Fibonacci or Quasi-symmetric? Simulating and detecting plant patterns

Christophe Golé

Mathematical Sciences department, Smith College
Thank you!

Work with:
Halley Wilkinson, Chelsea Fowler, Amelia Tarno, Emi Neuwalder, Yuhan Wang, Evelyn Gao (student picture)
Elaine Demetrion, Annie Karitonze, Maggie Hollis, Adara Williams, Xiaoman Xu, Yunxi Yan (students not shown, and many others...)
and Robin Belton (postdoc, picture), Stéphane Douady (Physics, CNRS Paris, France, not shown), Jacques Dumais (Biology, UAI Chile)
Fibonacci or Quasi-symmetric? Simulating plant patterns.
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Fibonacci or Quasi-symmetric? Simulating a
Fibonacci sequence:

\[ F_{n+1} = F_n + F_{n-1} \]

\[ F_0 = 1, \quad F_1 = 1 \]

Fibonacci-like sequence:

\[ F_{n+1} = F_n + F_{n-1} \]

\[ F_0 = \text{some #}, \quad F_1 = \text{another #} \]

Example:

\[ F_0 = 1, \quad F_1 = 3 \]

\[ 1, 3, 4, 7, 11, 18, \ldots \] (Lucas sequence)

Example:

\[ F_0 = 2, \quad F_1 = 2 \]

\[ 2, 2, 4, 6, 10, 16, \ldots \] ("bijugate phyllotaxis")

Classical phyllotaxis classification puts all plants in some Fibonacci-like sequence. Most plants satisfy this but not all...
Fibonacci sequence:

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*Example:*
\[ F_0 = 1, \; F_1 = 3 \text{ gives } 1, 3, 4, 7, 11, 18 \cdots \text{ (Lucas sequence)} \]
- Fibonacci sequence:
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- **Classical phyllotaxis classification** puts all plants in some Fibonacci like sequence. Most plants satisfy this *but not all*...
The “divergence” angle between successive organs *often* approaches the *golden angle*:

\[
137.51^\circ = 360^\circ \lim_{n \to \infty} \frac{F_n}{F_{n+2}}
\]

Fun fact: Divergence near the golden angle \(\Rightarrow\) Fibonacci phyllotaxis. But the converse is not true!
Hofmeister (1868): Primordia (nascent organs) form in the largest place left by previous ones around the meristem (growing tip)
Biology: Hofmeister confirmed (around year 2000)

- Diffusion of growth hormone auxin amplified by Pin protein
Biology: Hofmeister confirmed

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- Diffusion of growth hormone auxin amplified by Pin protein
- New primordia pump the auxin around them to form vasculature
- Primordia of roughly equal size form away from the newest ones, when there is enough auxin
Questions

- Why Fibonacci phyllotaxis?
- What happens when it fails?
- How to analyze plant data more systematically?
Leonardo: first classification

(See possible clue in “Do Plants Know Math?”, PUP 2024)
Bonnet/Calandrini (1754): first bio-math collaboration?

![Diagram of plant patterns](image-url)
Turing's computer simulation of Phyllotaxis (with Reaction-Diffusion PDE?)
Math-Physics-Computer models 1977-now

PDE:
Continuation of Turing's idea of reaction-diffusion (Meinhardt et. al., Newell-Shipman, etc.)

Threshold models (Veen-Lindemeyer, Douady-Couder, Rothen et. al. etc.):
1. Points on a cylinder are centers of inhibition potentials decaying with distance
2. Points move down on the cylinder
3. New points emerge at the top when/where the potential is low enough.

"Fixed plastochrone" models (Douady-Couder, Golé et. al. etc.):
Same as threshold model except points move down a definite amount before placing a new point.

Modeling at the cellular level (Prusinkiewicz et. al. etc.)
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Limitations of these models

- Focus on constant divergence angle and lattices
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- Focus on constant divergence angle and lattices
- Counting spirals is not computer friendly (transitions)
- Explanation of Fibonacci predominance relied on lattices
Go back in 1868: Disk stacking

hat about Fibonacci? Elaine Demetrion and Emi Neuwalder's app

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[Diagram of disk stacking with numbers 1 through 18, highlighting Fibonacci sequence]

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A front captures the geometry of organs morphogenesis.

Picture of picea by R. Rutishauer, Zurich
**Fronts and parastichies**

**Front**: zigzagging line between a point $P$ and its copy $P'$ one full rotation away, joining neighboring organs, as high as possible below segment $PP'$. 
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Numbers of parastichies = 8.5 = number of front segments = 8.5
**Quasi-symmetric**: front parastichy numbers are close to one another. Their ratio is *close* to, but statistically *not equal* to 1.
Why Fibonacci?

Figure by van Iterson (1907)

Transitions occur when the ratio:

\[ b = \frac{\text{disk radius}}{\text{cylinder circumference}} \]

decreases.
Why Fibonacci?

Front parastichy numbers: 5, 3: 5 up vectors; 3 down vectors
Why Fibonacci?

- Quadrilateral transitions $\rightarrow$ still 5, 3
Why Fibonacci?

- Quadrilateral transitions $\rightarrow$ still 5, 3
- But vectors are more horizontal!
Why Fibonacci?

5 up vectors $\rightarrow$ 5 up vectors + 5 down vectors

(Triangles occur on up vectors, because they’re flatter since more numerous here).
Why Fibonacci?

Hence Fibonacci rule:

\[ 5, 3 \rightarrow 5, 3 + 5 = 8 \]
Why Fibonacci?

Monotone Fibonacci: P. # increase 1 at a time, $F_n$ to $F_{n+2}$.
Why Fibonacci?

Fibonacci transitions in ornamental cabbage
Quasi-symmetry in simulations & Peace lilly

![Diagram of Quasi-symmetry in simulations & Peace lilly](image-url)
Simulations with decreasing radius, starting with (1, 1) front. The non Fibonacci-like patterns tend to be QS.
Stats on Magnolia flowers buds
Stats on Skunk Cabbage

Histogram of ratios of 450 front parastichy numbers
Quasi symmetric: Corn, strawberries, raspberries, peace lilly, skunk cabbage, banksia...
Hexagon is the parameter space of 3-fronts, colored by TDA distance between the orbit of each 3-front and its limiting rhombic tiling.
Thank you!