

Poster Session Blitz

April 25, 2023

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Preliminaries

Let $A \in \mathbb{Z}_+^{m \times n}$, $G \in \mathbb{Q}_+^{m \times p}$, and $b \in \mathbb{R}_+^m$. Assume cost vectors $c, h > 0$.

Consider the mixed-integer programming (MIP) problem:

$$z_{MIP}(b) := \max_{x \in \mathbb{Z}_+^n, y \in \mathbb{R}_+^p} \{c^\top x + h^\top y \mid Ax + Gy \leq b\}. \quad (\text{MIP})$$

And its LP relaxation:

$$z_{LPR}(b) := \max_{x \in \mathbb{R}_+^n, y \in \mathbb{R}_+^p} \{c^\top x + h^\top y \mid Ax + Gy \leq b\}. \quad (\text{LPR})$$

This is joint work with Dr. Tayo Ajayi and Dr. Andrew J. Schaefer. This research was supported by NSF grant CMMI-1933373.

Periodicity of Absolute MIP Gap Functions

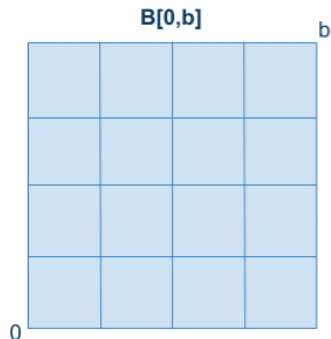
Definition

Given a set of RHSs, $\mathcal{B}[0, b] = \prod_{i=1}^m [0, b_i]$, the **Absolute Gap Function** for MIPs is defined as:

$$\Gamma : \mathcal{B}[0, b] \rightarrow \mathbb{R}_+ \cup \{\infty\}, \quad \Gamma(\hat{\beta}) := z_{LPR}(\hat{\beta}) - z_{MIP}(\hat{\beta}).$$

Theorem

Let a_j be the j^{th} column of A and g_k be the k^{th} column of G .
Then, **under certain conditions**, $\Gamma(\hat{\beta} - \eta a_j - \lambda g_k) = \Gamma(\hat{\beta})$.



Enhancing classification and regression trees via mathematical optimization

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Trends in Computational Discrete Optimization at Providence, USA

April 25th 2023



Mathematical
Optimization

Machine
Learning

Optimal Classification and
Regression Trees

- We develop a Continuous Optimization formulation.
- We model probabilistic (as opposed to deterministic) splitting rules.

Prediction accuracy

Sparsity

Cost-sensitivity

Fairness

Local explainability

Data complexity

Thank you for your attention!

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All this research is available at:

https://www.researchgate.net/profile/Cristina_Molero-Rio

An analysis of alternative perspective reformulations for piecewise-convex optimization

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1 Introduction

- Sequential Convex MINLP

2 Formulations for SC-MINLP

- Incremental Model (IM)
- Multiple Choice Model (MCM)
- Convex Combination Model (CCM)

3 Comparison

- Theoretical
- Computational results

Thank you!



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IP PARIS

Pre-Trained Solution Methods for Unit Commitment

Nagisa Sugishita, Andreas Grothey, Ken McKinnon
University of Edinburgh

April 2023

Pre-Trained Solution Methods for Unit Commitment

- ▶ Find the optimal operation of generators given demand ω :

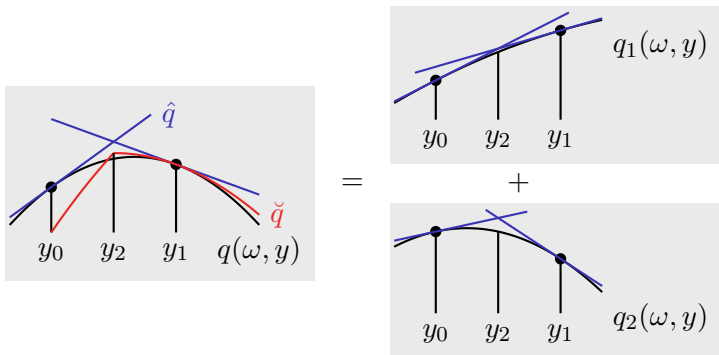
$$x = \left(\begin{array}{c} \text{on-off status} \\ \begin{array}{|c|c|c|c|} \hline 0.0 & 0.0 & 1.0 & 0.0 \\ \hline 1.0 & 1.0 & 1.0 & 0.0 \\ \hline 1.0 & 1.0 & 1.0 & 1.0 \\ \hline \end{array} \\ \leftarrow \text{time period} \rightarrow \end{array} \right) , \left(\begin{array}{c} \text{power output} \\ \begin{array}{|c|c|c|c|} \hline 0.0 & 0.0 & 0.4 & 0.0 \\ \hline 0.2 & 0.5 & 0.6 & 0.0 \\ \hline 0.8 & 1.0 & 1.0 & 0.8 \\ \hline \end{array} \\ \leftarrow \text{time period} \rightarrow \end{array} \right)$$

- ▶ The dual problem is decomposable by generators

$$\max \left\{ q(\omega, y) := \frac{1}{G} \sum_{g=1}^G q_g(\omega, y) \right\},$$

where G is the number of generators.

Improvement on Regularised Cutting-Plane Method



1. How should initialise the algorithm, i.e. get y_0 ?
 - ▶ Use machine learning to find a good point.
2. How should we accelerate the progress of the algorithm?
 - ▶ Only evaluate a subset of $q_g(\omega, y)$ and take incremental steps.

Question

What is the **smallest** n such that every k -coloring of $\{1, 2, \dots, n\}$ produces a **monochromatic** solution to $x + y = z$?

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- $S(3) = 14$:

1 2 3 4 5 6 7 8 9 10 11 12 13

Schur numbers

Table of all known Schur numbers:

k	$S(k)$	proof author/year	proof computation time
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- Longest proof in history!
- Our goal: study generalizations of Schur numbers called **Rado numbers** using SAT solvers