Combinatorics and Optimization
Poster Session Abstracts
Tuesday, March 28, 2023

The Polyhedral Combinatorics of Pivot Rules

Alexander Black, UC Davis

The existence of a pivot rule for the simplex method that guarantees a polynomial run-time is a fundamental open problems in the theory of linear programming. A major challenge for this task is the sheer diversity of potential pivot rules. In this work, we parametrize the space of all possible pivot rules for the simplex method by introducing a fully geometric interpretation of a pivot rule. From this interpretation, we model the behavior of a pivot rule on a fixed linear program or fixed polytope using two polytopal constructions that we call the pivot rule polytope and neighbotope respectively. Using these constructions, we provide new interpretations of polytopes in algebraic combinatorics such as flag matroid polytopes, permutahedra, associahedra, and multiplihedra all grounded in a generalization of the shadow-vertex pivot rule that we introduce called the max-slope pivot rule.

This is based on joint work with Jesús De Loera, Niklas Lütjeharms, and Raman Sanyal.

Quasi-kernels in directed graphs

Hélène Langlois, CERMICS

In a digraph, a quasi-kernel is a subset of vertices that is independent and such that every vertex can reach some vertex in that set via a directed path of length at most two. Whereas Chvátal and Lovász proved in 1974 that every digraph has a quasi-kernel, very little is known so far about the complexity of finding small quasi-kernels. In 1976 Erdös and Székely conjectured that every sink-free digraph D = (V, A) has a quasi-kernel of size at most |V|/2. Obviously, if D has two disjoint quasi-kernels then it has a quasi-kernel of size at most |V|/2, and in 2001 Gutin, Koh, Tay and Yeo conjectured that every sink-free digraph has two disjoint quasi-kernels. Yet, they constructed in 2004 a counterexample, thereby disproving this stronger conjecture. We show that, not only sink-free digraphs occasionally fail to contain two disjoint quasi-kernels, but it is computationally hard to distinguish those that do from those that do not. We also prove that the problem of computing a small quasi-kernel is polynomial time solvable for orientations of trees but is computationally hard in most other cases (and in particular for restricted acyclic digraphs).

Simple and Optimal Greedy Online Contention Resolution Schemes

Vasilis Livanos, UIUC

Matching based markets, like ad auctions, ride-sharing, and eBay, are inherently online and combinatorial, and therefore have been extensively studied under the lens of online

stochastic combinatorial optimization models. The general framework that has emerged uses Contention Resolution Schemes (CRSs) introduced by Chekuri, Vondrák, and Zenklusen for combinatorial problems, where one first obtains a fractional solution to a (continuous) relaxation of the objective, and then proceeds to round it. When the order of rounding is controlled by an adversary, it is called an Online Contention Resolution Scheme (OCRSs), which has been successfully applied in online settings such as posted-price mechanisms, prophet inequalities and stochastic probing.The study of greedy OCRSs against an almighty adversary has emerged as one of the most interesting problems since it gives a simple-to-implement scheme against the worst possible scenario. Intuitively, a greedy OCRS has to make all its decisions before the online process starts. We present simple 1/e-selectable greedy OCRSs for the single-item setting, partition matroids, and transversal matroids. This improves upon the previous state-of-the-art greedy OCRSs of [FSZ16] that achieves 1/4 for these constraints. Finally, we show that no better competitive ratio than 1/e is possible, making our greedy OCRSs the best possible.

Half integral polytopes and the Hirsch conjecture
Moira MacNeil, University of Toronto

Finding a good bound on the maximal edge-diameter of a polytope in terms of its dimension d and the number of its facets n is not only a natural question of discrete geometry, but also historically closely connected with the theory of the simplex method, as the diameter is a lower bound for the number of pivots required in the worst case. Hirsch conjectured in 1953 that the diameter of a bounded polytope is at most n-d. Santos provided in counterexample in 2012.

We investigate this question considering bounded polytopes whose vertices are integer-valued. A lattice (d,k)-polytope is the convex hull of a set of points in dimension d whose coordinates are integers between 0 and k. Let $\delta(d,k)$ denote the largest possible diameter of a lattice (d,k)-polytope. Naddef showed in 1989 that $\delta(d,1)=d$ and that lattice (d,1)-polytopes satisfy Hirsch conjecture. Kleinschmidt and Onn generalized this result in 1992 showing that $\delta(d, k) \leq kd$. In 2016, Del Pia and Michini strengthened the upper bound to $\delta(d,k) \leq kd-\lceil d/2\rceil$ for $k \geq 2$ before Deza and Pournin further strengthened the upper bound to $\delta(d,k) \leq kd-\lceil 2d/3\rceil-(k-2)$ for $k \geq 4$. Del Pia and Michini showed that $\delta(d,2)=\lfloor 3d/2\rfloor$and asked whether half-integral polytopes, or equivalently lattice (d,2)-polytopes satisfy the Hirsch conjecture. The first dimension for which the answer to the question of Del Pia and Michini is not known is dimension 6.

We investigate the diameter of lattice (6,2)-polytopes and show that the search space can be reduced to the computation of the largest diameter of lattice (4,2)-polytopes having 9, 10 or 11 facets. A novel computational framework is introduced to tackle these 3 previously intractable instances.

The Tropical and Zonotopal Geometry of Periodic Timetables

Berenike Masing and Enrico Bortoletto; Zuse Institute Berlin

The Periodic Event Scheduling Problem (PESP) is the standard mathematical tool for optimizing periodic timetabling problems in public transport. A solution to PESP consists of three parts: a periodic timetable, a periodic tension, and integer periodic offset values.
In our work we provide new geometric perspectives on them, using tools from tropical geometry and zonotope theory. We show that the space of timetables can be interpreted as a finite collection of specially arranged polytropes on a torus, and derive a tropical neighbourhood search. Furthermore, we study the linear relaxation of cycle offsets, recognise its zonotopal structure, and establish duality relations between its tiles and the polytropes in the torus.

Aggregations of quadratic inequalities and hidden hyperplane convexity

Shengding Sun, Georgia Tech

We study properties of the convex hull of a set S described by quadratic inequalities. A simple way of generating inequalities valid on S is to to take nonnegative linear combinations of the defining inequalities of S. We call such inequalities \emph{aggregations}. Special aggregations naturally contain the convex hull of S, and we give sufficient conditions for intersection of such aggregations to define the convex hull. We introduce the notion of hidden hyperplane convexity (HHC), which is related to the classical notion of hidden convexity of quadratic maps. We show that if the quadratic map associated with S satisfies HHC, then the convex hull of S is defined by special aggregations.
To the best of our knowledge, this result generalizes all known results regarding aggregations defining convex hulls. Using this sufficient condition, we are able to recognize previously unknown classes of sets where aggregations lead to convex hull. We show that the condition known as positive definite linear combination for every triple of inequalities, together with hidden hyperplane convexity is sufficient for finitely many aggregations to define the convex hull, answering a question raised by Dey, Munoz and Serrano. All the above results are for sets defined using open quadratic inequalities. For closed quadratic inequalities, we prove a new result regarding aggregations giving the convex hull, without topological assumptions on S.

The Number of Differing Columns of a Δ-modular Matrix

Luze Xu, University of California, Davis

We study integer-valued matrices with bounded determinants. Such matrices appear in the theory of integer programs (IP) with bounded determinants, where it is known that determinants in the constraint matrix are one measure of complexity. One of the first works to quantify the complexity of IPs with bounded determinants was that of Heller, who identified the maximum number of differing columns in a totally unimodular matrix. We provide two column bounds generalizing Heller's bound that are polynomial in both the

determinants or the number of equations. Furthermore, we show a tight bound on the number of differing columns in a bimodular matrix; this is the first tight bound since Heller. Our analysis reveals combinatorial properties of bimodular IPs that may be of independent interest.

Approximating TSP walks in subcubic graphs
Youngho Yoo, Texas A&M University

The Graphic Travelling Salesman Problem is the problem of finding a spanning closed walk (a TSP walk) of minimum length in a given connected graph. The special case of the Graphic TSP on subcubic graphs has been studied extensively due to their worst-case behaviour in the famous $\frac{4}{3}$-integrality-gap conjecture on the ""subtour elimination"" linear programming relaxation of the Metric TSP.

We prove that every simple 2-connected subcubic graph on n vertices with n_2 vertices of degree 2 has a TSP walk of length at most $\frac{5n+n_2}{4}-1$, confirming a conjecture of Dvo\v r\'ak, Kr{\'a}l', and Mohar. This bound is best possible and we characterize the extremal subcubic examples meeting this bound. We also give a quadratic time combinatorial algorithm to find such a TSP walk. In particular, we obtain a $\frac{5}{4}$-approximation algorithm for the Graphic TSP on cubic graphs.

Joint work with Michael Wigal and Xingxing Yu.

The Base Augmentation Problem
Michael Zlatin, Carnegie Mellon University

We define a natural generalization of the Tree Augmentation Problem to matroids. We are given a matroid M=(E,I) a base B, and costs on elements of E\B. The goal is to select a minimum cost subset of elements F \subset (E\B) so that the set (B \cup F) is full rank after any element is deleted. Note that the classic Tree Augmentation Problem is the case where M is a graphic matroid.

We characterize the approximability of the Base Augmentation Problem for some natural matroid classes including graphic, co-graphic, transversal, binary, and laminar matroids, and conjecture an approximation ratio of 2 for regular matroids.