Fibonacci or Quasi-symmetric?
Simulating and detecting plant patterns

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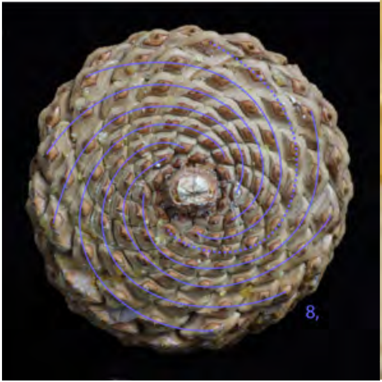
Thank you!

Work with:
Halley Wilkinson, Chelsea Fowler, Amelia Tarno, Emi Neuwalder, Yuhan Wang, Evelyn Gao (student picture)
Elaine Demetrion, Annie Karitonze, Maggie Hollis, Adara Williams, Xiaoman Xu, Yunxi Yan (students not shown, and many others...)
and Robin Belton (postdoc, picture), Stéphane Douady (Physics, CNRS Paris, France, not shown), Jacques Dumais (Biology, UAI Chile)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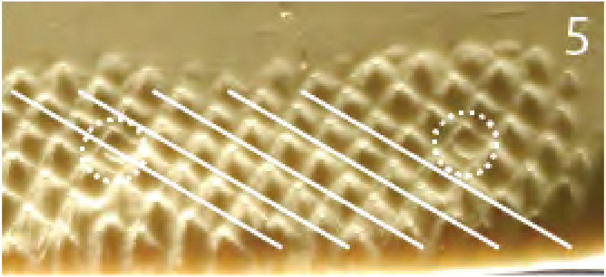
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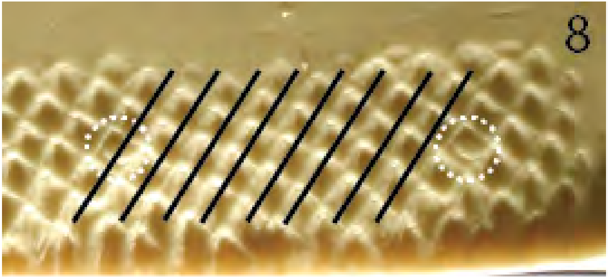
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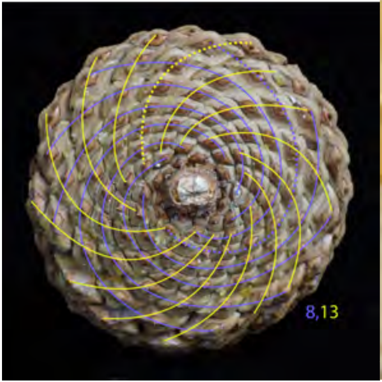
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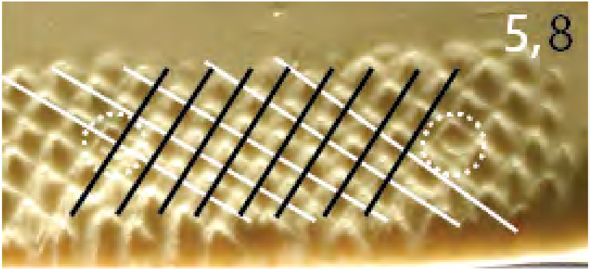
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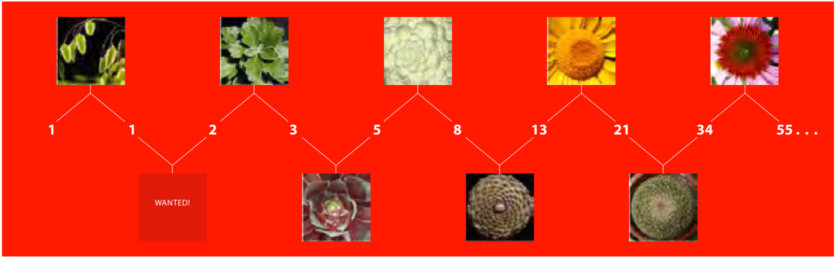
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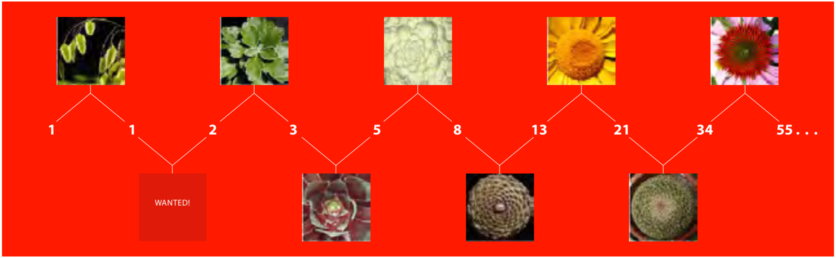
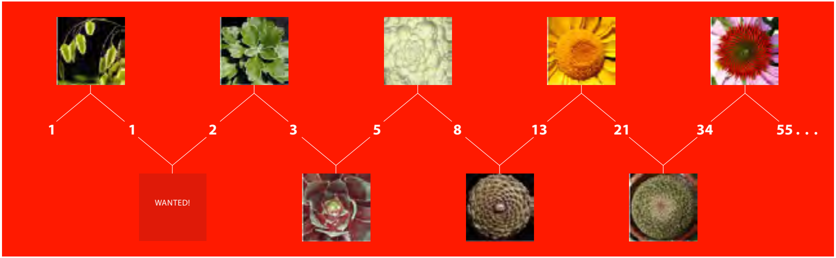


- Fibonacci sequence:
  $F_{n+1} = F_n + F_{n-1}, \quad F_0 = 1, F_1 = 1$



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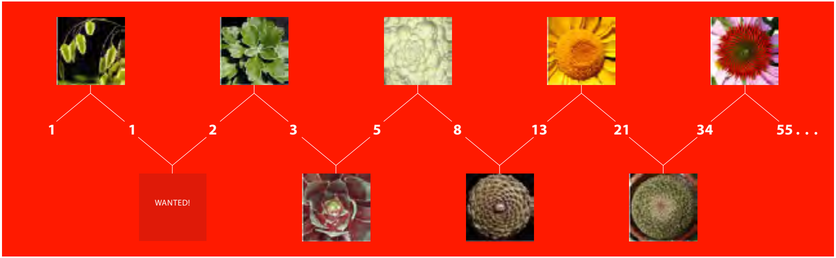

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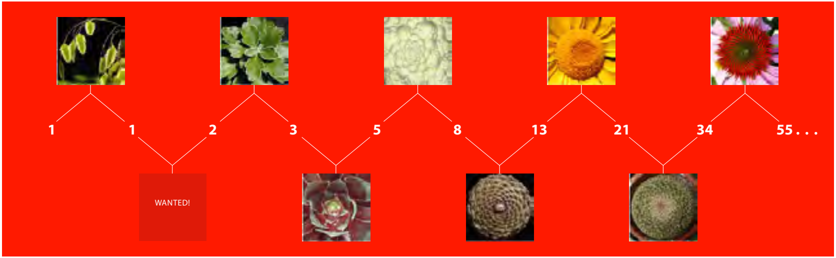
  - Example:
    $F_0 = 1, F_1 = 3$ gives $1, 3, 4, 7, 11, 18 \cdots$ (Lucas sequence)



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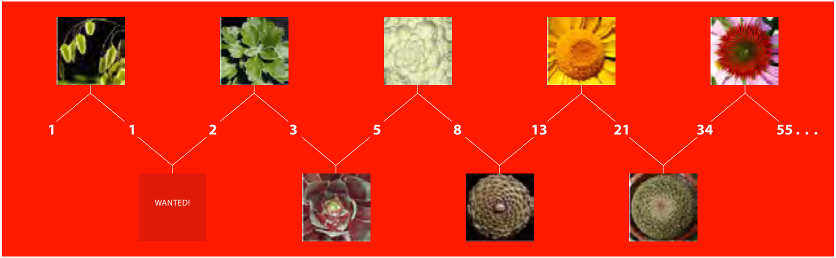
  - Example:
    $F_0 = 2, F_1 = 2$ gives $2, 2, 4, 6, 10, 16 \cdots$ ("bijugate phyllotaxis")



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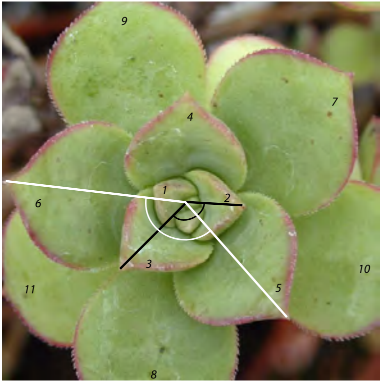
- *Classical* phyllotaxis classification puts all plants in some Fibonacci like sequence. Most plants satisfy this *but not all...*

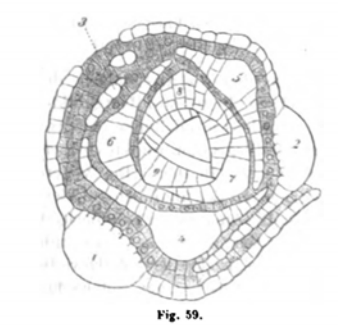


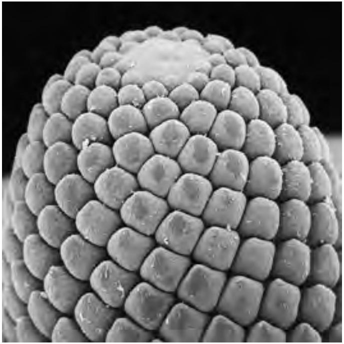
The "divergence" angle between successive organs *often* approaches *the golden angle:*

$$137^o.51 = 360^o \lim_{n\to\infty} \frac{F_n}{F_{n+2}}$$

Fun fact: Divergence near the golden angle $\Rightarrow$ Fibonacci phyllotaxis. But the converse is not true!

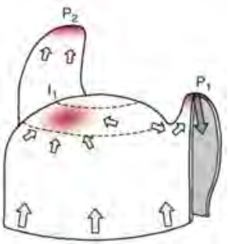
Biology

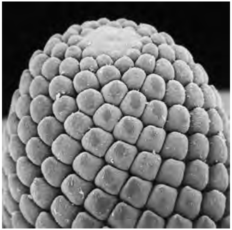




Hofmeister (1868): Primordia (nascent organs) form in the in the largest place left by previous ones around the meristem (growing tip)

Biology: Hofmeister confirmed



(around year 2000)

- Diffusion of growth hormone auxin amplified by Pin protein

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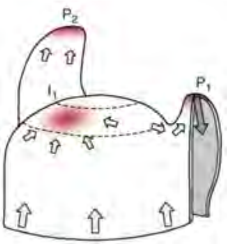


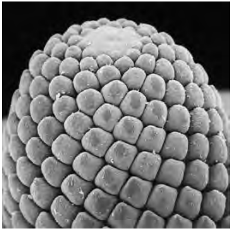

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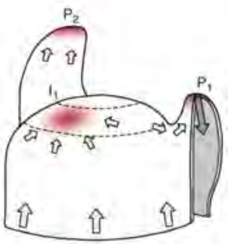
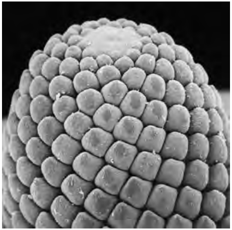


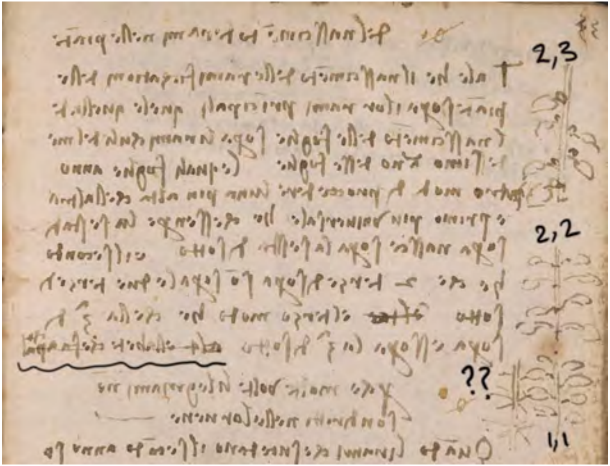
(around year 2000)

- Diffusion of growth hormone auxin amplified by Pin protein
- New primordia pump the auxin around them to form vasculature
- Primordia of roughly equal size form away from the newest ones, when there is enough auxin

Questions

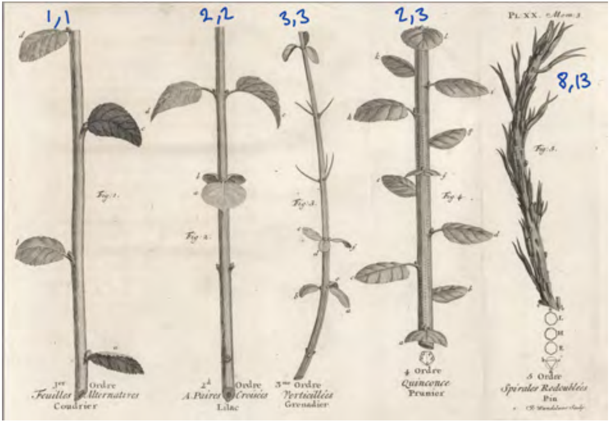
- Why Fibonacci phyllotaxis?
- What happens when it fails?
- How to analyze plant data more systematically?

Leonardo: first classification



(See possible clue in "Do Plants Know Math?", PUP 2024)

Bonnet/Calandrini (1754): first bio-math collaboration?



Turing : first computer experiments (1951-54)

Turing's computer simulation of Phyllotaxis (with Reaction-Diffusion PDE?)

Math-Physics-Computer models 1977-now

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  1. Points on a cylinder are centers of inhibition potentials decaying with distance
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- "Fixed plastochrone" models (Douady-Couder, Golé et. al. etc.): Same as threshold model except points move down a definite amount before placing a new point.
- Modeling at the cellular level (Prusinkiewicz et. al. etc.)

Limitations of these models

- Focus on constant divergence angle and lattices

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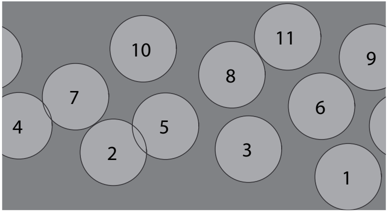
- Counting spirals is not computer friendly (transitions)
- Explanation of Fibonacci predominance relied on lattices

Going back in 1868: Disk stacking



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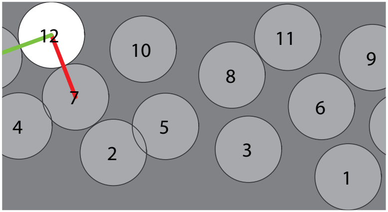

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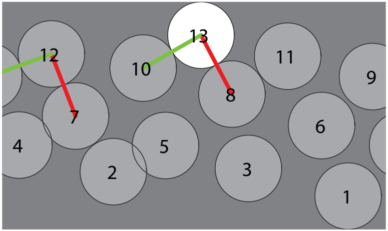

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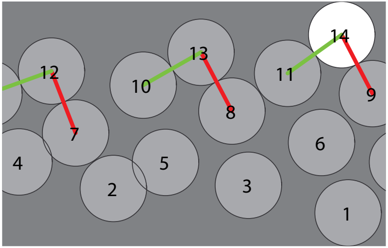

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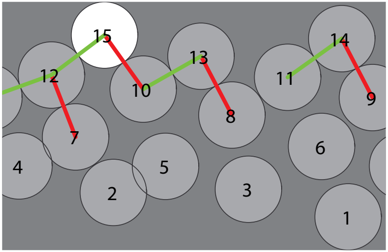

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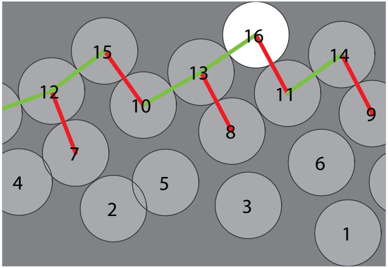

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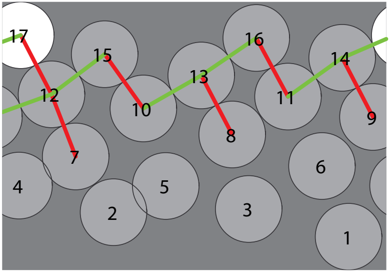

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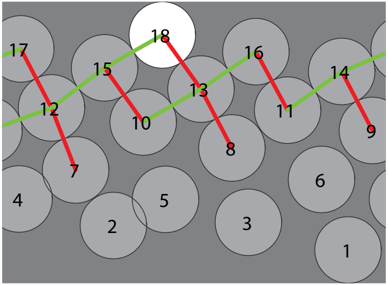

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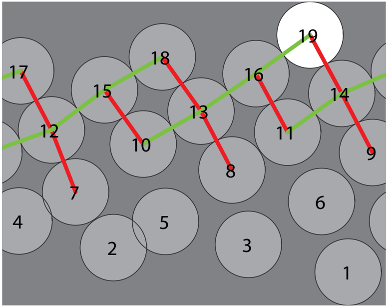

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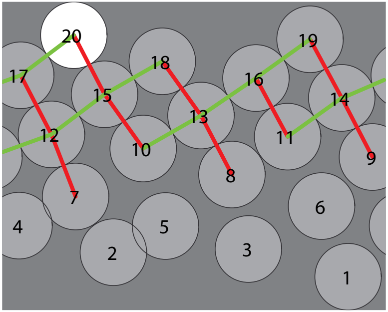

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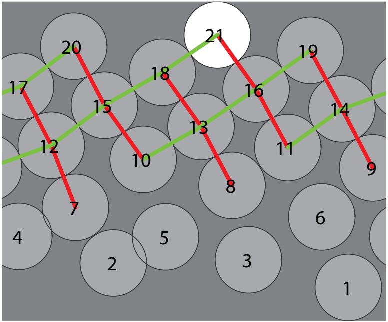

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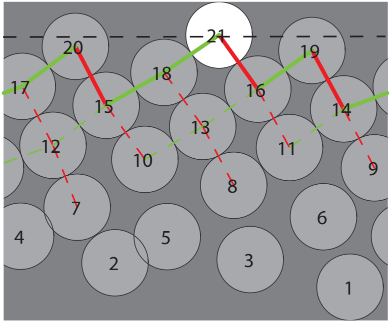
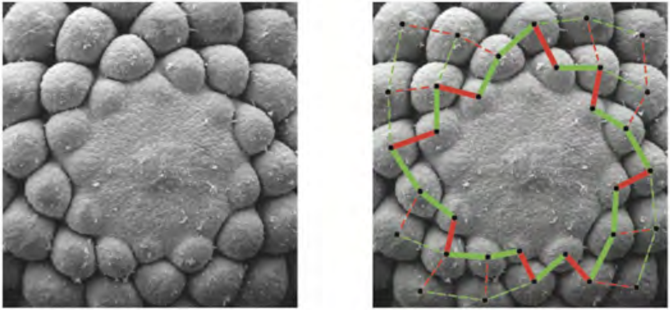


Going back in 1868: Disk stacking

What about Fibonacci? Elaine Demetrion and Emi Neuwalder's app

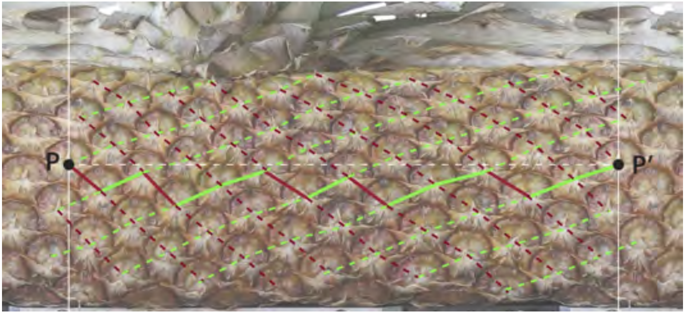
Fronts: in real plants too!



[1] A front captures the geometry of organs morphogenesis.

[1]Picture of picea by R. Rutishauser, Zurich

Fronts and parastichies



Front: zigzagging line between a point P and its copy P' one full rotation away, joining neighboring organs, as high as possible below segment PP'.

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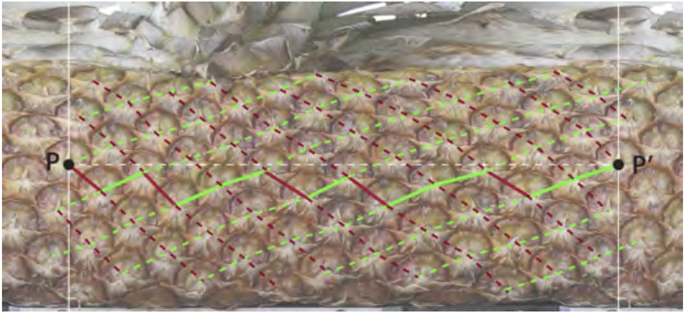

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Numbers of **parastichies** = 8,5 = number of **front** segments = 8,5

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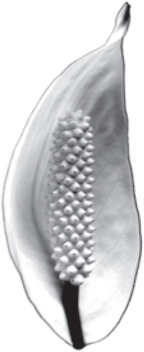
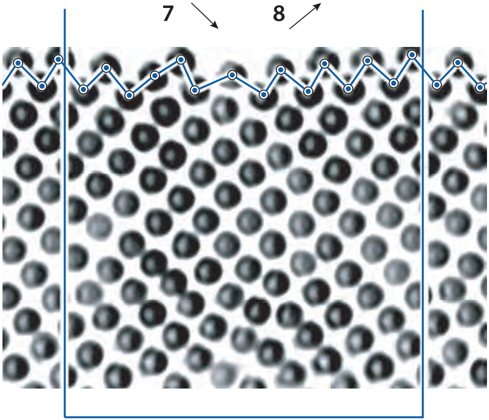


(courtesy Scott Hotton)



Quasi-symmetric: front parastichy numbers are close to one another. Their ratio is *close* to, but statistically *not equal* to 1.

Why Fibonacci?

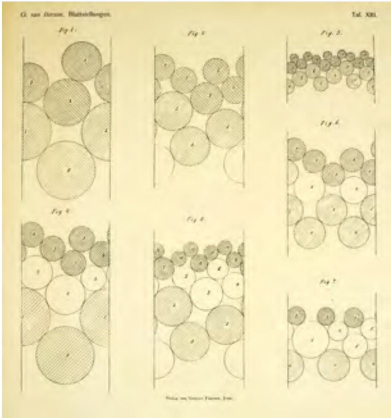


Figure by van Iterson (1907)

Transitions occur when the ratio:

$$b = (\text{disk radius})/(\text{cylinder circumference})$$

decreases.

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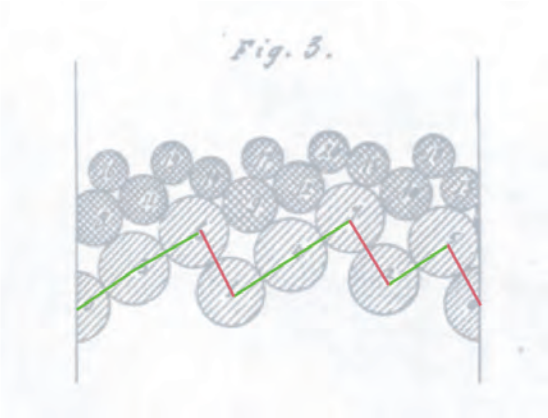
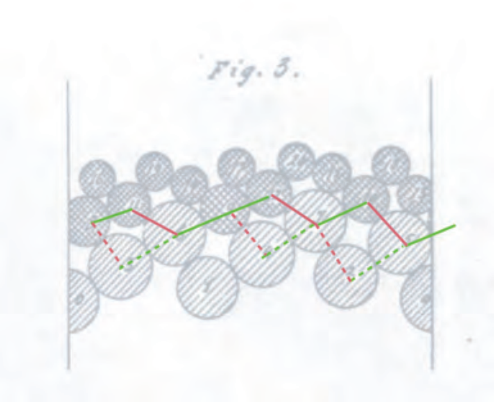


Front parastichy numbers: 5, 3: 5 up vectors; 3 down vectors

Why Fibonacci?



- Quadrilateral transitions → still 5 , 3

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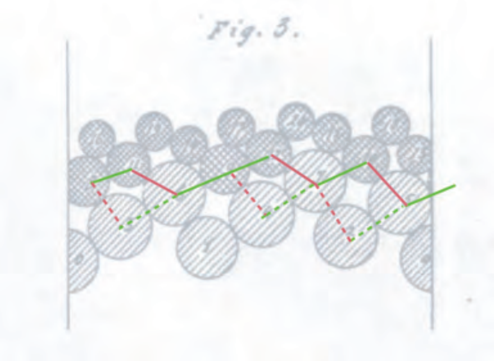
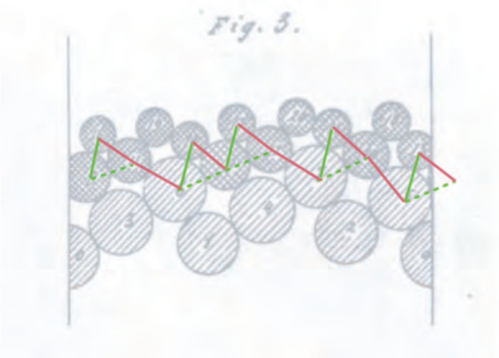


- Quadrilateral transitions → still 5 , 3
- But vectors are more horizontal!

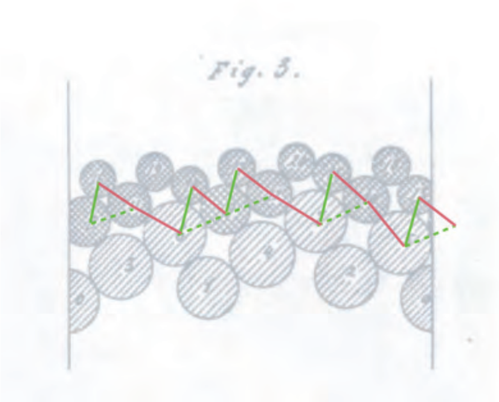
Why Fibonacci?



5 up vectors $\rightarrow$ 5 up vectors + 5 down vectors

(Triangles occur on up vectors, because they're flatter since more numerous here).

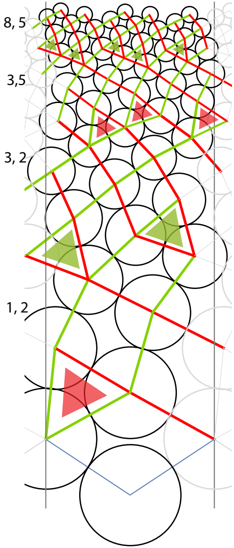
Why Fibonacci?

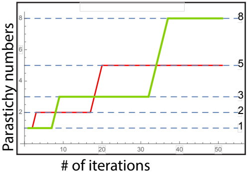


Hence Fibonacci rule:

$5\ ,\ 3 \rightarrow 5\ ,\ 3{+}5 = 8$

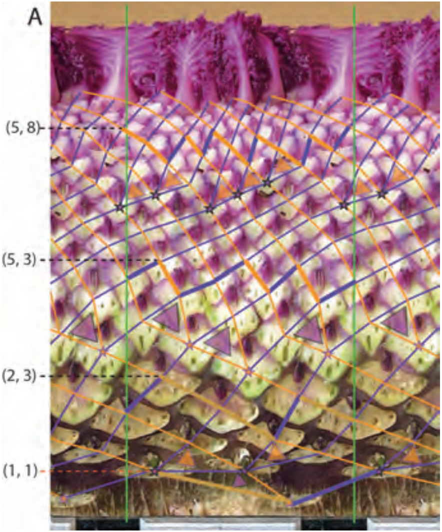
Why Fibonacci?





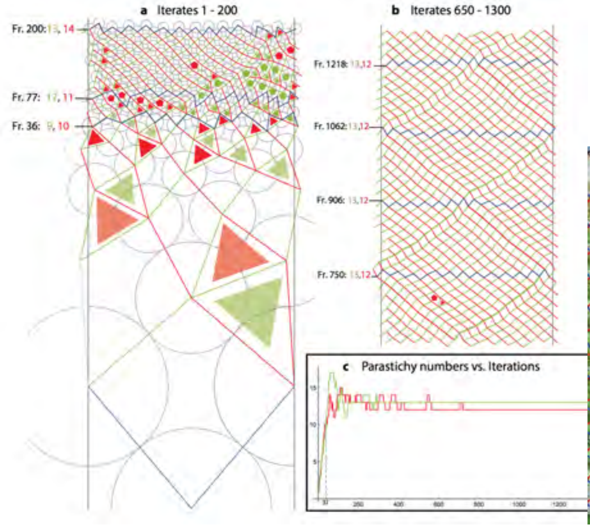
Monotone Fibonacci: P. # increase 1 at a time, F_n to F_{n+2}.

Why Fibonacci?

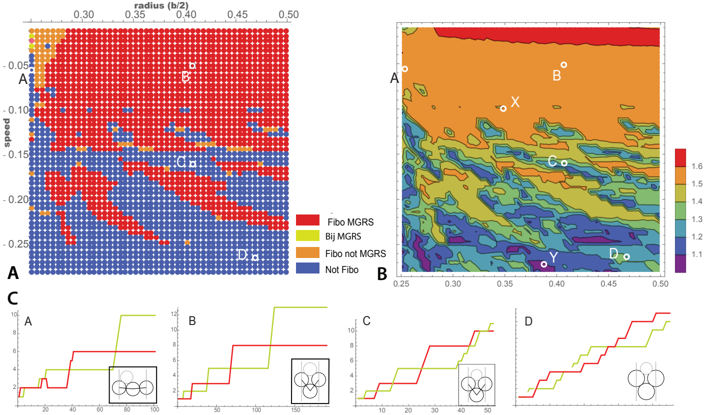


Fibonacci transitions in ornamental cabbage

Quasi-symmetry in simulations & Peace lilly

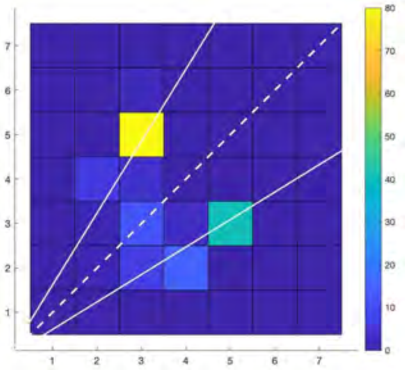


Universe of plants partitioned

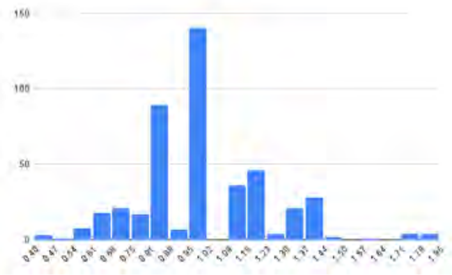


Simulations with decreasing radius, starting with (1, 1) front. The non Fibonacci-like patterns tend to be QS.

Stats on Magnolia flowers buds



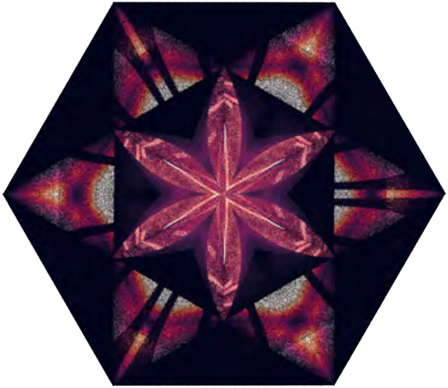
Stats on Skunk Cabbage



Histogram of ratios of 450 front parastichy numbers

Quasi symmetric: Corn, strawberries, raspberries, peace lilly, skunk cabbage, banksia...

TDA Teaser



Hexagon is the parameter space of 3-fronts, colored by TDA distance between the orbit of each 3-front and its limiting rhombic tiling.

Thank you!