Mathematical Optimization for Traffic Management in Urban Air Mobility

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Chair “Integrated Urban Mobility"

Trends in Computational Discrete Optimization
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Outline

Outline

Air Traffic Management

- Flights share the same resources, i.e., the airport parking, the runway, the air space, ...
- How to affect the resources so as to guarantee **safety** ?
- **Air space management**:
  - Planning $\rightarrow$ Strategic Deconfliction
  - Online $\rightarrow$ Tactical Deconfliction
  - Last minute $\rightarrow$ Conflict Avoidance

Classic Air Traffic Management

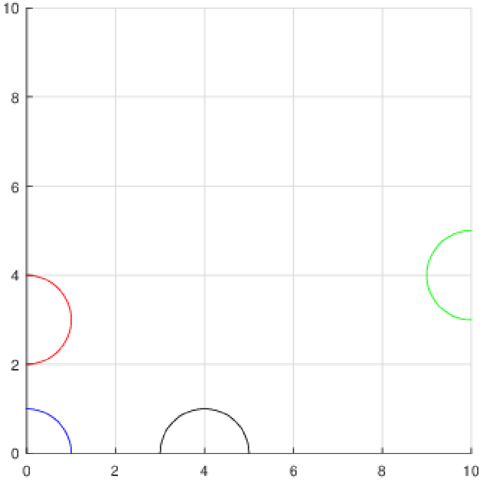
Separation constraint in classic **ATM** :

$$\forall i < j \in A, t \in T \quad \|x_i(t) - x_j(t)\| \geq D$$

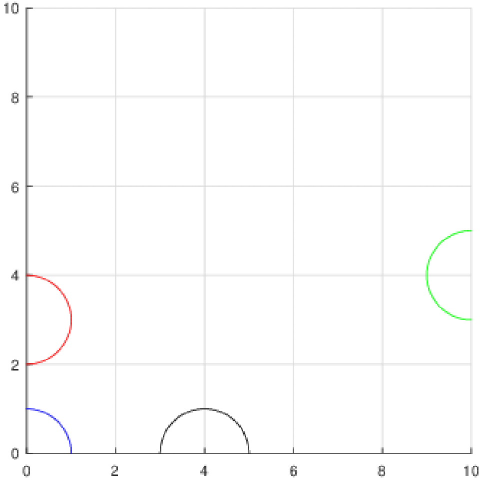
where

- A is the set of aircraft,
- T is the considered time horizon,
- $x_i(t)$ the position in the sky of aircraft i at time t,
- D is the safety distance to guarantee between pairs of aircraft.

Classic Air Traffic Management



Classic Air Traffic Management



Classic Air Traffic Management

Outline

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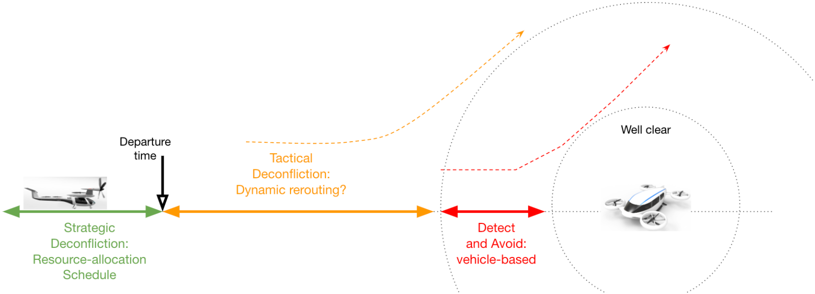
Urban Air Mobility – eVTOLs

Urban Air Mobility

Source `https://www.objetconnecte.com/`

- Urban Air Mobility (**UAM**)
- Electric vertical take-off and landing (**eVTOLs**) aircraft
- Decision-makers tool to guarantee **safety and efficiency**

Urban Air Mobility



Our focus: UAM **tactical deconfliction**

Differences w.r.t. classic ATM?

Outline

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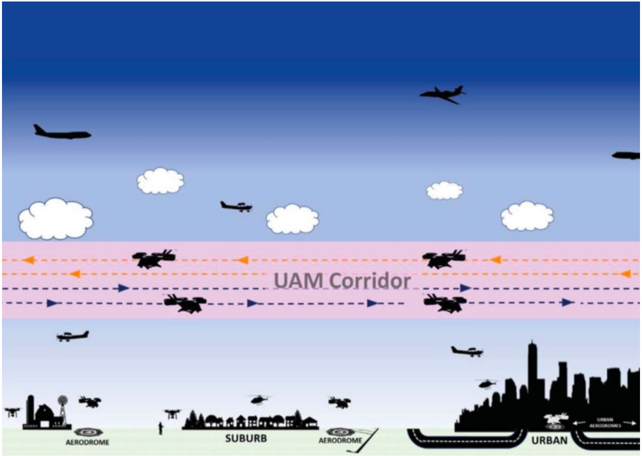
Problem Definition

Well defined **skylane network** :

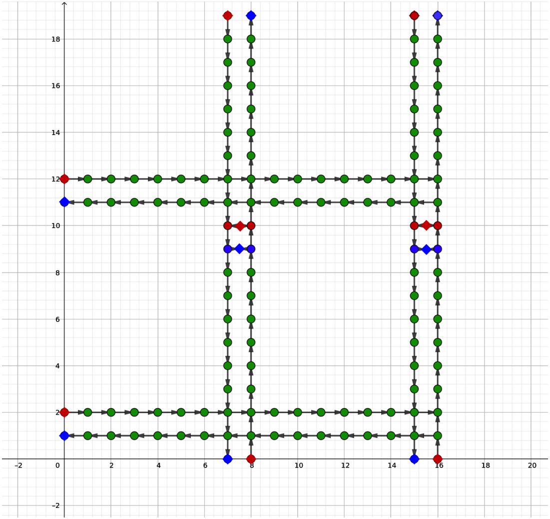


Source: FAA

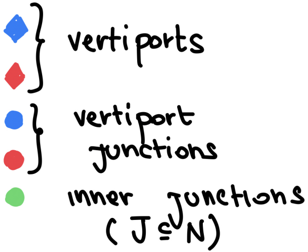
- connecting **vertiports**
- virtual 3D **corridors**
- corridors can intersect at exclusive **junctions**

Notation

Network infrastructure:

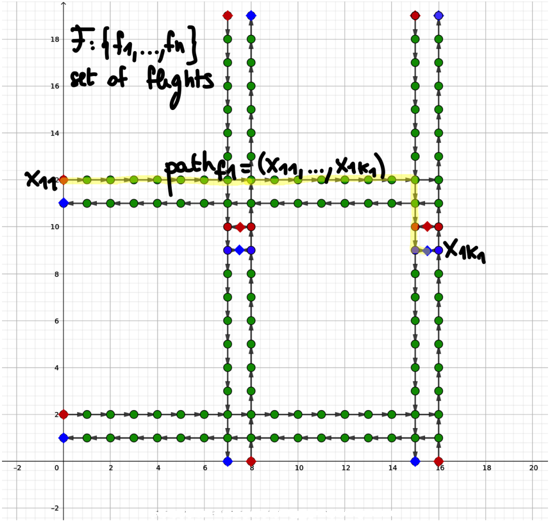


$G = (N, A)$
N nodes set
A arcs set (skylanes)



Notation

Flights and schedule:



$(x_h, x_m) \in A$ belong to $path_{f_i}$ if x_h and x_m are consecutive nodes in $path_{f_i}$

$\hat{t}_{im}$ time at which f_i arrives at/traverses x_m

v_{im} speed at which f_i traverses x_m (constant through the arc)

Considered problem

Given a nominal planning, consider the following uncertainties.

Disruption

1. For strategically deconflicted **scheduled traffic**
2. To accommodate a **new, priority, operation**
3. In reaction to some **unexpected traffic** e.g., intruder crossing a UAM corridor.

Outline

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Mathematical optimization formulation

- **Degrees of freedom**: speed changes manoeuvres at junctions (implicitly) + departure time reschedule
- **Output** : new schedule of the flights with safe arrival times at the waypoints of their paths
- Nominal and the deconflicted schedules are made of **safe time intervals** at which the flights can traverse each waypoint without incurring in conflicts
- **Minimum width** of the time intervals

Mathematical optimization formulation

Decision Variables:

- $t_{im}^{ear}, t_{im}^{lat}$ **new scheduled times**

  Safe time interval $[t_{im}^{ear}, t_{im}^{lat}]$ for every $f_i \in \mathcal{F}$ to traverse each $x_m \in path_{f_i}$.

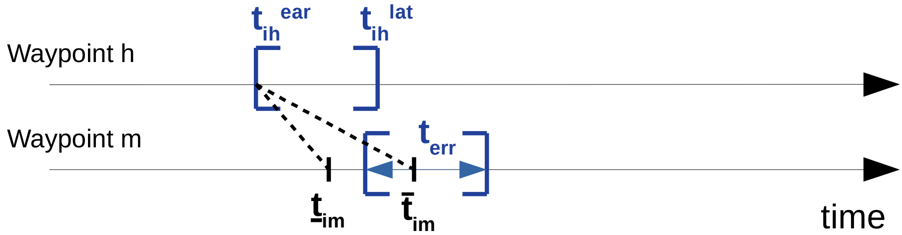
$$t_{im}^{lat} = t_{im}^{ear} + (\hat{t}_{im}^{lat} - \hat{t}_{im}^{ear})$$

- $\underline{t}_{im}, \overline{t}_{im}$ **bounding times**

  Bounding interval $[\underline{t}_{im}, \overline{t}_{im}]$ for every $f_i \in \mathcal{F}$ to traverse each $x_m \in path_{f_i}$ **without violating speed and/or operating limits** .

$$\underline{t}_{im} \leq t_{im}^{ear} \leq \overline{t}_{im}$$

Mathematical optimization formulation



For each arc (h, m), $\underline{t}_{im}$ depends on t_{ih}^{ear}, $\text{dist}(x_h, x_m)$, and $\underline{v}_{im}$

For each arc (h, m), $\bar{t}_{im}$ depends on t_{ih}^{ear}, $\text{dist}(x_h, x_m)$, and $\bar{v}_{im}$

Mathematical optimization formulation

Objective function:

$$\min \sum_{f_i \in \mathcal{F}} \sum_{x_m \in path_{f_i}} |t^{ear}_{im} - \hat{t}^{ear}_{im}| + \sum_{f_i i \in \mathcal{F}^{prior}} M \cdot (t^{ear}_{i,x_{i1}} - \hat{t}^{ear}_{i,x_{i1}})$$

M large number (to prioritize minimization of deviation of priority flights)

Outline

Outline

Modeling of aircraft separation

Given a conflict $(i, j, m) \in Conf$, UAM separation constraints:

$$|t_{im} - t_{jm}| \geq \text{“safety time lapse”}.$$

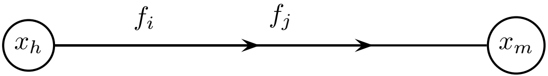
Time lapse depends on a **minimum safety distance** D, see classic ATM.

Two main families of conflicts:

- Trailing
- Intersection

Trailing conflicts

Two flights $f_i, f_j \in \mathcal{F}$ travel through the same arc (x_h, x_m) at the same time



- overtaking is **forbiden**
- hp: **constant speed** on each arc
- sufficient to impose **separation at x_h and x_m**

Trailing conflicts: MP formulation

$$v_{ih}(t_{jh} - t_{ih}) \geq D \qquad \forall (i,j,h,m) \in Trail$$
$$v_{jm}(t_{jm} - t_{im}) \geq D \qquad \forall (i,j,h,m) \in Trail.$$

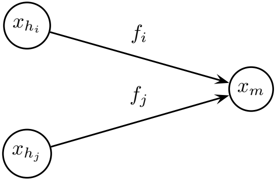
- $Trail := \{(i,j,h,m) : (i,j,m) \in Conf,\ (x_h, x_m) \in path_{f_i} \cap path_{f_j}\}$, set of potential trailing conflicts.

Constraints:

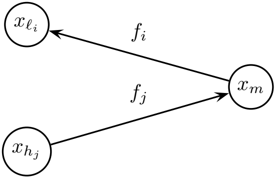
$$t^{ear}_{jh} - t^{lat}_{ih} \geq \frac{D}{\underline{v}_{ih}} \qquad \forall (i,j,h,m) \in Trail$$
$$t^{ear}_{jm} - t^{lat}_{im} \geq \frac{D}{\underline{v}_{jm}} \qquad \forall (i,j,h,m) \in Trail$$

Intersection conflicts

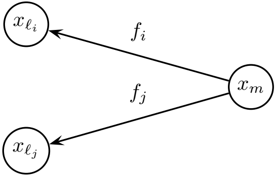
Given a conflict $(i, j, m) \in Conf$, three types of **intersection conflicts**



In/in



Out/in



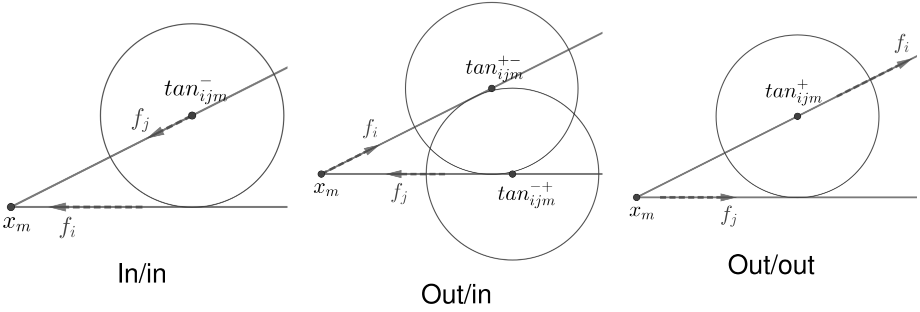
Out/out

Intersection conflicts

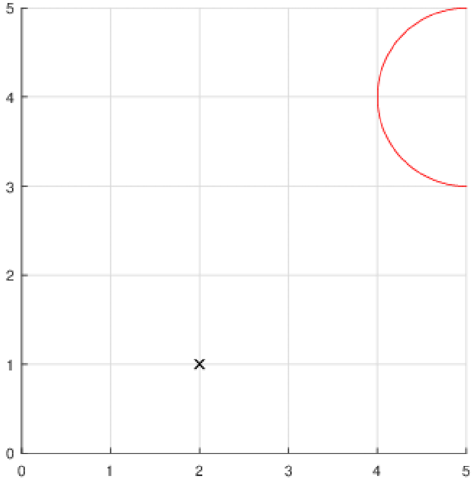
- **Safety disks** of radius D around both f_i and f_j along their trajectories
- **Conflict zones** : when one trajectory is tangent to the safety disk of the other flight
- **Time separation** of passage through conflict zones
- Link **ATM strategies** : ensures a minimum distance of D between the flights all their way throughout their paths.
- **Arbitrating rule** (hp. $t_{im} < t_{jm}$):

  "f_j does not enter its conflict zone until
  f_i does not leave its own"

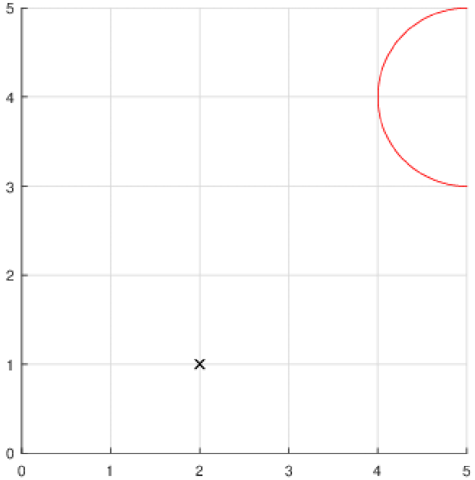
Intersection conflicts



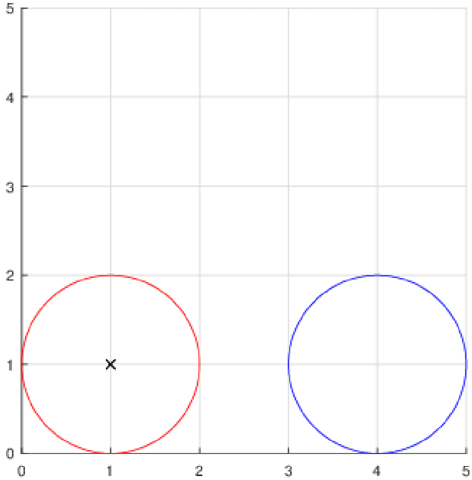
- Case in/in: $t_{im} \leq time(j, tan^{-}_{ijm})$,
  where $time(i, x)$ = the arrival time of f_i at the point x.
- Case out/in: $time(i, tan^{+-}_{ijm}) \leq time(j, tan^{-+}_{ijm})$
- Case out/out: $time(i, tan^{+}_{ijm}) \leq t_{jm}$.


0
1
2
3
4
5
0
1
2
3
4
5

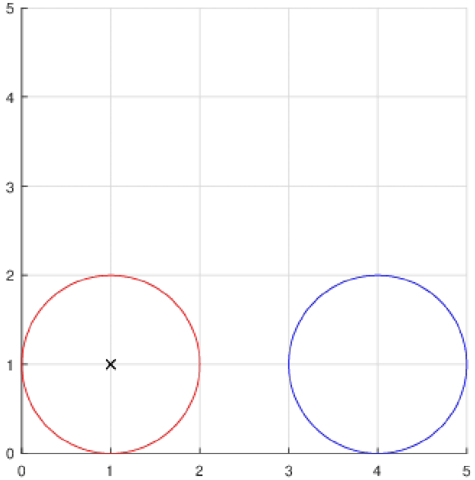
Classic Air Traffic Management



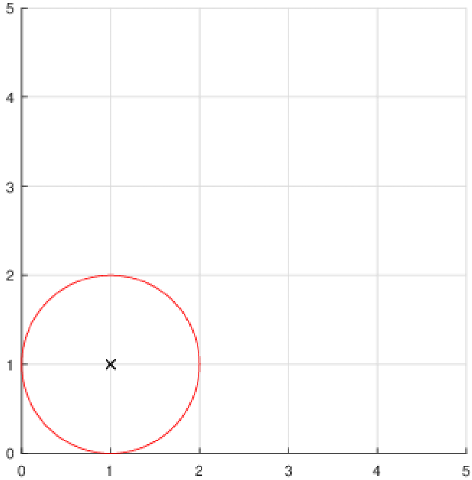
Classic Air Traffic Management



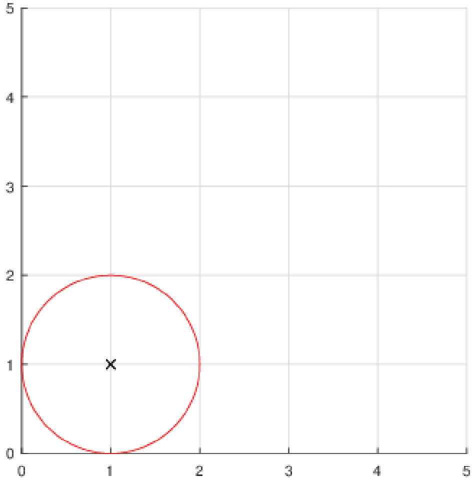
Classic Air Traffic Management



Classic Air Traffic Management



Classic Air Traffic Management



Intersection conflicts: MP formulation

Constraints:

$$t^{ear}_{jm} - t^{lat}_{im} \geq S(\alpha^{-}_{ijm}) \frac{D}{\underline{v}_{jm}} \qquad \forall (i,j,m,h_i,h_j) \in Cross^{-}$$

$$t^{ear}_{jm} - t^{lat}_{im} \geq S(\alpha^{+-}_{ijm}) \left(\frac{D}{\underline{v}_{jm}} + \frac{D}{\underline{v}_{i\ell_i}} \right) \qquad \forall (i,j,m,\ell_i,h_j) \in Cross^{+-}$$

$$t^{ear}_{jm} - t^{lat}_{im} \geq S(\alpha^{+}_{ijm}) \frac{D}{\underline{v}_{i\ell_i}} \qquad \forall (i,j,m,\ell_i,\ell_j) \in Cross^{+}$$

- $Cross^{-}/Cross^{+-}/Cross^{+} := \{(i,j,m,h_i,h_j) : (i,j,m) \in Conf,\ (x_{h_i}, x_m) \in path_{f_i},\ (x_{h_j}, x_m) \in path_{f_j}\}$, set of potential **in/in** /**out/in**/**out/out** intersection conflicts.
- $\alpha_{xyz} := \angle(x,y),(y,z)$ is the angle made by the arcs (x,y) and (y,z) at the junction y, $x,y,x \in X$.
- $S(\alpha) := \frac{1}{\sin\alpha}$ if $\alpha < \pi/2$, 1 otherwise.

Modeling some special cases or generalization

Further modeling details:

- When allow a change in passing order at a node: **binary variables** needed
- Departure times: integer values $\rightarrow$ **general integer variables**
- Climbing arcs: **additional conditions**
- etc.

Outline

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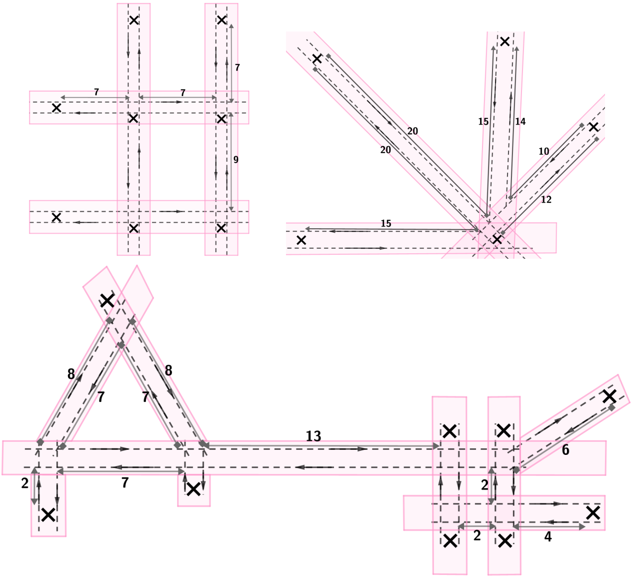
Computational experience



Computational experience

Intel Core i9-9880H CPU, 2.30GHz $\times$ 16, Ubuntu 18.04.4 LTS
CPLEX v. 12.10 (AMPL environment)

- **Scenario 1** (pre-tactical): 20 instances per topology (60 instances)
- **Scenario 2** (priority flight): 20 instances per topology (60 instances)
- **Scenario 3** (intruder): 180 instances ($r \in \{1, 5, 10\}$)

Computational experience: Scenario 1

		Grid	Airport	Metroplex	Overall
cpu (s.)	mean	0.18	0.35	0.29	0.27
	max	0.73	1.65	1.06	1.65
	num. infeasible	0	0	1	1
deviation	total	137.24	225.10	151.82	171.72
	mean (per f_i, x)	0.16	0.31	0.19	0.22
delay at O	% trips	5.35	19.88	7.02	10.81
	total	10.20	13.55	13.26	12.32
	mean (per f_i)	0.14	0.33	0.16	0.21
	max (per f_i)	5.50	2.85	5.68	4.66
delay at D	% trips	5.75	27.37	7.89	13.77
	total	7.56	11.67	10.00	9.74
	mean (per f_i)	0.10	0.28	0.13	0.17
	max (per f_i)	5.26	2.14	5.30	4.22
speed changes	% trips	11.34	33.71	12.56	19.32
	% waypoints	9.97	29.53	11.83	17.20
	max (per f_i)	15.10	18.10	14.32	15.86
% obj. improv.	mean	43.71	51.77	41.81	45.90
	max	75.23	75.23	75.23	75.23

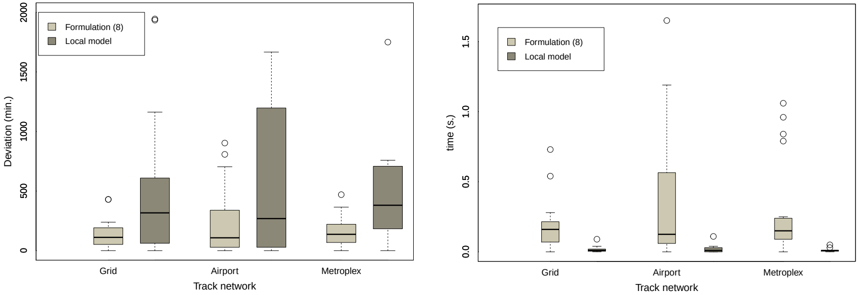
Computational experience: Scenario 2

		Grid	Airport	Metroplex	Overall
cpu (s.)	mean	0.29	0.31	0.36	0.32
	max	0.46	0.47	1.81	1.81
	num. infeasible	0	0	0	0
deviation	total	157.66	763.94	775.54	565.71
	mean (per f_i, x)	0.11	0.93	0.54	0.53
delay prior.	% instances	60	45	60	55
	total	3.95	0.70	2.70	2.45
	max (among instances)	14	5	8	14
delay at O	% trips	9.79	56.40	18.59	28.26
	total	12.55	38.95	66.00	39.17
	mean (per f_i)	0.13	0.89	0.57	0.53
	max (per f_i)	2.00	2.20	1.55	1.92
delay at D	% trips	11.26	56.46	19.60	29.11
	total	7.83	43.24	56.71	35.93
	mean (per f_i)	0.08	0.98	0.49	0.52
	max (per f_i)	1.50	1.92	1.30	1.57
speed changes	% trips	15.13	61.95	20.99	32.69
	% waypoints	12.92	63.77	19.70	32.13
	max (per f_i)	19.30	21.85	8.70	16.62
% obj. improv.	mean	48.15	18.64	7.29	24.70
	max	99.82	99.43	32.73	99.82

Computational experience: Scenario 3

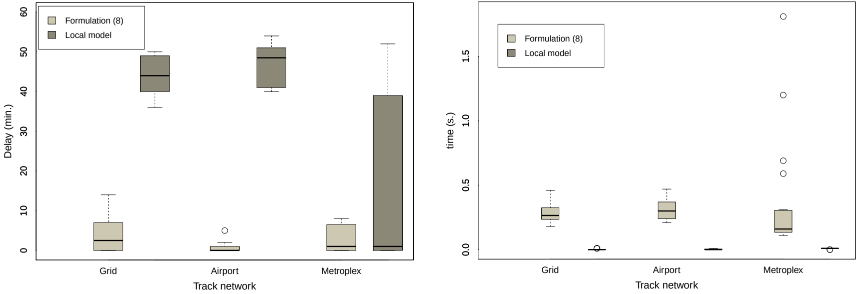
		Grid	Airport	Metroplex	Overall
cpu (s.)	mean	0.60	0.05	0.28	0.17
	max	0.20	0.08	0.37	0.37
	num. infeasible	41	46	41	128
	inf. $r = 1$	16	20	19	55
	inf. $r = 5$	15	16	14	45
	inf. $r = 10$	10	10	8	28
deviation	total	303.83	83.11	1162.59	558.19
	mean (per f_i, x)	0.21	0.14	0.79	0.40
delay at O	% trips	12.75	7.80	44.65	23.08
	total	21.89	4.57	101.68	46.38
	mean (per f_i)	0.22	0.12	0.88	0.44
	max (per f_i)	2.74	1.57	3.63	2.75
delay at D	% trips	17.40	18.75	43.06	27.14
	total	16.57	3.95	77.49	35.43
	mean (per f_i)	0.17	0.10	0.67	0.33
	max (per f_i)	1.94	0.60	3.21	2.04
speed changes	% trips	20.83	40.44	56.63	39.19
	% waypoints	18.09	17.61	47.47	28.69
	max (per f_i)	21.63	20.07	21.58	21.19
% obj. improv.	mean	29.55	49.08	63.31	46.38
	max	44.66	71.69	88.93	88.93
	become feas.	3	2	5	10

Computational experience: global vs. local approach



Comparison of performance against the local model for Scenario 1

Computational experience: global vs. local approach



Comparison of performance against the local model for Scenario 2

Scenario 3: only 3 instances solved over 180 for the local approach!

Outline

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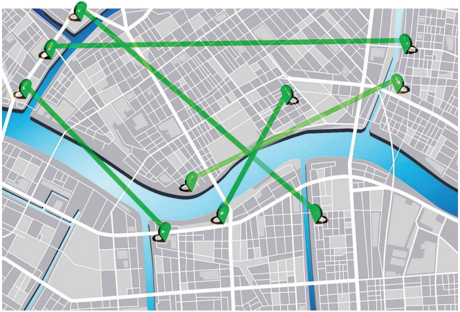
Conclusions and future perspectives

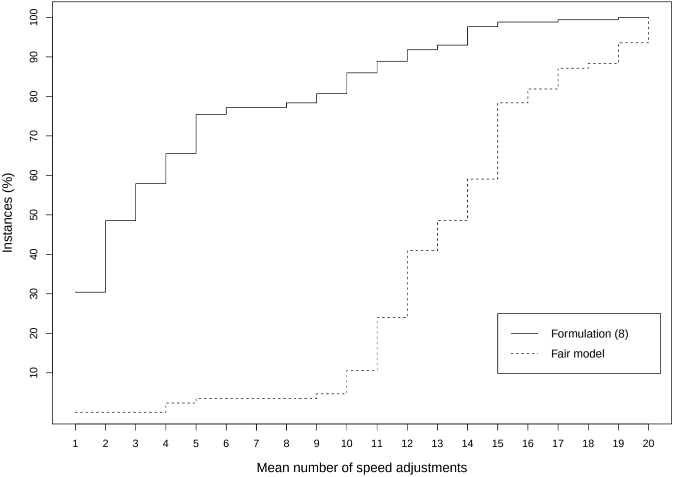


- Definition of a **new problem in UAM**
- Mathematical Optimization **formulation**
- **Promising** computational results
- **Fairness**
- **Ongoing research**:
    - Study robustness approaches at strategic level (with **Tom Portoleau**)

Questions

The cost of fairness

Spread equally the delay among flights



The cost of fairness: Mean number of speed adjustments made per trip

The cost of fairness

Spread equally the delay among flights

		Scenario 1		Scenario 2		Scenario 3	
		non-fair	fair	non-fair	fair	non-fair	fair
fair o.f. value	mean	39.36	23.99	36.72	19.02	38.86	19.99
	max	272.62	255.40	795.50	411.60	210.71	127.70
	% gain	-	35.06	-	39.87	-	45.29
delay at D	mean	9.74	37.62	35.93	65.39	35.43	77.50
	max	49.80	191.76	627.15	710.20	221.63	295.53
	% degrad.	-	478.76	-	430.39	-	397.72
min delay at D	mean	0.00	0.07	0.00	0.02	0.00	0.04
	max	0.00	1.11	0.00	0.42	0.00	0.37
max delay at D	mean	4.22	3.30	1.57	1.58	2.04	2.04
	max	15.52	15.52	8.00	8.00	6.76	6.76

Preliminary results

Instance	Model	MILP			CP		
	#trips	10	100	500	10	100	500
Grid	avg time(s)	<0.1	0.19	0.82	<0.1	<0.1	<0.1
Airport	avg time(s)	<0.1	0.35	0.79	<0.1	<0.1	<0.1
Metroplex	avg time(s)	<0.1	0.29	0.86	<0.1	<0.1	<0.1

Table: Computation time comparison between MILP and CP formulation