Mathematical Optimization for Traffic Management in Urban Air Mobility

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joint work with  
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Chair “Integrated Urban Mobility"

Trends in Computational Discrete Optimization  
ICERM – 24/04/23-28/04/23
Outline

1. Introduction
   - Mathematical Optimization for Air Traffic Management
   - Urban Air Mobility

2. Problem Definition

3. Mathematical Optimization formulation

4. Modeling of aircraft separation

5. Computational experience

6. Conclusions and future perspectives
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Air Traffic Management

- Flights share the same resources, i.e., the airport parking, the runway, the air space, ...

- How to affect the resources so as to guarantee safety?

- **Air space management:**
  - Planning → Strategic Deconfliction
  - Online → Tactical Deconfliction
  - Last minute → Conflict Avoidance
Separation constraint in classic ATM:

$$\forall i < j \in A, t \in T \quad \|x_i(t) - x_j(t)\| \geq D$$

where

- $A$ is the set of aircraft,
- $T$ is the considered time horizon,
- $x_i(t)$ the position in the sky of aircraft $i$ at time $t$,
- $D$ is the safety distance to guarantee between pairs of aircraft.
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Urban Air Mobility – eVTOLs

[Images of eVTOLs in urban and scenic settings]
Urban Air Mobility (UAM)

Electric vertical take-off and landing (eVTOLs) aircraft

Decision-makers tool to guarantee safety and efficiency

Source https://www.objetconnecte.com/
Our focus: UAM tactical deconfliction

Differences w.r.t. classic ATM?
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Problem Definition

Well defined **skylane network**:

- connecting **vertiports**
- virtual **3D corridors**
- corridors can intersect at exclusive **junctions**

Source: FAA
Notation

Network infrastructure:

\[ G = (N, A) \]

- \( N \): nodes set
- \( A \): arcs set (skylanes)

Diagram:

- Vertiports
- Vertiport junctions
- Inner junctions (\( J \subseteq N \))
Notation

Flights and schedule:

\( F: \{f_1, \ldots, f_n\} \)

\( \text{set of flights} \)

\( \text{path}_{f_i} = (x_{i1}, \ldots, x_{ik}) \)

\( (x_h, x_m) \in A \) belong to \( \text{path}_{f_i} \)

if \( x_h \) and \( x_m \) are consecutive nodes in \( \text{path}_{f_i} \)

\( \hat{t}_{im} \) time at which \( f_i \) arrives at/traverses \( x_m \)

\( \nu_{im} \) speed at which \( f_i \) traverses \( x_m \) (constant through the arc)
Considered problem

Given a nominal planning, consider the following uncertainties.

**Disruption**

1. For strategically deconflicted **scheduled traffic**

2. To accommodate a **new, priority, operation**

3. In reaction to some **unexpected traffic** e.g., intruder crossing a UAM corridor.
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<td>2. Problem Definition</td>
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<tr>
<td>3. <strong>Mathematical Optimization formulation</strong></td>
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<tr>
<td>4. Modeling of aircraft separation</td>
</tr>
<tr>
<td>5. Computational experience</td>
</tr>
<tr>
<td>6. Conclusions and future perspectives</td>
</tr>
</tbody>
</table>
Mathematical optimization formulation

- **Degrees of freedom**: speed changes manoeuvres at junctions (implicitly) + departure time reschedule

- **Output**: new schedule of the flights with safe arrival times at the waypoints of their paths

- Nominal and the deconflicted schedules are made of **safe time intervals** at which the flights can traverse each waypoint without incurring in conflicts

- **Minimum width** of the time intervals
Mathematical optimization formulation

Decision Variables:

- \( t_{im}^{ear}, t_{im}^{lat} \) new scheduled times

Safe time interval \([t_{im}^{ear}, t_{im}^{lat}]\) for every \( f_i \in \mathcal{F} \) to traverse each \( x_m \in \text{path}_{f_i} \).

\[
t_{im}^{lat} = t_{im}^{ear} + (\hat{t}_{im}^{lat} - \hat{t}_{im}^{ear})
\]

- \( t_{im}, \bar{t}_{im} \) bounding times

Bounding interval \([t_{im}, \bar{t}_{im}]\) for every \( f_i \in \mathcal{F} \) to traverse each \( x_m \in \text{path}_{f_i} \) without violating speed and/or operating limits.

\[
t_{im} \leq t_{im}^{ear} \leq \bar{t}_{im}
\]
Mathematical optimization formulation

For each arc \((h, m)\), \(t_{im}\) depends on \(t_{ih}^{ear}\), dist\((x_h, x_m)\), and \(v_{im}\)

For each arc \((h, m)\), \(\bar{t}_{im}\) depends on \(t_{ih}^{ear}\), dist\((x_h, x_m)\), and \(\bar{v}_{im}\)
Mathematical optimization formulation

Objective function:

$$\min \sum_{f_i \in F} \sum_{x_m \in \text{path}_{f_i}} |t_{im}^{\text{ear}} - \hat{t}_{im}^{\text{ear}}| + \sum_{f_i \in F^{\text{prior}}} M \cdot (t_{i,x_1}^{\text{ear}} - \hat{t}_{i,x_1}^{\text{ear}})$$

$M$ large number (to prioritize minimization of deviation of priority flights)
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Given a conflict \((i, j, m) \in \text{Conf}\), UAM separation constraints:

\[ |t_{im} - t_{jm}| \geq \text{“safety time lapse”}. \]

Time lapse depends on a **minimum safety distance** \(D\), see classic ATM.

**Two main families of conflicts:**

- Trailing
- Intersection
Trailing conflicts

Two flights \( f_i, f_j \in \mathcal{F} \) travel through the same arc \((x_h, x_m)\) at the same time.

- overtaking is **forbidden**
- hp: **constant speed** on each arc
- sufficient to impose *separation at \( x_h \) and \( x_m \)*
Trailing conflicts: MP formulation

\[ v_{ih}(t_{jh} - t_{ih}) \geq D \quad \forall (i, j, h, m) \in \text{Trail} \]
\[ v_{jm}(t_{jm} - t_{im}) \geq D \quad \forall (i, j, h, m) \in \text{Trail}. \]

• \text{Trail} := \{ (i, j, h, m) : (i, j, m) \in \text{Conf}, (x_h, x_m) \in \text{path}_i \cap \text{path}_j \}, \text{ set of potential trailing conflicts.} \]

Constraints:

\[ t_{ear} - t_{i \text{lat}} \geq \frac{D}{v_{ih}} \quad \forall (i, j, h, m) \in \text{Trail} \]
\[ t_{ear} - t_{i \text{lat}} \geq \frac{D}{v_{jm}} \quad \forall (i, j, h, m) \in \text{Trail} \]
Given a conflict \((i, j, m) \in \text{Conf}\), three types of **intersection conflicts**

\[
\begin{align*}
&\text{In/in} & &\text{Out/in} & &\text{Out/out} \\
&x_{h_i} \quad f_i \quad x_{m} & &x_{\ell_i} \quad f_i \quad x_{m} & &x_{\ell_i} \quad f_i \quad x_{m} \\
&f_j \quad x_{h_j} & &f_j \quad x_{h_j} & &f_j \quad x_{\ell_j}
\end{align*}
\]
Intersection conflicts

- **Safety disks** of radius $D$ around both $f_i$ and $f_j$ along their trajectories.

- **Conflict zones**: when one trajectory is tangent to the safety disk of the other flight.

- **Time separation** of passage through conflict zones.

- Link **ATM strategies**: ensures a minimum distance of $D$ between the flights all their way throughout their paths.

- **Arbitrating rule** (hp. $t_{im} < t_{jm}$):
  
  "$f_j$ does not enter its conflict zone until $f_i$ does not leave its own"
Intersection conflicts

**Case in/in:** \( t_{im} \leq \text{time}(j, \tan_{ijm}^-) \)

where \( \text{time}(i, x) = \) the arrival time of \( f_i \) at the point \( x \).

**Case out/in:** \( \text{time}(i, \tan_{ijm}^-) \leq \text{time}(j, \tan_{ijm}^+) \)

**Case out/out:** \( \text{time}(i, \tan_{ijm}^+) \leq t_{jm} \)
Classic Air Traffic Management

Diagram with a coordinate system and a red curve.
Classic Air Traffic Management

![Diagram of two circles on a grid]

C. D'Ambrosio (CNRS & X)

ATM in UAM

ICERM – 24-28/04/23

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Classic Air Traffic Management
Classic Air Traffic Management
Intersection conflicts: MP formulation

Constraints:

\[ t^{\text{ear}}_{jm} - t^{\text{lat}}_{im} \geq S(\alpha_{ijm}^-) \frac{D}{v_{jm}} \quad \forall (i, j, m, h_i, h_j) \in \text{Cross}^- \]

\[ t^{\text{ear}}_{jm} - t^{\text{lat}}_{im} \geq S(\alpha_{ijm}^{+-}) \left( \frac{D}{v_{jm}} + \frac{D}{v_{i\ell_i}} \right) \quad \forall (i, j, m, \ell_i, h_j) \in \text{Cross}^{+-} \]

\[ t^{\text{ear}}_{jm} - t^{\text{lat}}_{im} \geq S(\alpha_{ijm}^+) \frac{D}{v_{i\ell_i}} \quad \forall (i, j, m, \ell_i, \ell_j) \in \text{Cross}^+ \]

- \text{Cross}^- / \text{Cross}^{+-} / \text{Cross}^+ := \{(i, j, m, h_i, h_j) : (i, j, m) \in \text{Conf}, (x_{h_i}, x_m) \in \text{path}_{f_i}, (x_{h_j}, x_m) \in \text{path}_{f_j}\}, \text{ set of potential in/in/out/in/out/out intersection conflicts.} \\
- \alpha_{xyz} := \angle(x, y), (y, z) \text{ is the angle made by the arcs } (x, y) \text{ and } (y, z) \text{ at the junction } y, x, y, x \in X. \\
- S(\alpha) := \frac{1}{\sin \alpha} \text{ if } \alpha < \pi/2, 1 \text{ otherwise.}
Further modeling details:

- When allow a change in passing order at a node: **binary variables** needed
- Departure times: integer values $\rightarrow$ **general integer variables**
- Climbing arcs: **additional conditions**
- etc.
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Computational experience

Intel Core i9-9880H CPU, 2.30GHz × 16, Ubuntu 18.04.4 LTS
CPLEX v. 12.10 (AMPL environment)

- **Scenario 1** (pre-tactical): 20 instances per topology (60 instances)

- **Scenario 2** (priority flight): 20 instances per topology (60 instances)

- **Scenario 3** (intruder): 180 instances ($r \in \{1, 5, 10\}$)
### Computational experience: Scenario 1

<table>
<thead>
<tr>
<th></th>
<th>Grid</th>
<th>Airport</th>
<th>Metroplex</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>cpu (s.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.18</td>
<td>0.35</td>
<td>0.29</td>
<td>0.27</td>
</tr>
<tr>
<td>max</td>
<td>0.73</td>
<td>1.65</td>
<td>1.06</td>
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<td>1</td>
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<tr>
<td><strong>deviation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>137.24</td>
<td>225.10</td>
<td>151.82</td>
<td>171.72</td>
</tr>
<tr>
<td>mean (per ( f_i ), ( x ))</td>
<td>0.16</td>
<td>0.31</td>
<td>0.19</td>
<td>0.22</td>
</tr>
<tr>
<td><strong>delay at O</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% trips</td>
<td>5.35</td>
<td>19.88</td>
<td>7.02</td>
<td>10.81</td>
</tr>
<tr>
<td>total</td>
<td>10.20</td>
<td>13.55</td>
<td>13.26</td>
<td>12.32</td>
</tr>
<tr>
<td>mean (per ( f_i ))</td>
<td>0.14</td>
<td>0.33</td>
<td>0.16</td>
<td>0.21</td>
</tr>
<tr>
<td>max (per ( f_i ))</td>
<td>5.50</td>
<td>2.85</td>
<td>5.68</td>
<td>4.66</td>
</tr>
<tr>
<td><strong>delay at D</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% trips</td>
<td>5.75</td>
<td>27.37</td>
<td>7.89</td>
<td>13.77</td>
</tr>
<tr>
<td>total</td>
<td>7.56</td>
<td>11.67</td>
<td>10.00</td>
<td>9.74</td>
</tr>
<tr>
<td>mean (per ( f_i ))</td>
<td>0.10</td>
<td>0.28</td>
<td>0.13</td>
<td>0.17</td>
</tr>
<tr>
<td>max (per ( f_i ))</td>
<td>5.26</td>
<td>2.14</td>
<td>5.30</td>
<td>4.22</td>
</tr>
<tr>
<td><strong>speed changes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% trips</td>
<td>11.34</td>
<td>33.71</td>
<td>12.56</td>
<td>19.32</td>
</tr>
<tr>
<td>% waypoints</td>
<td>9.97</td>
<td>29.53</td>
<td>11.83</td>
<td>17.20</td>
</tr>
<tr>
<td>max (per ( f_i ))</td>
<td>15.10</td>
<td>18.10</td>
<td>14.32</td>
<td>15.86</td>
</tr>
<tr>
<td><strong>% obj. improv.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>mean</td>
<td>43.71</td>
<td>51.77</td>
<td>41.81</td>
<td>45.90</td>
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<tr>
<td>max</td>
<td>75.23</td>
<td>75.23</td>
<td>75.23</td>
<td>75.23</td>
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</tbody>
</table>
## Computational experience: Scenario 2

<table>
<thead>
<tr>
<th></th>
<th>Grid</th>
<th>Airport</th>
<th>Metroplex</th>
<th>Overall</th>
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</thead>
<tbody>
<tr>
<td><strong>cpu (s.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.29</td>
<td>0.31</td>
<td>0.36</td>
<td>0.32</td>
</tr>
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<td>0</td>
</tr>
<tr>
<td>deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>157.66</td>
<td>763.94</td>
<td>775.54</td>
<td>565.71</td>
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<td>mean (per $f_i$, $x$)</td>
<td>0.11</td>
<td>0.93</td>
<td>0.54</td>
<td>0.53</td>
</tr>
<tr>
<td>% instances</td>
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<td>45</td>
<td>60</td>
<td>55</td>
</tr>
<tr>
<td>total</td>
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<td>0.70</td>
<td>2.70</td>
<td>2.45</td>
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<td>8</td>
<td>14</td>
</tr>
<tr>
<td><strong>delay prior.</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% trips</td>
<td>9.79</td>
<td>56.40</td>
<td>18.59</td>
<td>28.26</td>
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<tr>
<td>total</td>
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<tr>
<td>mean (per $f_i$)</td>
<td>0.13</td>
<td>0.89</td>
<td>0.57</td>
<td>0.53</td>
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<tr>
<td>max (per $f_i$)</td>
<td>2.00</td>
<td>2.20</td>
<td>1.55</td>
<td>1.92</td>
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<tr>
<td><strong>delay at O</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% trips</td>
<td>11.26</td>
<td>56.46</td>
<td>19.60</td>
<td>29.11</td>
</tr>
<tr>
<td>total</td>
<td>7.83</td>
<td>43.24</td>
<td>56.71</td>
<td>35.93</td>
</tr>
<tr>
<td>mean (per $f_i$)</td>
<td>0.08</td>
<td>0.98</td>
<td>0.49</td>
<td>0.52</td>
</tr>
<tr>
<td>max (per $f_i$)</td>
<td>1.50</td>
<td>1.92</td>
<td>1.30</td>
<td>1.57</td>
</tr>
<tr>
<td><strong>delay at D</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% trips</td>
<td>15.13</td>
<td>61.95</td>
<td>20.99</td>
<td>32.69</td>
</tr>
<tr>
<td>% waypoints</td>
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<td>63.77</td>
<td>19.70</td>
<td>32.13</td>
</tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% obj. improv.</td>
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<td></td>
<td></td>
<td></td>
</tr>
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<tr>
<td>max</td>
<td>99.82</td>
<td>99.43</td>
<td>32.73</td>
<td>99.82</td>
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</table>
## Computational experience: Scenario 3

<table>
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<tr>
<th></th>
<th>Grid</th>
<th>Airport</th>
<th>Metroplex</th>
<th>Overall</th>
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</thead>
<tbody>
<tr>
<td><strong>CPU (s.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.60</td>
<td>0.05</td>
<td>0.28</td>
<td>0.17</td>
</tr>
<tr>
<td>max</td>
<td>0.20</td>
<td>0.08</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>num. infeasible</td>
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<td>46</td>
<td>41</td>
<td>128</td>
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<td>inf. $r = 1$</td>
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<td>20</td>
<td>19</td>
<td>55</td>
</tr>
<tr>
<td>inf. $r = 5$</td>
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<td>16</td>
<td>14</td>
<td>45</td>
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<td>inf. $r = 10$</td>
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<td>10</td>
<td>8</td>
<td>28</td>
</tr>
<tr>
<td><strong>Deviation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>303.83</td>
<td>83.11</td>
<td>1162.59</td>
<td>558.19</td>
</tr>
<tr>
<td>mean (per $f_i, x$)</td>
<td>0.21</td>
<td>0.14</td>
<td>0.79</td>
<td>0.40</td>
</tr>
<tr>
<td><strong>Delay at O</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% trips</td>
<td>12.75</td>
<td>7.80</td>
<td>44.65</td>
<td>23.08</td>
</tr>
<tr>
<td>total</td>
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C. D'Ambrosio (CNRS & X)
Comparison of performance against the local model for Scenario 1
Computational experience: global vs. local approach

Comparison of performance against the local model for Scenario 2

Scenario 3: only 3 instances solved over 180 for the local approach!
Outline

1. Introduction
   - Mathematical Optimization for Air Traffic Management
   - Urban Air Mobility

2. Problem Definition

3. Mathematical Optimization formulation

4. Modeling of aircraft separation

5. Computational experience

6. Conclusions and future perspectives
1 Introduction
   • Mathematical Optimization for Air Traffic Management
   • Urban Air Mobility

2 Problem Definition

3 Mathematical Optimization formulation

4 Modeling of aircraft separation

5 Computational experience

6 Conclusions and future perspectives
Conclusions and future perspectives

- Definition of a new problem in UAM
- Mathematical Optimization formulation
- Promising computational results
- Fairness

Ongoing research:
- Study robustness approaches at strategic level (with Tom Portoleau)
Questions
The cost of fairness

Spread equally the delay among flights

The cost of fairness: Mean number of speed adjustments made per trip
The cost of fairness

Spread equally the delay among flights

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## Preliminary results

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**Table**: Computation time comparison between MILP and CP formulation