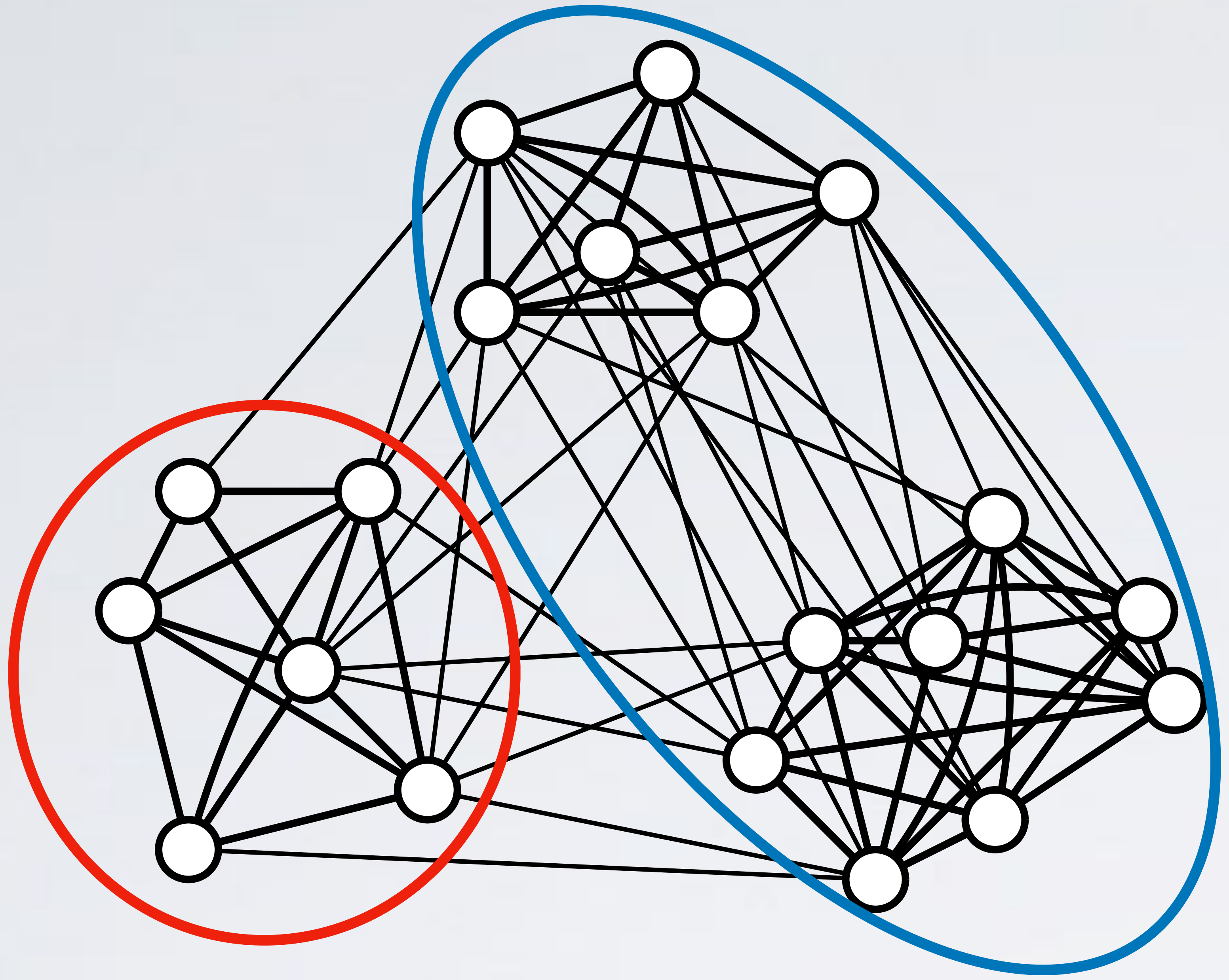

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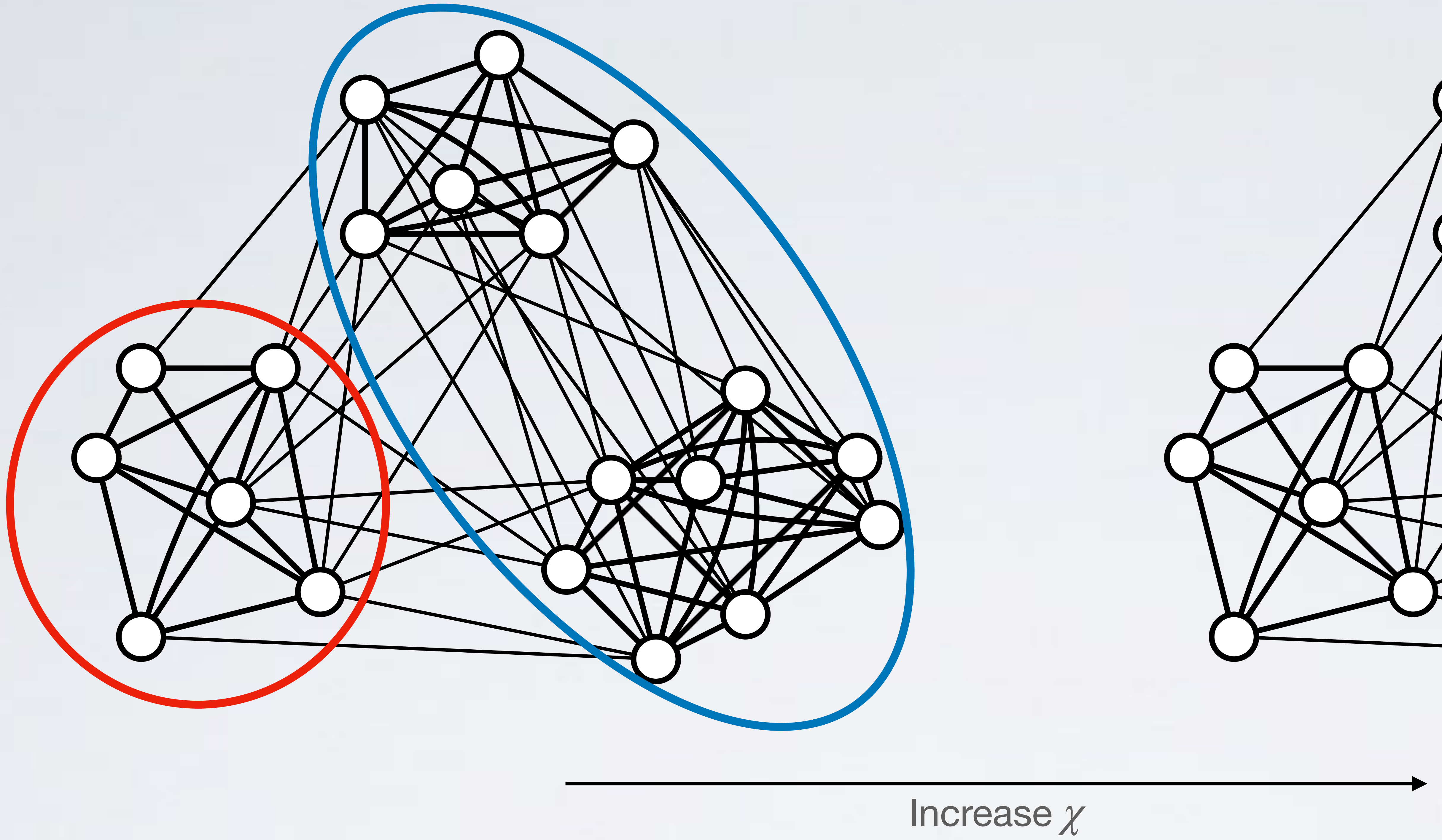

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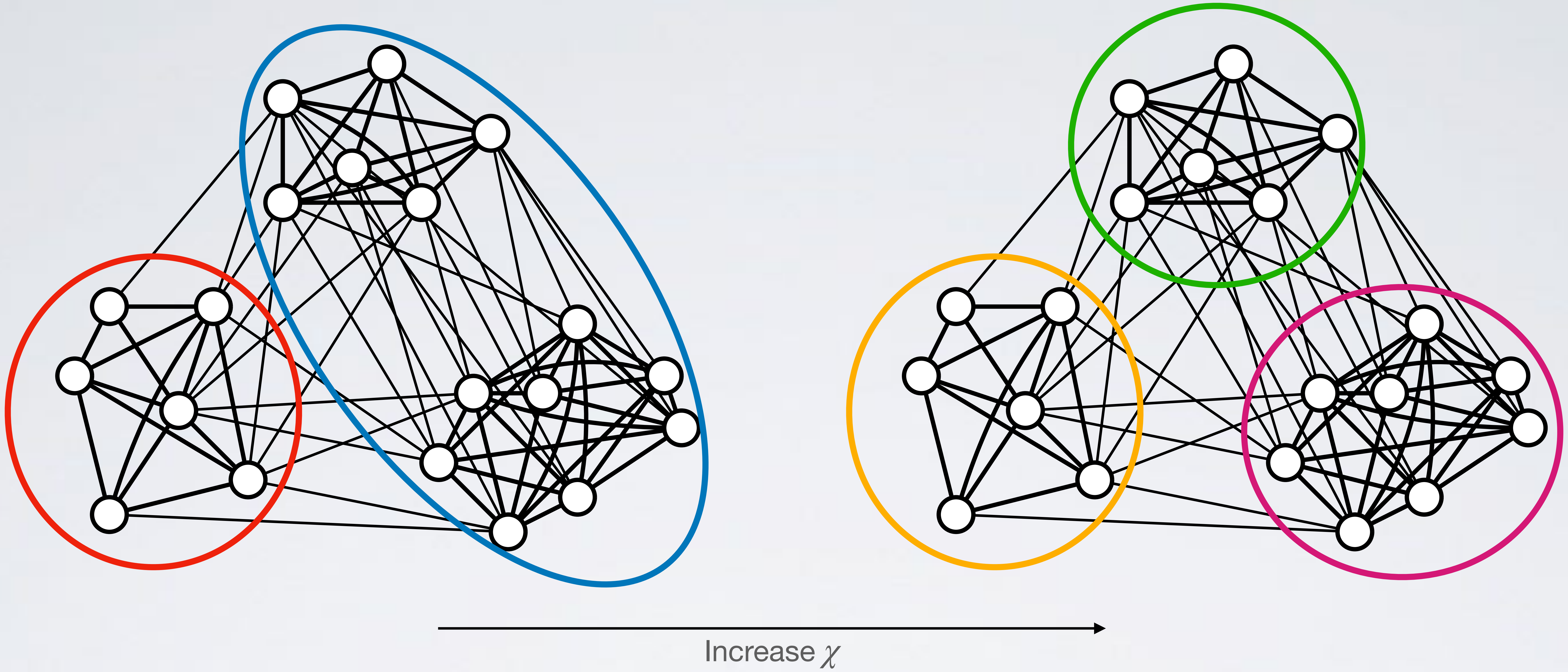
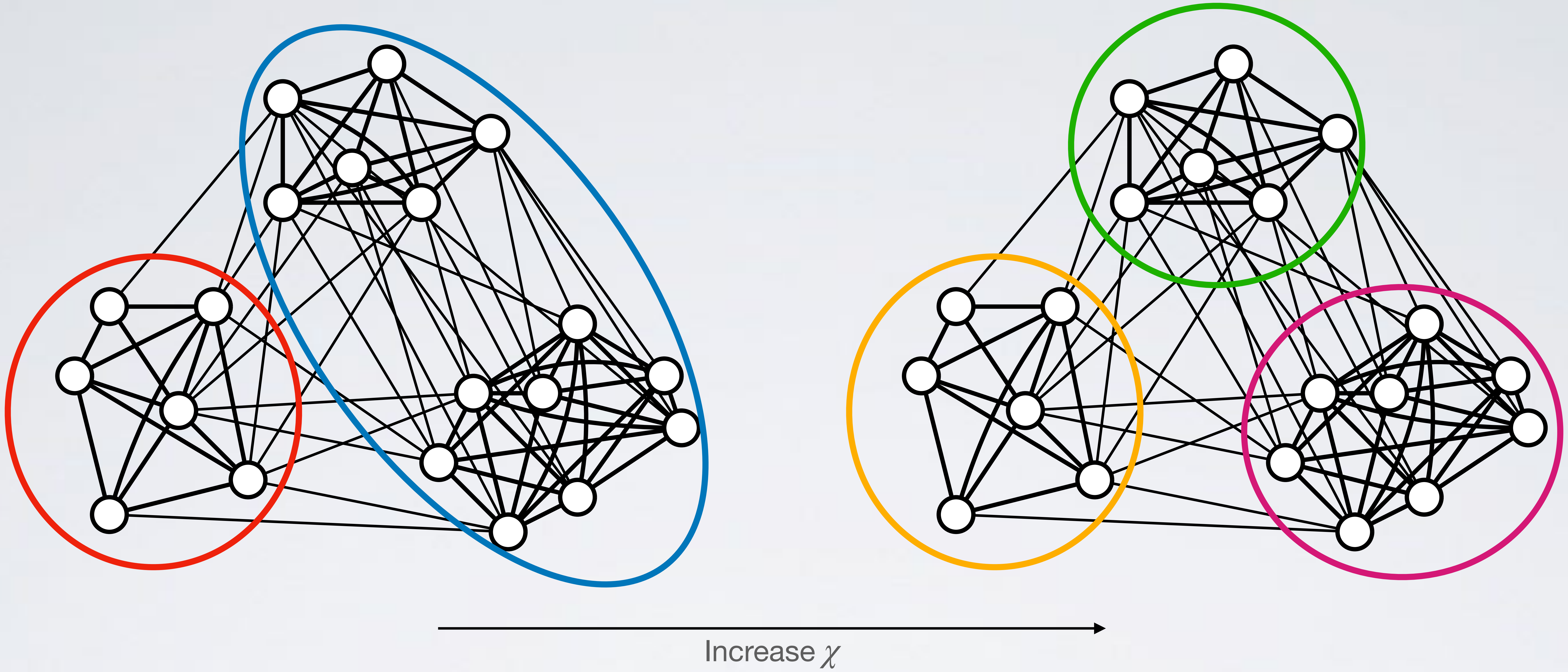
Community Detection



$$Q_g = \frac{1}{2m} \sum_c \left(2m_c - \frac{K_c^2}{2m} \right) \rho_c^{\chi}$$

Community Detection



$$Q_g = \frac{1}{2m} \sum_c \left(2m_c - \frac{K_c^2}{2m} \right) \rho_c^{\chi}$$

RenEEL algorithm (Guo et al 2019)

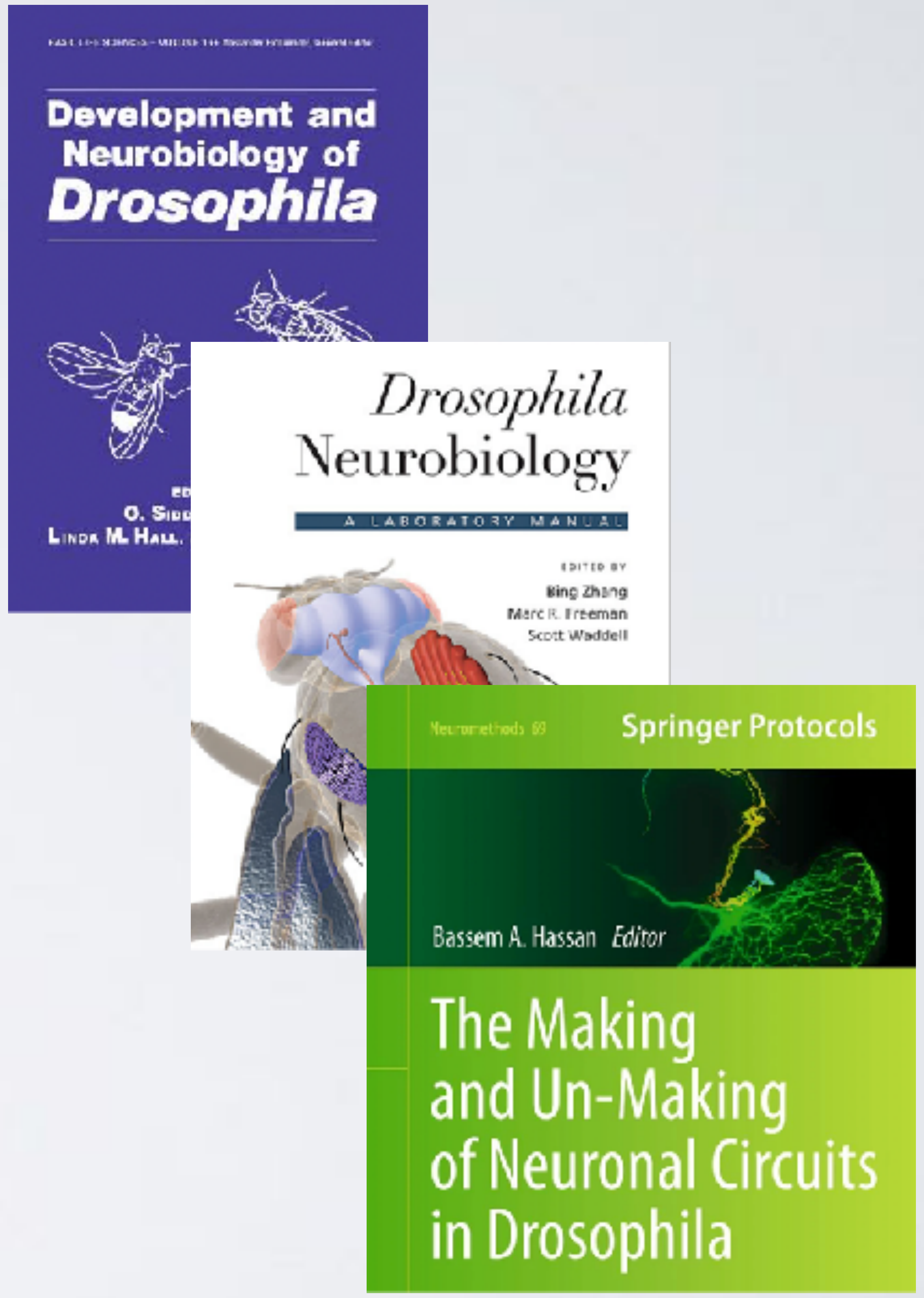
Input: weighted undirected graph, control parameter χ

Output: list of communities c maximizing Q_g

Why apply community detection to the connectome?

1. Complement what fly physiologists have discovered about the anatomy of the Drosophila brain.

2. Discover potentially meaningful structures that fly physiologists may not know about yet.



$\chi = 0$

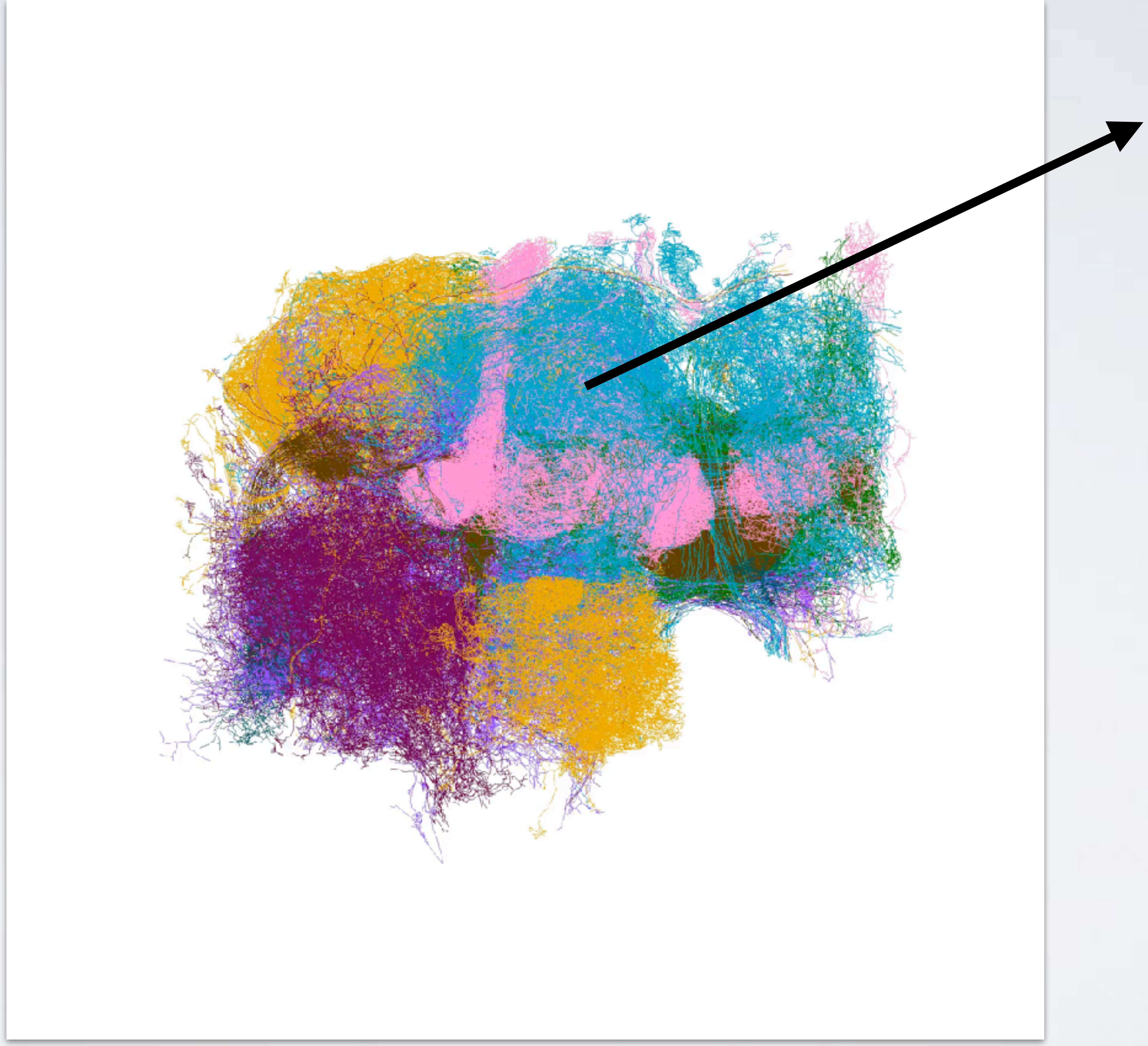
8 clusters

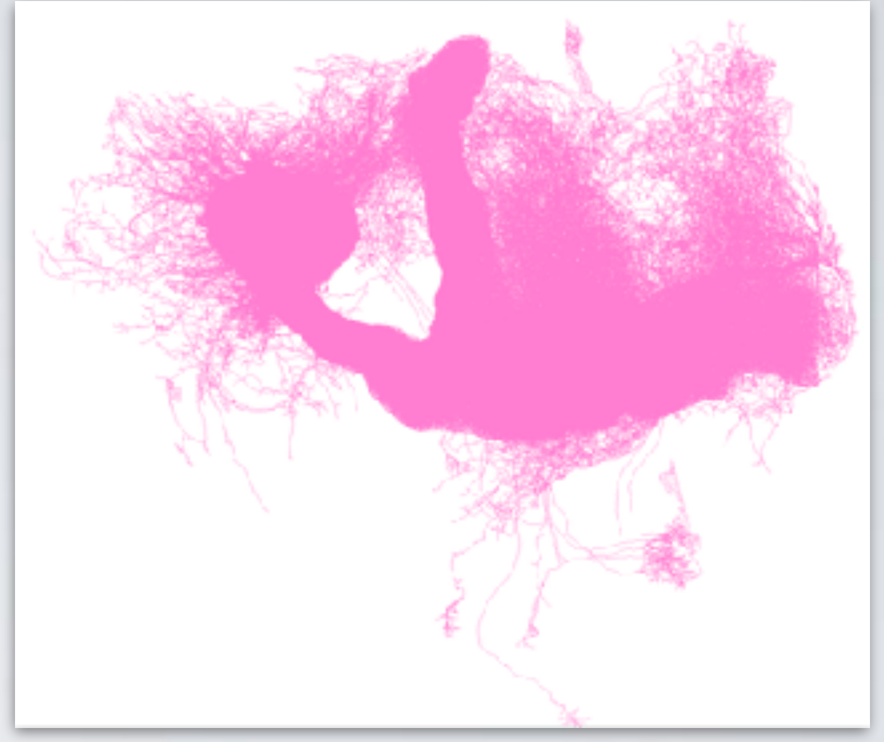
$\chi = 0$

8 clusters

$\chi = 0$

8 clusters

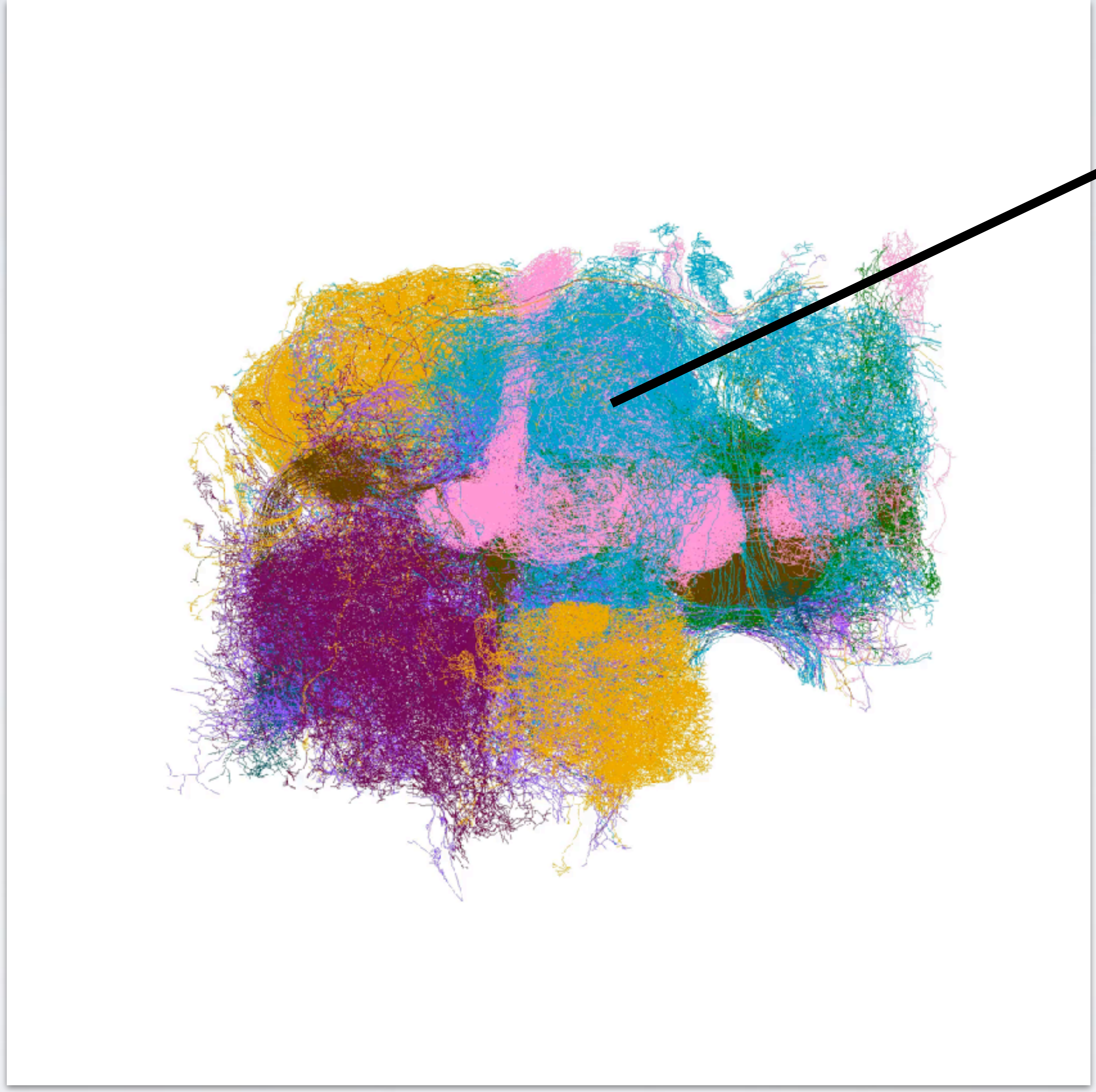


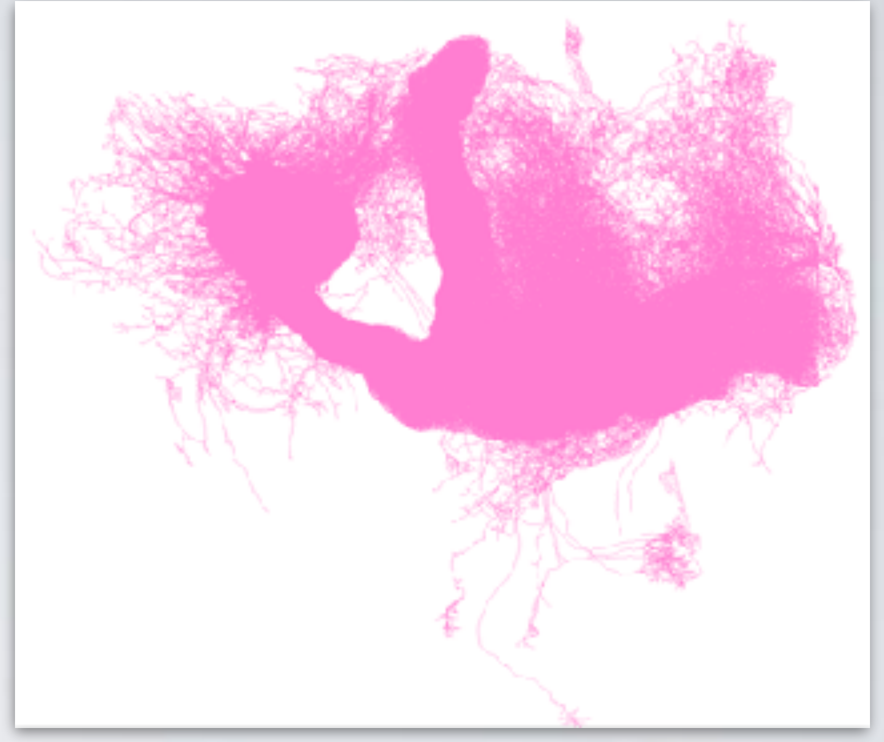


Mushroom body

$\chi = 0$

8 clusters





Mushroom body

chi = 0.0

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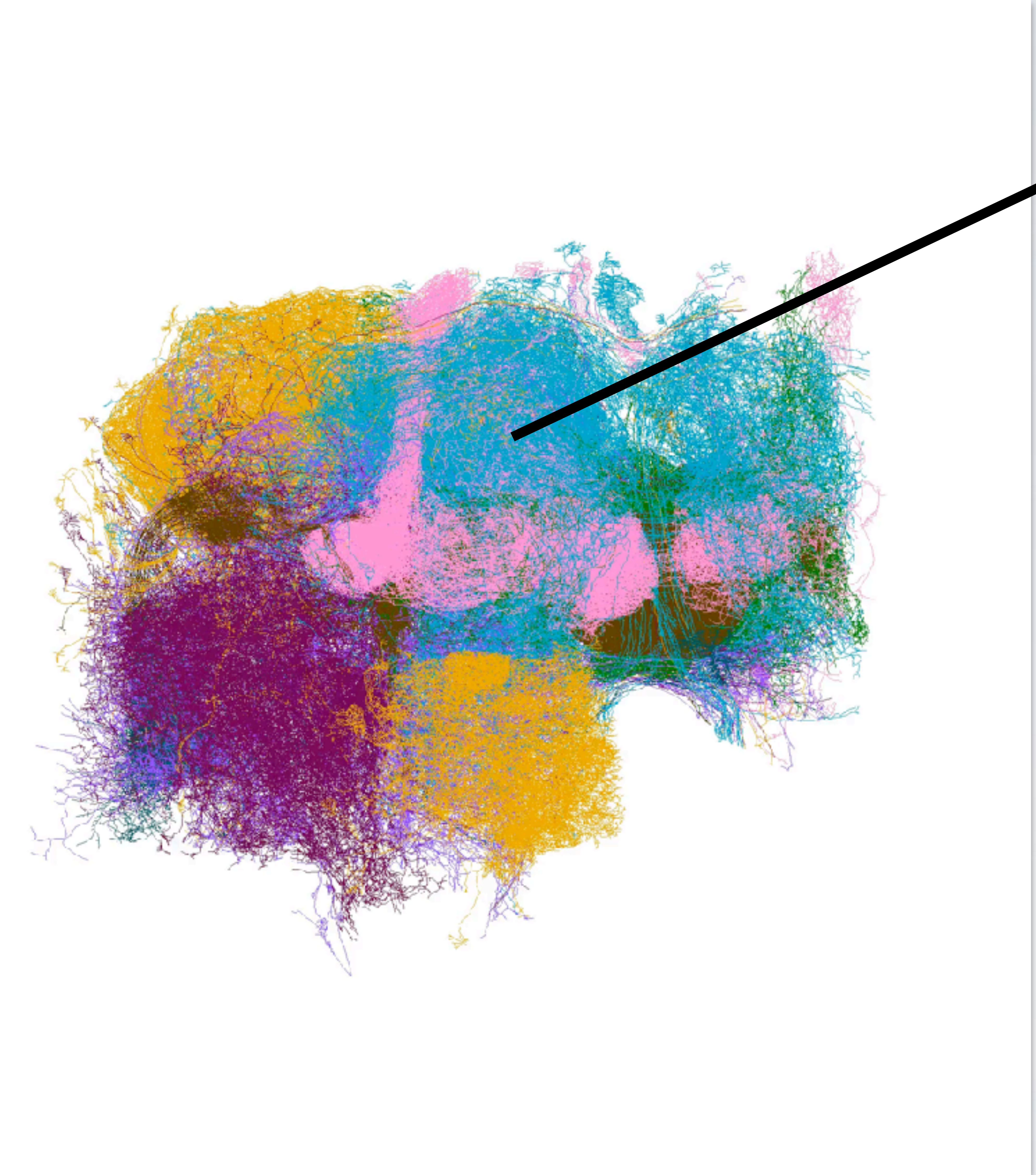
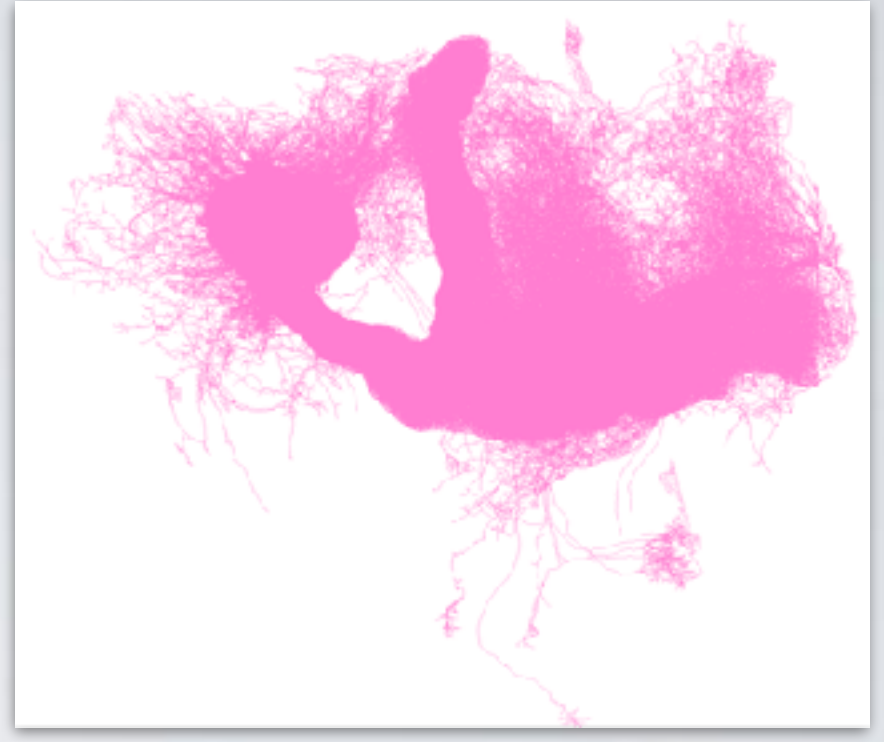
8 clusters



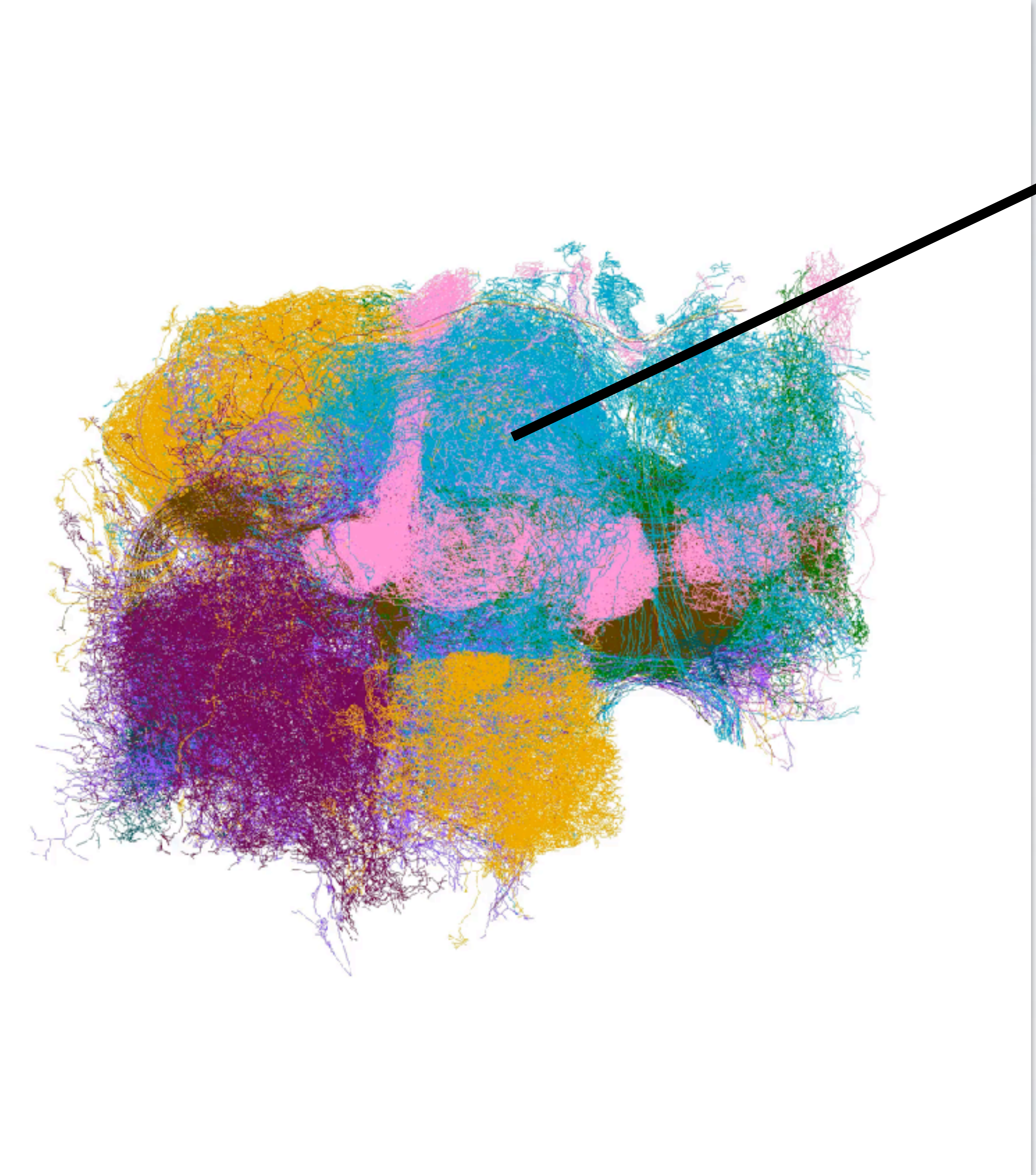


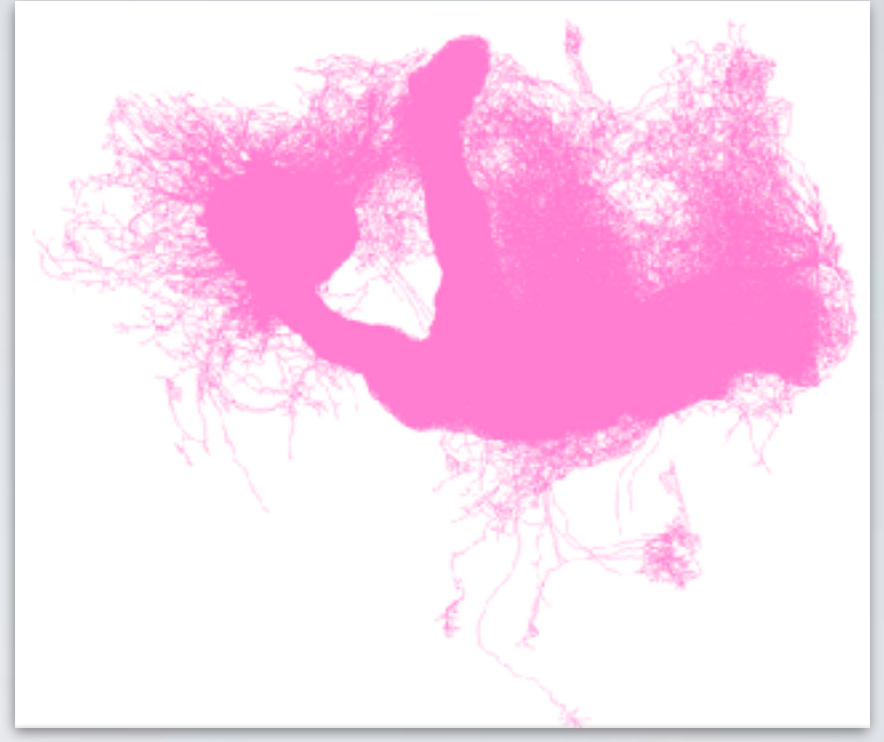
Mushroom body

chi = 0.05

$\chi = 0$

8 clusters



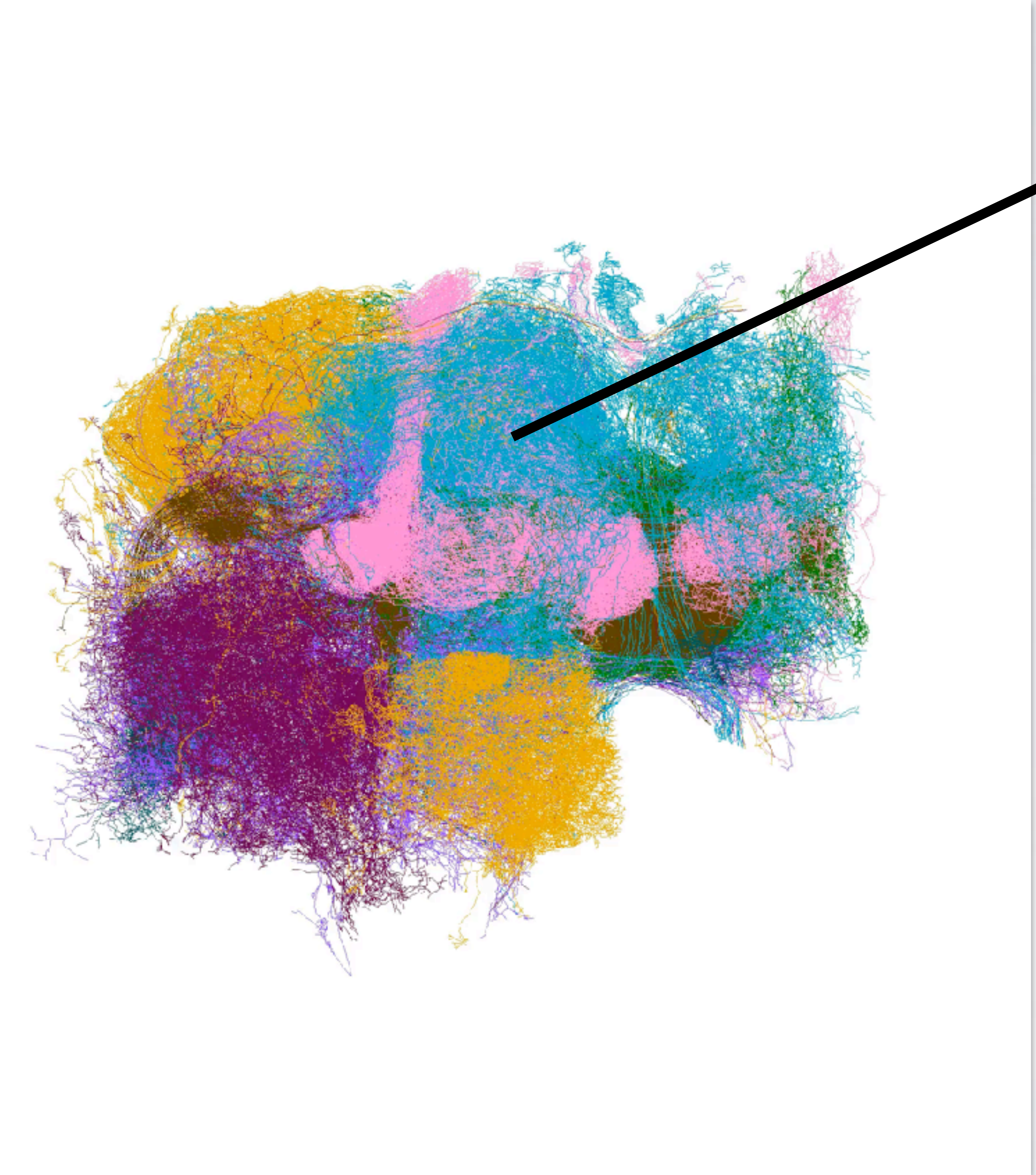


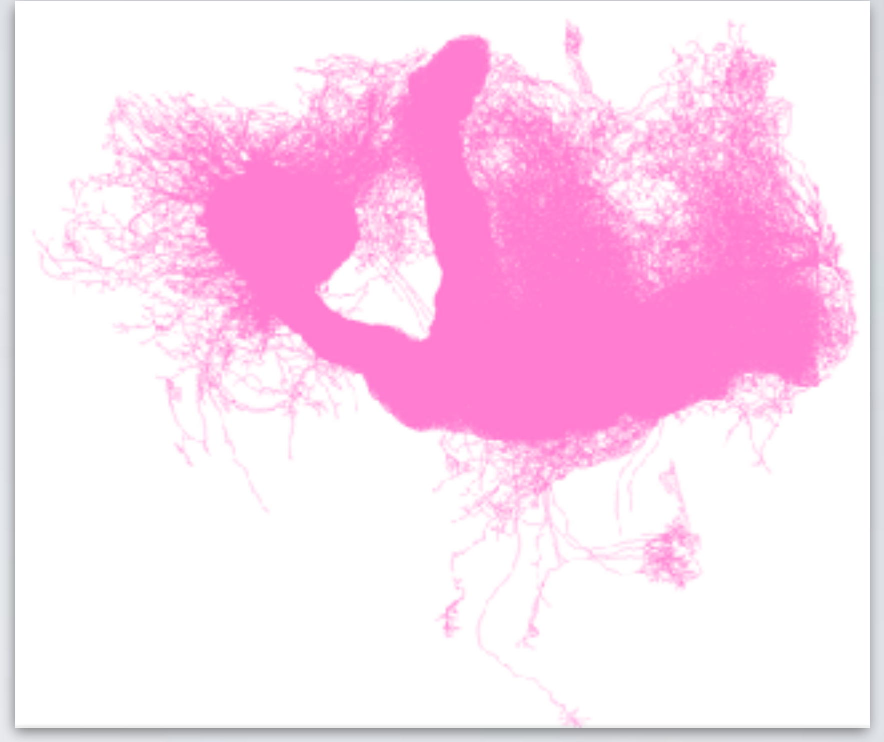
Mushroom body

chi = 0.1

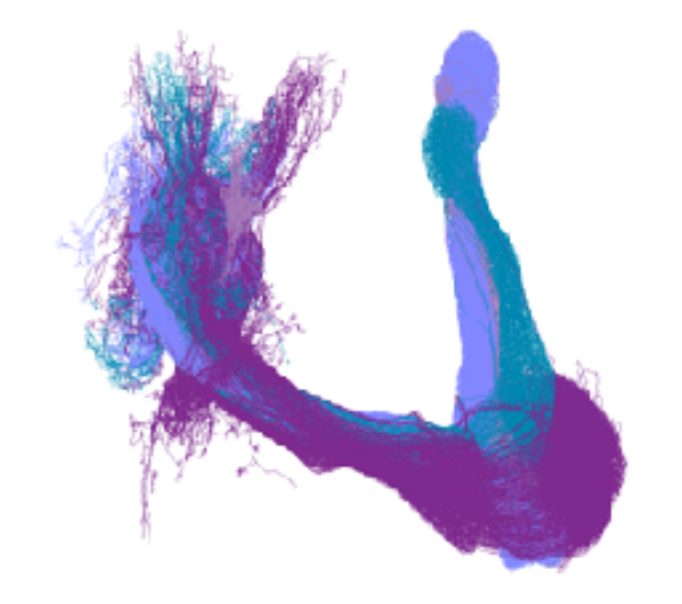
$\chi = 0$

8 clusters





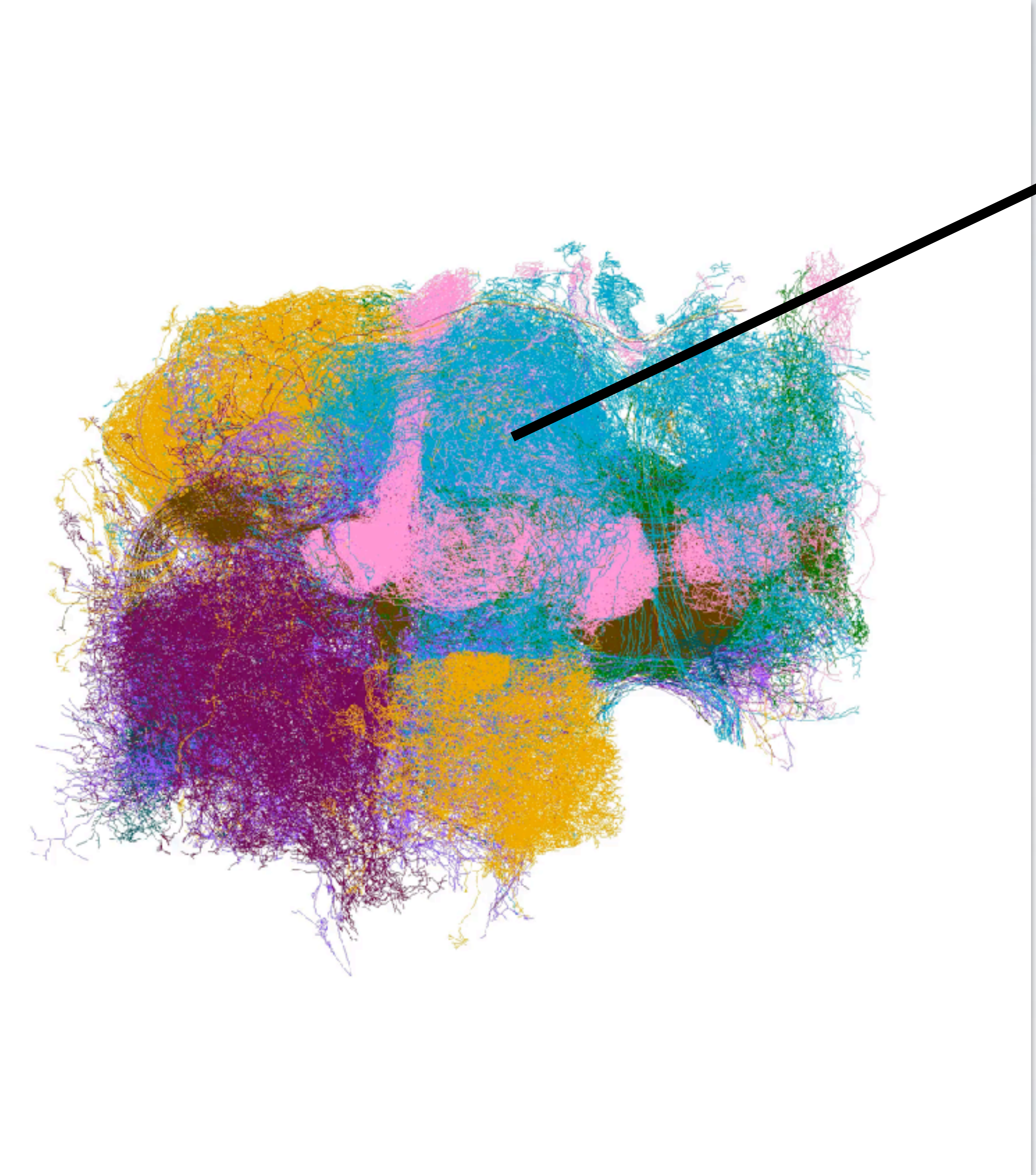
Mushroom body

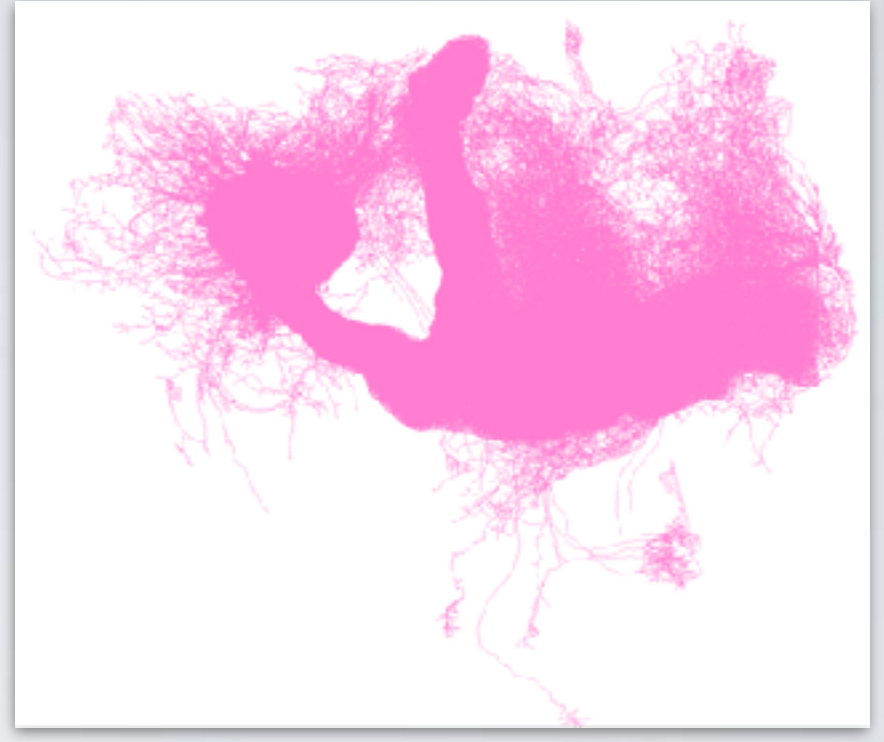


chi = 0.25

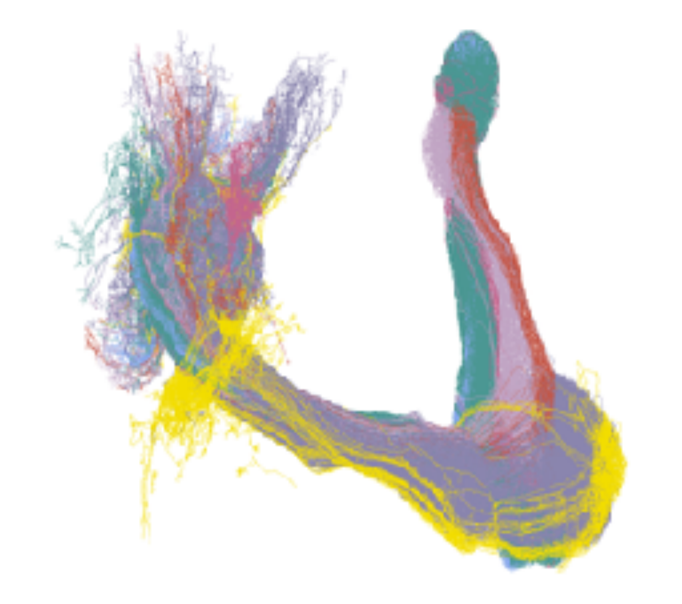
$\chi = 0$

8 clusters





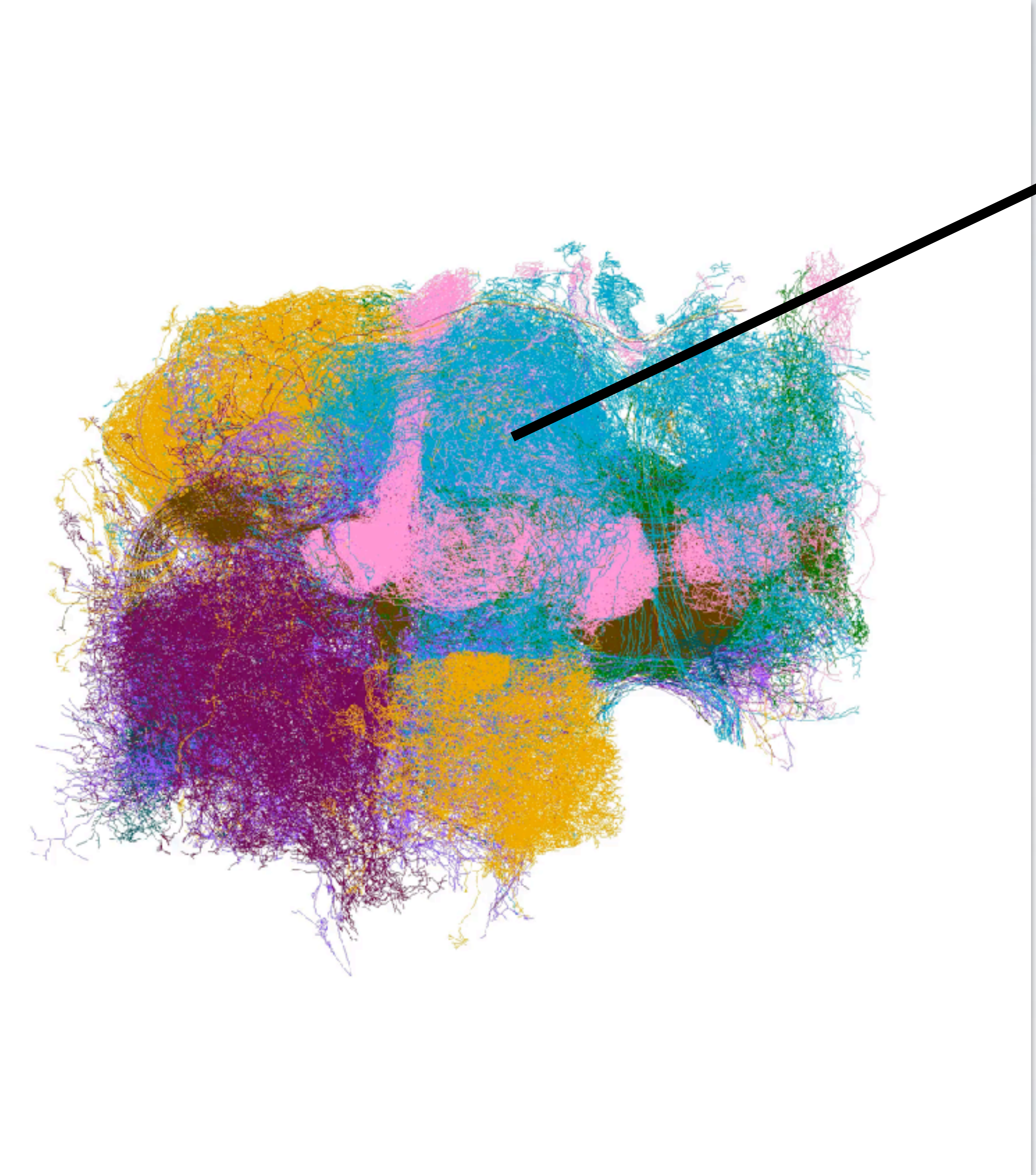
Mushroom body

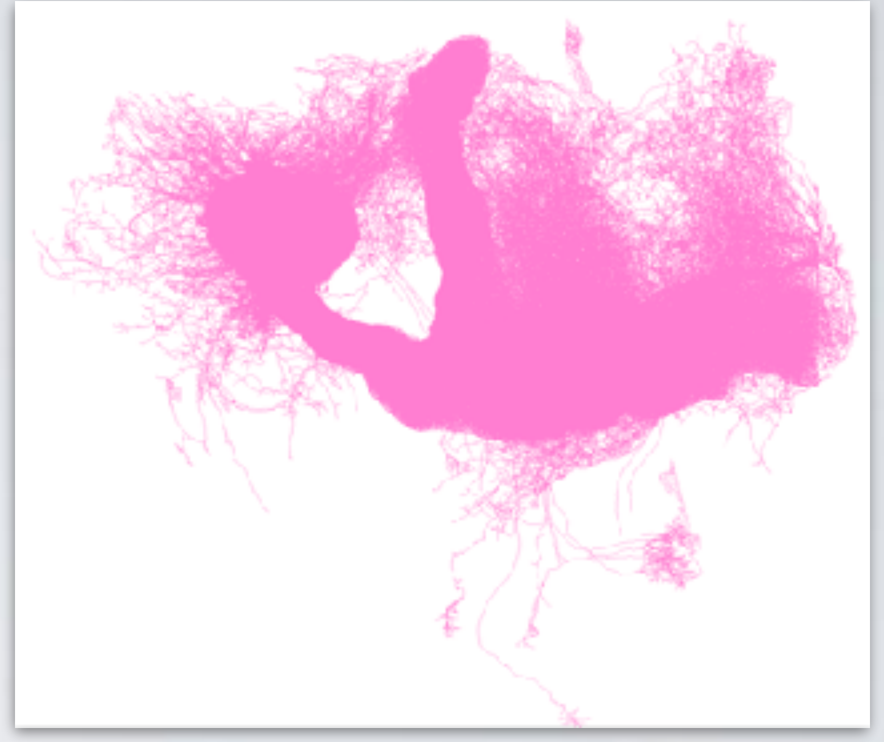


chi = 0.5

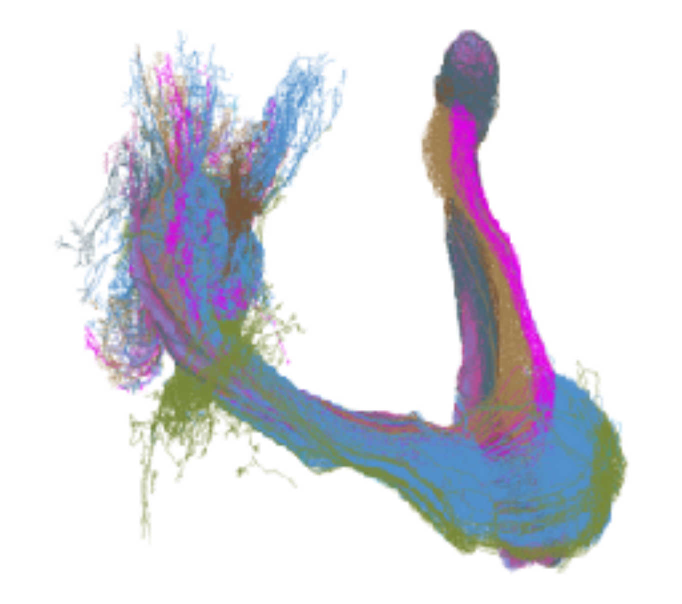
$\chi = 0$

8 clusters





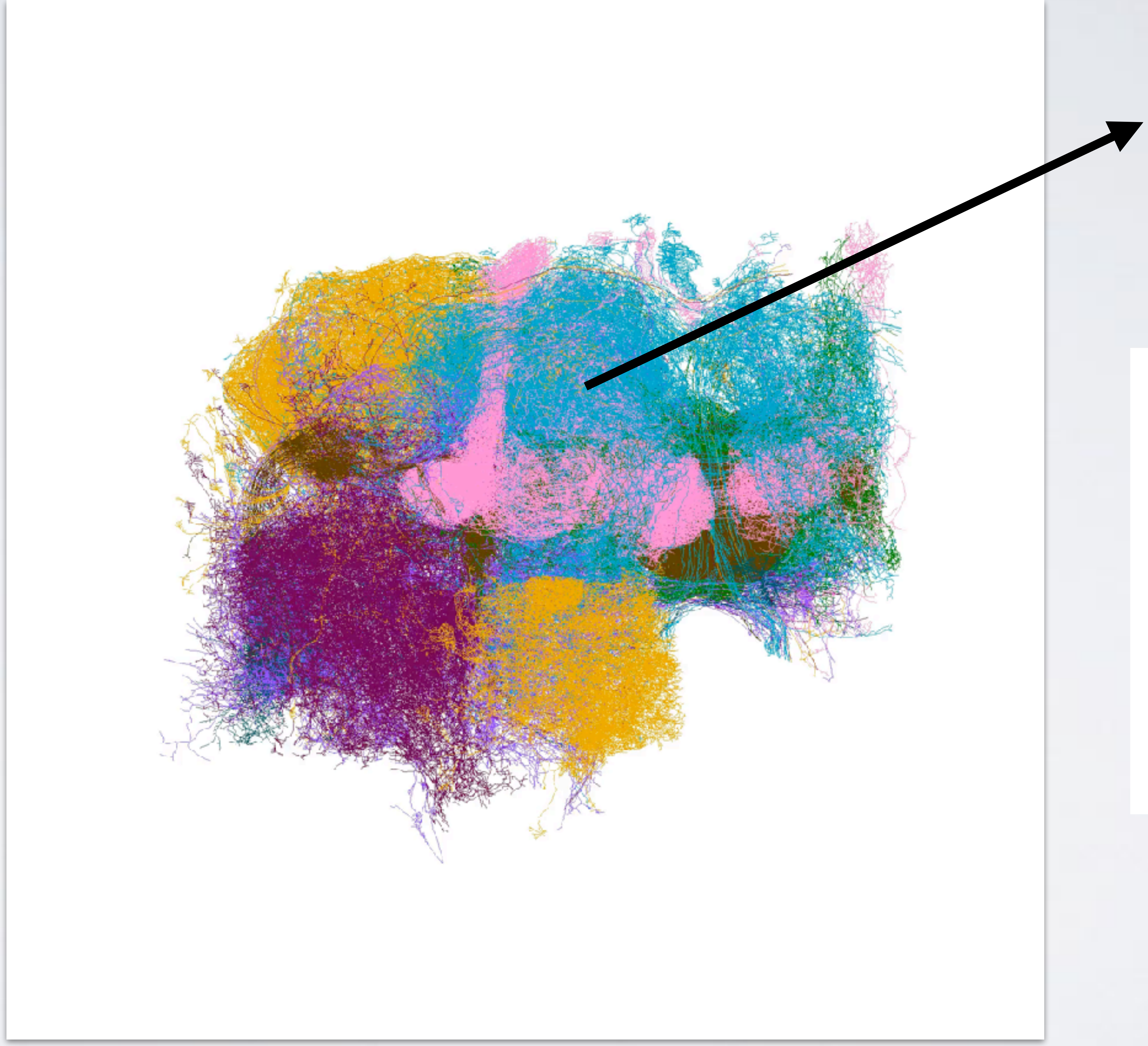
Mushroom body

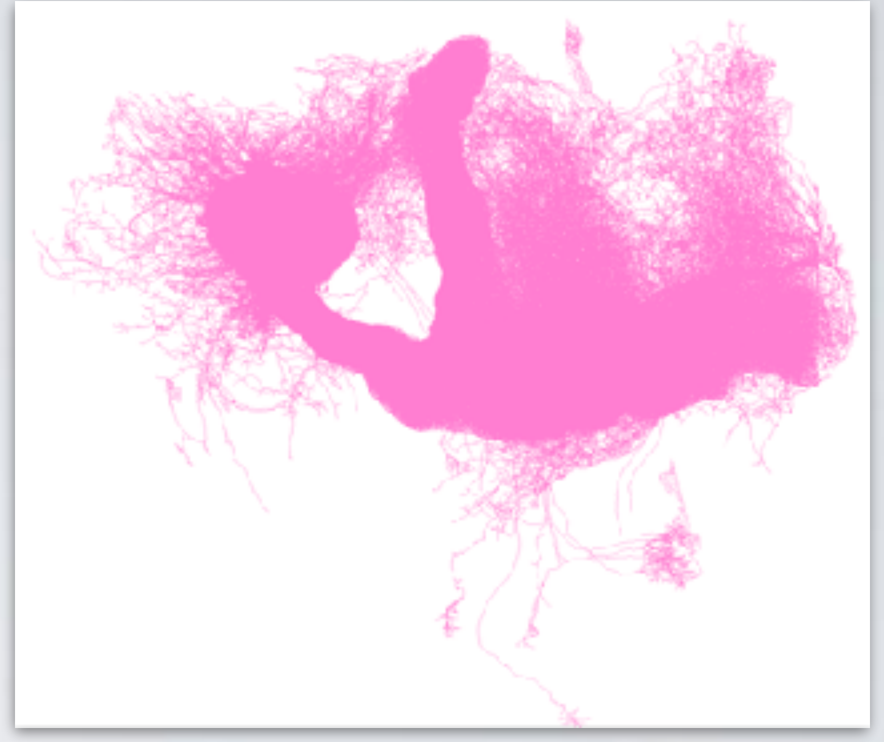


chi = 0.75

$\chi = 0$

8 clusters





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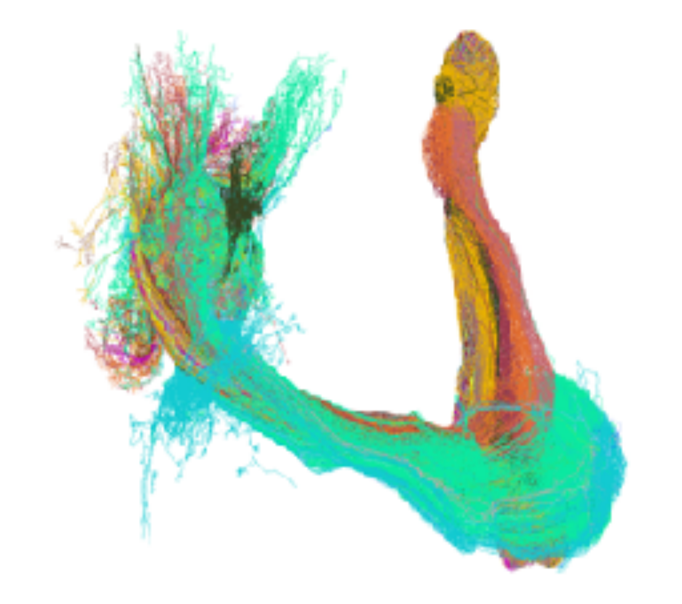
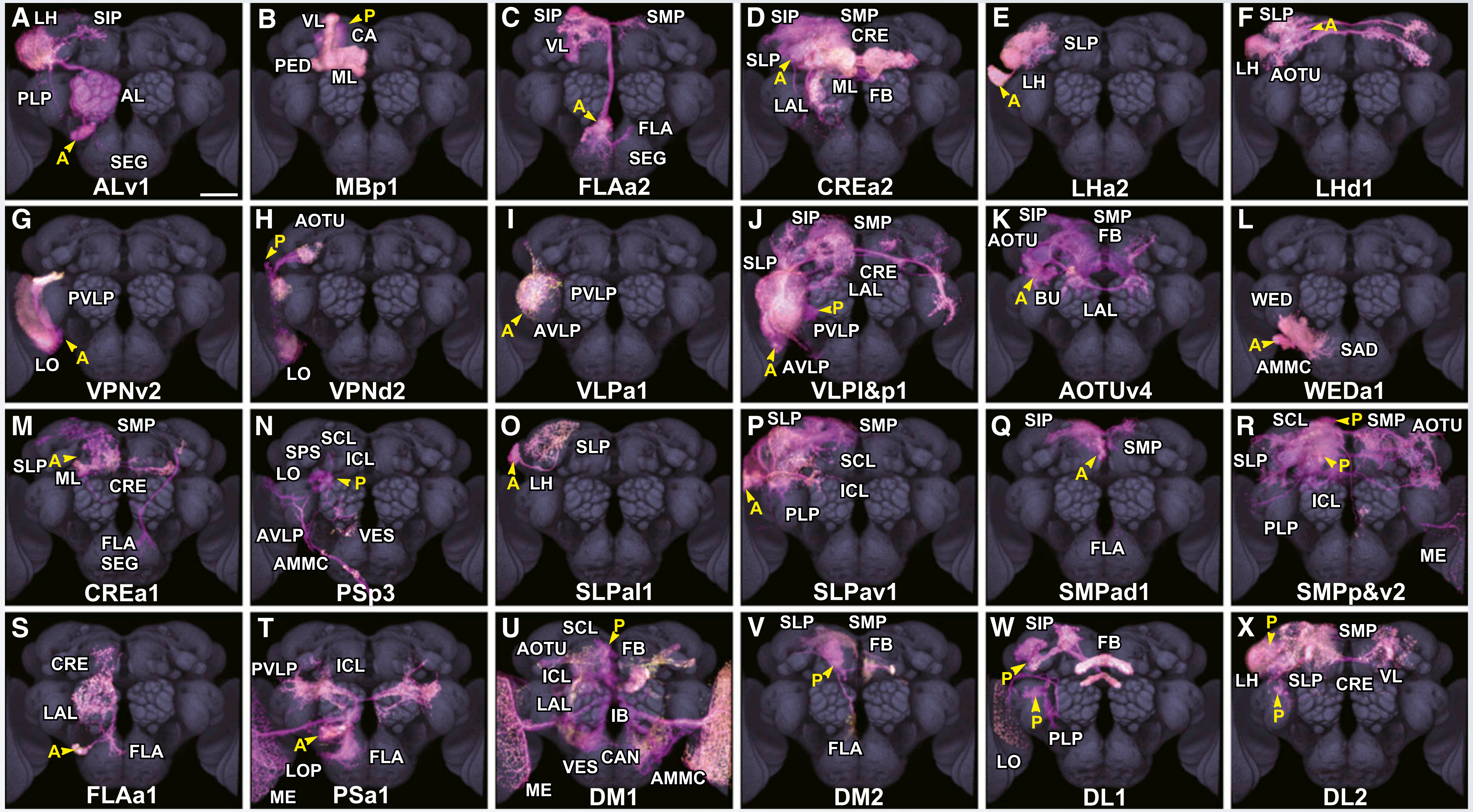


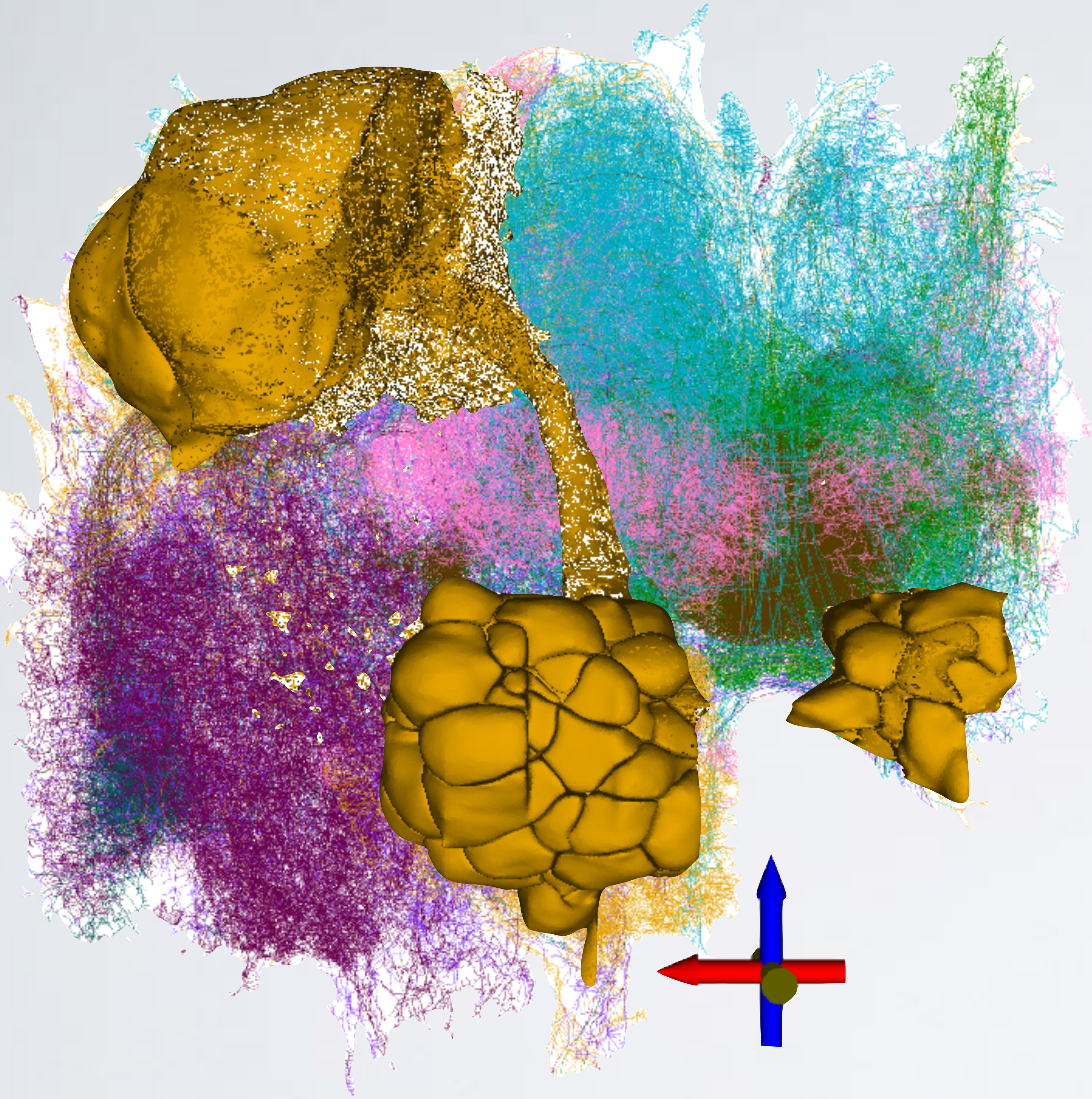
chi = 1.0

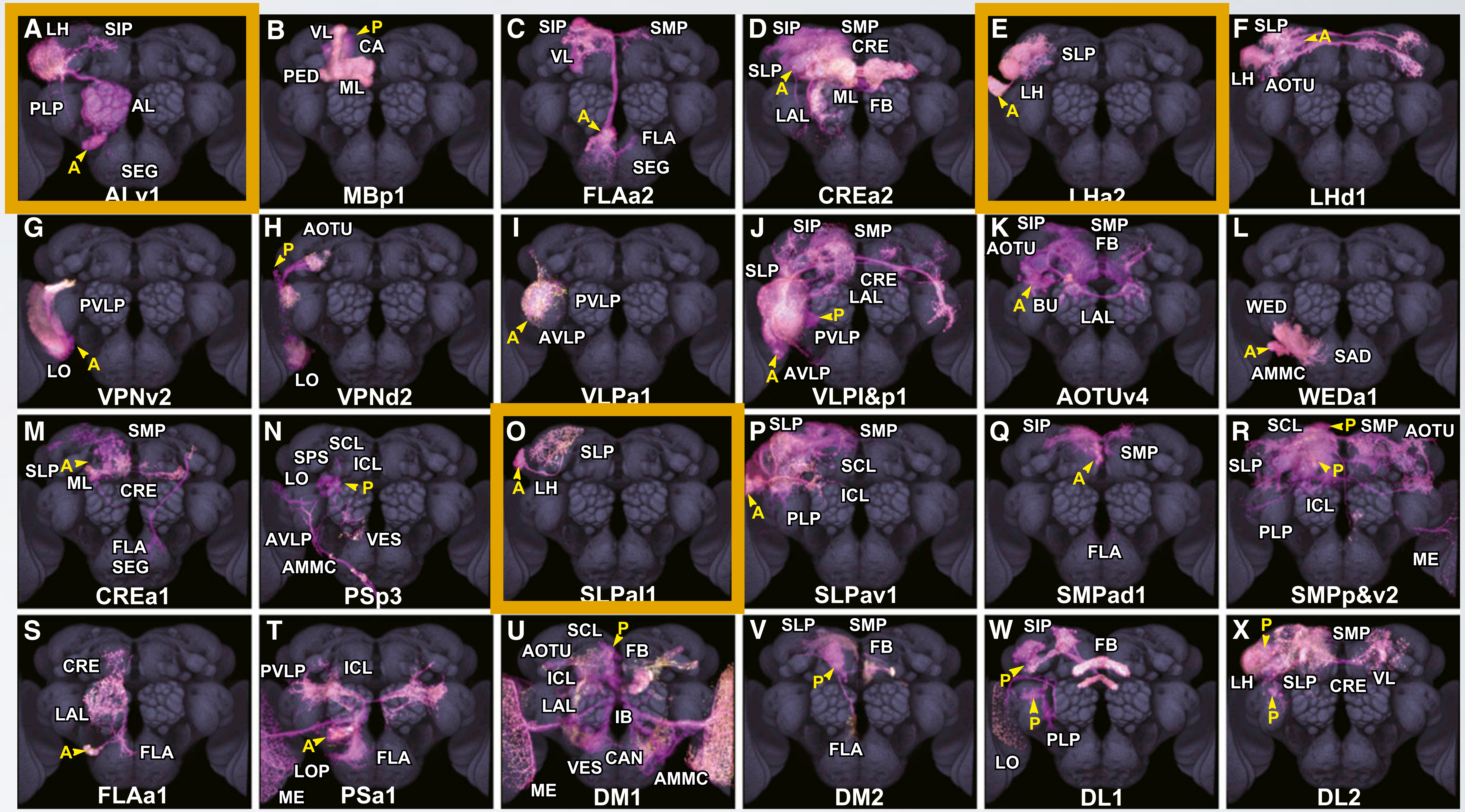
Clonal Units



Ito, M., et al (2013). *Current Biology*, *23*(8), 644–655.

Clonal Units





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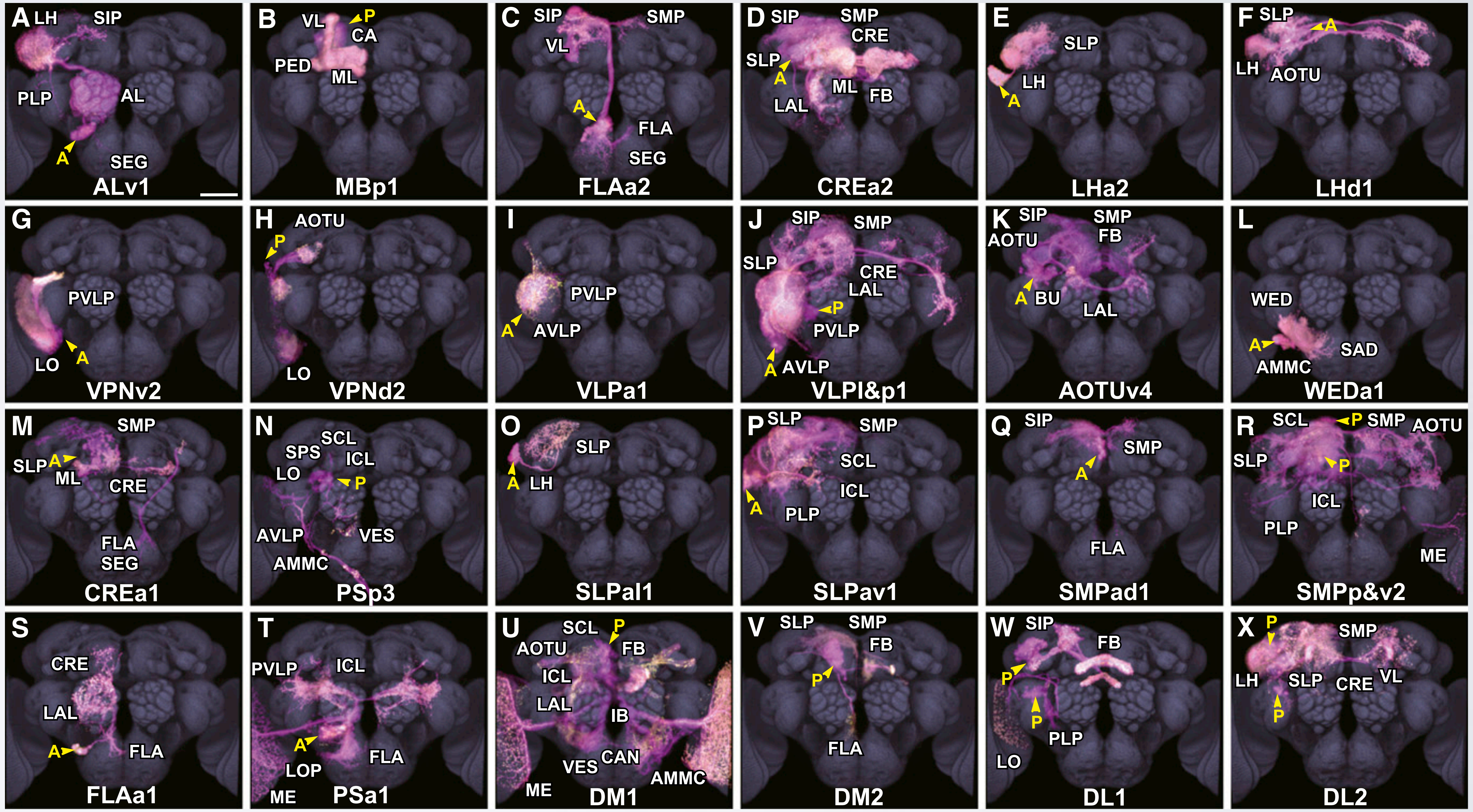
Clonal Units



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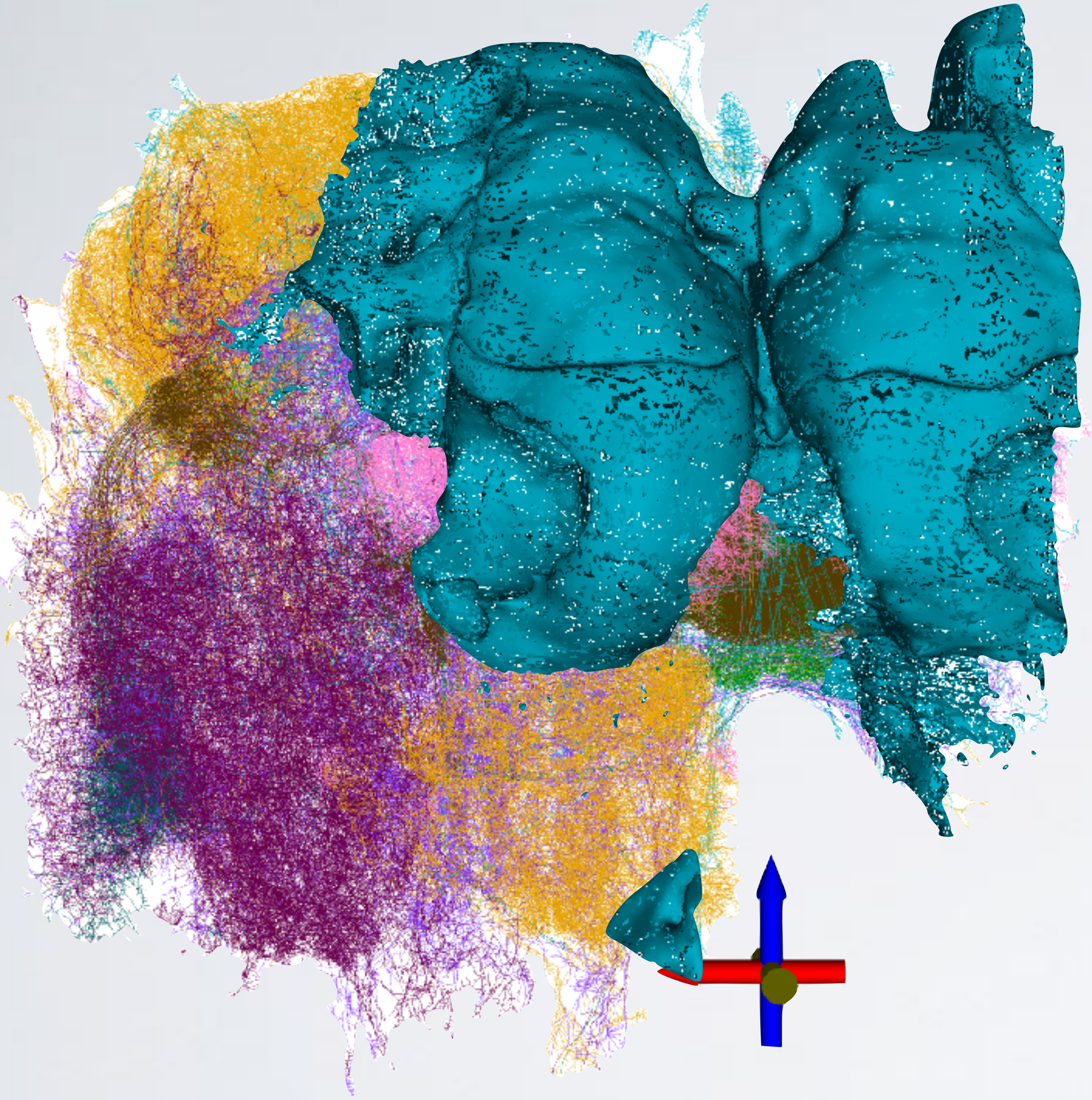

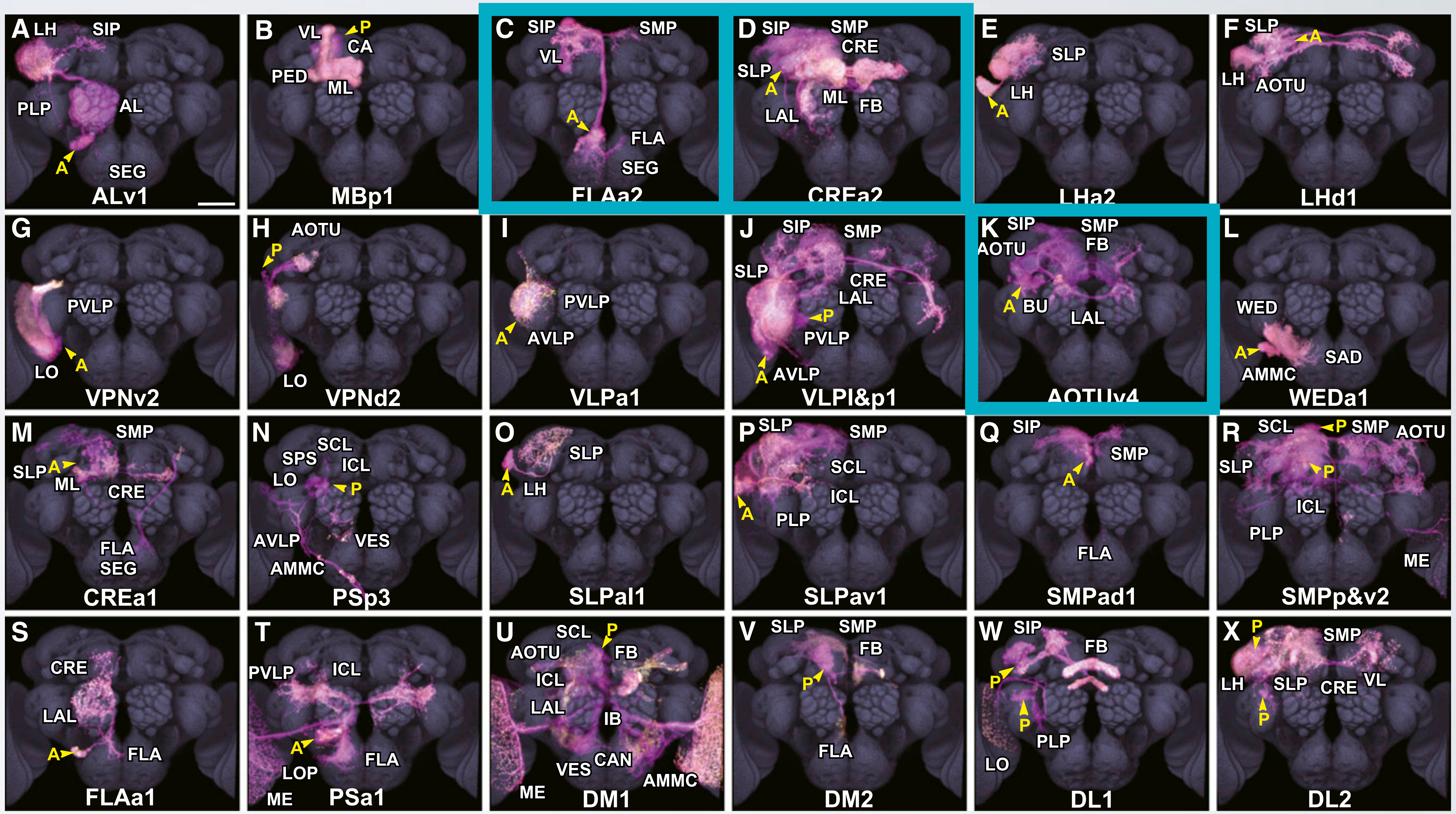

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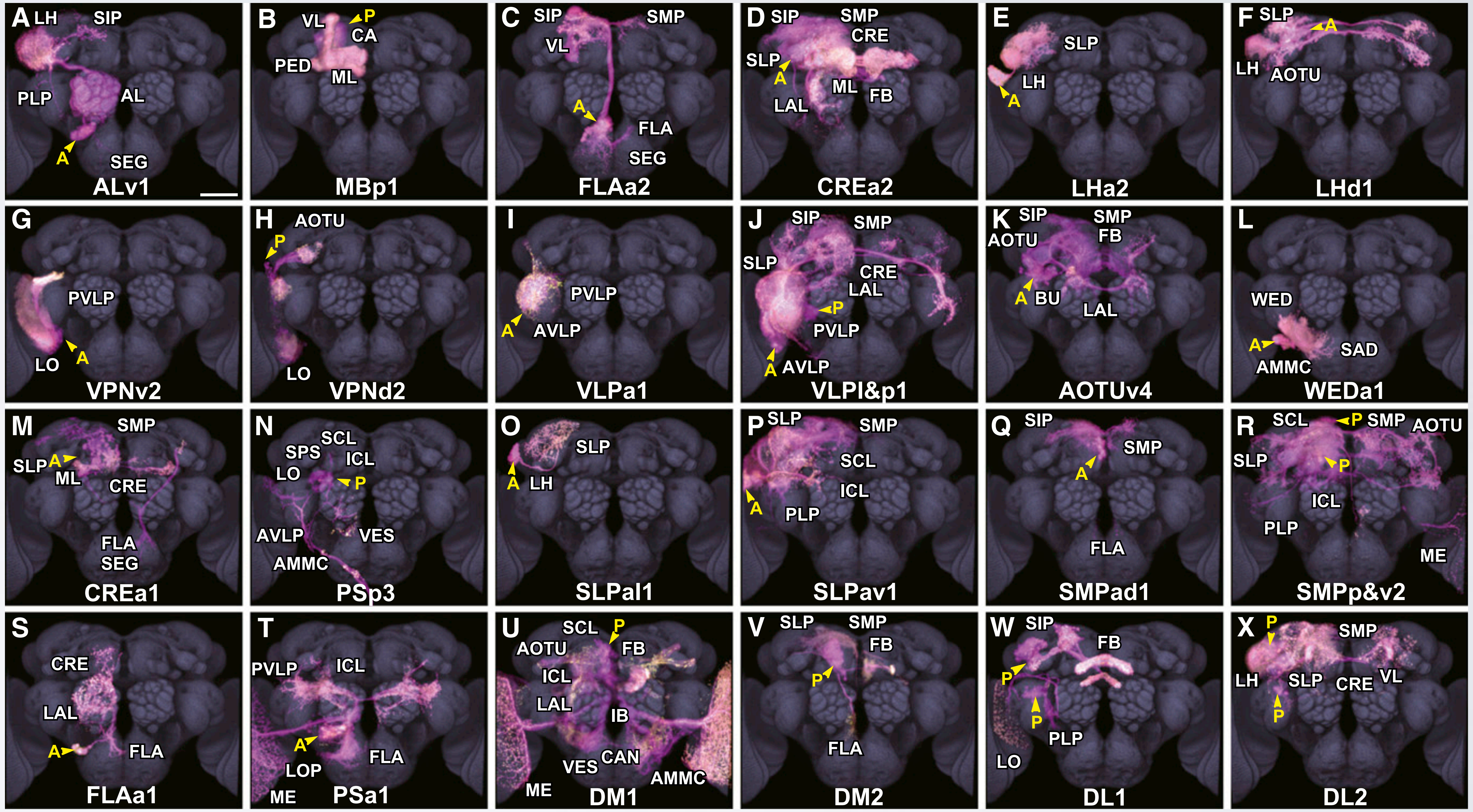

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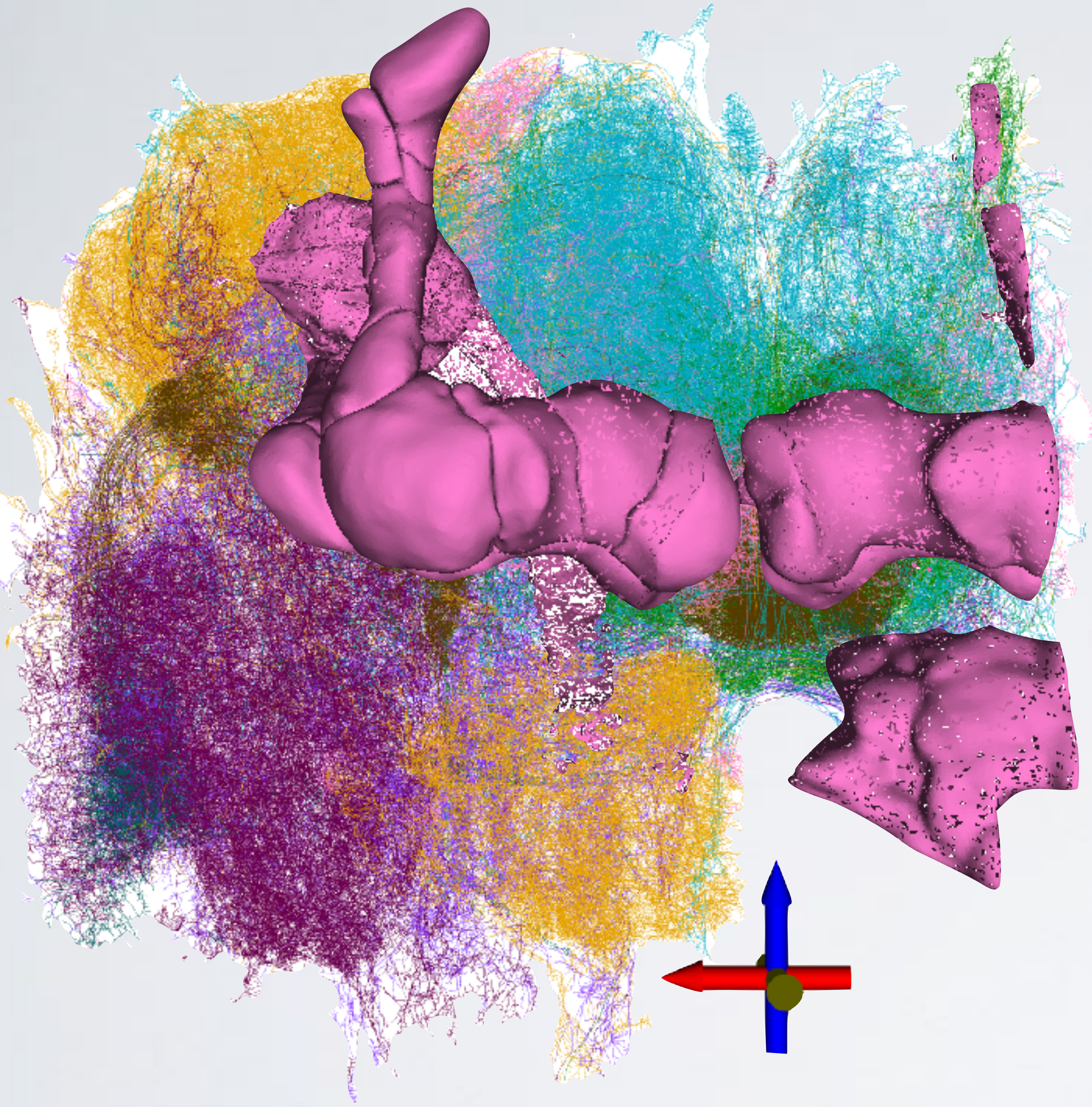

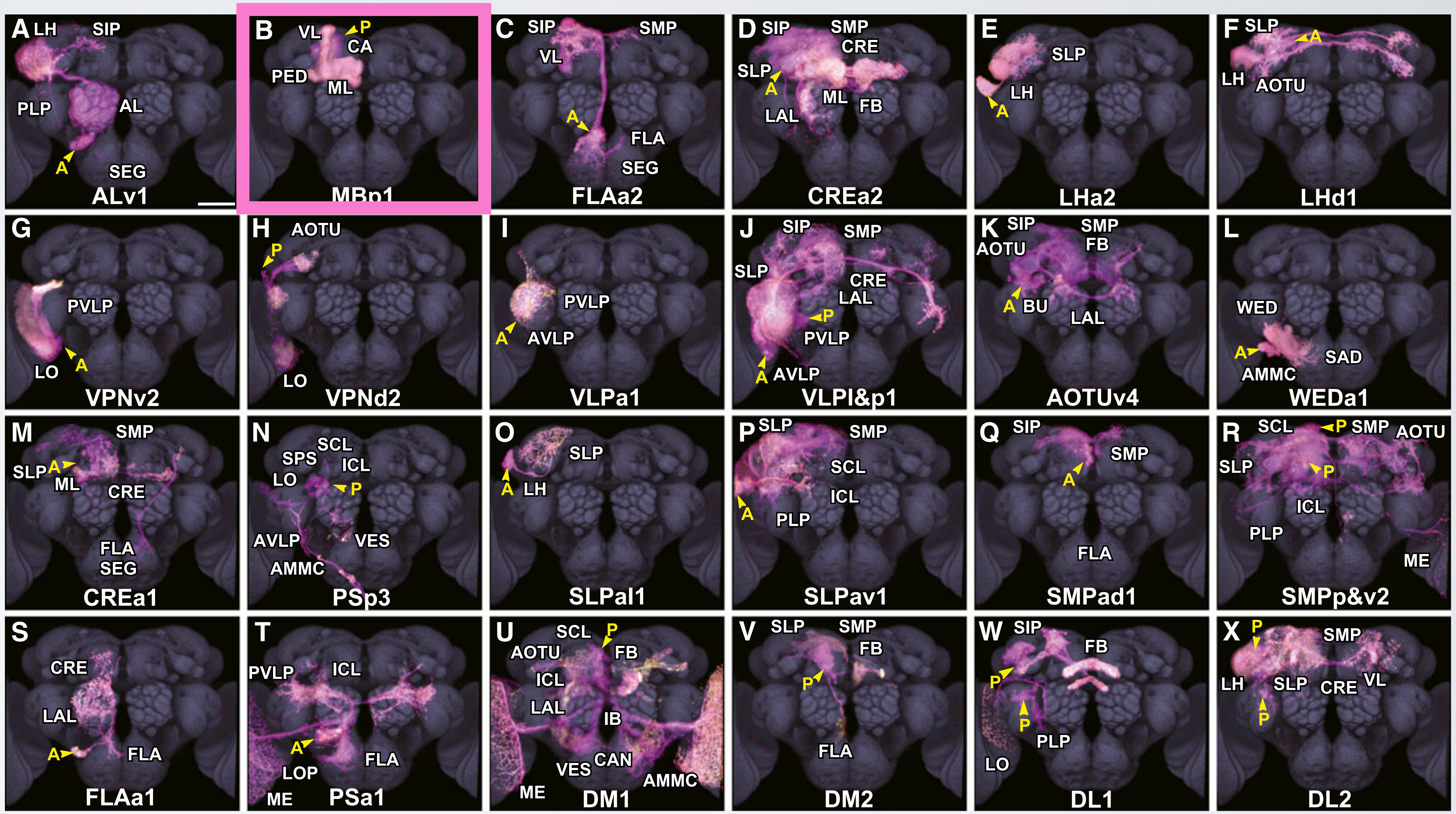

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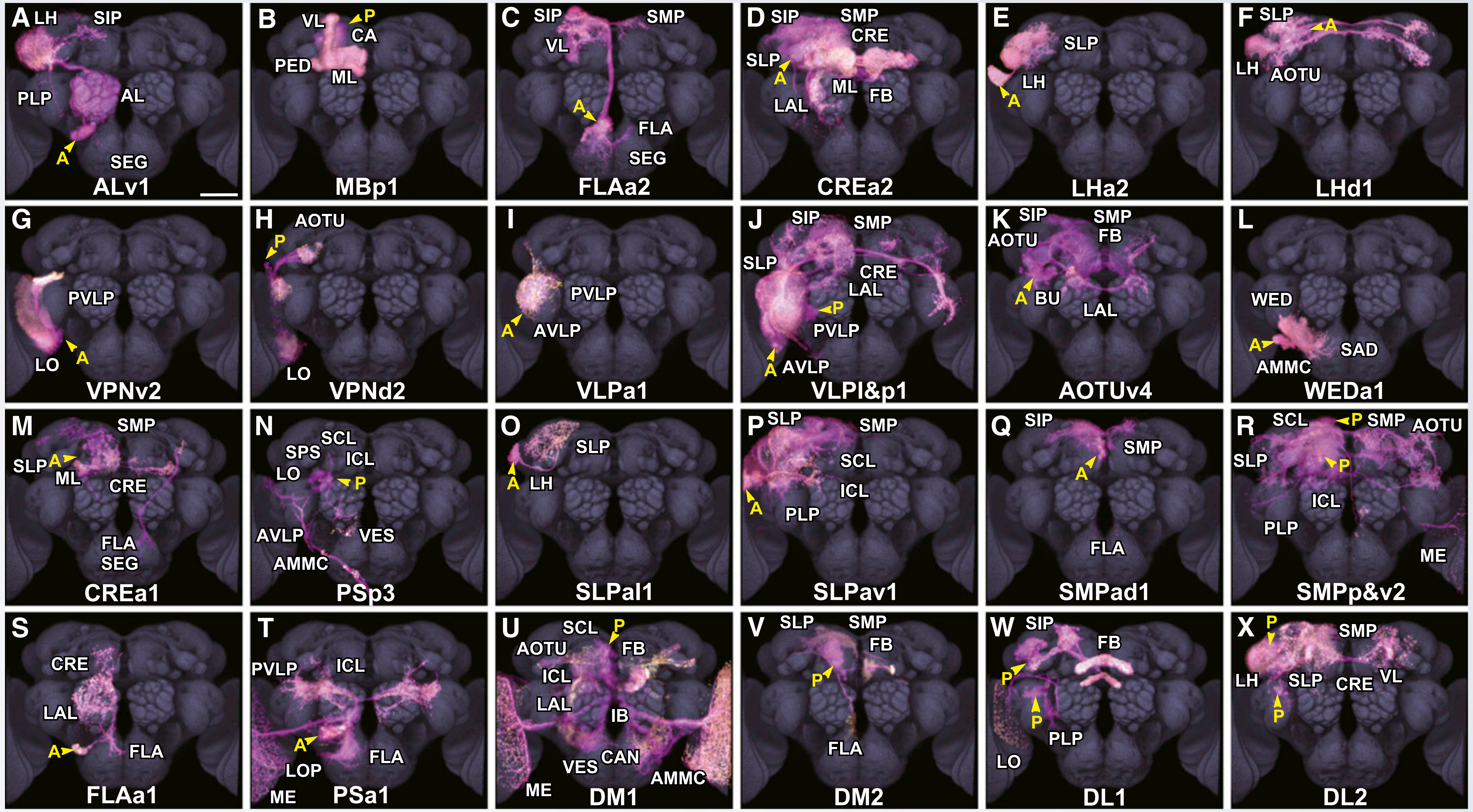

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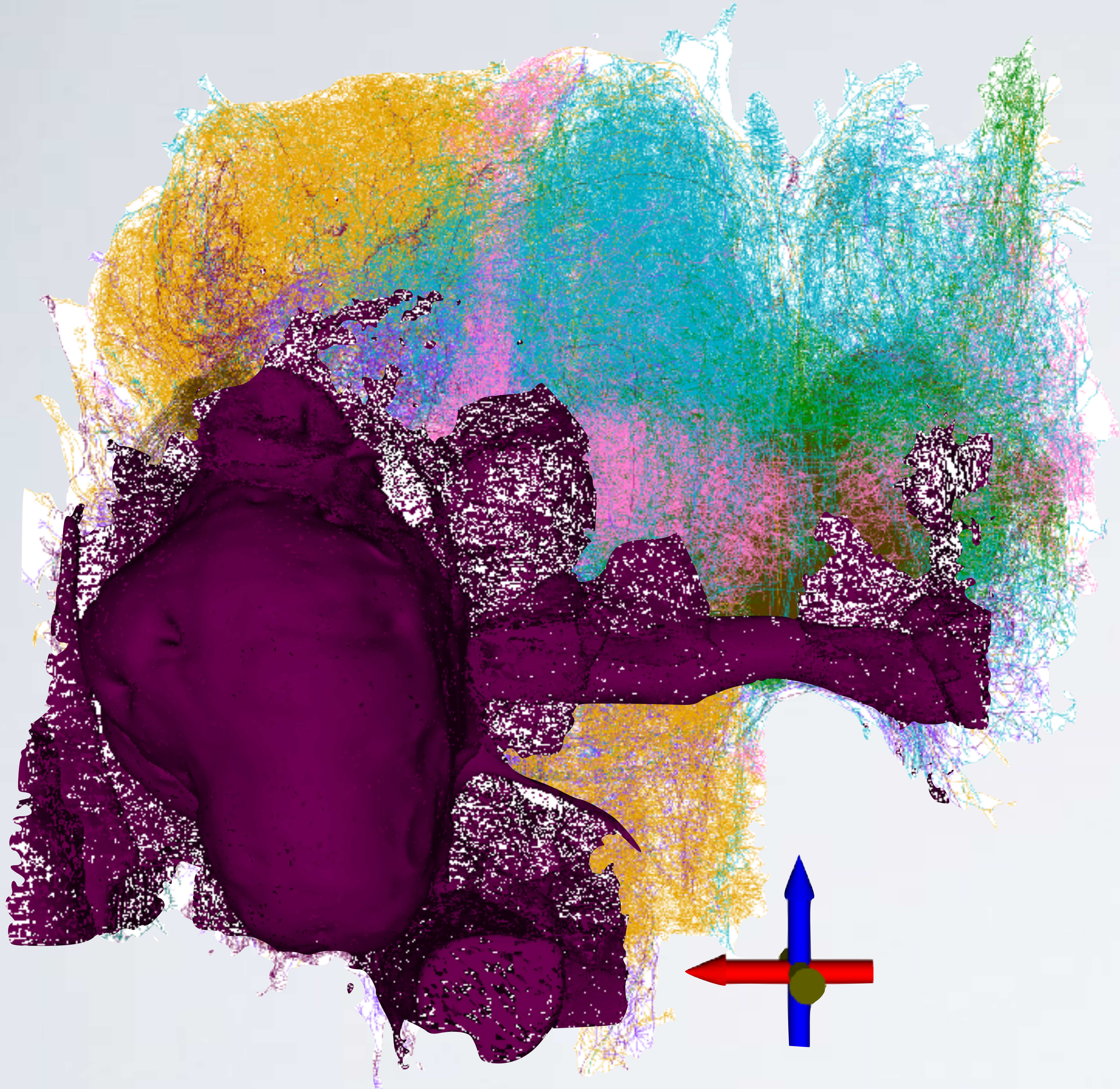

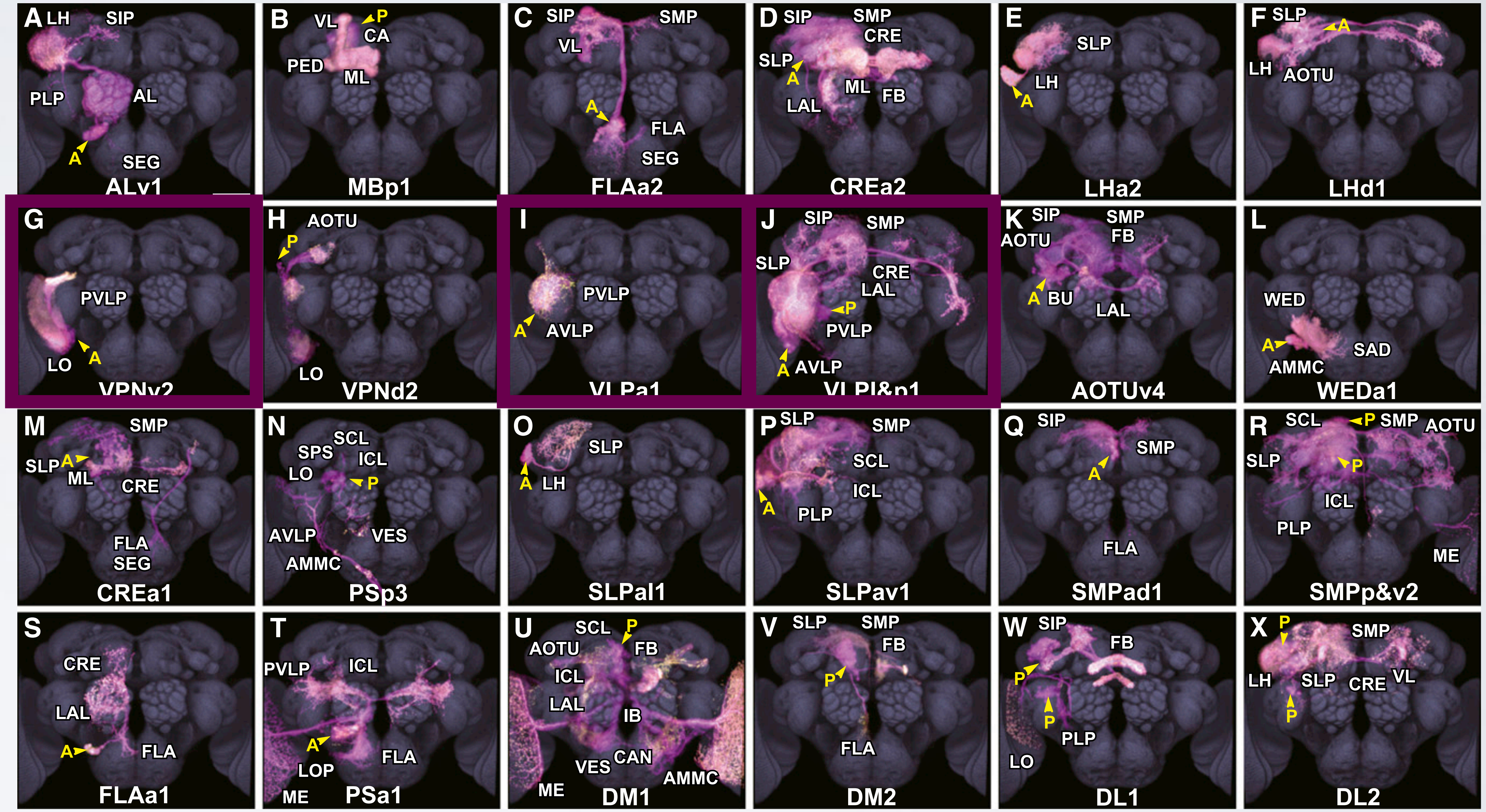

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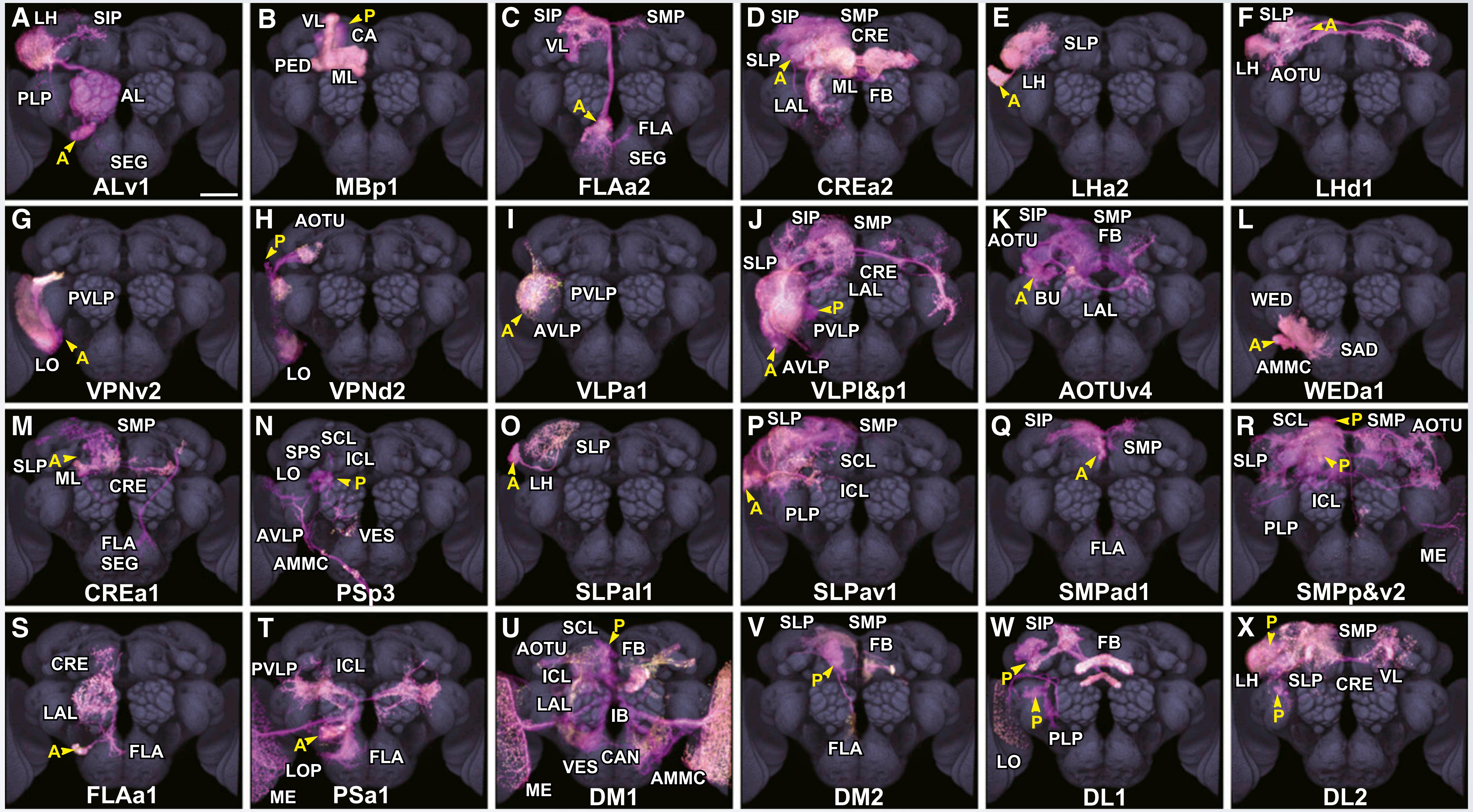


Ito, M., et al (2013). *Current Biology*, *23*(8), 644–655.

$\chi = 0.1$

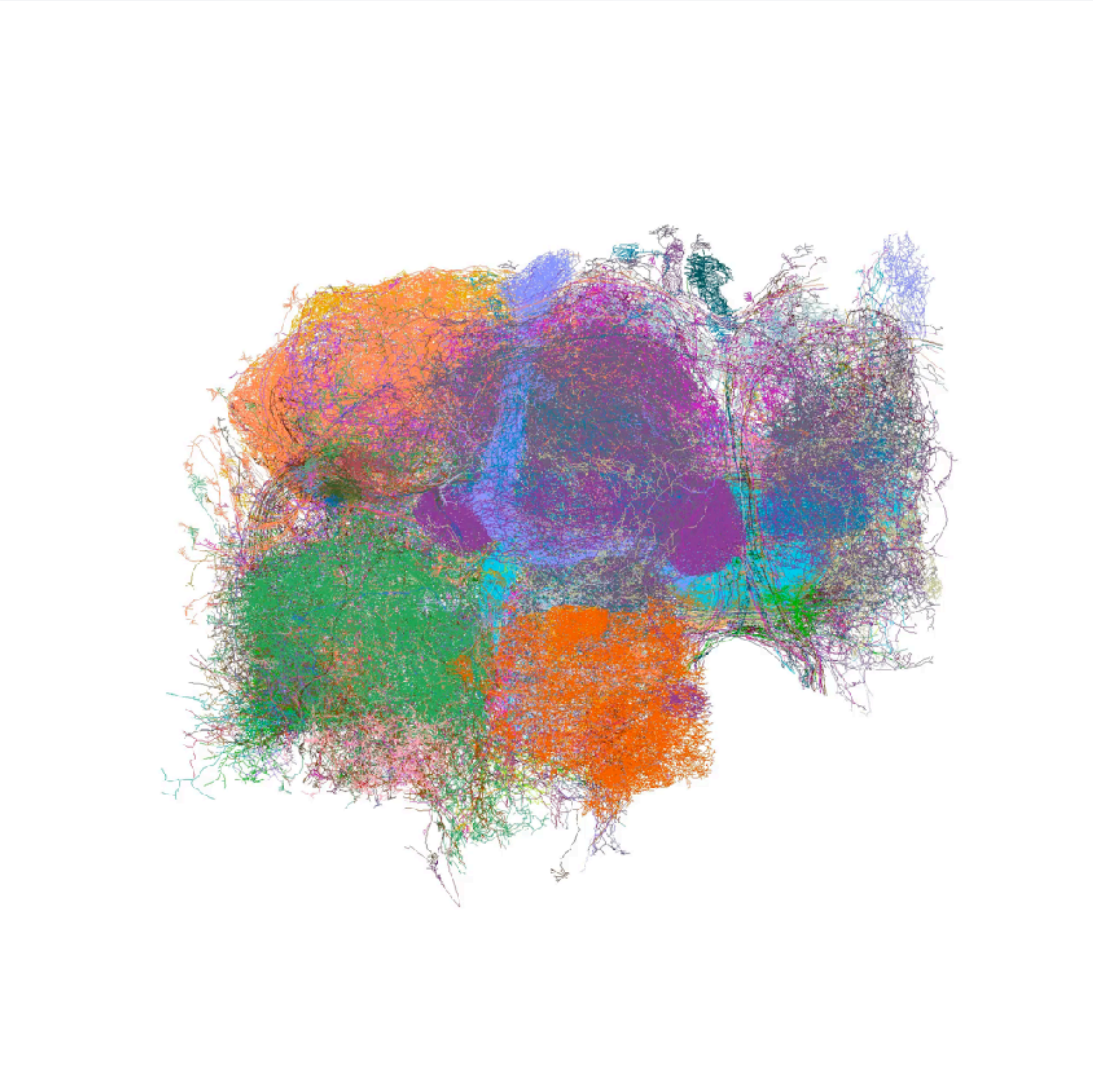
175 clusters

$\chi = 0.1$

175 clusters

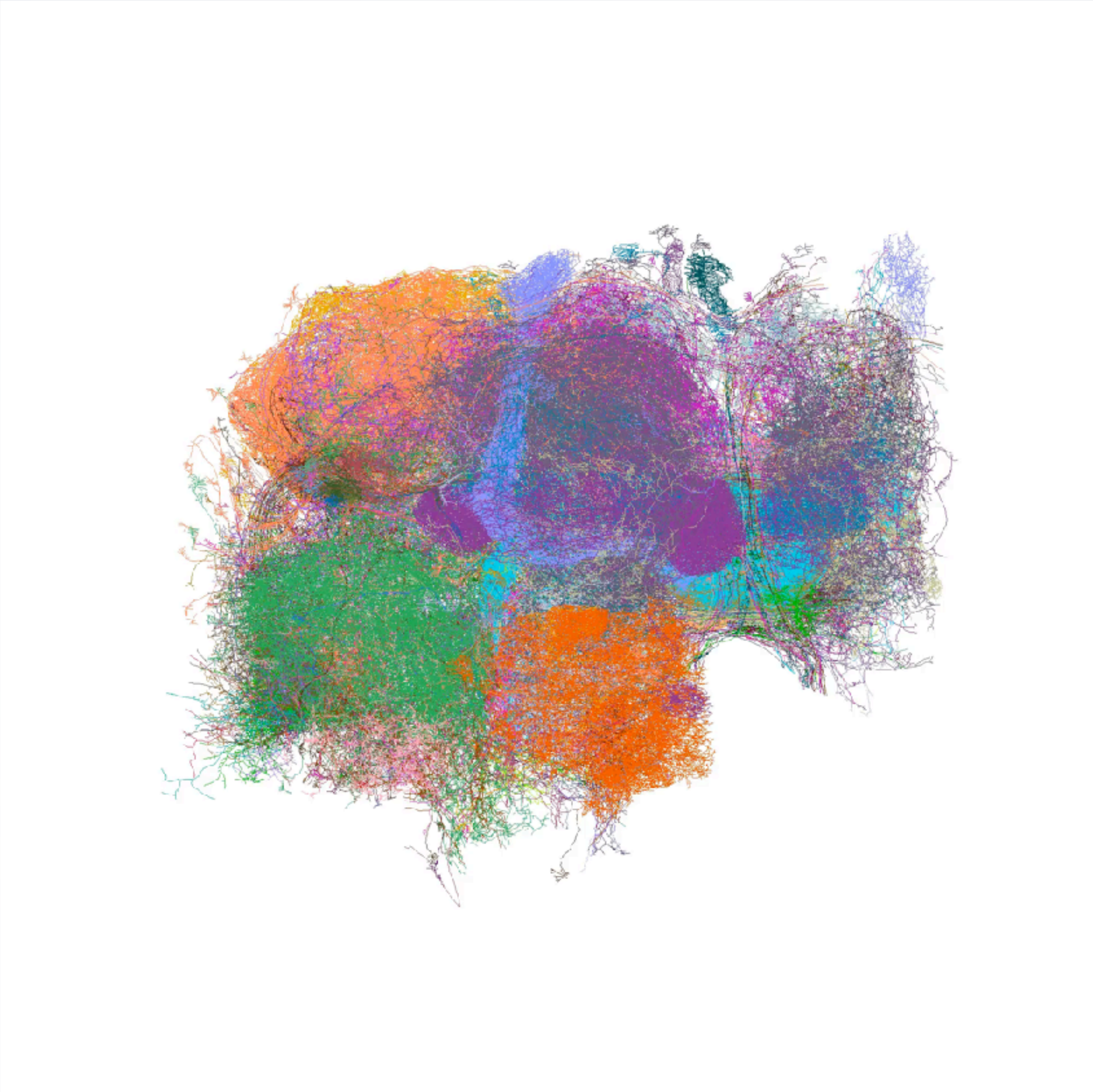
$\chi = 0.25$

440 clusters



$\chi = 0.25$

440 clusters



Partitioning is Largely Hierarchical

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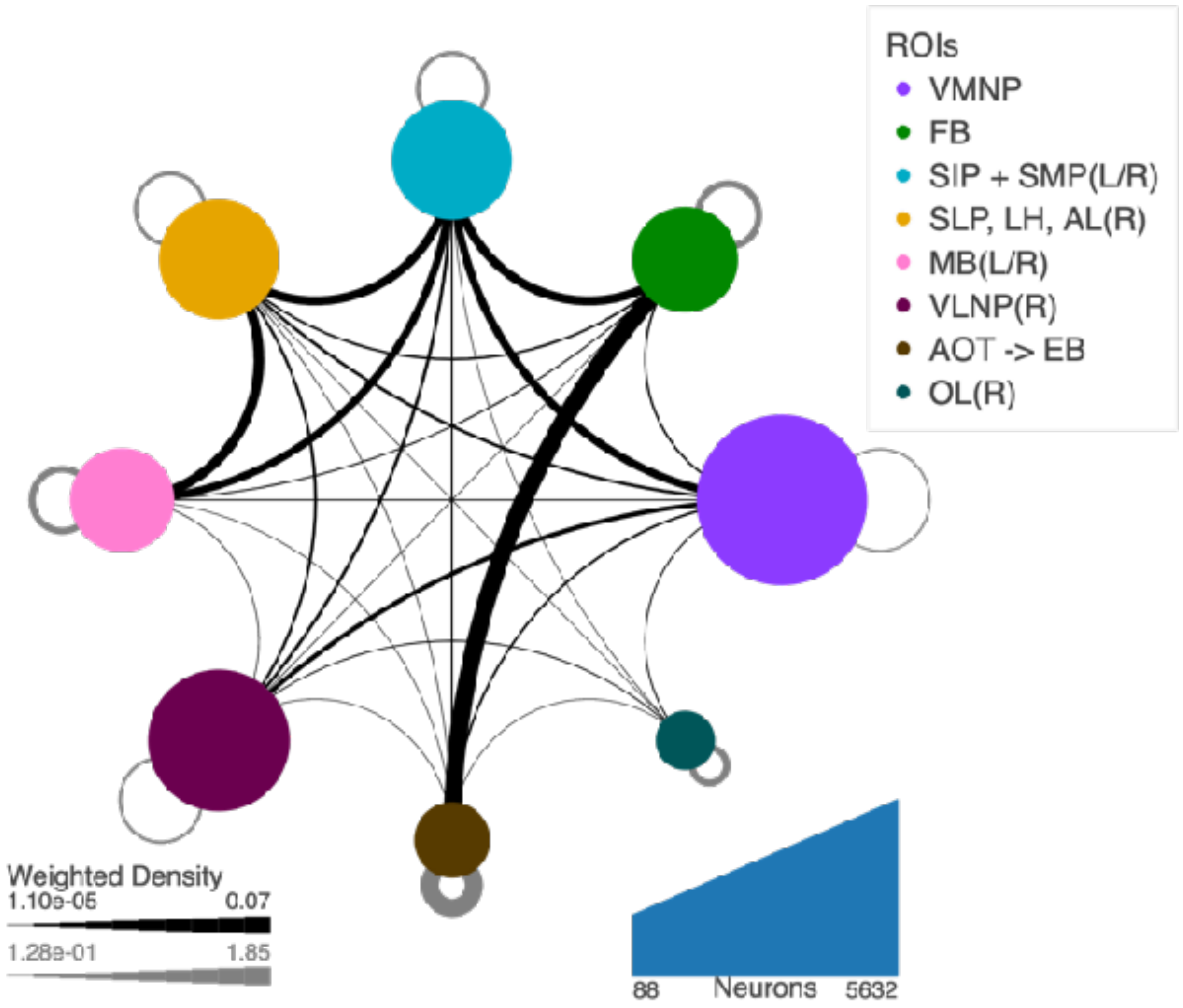
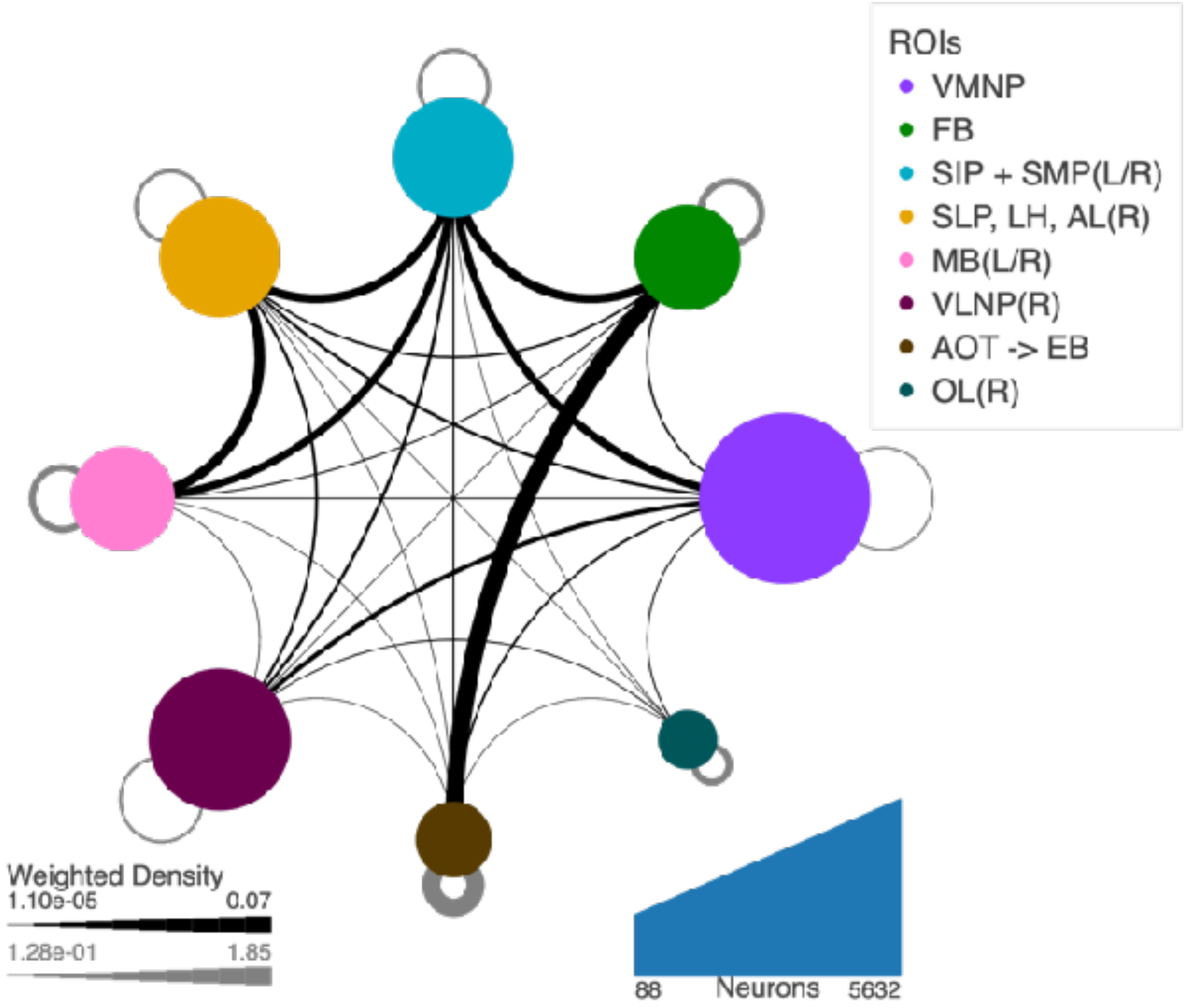
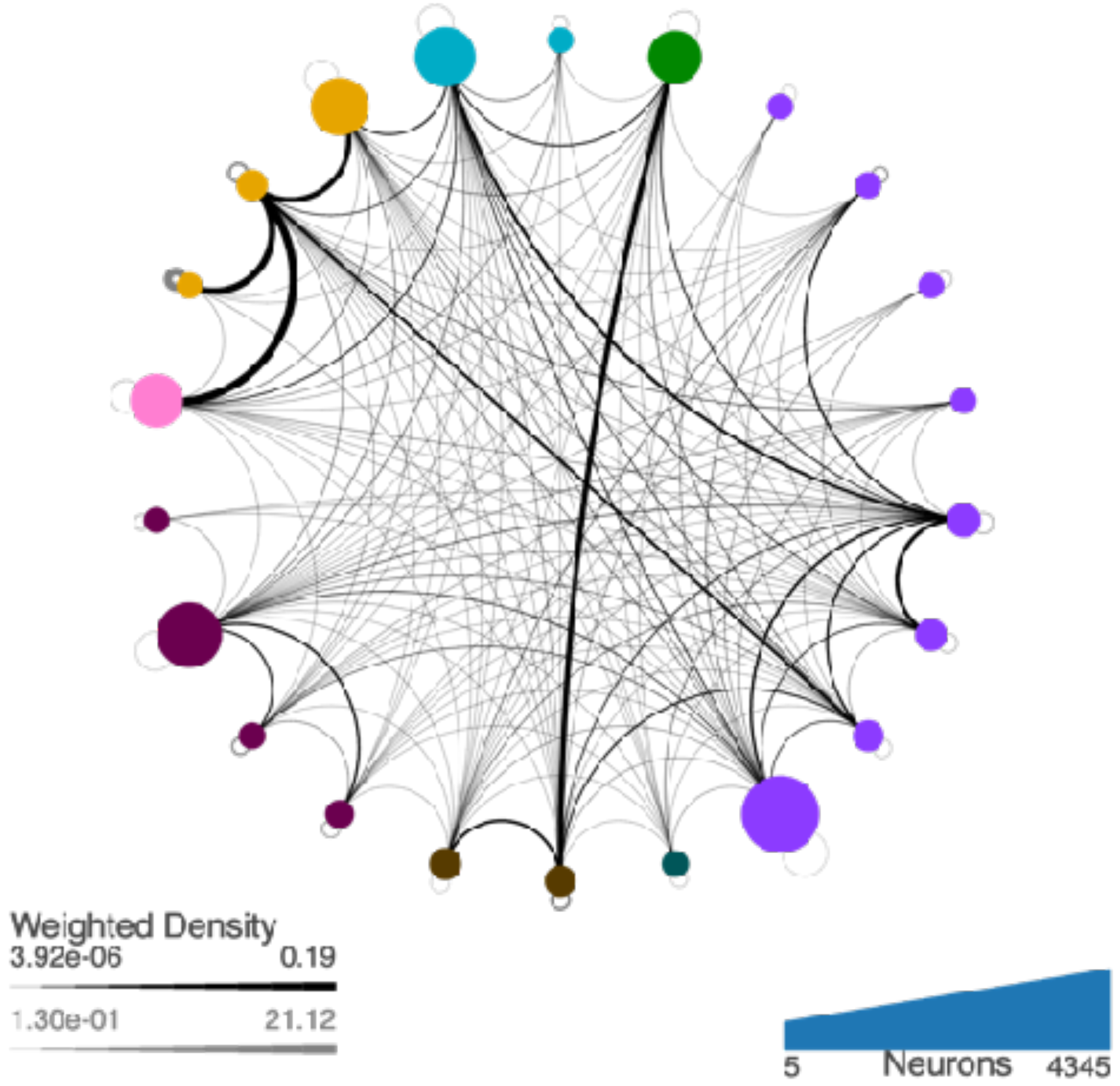


Chi = 0.0. n = 8, m = 26

Partitioning is Largely Hierarchical

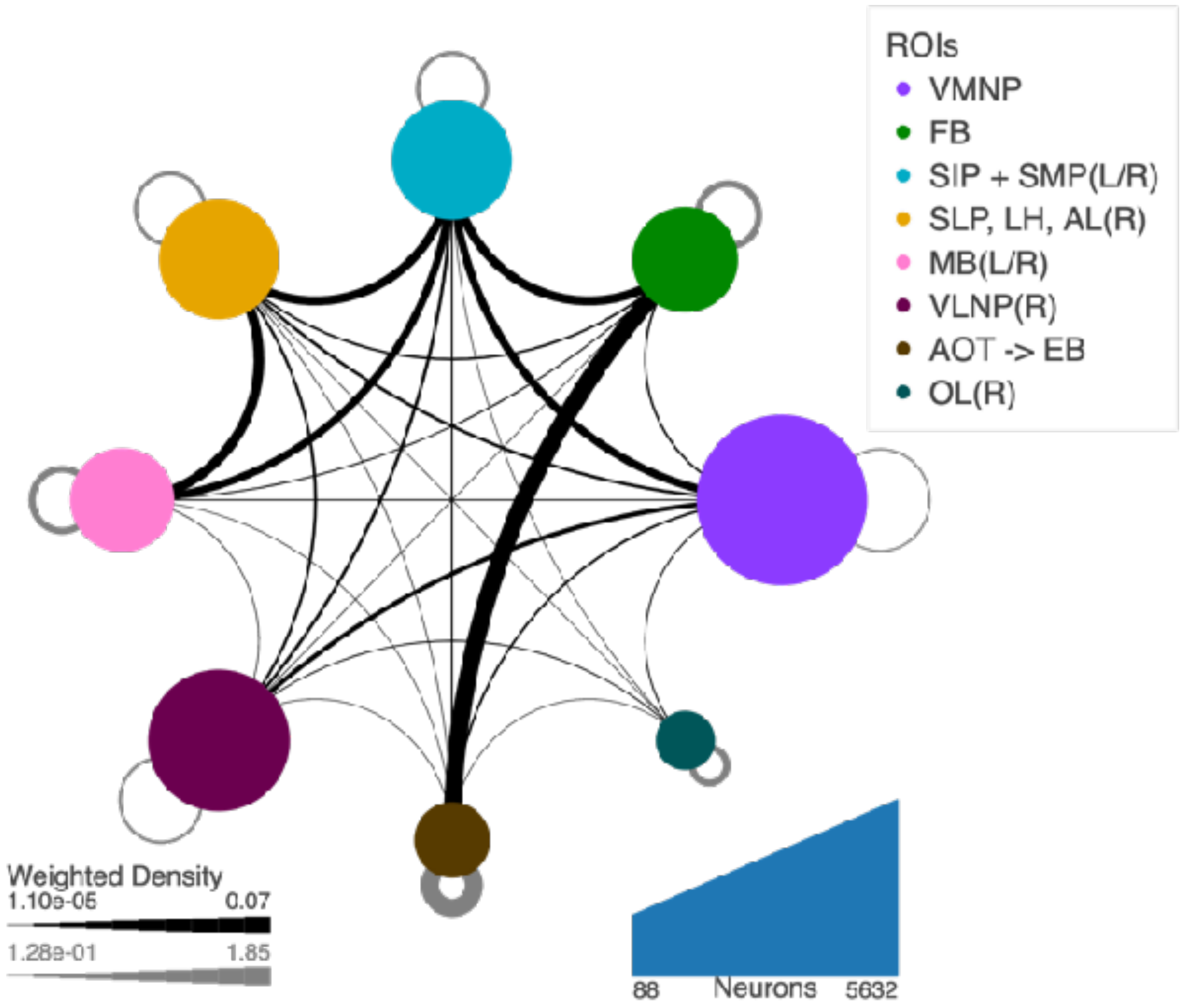


Chi = 0.0. n = 8, m = 26

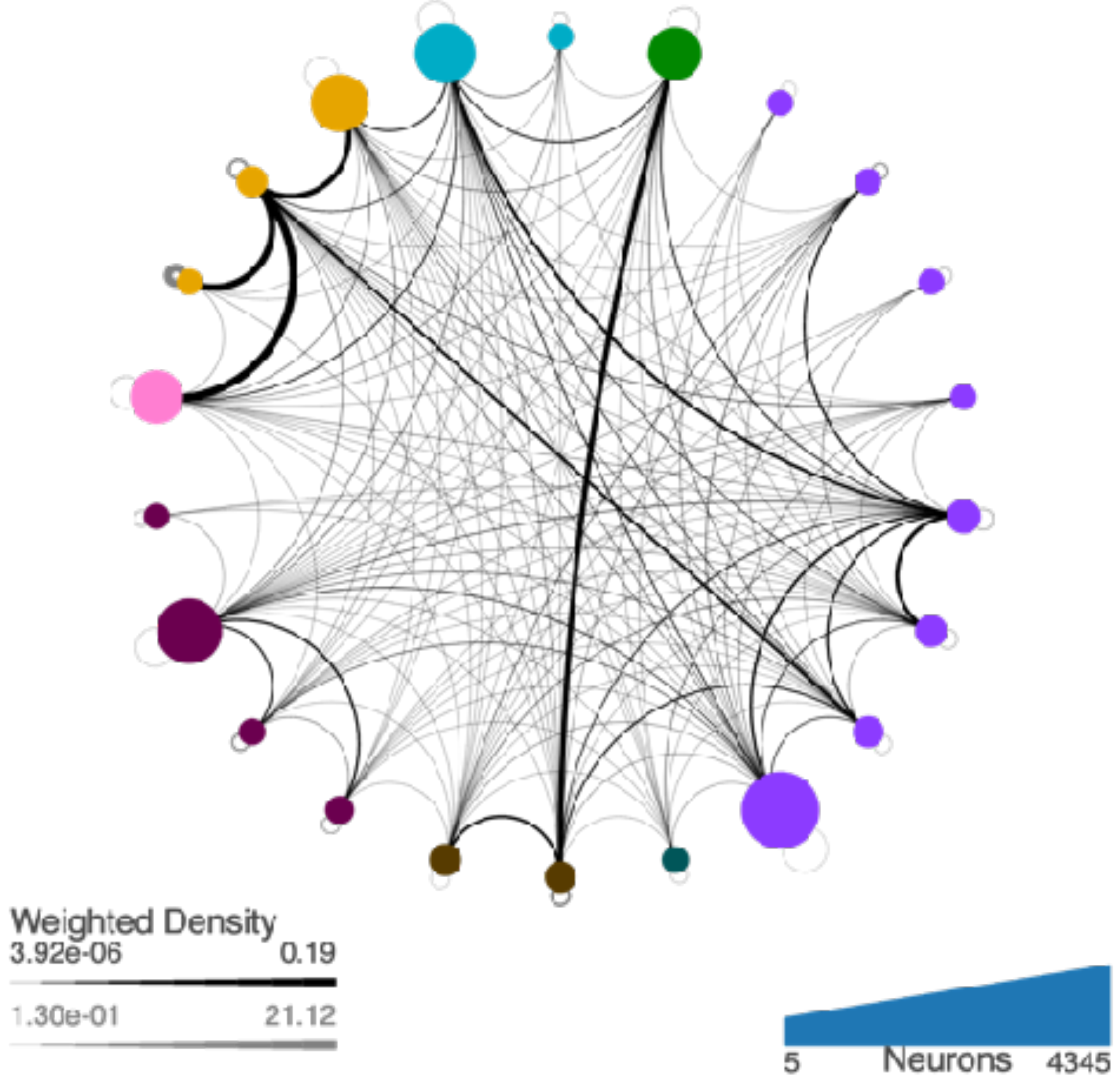


Chi = 0.05. n = 68 (22 shown), m = 307

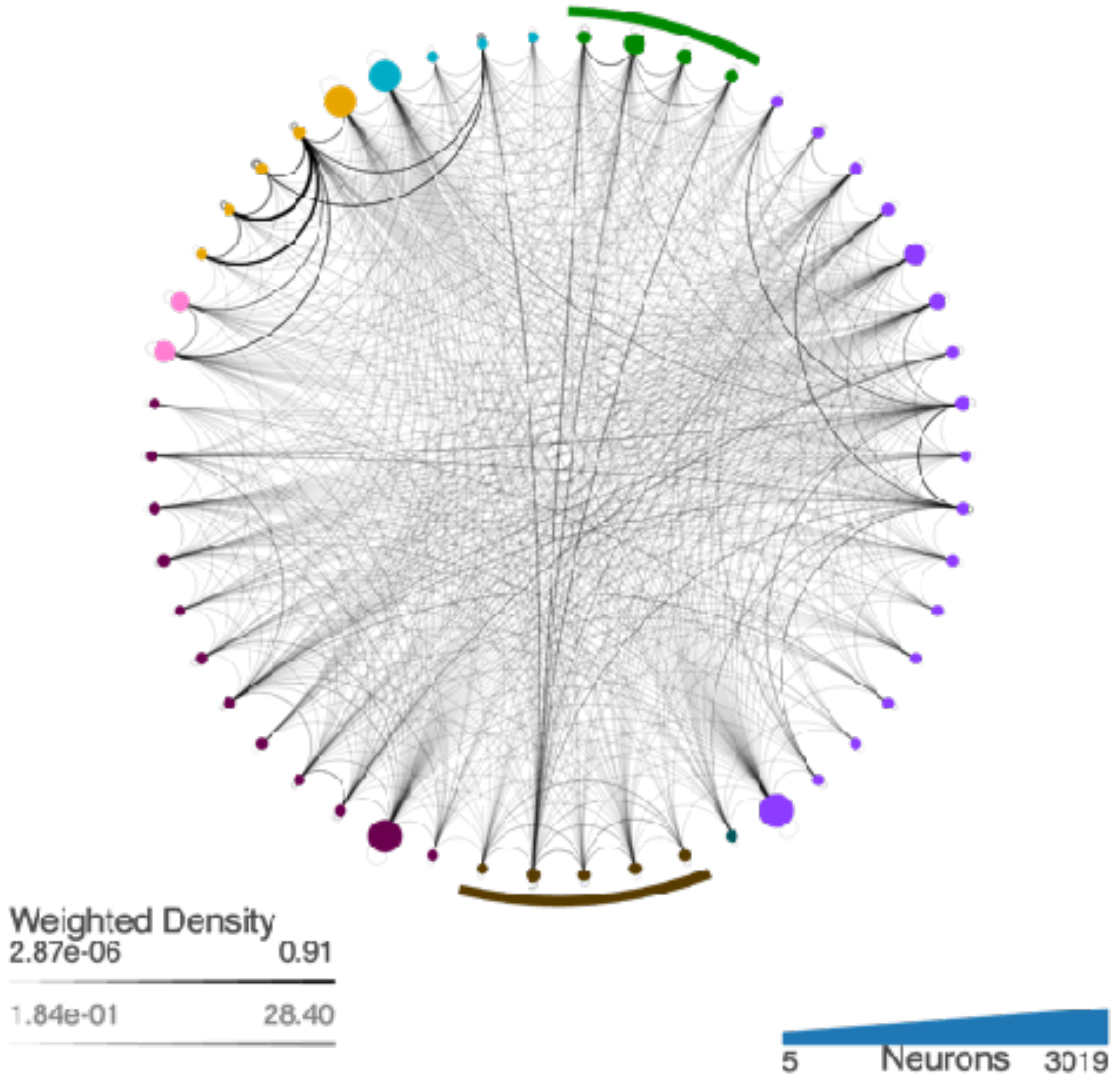
Partitioning is Largely Hierarchical



Chi = 0.0. n = 8, m = 26

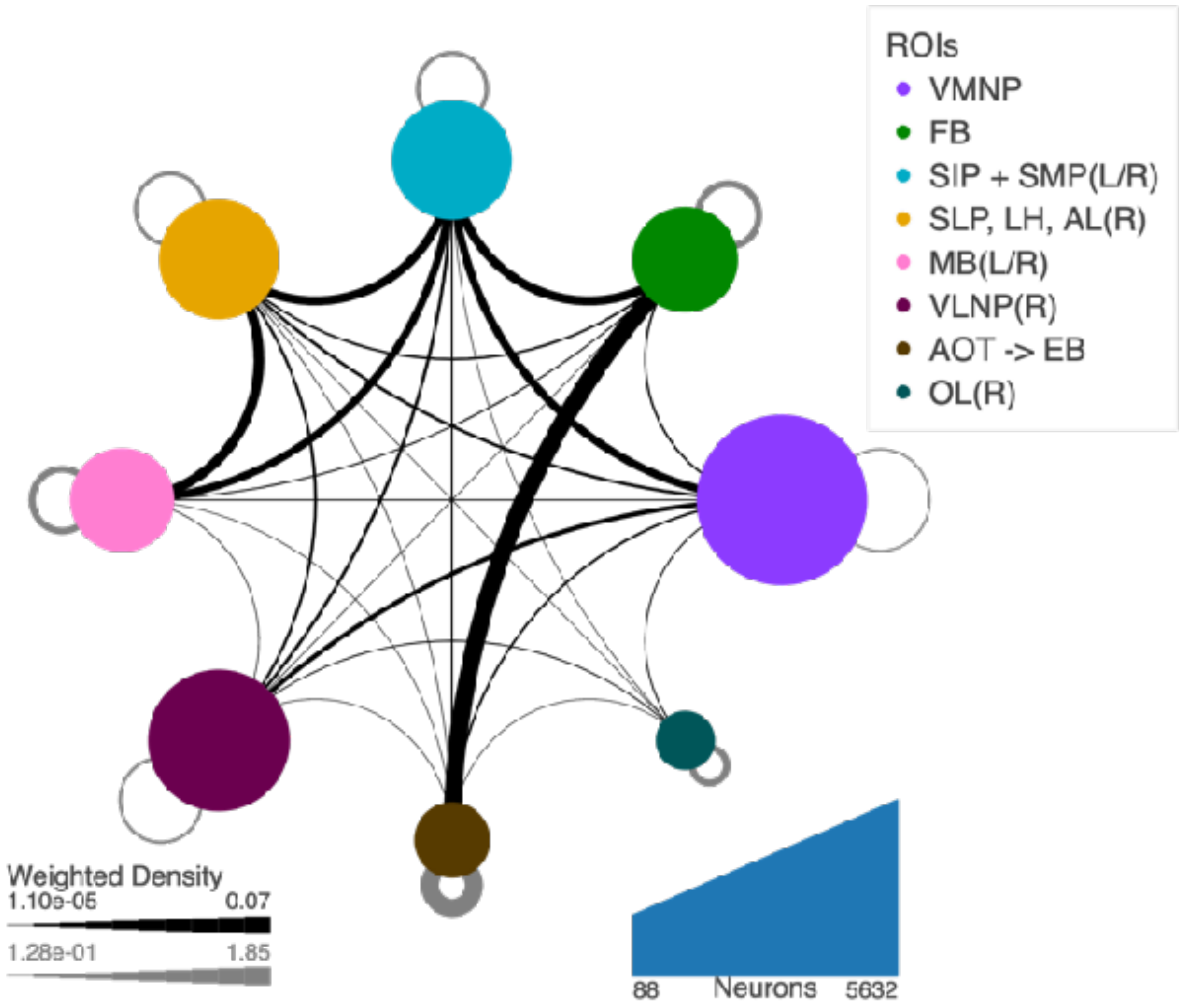


Chi = 0.05. n = 68 (22 shown), m = 307

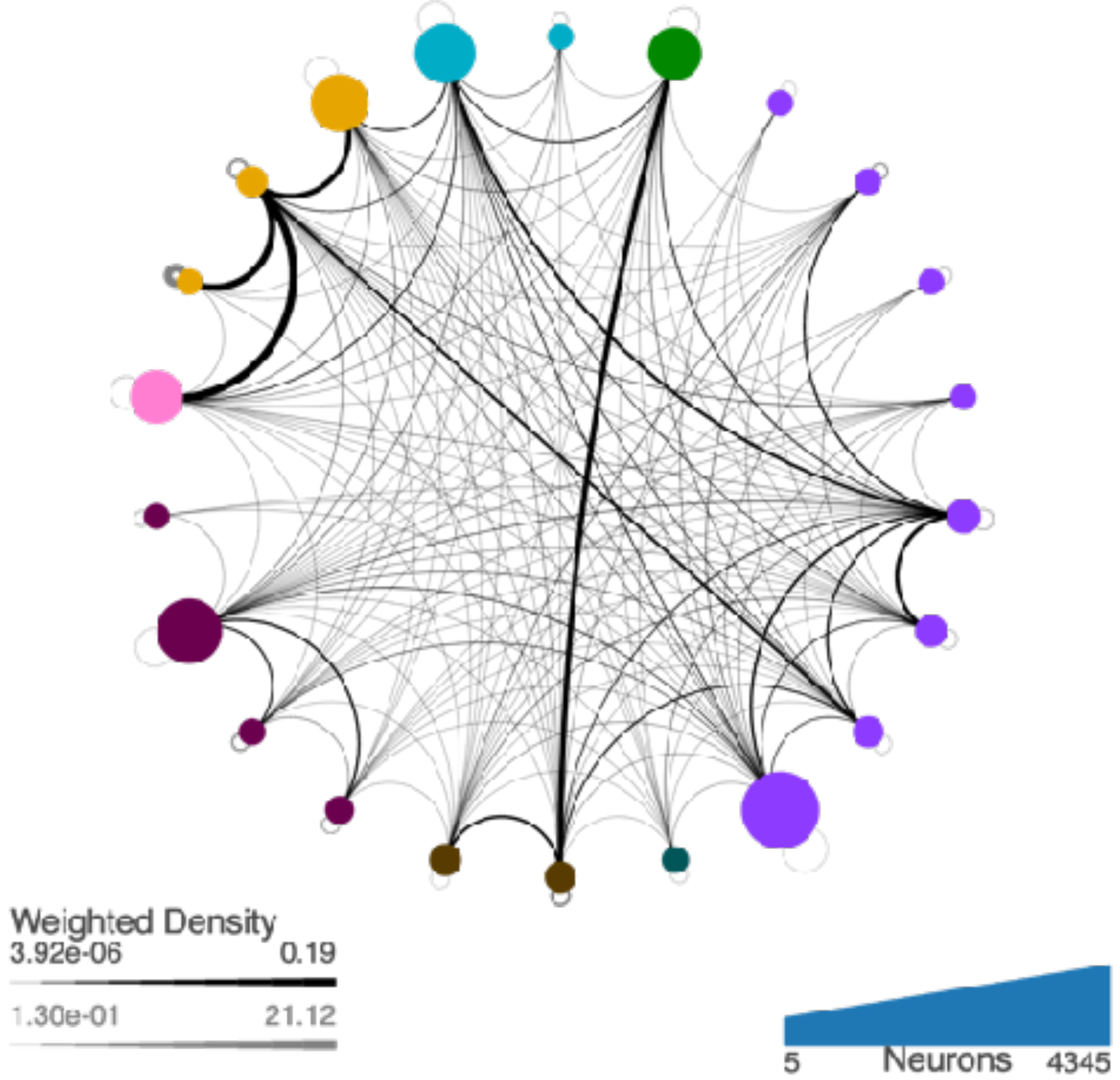


Chi = 0.1. n = 175 (50 shown), m = 1,266

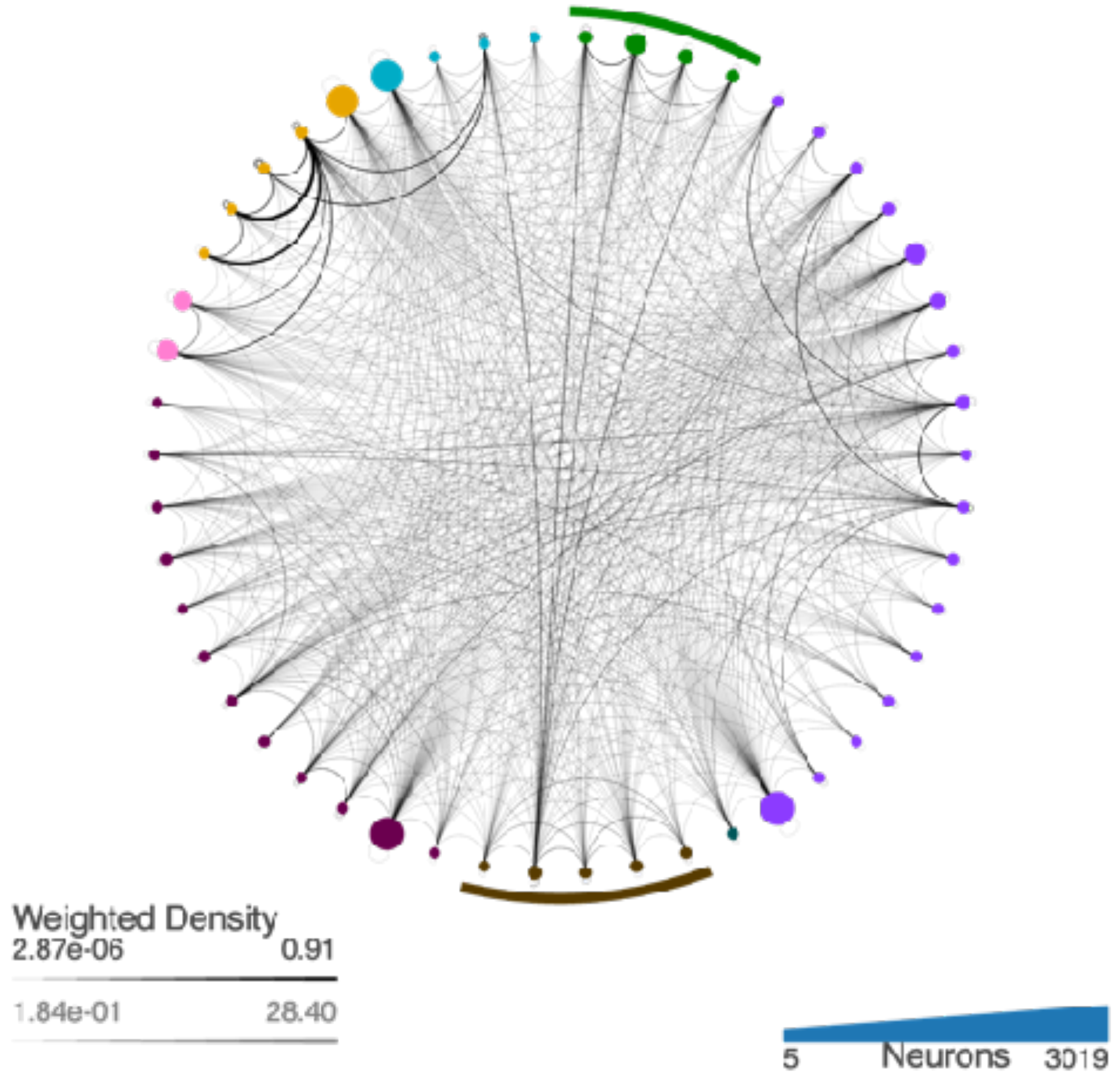
Partitioning is Largely Hierarchical



Chi = 0.0. n = 8, m = 26



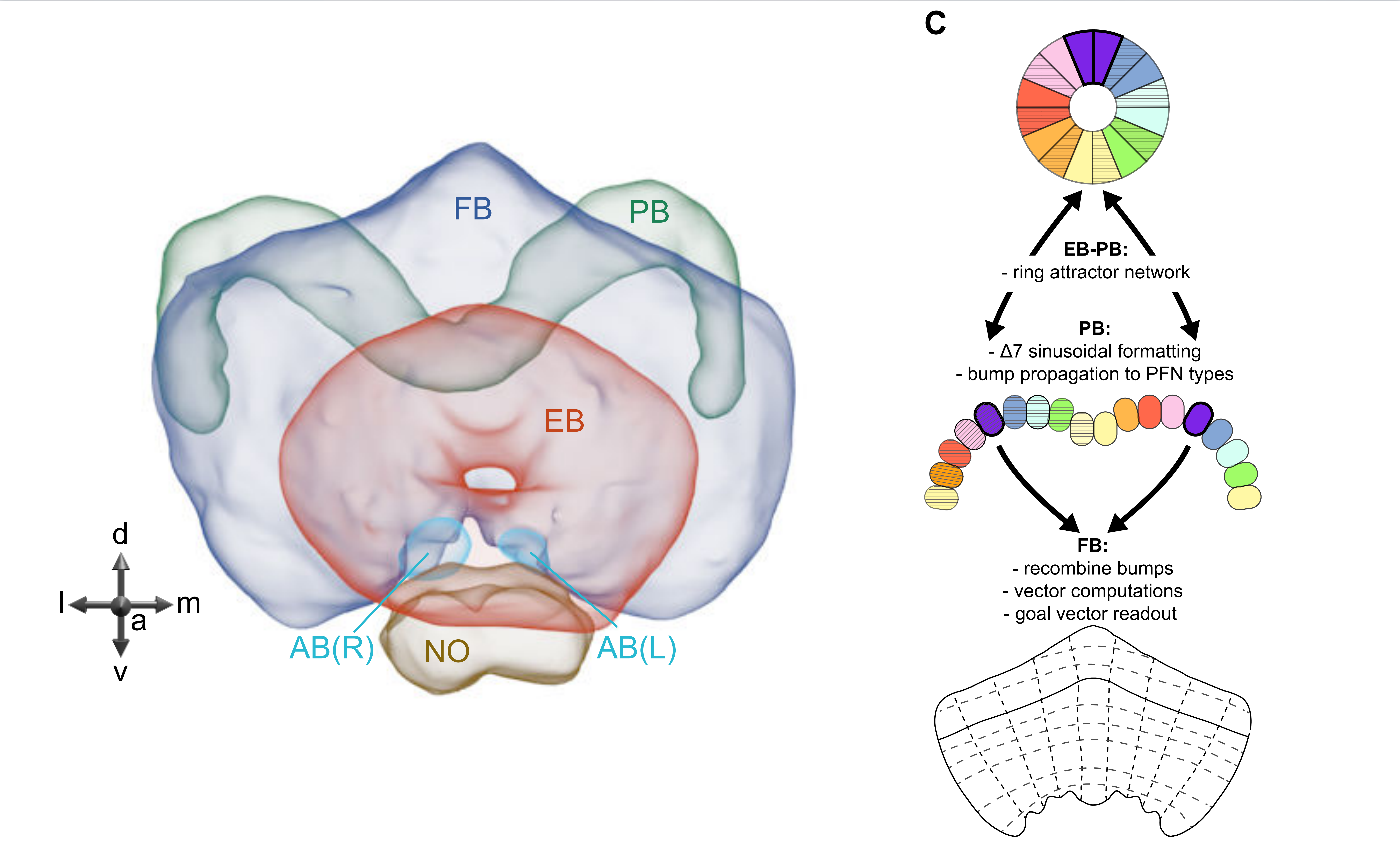
Chi = 0.05. n = 68 (22 shown), m = 307



Chi = 0.1. n = 175 (50 shown), m = 1,266

Majority of clusters identified at resolutions $\chi > 0$ were at least 85% contained in one of the 8 clusters found with $\chi = 0$.

The central complex



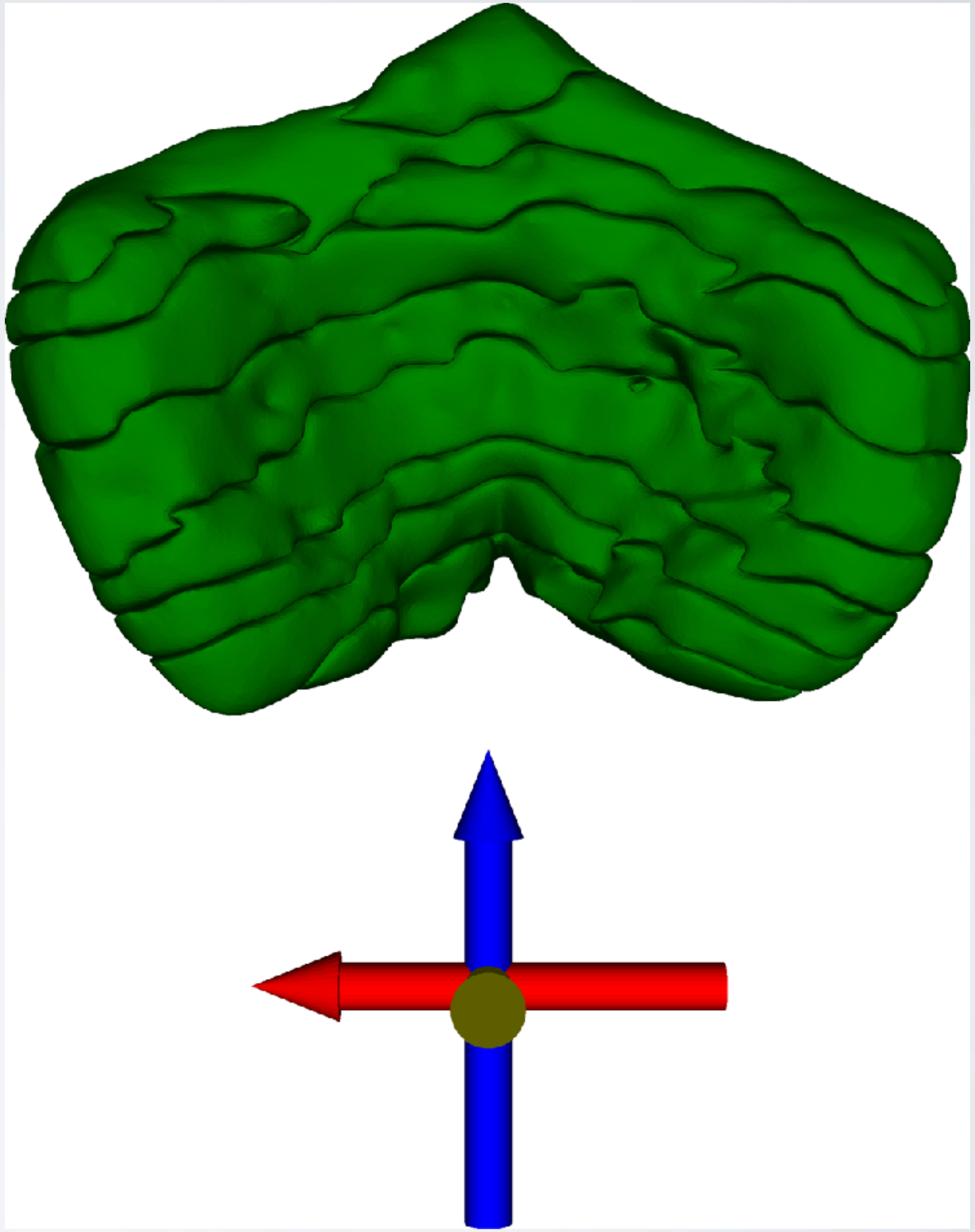
Hulse et al, 2021

Fan Shaped Body

Nine anatomically-identified layers

Our clustering is meaningful:
Clusters identify layers

New finding:
Ventral layers form a separate module at low values of χ



Chi = 0

Fan Shaped Body

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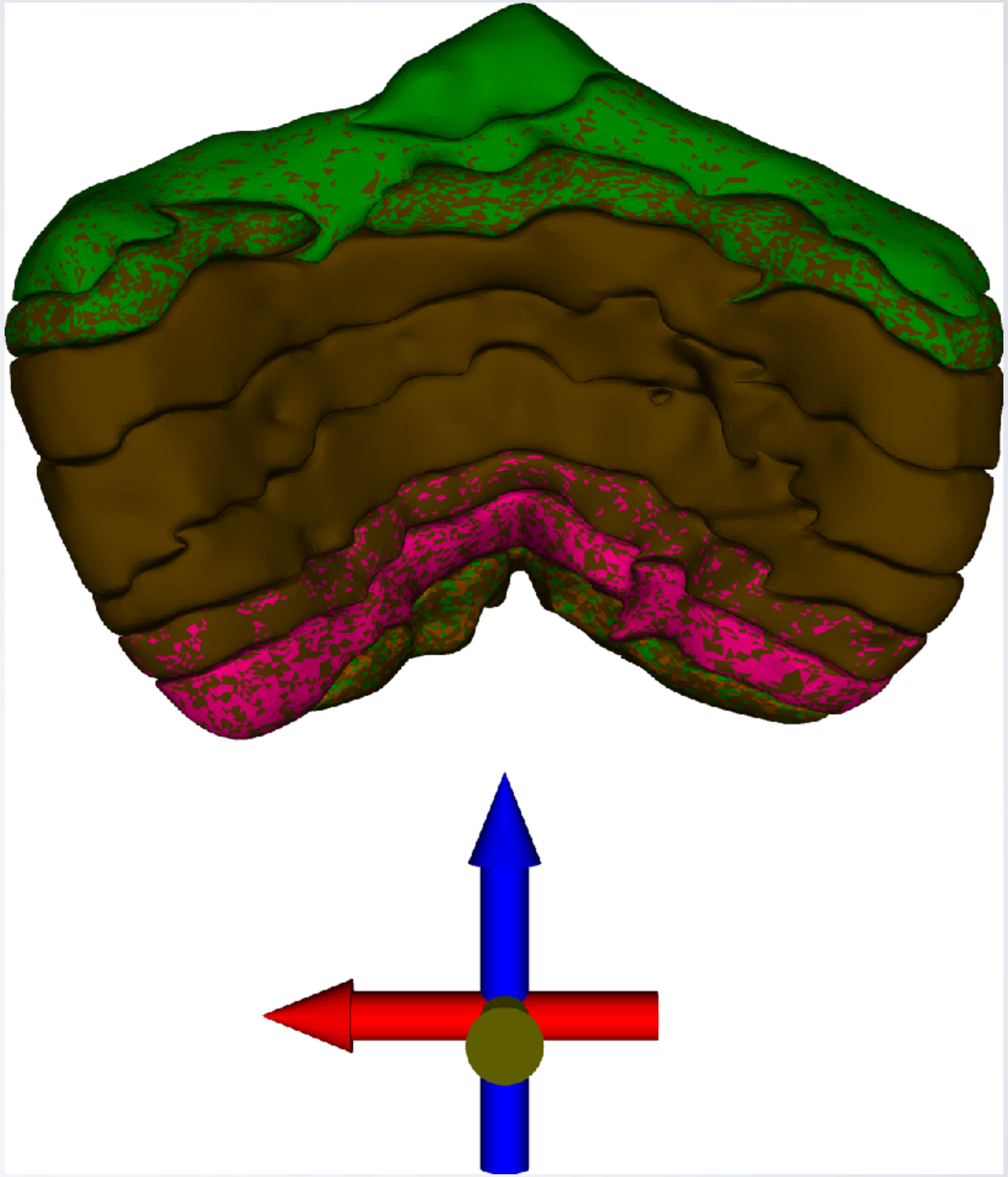

Chi = 0.1

Fan Shaped Body

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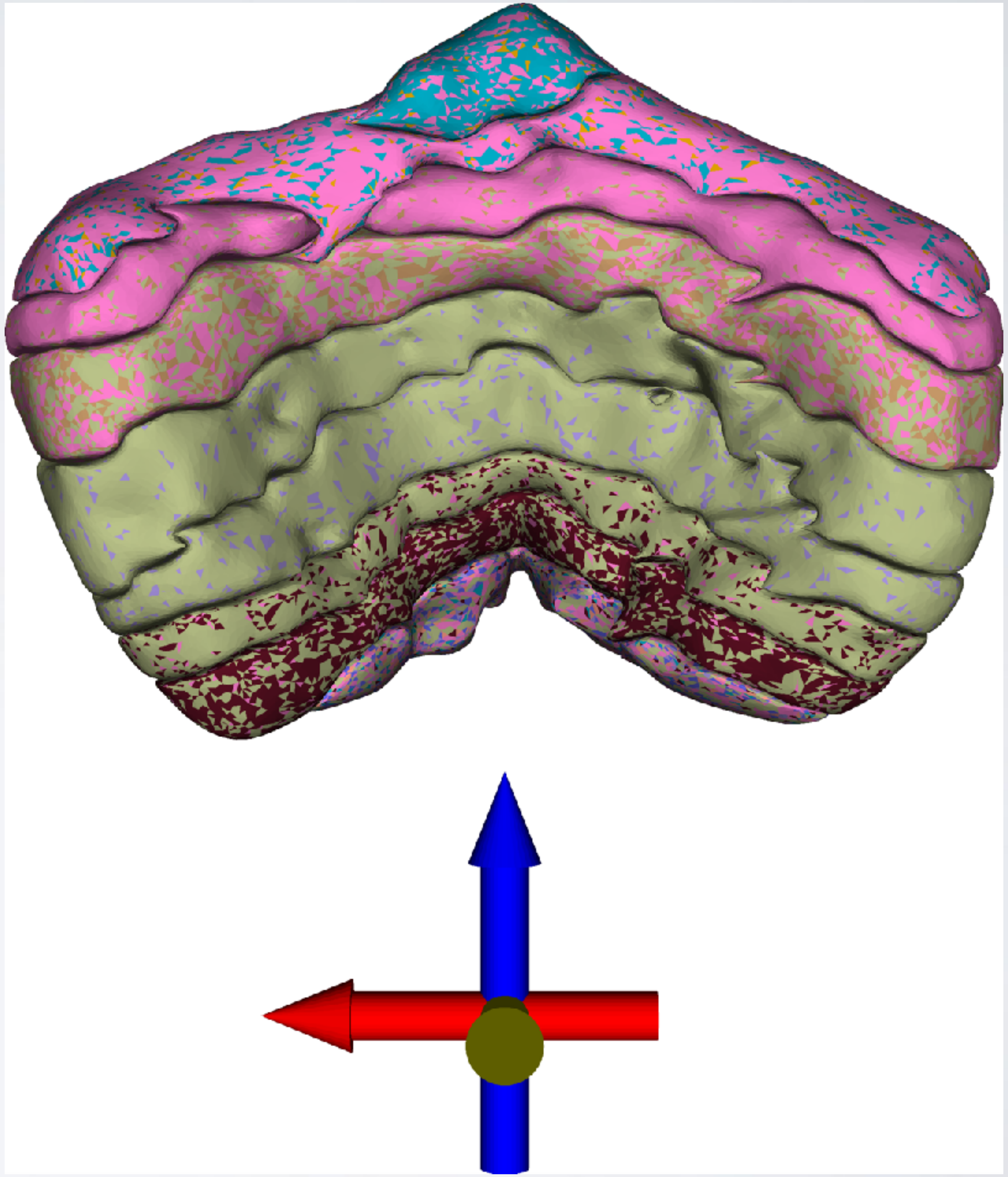

Chi = 0.25

Fan Shaped Body

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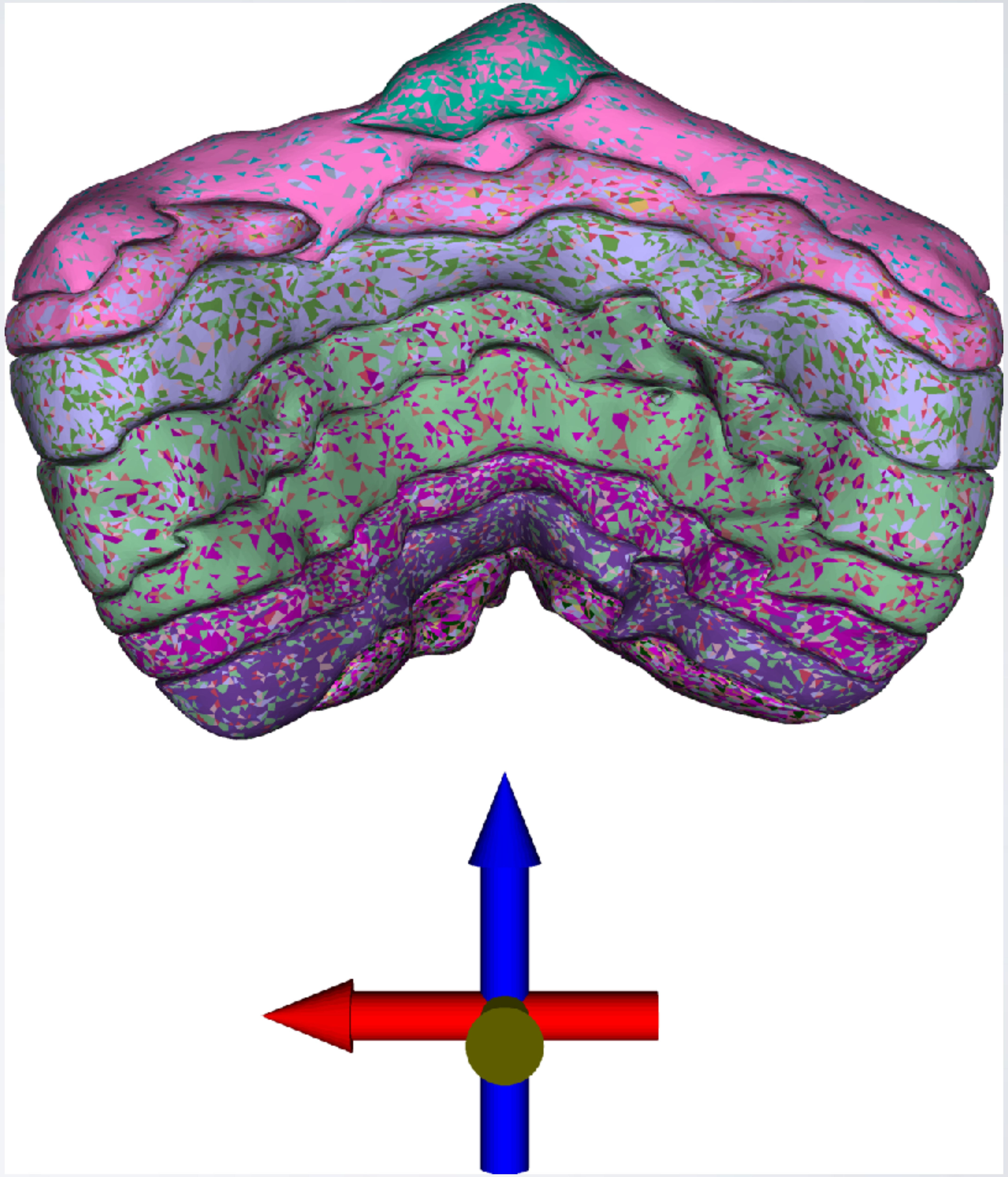

Chi = 0.5

Fan Shaped Body

Nine anatomically-identified layers

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New finding:
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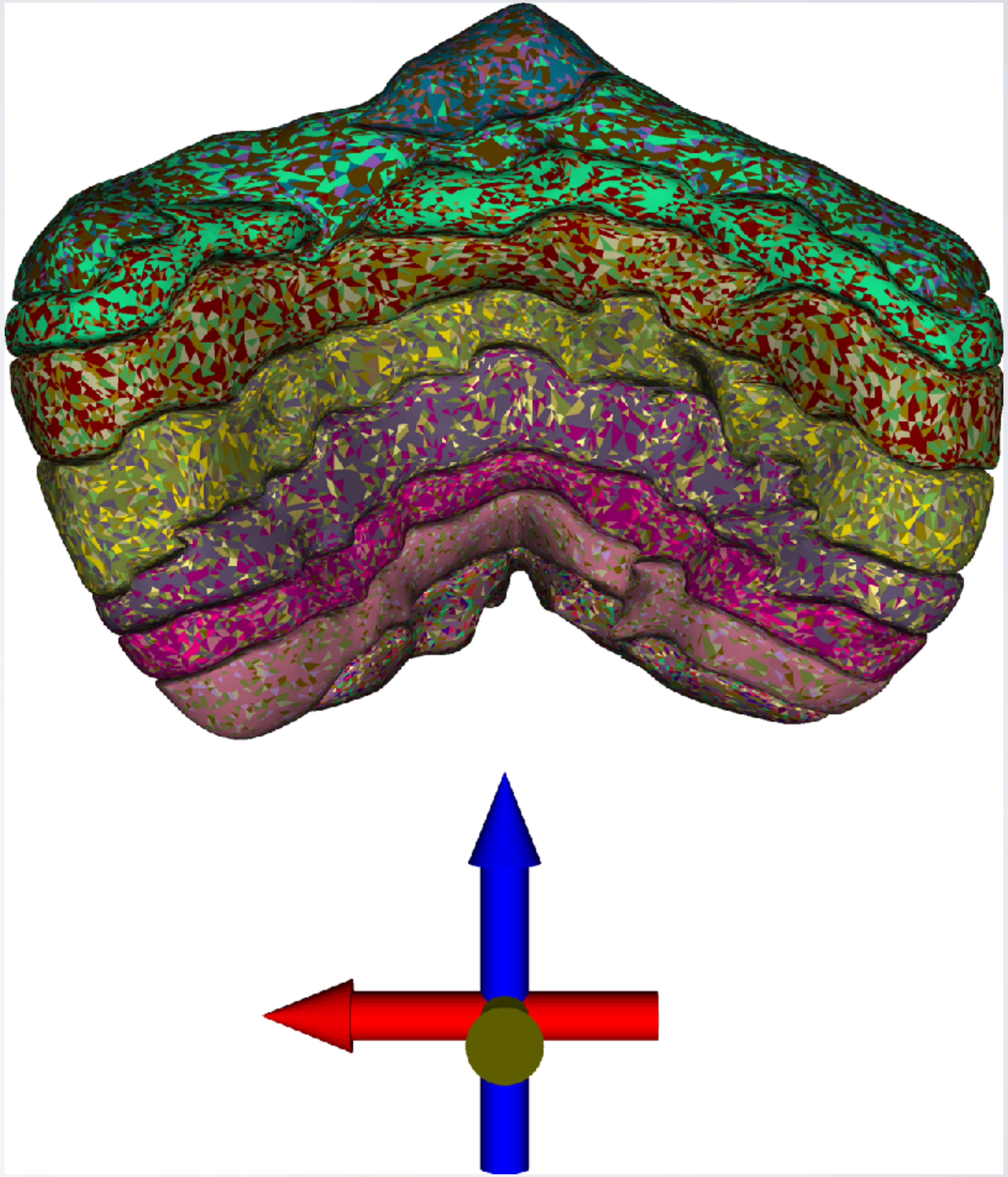


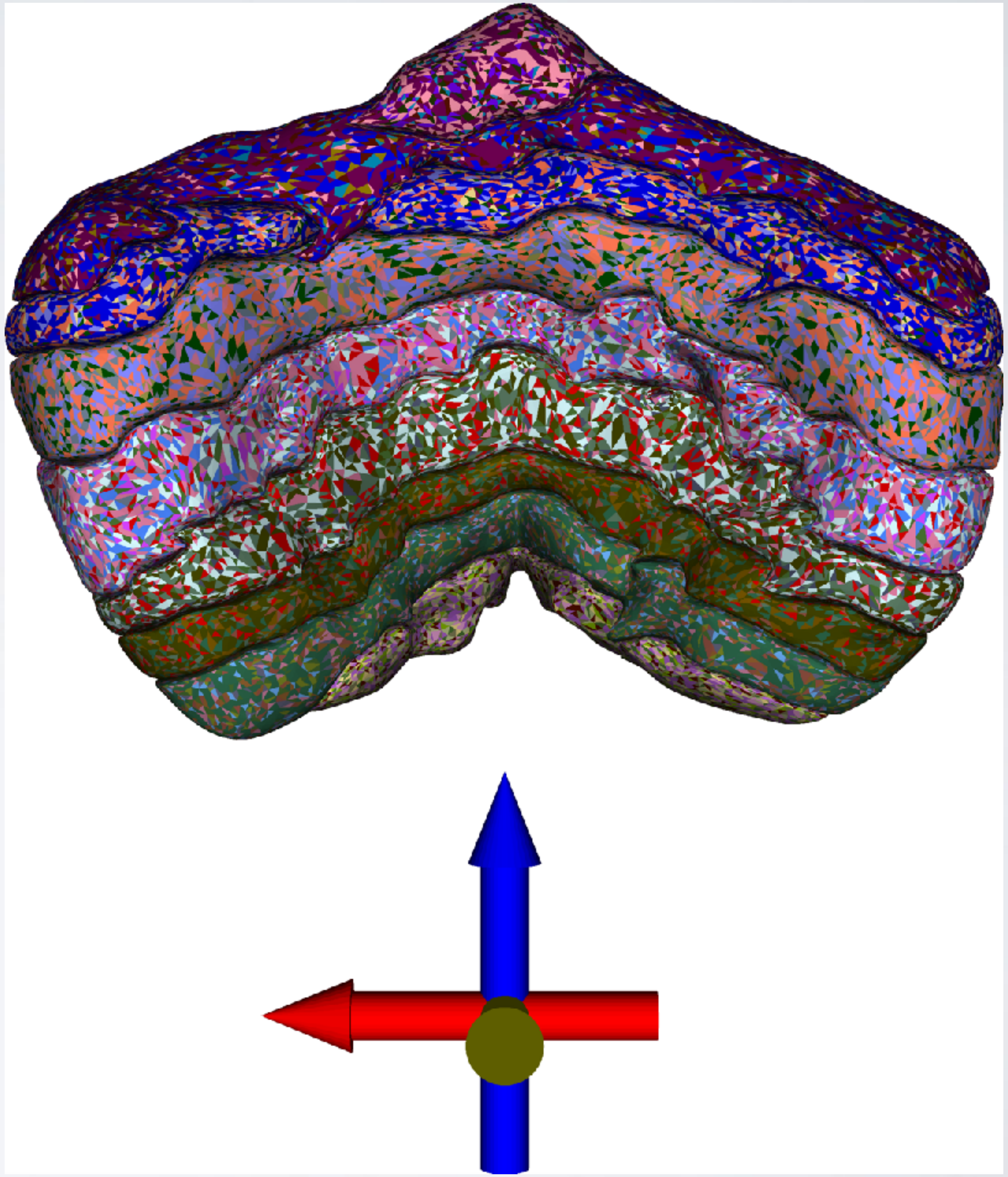
Chi = 0.75

Fan Shaped Body

Nine anatomically-identified layers

Our clustering is meaningful:
Clusters identify layers

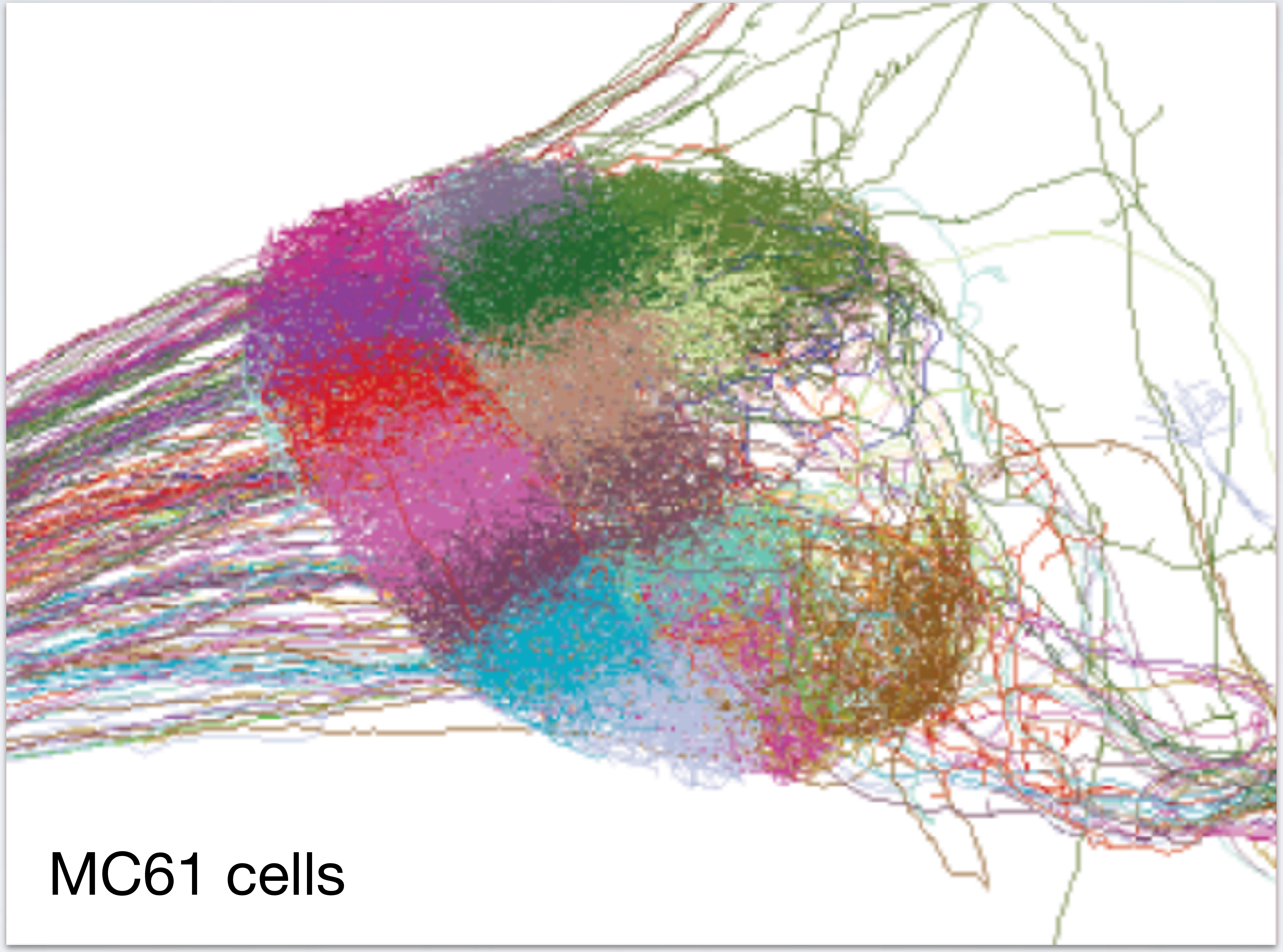
New finding:
Ventral layers form a separate module at low values of χ



Chi = 1.0

Including cell type

Including cell type



MC61 cells

Conclusion and Outlook

- A *general* framework to model the effects of STDP on network dynamics. It predicts the state of the network, and average quantities.
- Does not capture higher order interactions (eg triplets) which are important in some rules.
- Homeostatic processes observed in waking and resting state can be explained using fast changes in inhibitory synapses.
- Automated detection of the large scale organization of neuronal networks can provide experimentally testable hypotheses.
- The reconstructed connectome is noisy. We need new statistical methods to make inferences about structures, both small and large.

Plasticity and Balance

University of Houston
Alan Akil

University of Notre Dame
Robert Rosenbaum

UT, Houston
Ariana Andrei
Valentin Dragoi

Fly brain community detection

University of Houston
Kevin Bassler
Jiahao Guo

Baylor College of Medicine
Xaq Pitkow
Alex Kunin

Janelia
Brad Hulse
Romain Franconville

Funding: NSF NeuroNex, NIH

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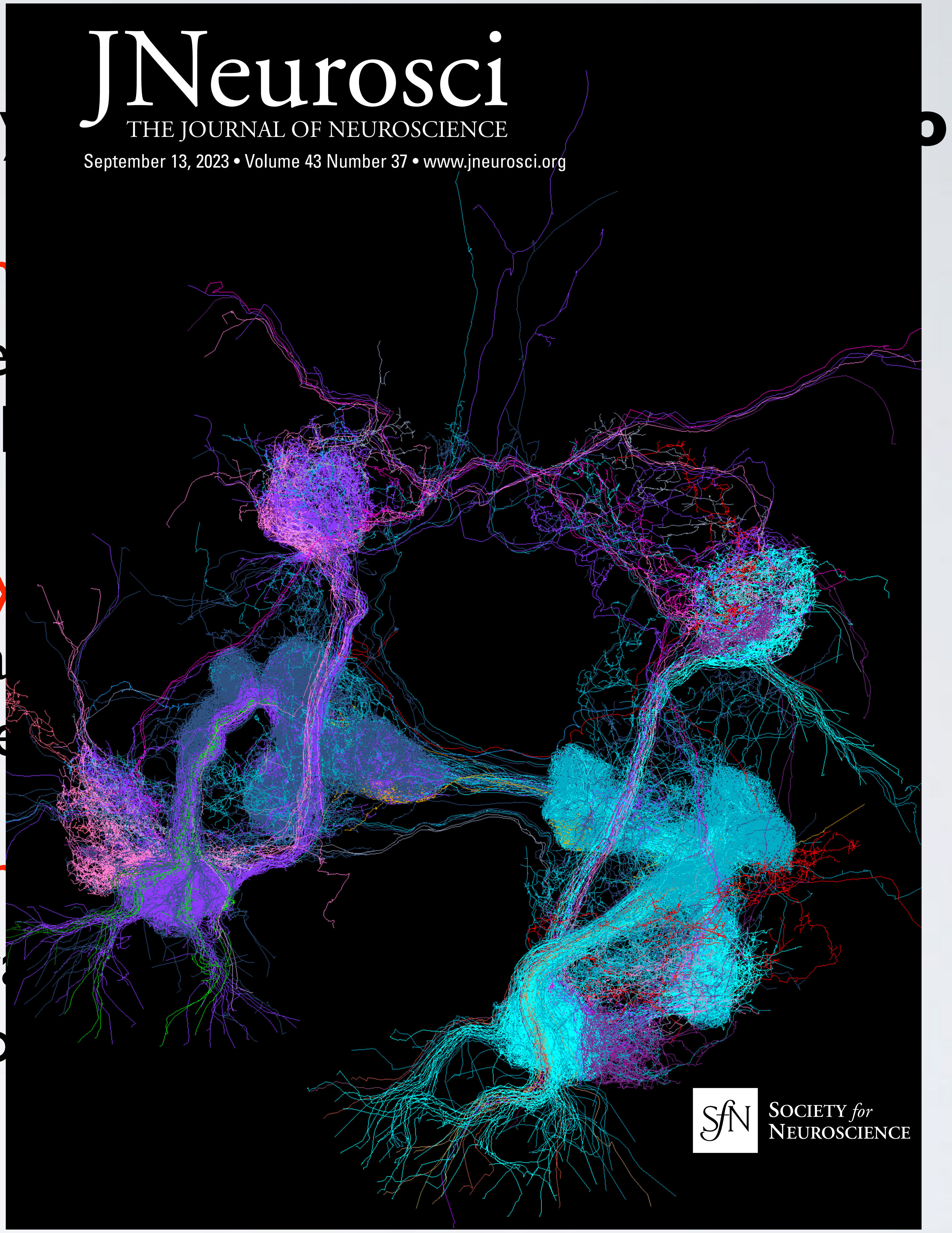

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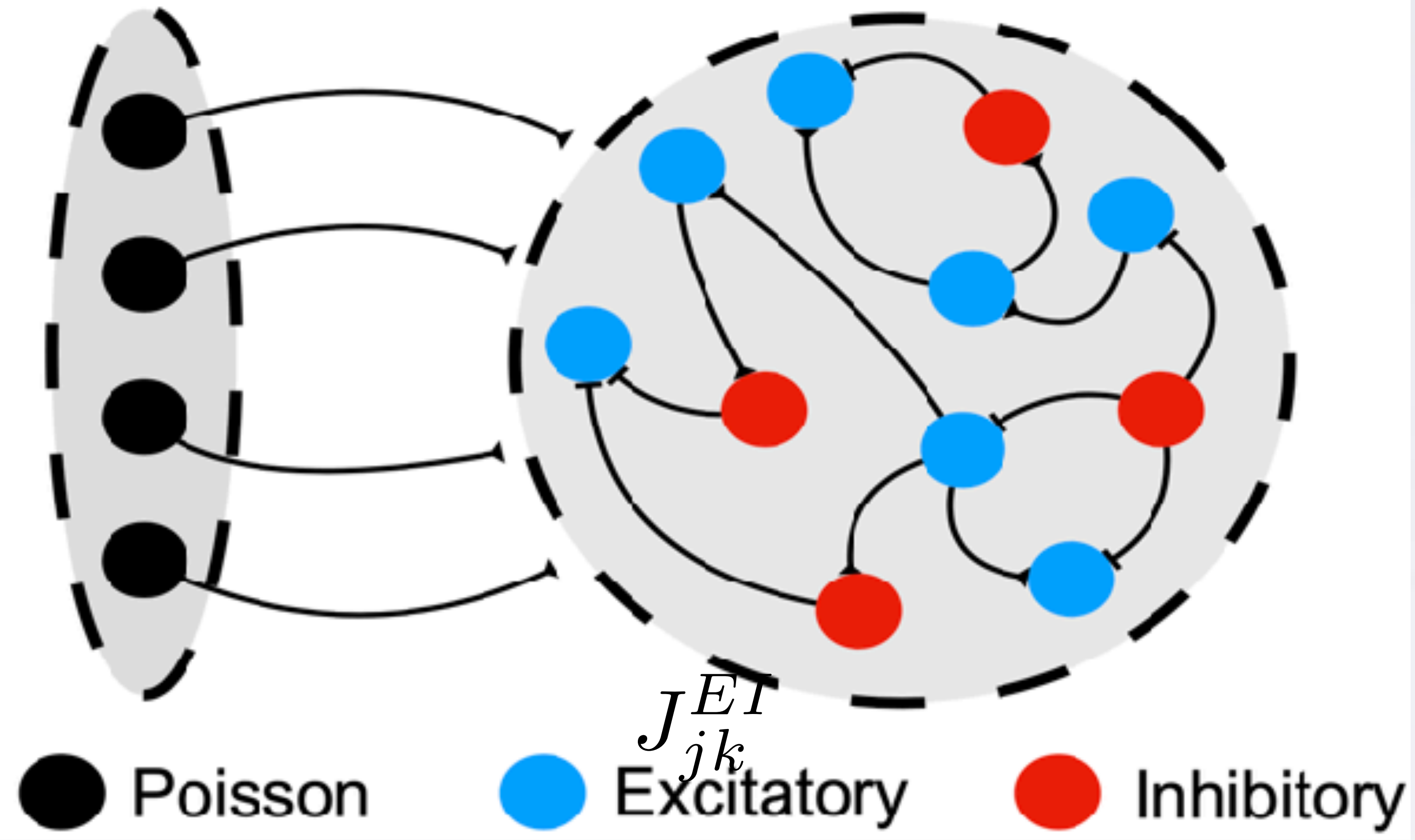
SfN Society for Neuroscience

Funding: NSF NeuroNex, NIH

Mean Field Theory

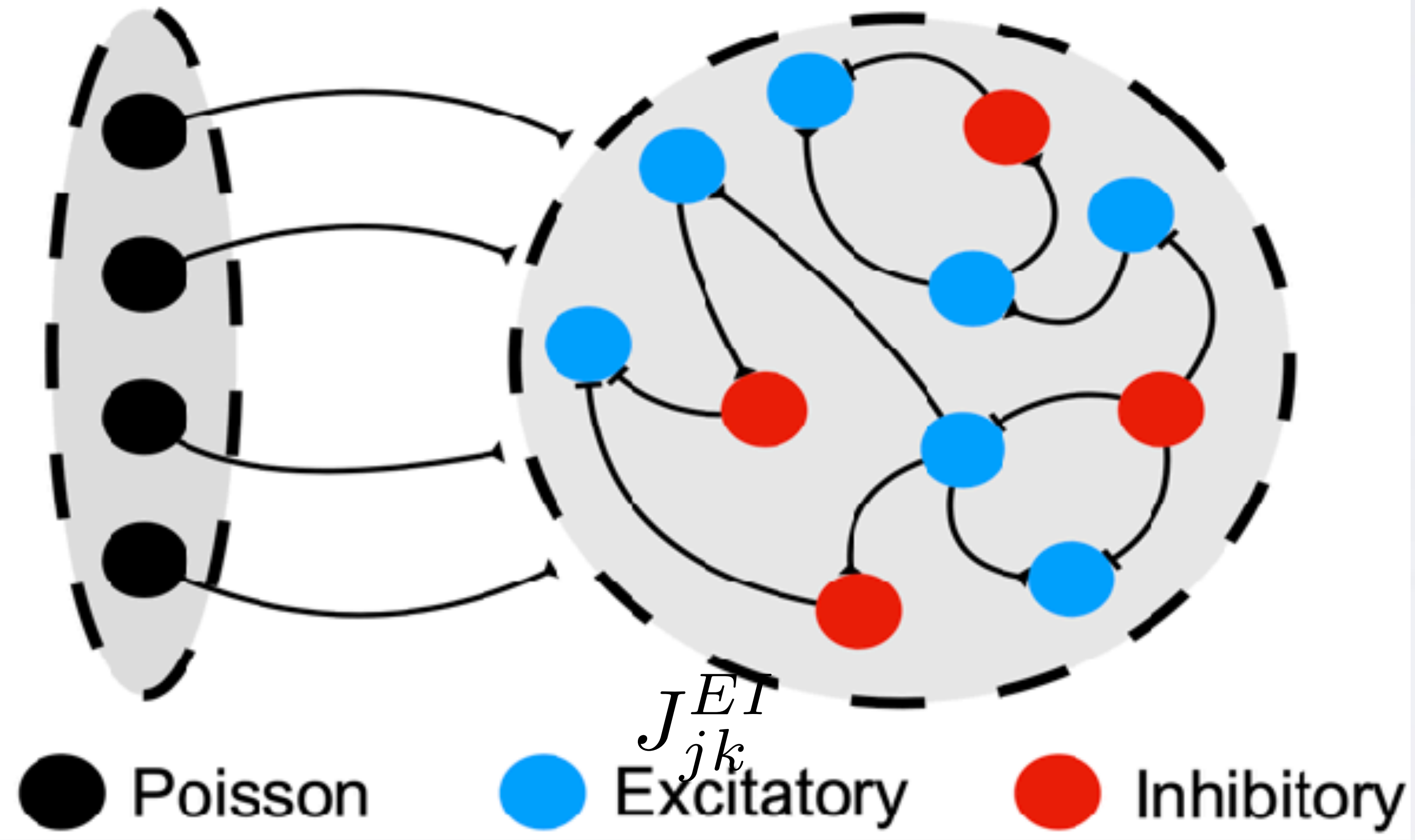


$$\overline{W} = \begin{bmatrix} \overline{w}_{\mathrm{ee}} & \overline{w}_{\mathrm{ei}} \\ \overline{w}_{\mathrm{ie}} & \overline{w}_{\mathrm{ii}} \end{bmatrix}$$

$$\overline{W}_x = \begin{bmatrix} \overline{w}_{\mathrm{ex}} \\ \overline{w}_{\mathrm{ix}} \end{bmatrix}$$

$$\mathbf{r} = \begin{bmatrix} \overline{r}_{\mathrm{e}} \\ \overline{r}_{\mathrm{i}} \end{bmatrix}$$

Mean Field Theory

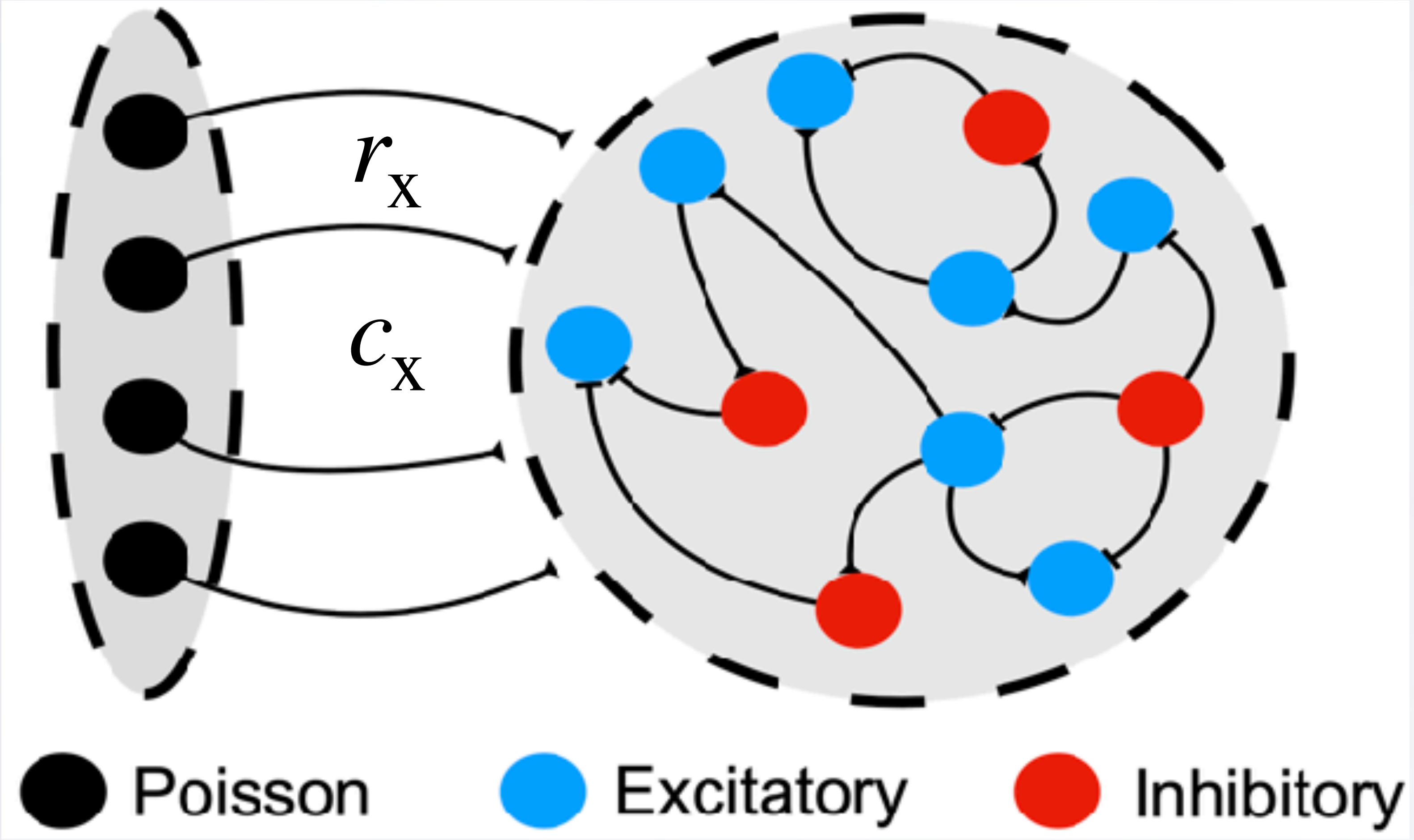


The average input to the two cell types

$$\overline{\boldsymbol{I}} = \sqrt{N}\left[\overline{W}\boldsymbol{r} + \overline{W}_{\mathrm{x}} r_{\mathrm{x}}\right]$$

$$\overline{W} = \begin{bmatrix} \overline{w}_{\mathrm{ee}} & \overline{w}_{\mathrm{ei}} \\ \overline{w}_{\mathrm{ie}} & \overline{w}_{\mathrm{ii}} \end{bmatrix} \qquad \overline{W}_x = \begin{bmatrix} \overline{w}_{\mathrm{ex}} \\ \overline{w}_{\mathrm{ix}} \end{bmatrix} \qquad \mathbf{r} = \begin{bmatrix} \overline{r}_{\mathrm{e}} \\ \overline{r}_{\mathrm{i}} \end{bmatrix}$$

Correlated Balanced State

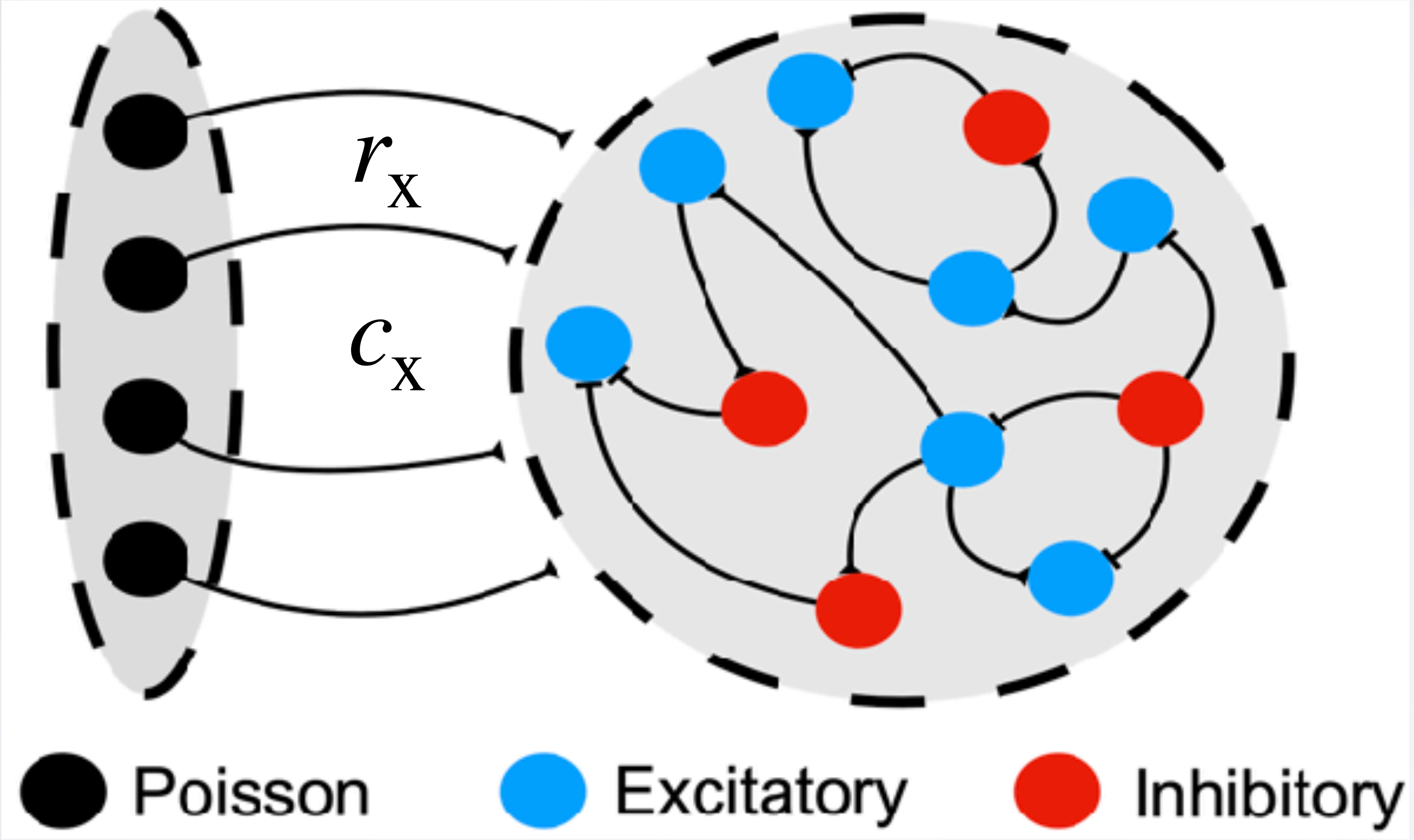


Spike count covariance matrix in window T_{win}

$$C = \begin{bmatrix} C_{\mathrm{ee}} & C_{\mathrm{ei}} \\ C_{\mathrm{ie}} & C_{\mathrm{ii}} \end{bmatrix}$$

Baker, et al. 2019

Correlated Balanced State



Spike count covariance matrix in window T_{win}

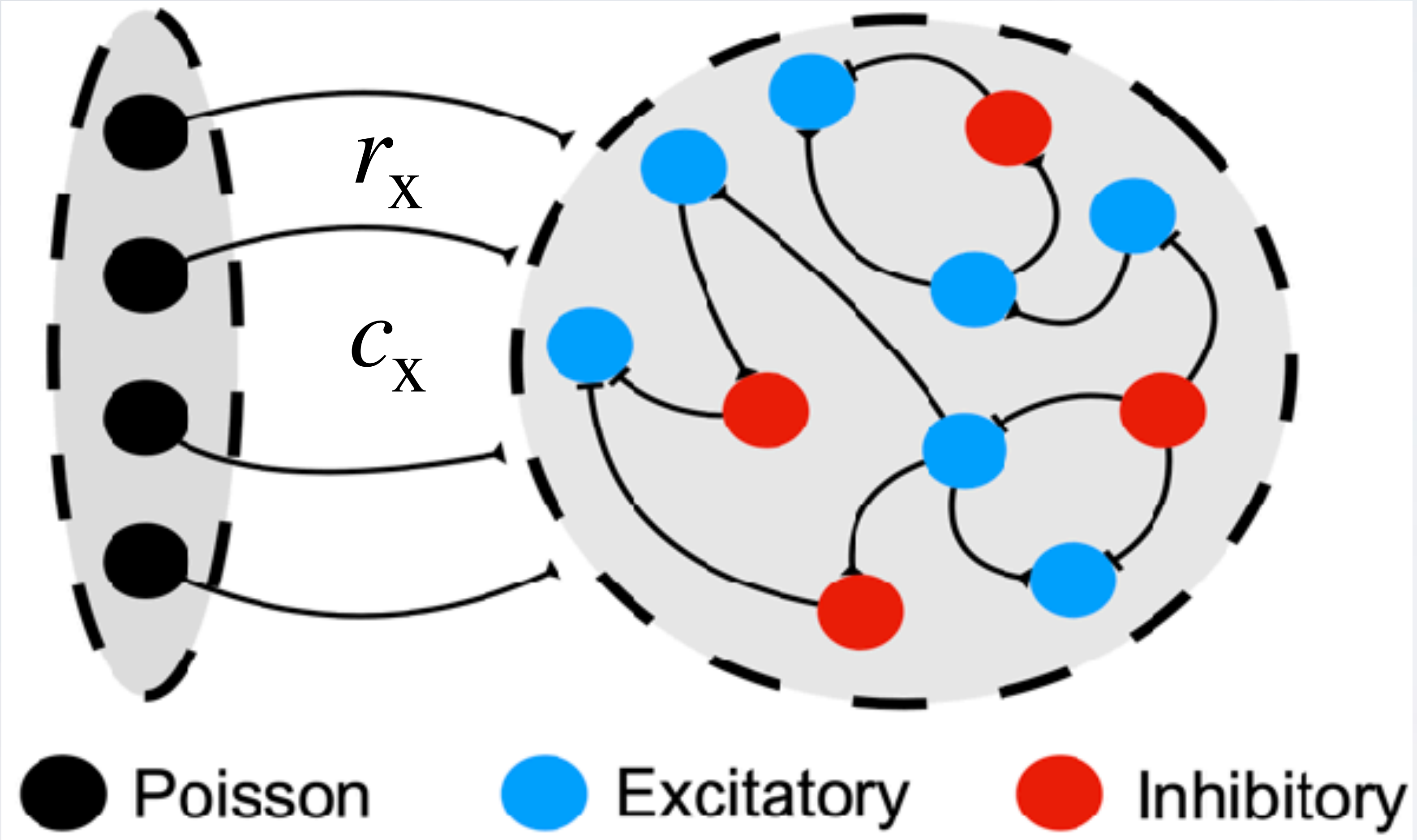
$$C = \begin{bmatrix} C_{\text{ee}} & C_{\text{ei}} \\ C_{\text{ie}} & C_{\text{ii}} \end{bmatrix}$$

If $c_{\text{x}} \neq 0$, then *C* remains *O(1)*, and the network is in a correlated state.

$$C \approx T_{\text{win}} c_{\text{x}} r_{\text{x}} \overline{W}^{-1} \overline{W}_{\text{x}} \overline{W}_{\text{x}}^{T} \overline{W}^{-T}$$

Baker, et al. 2019

Correlated Balanced State

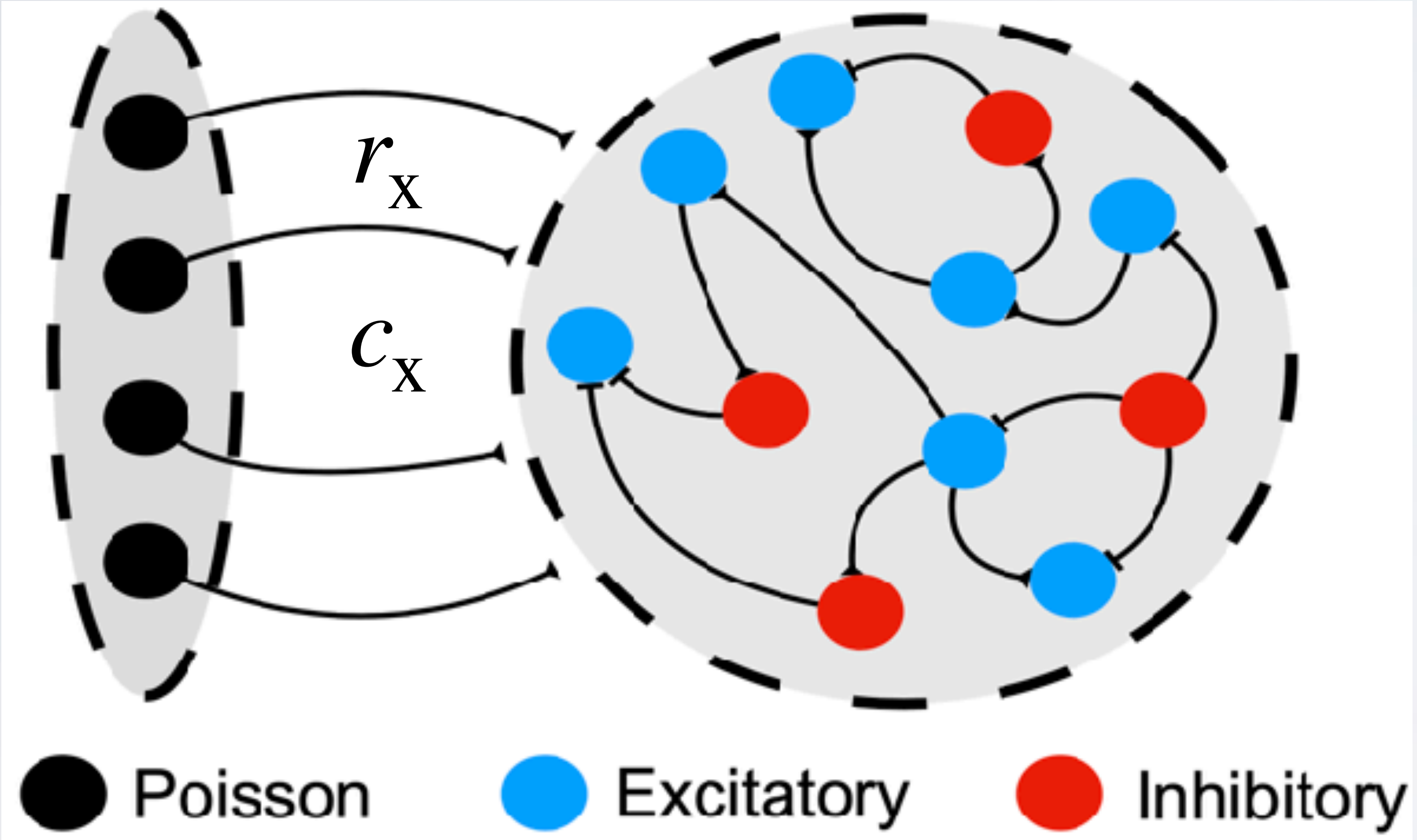


Spike count covariance matrix in window T_{win}

$$C = \begin{bmatrix} C_{\text{ee}} & C_{\text{ei}} \\ C_{\text{ie}} & C_{\text{ii}} \end{bmatrix}$$

Baker, et al. 2019

Correlated Balanced State



Spike count covariance matrix in window T_{win}

$$C = \begin{bmatrix} C_{\text{ee}} & C_{\text{ei}} \\ C_{\text{ie}} & C_{\text{ii}} \end{bmatrix}$$

Covariance in the balanced state

$$C \approx \frac{1}{N} T_{\text{win}} \overline{W}^{-1} \Gamma \overline{W}^{-T}$$

If $c_{\text{x}} \neq 0$, then the network is in a correlated state

$$\Gamma = N \overline{W}_{\text{x}} \overline{W}_{\text{x}}^{T} c_{\text{x}} r_{\text{x}}$$

Baker, et al. 2019