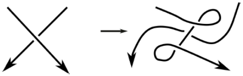
An Unknotting Number for Transverse Knots

Lisa Traynor

Bryn Mawr College



April 2022

Motivation/Goals

1. What are some projects for undergraduates in contact/symplectic topology?

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1. What are some projects for undergraduates in contact/symplectic topology?

2. Today discuss a student project on transverse knots.

Braids = Transverse Knots

Braids have close connections to tranverse knots!

Today: Results from Senior Honors Thesis work of
Blossom Jeong, BMC 2020

Comparative Literature + Math Double Major

Unknotting Number for Smooth Knots

Given a smooth knot K, the **unknotting number** $u(K)$ measures the minimal number of times that a knot must cross through itself in order to become an unknot

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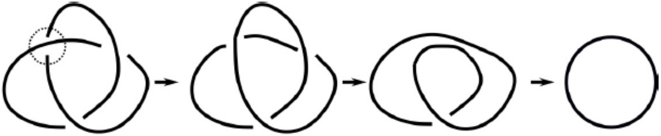

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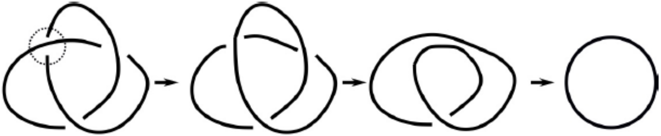
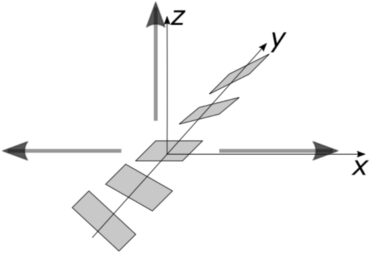


$u(K) = n \implies \exists$ projection of K such that
changing n crossings turns projection into projection of unknot

Contact World

Standard Contact Structure:



$$\xi_{std} = \ker\alpha, \qquad \alpha = dz - y\,dx$$
$$= \langle \vec{j}, \vec{i} + y\vec{k} \rangle$$

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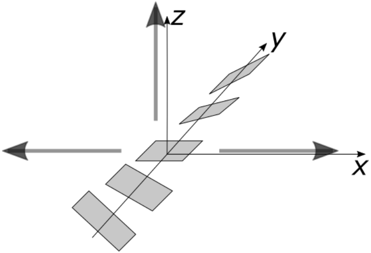

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Natural Curves:

- **Legendrian Curves**: $\Lambda(t)$ s.t. $\alpha\left(\frac{d}{dt}\Lambda(t)\right) = 0$

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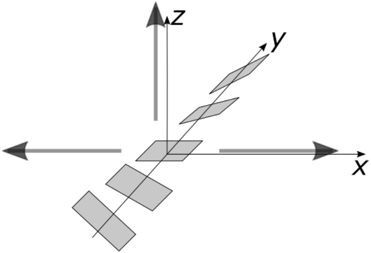

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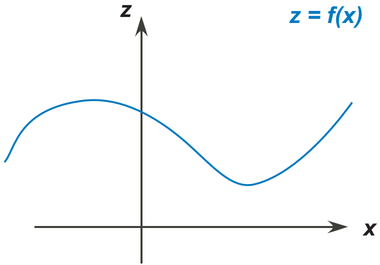
Natural Curves:

- **Legendrian Curves**: $\Lambda(t)$ s.t. $\alpha\left(\frac{d}{dt}\Lambda(t)\right) = 0$
- **Transverse Curves**: $T(t)$ s.t. $\alpha\left(\frac{d}{dt}T(t)\right) > 0$

Constructing Legendrian Curves

There are many smooth curves in 3-dimensional space that project to this curve:



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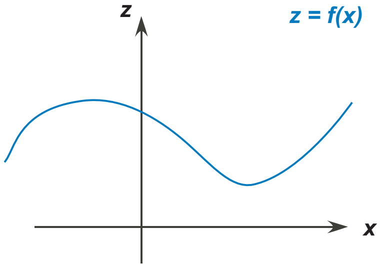

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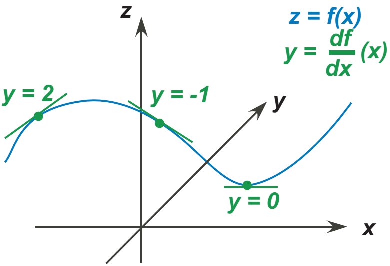

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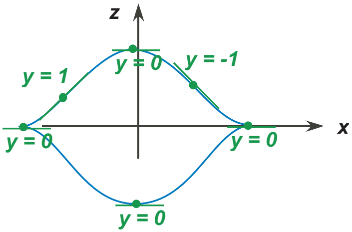
Constructing Legendrian Knots

Front Projection:



Cusped curve in the plane (without vertical tangents) can be lifted to 3-space using slope as the third coordinate: $dz - y dx = 0 \implies y = dz/dx$.

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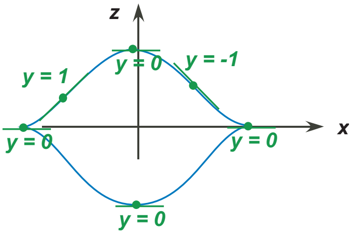

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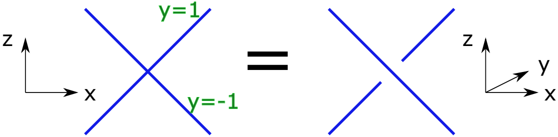
Projection crossing resolved by slope:



Constructing Transverse Curves

Transverse curves are more flexible:

$T(t) = (x(t), y(t), z(t))$

$$z'(t) - y(t)x'(t) > 0 \implies z'(t) > y(t)x'(t)$$

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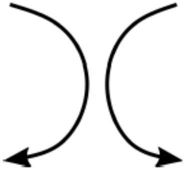
- There are **no downward vertical tangencies**:



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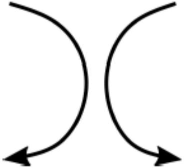
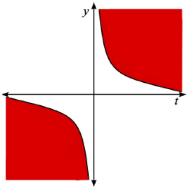


- There are **no +-down-down crossings:**

Forbidding Downward Vertical Tangencies



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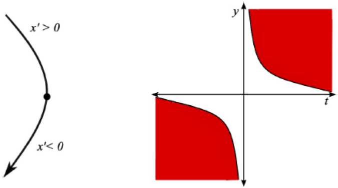

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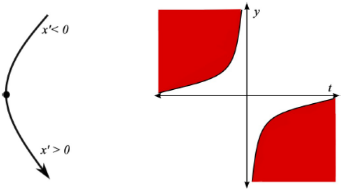
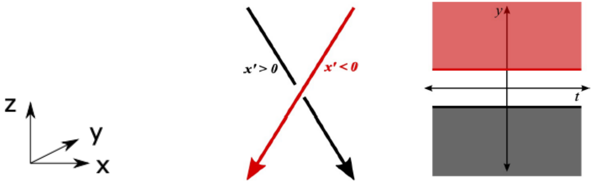


Forbidding +-Down-Down Crossings



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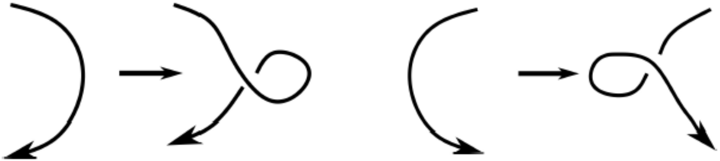

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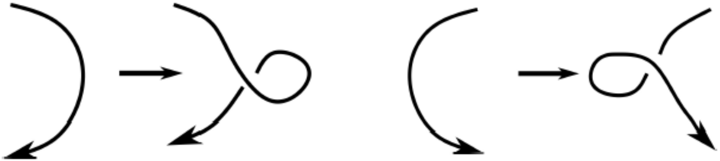

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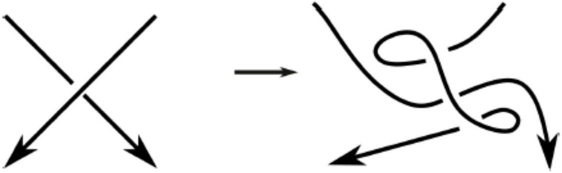


Moving Transverse Knots

Transverse Reidemeister Moves: no +-down-down-crossings:

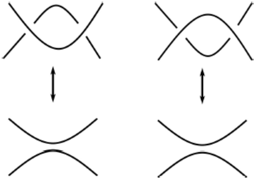


FIGURE 24. Transverse Reidemeister II moves.

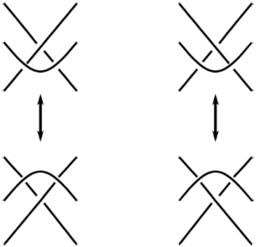
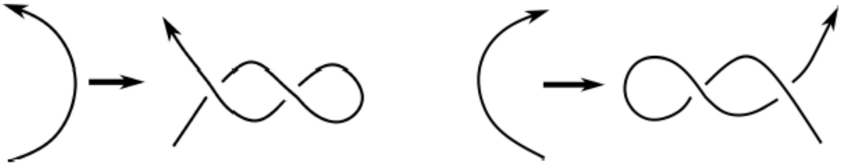


FIGURE 25. Transverse Reidemeister III moves.

Stabilization

Transverse Modification:



Stabilization Operation

Stabilization

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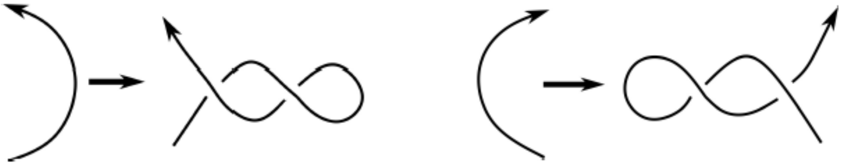

Stabilization Operation

Classical Transverse Invariant: **Self-Linking Number**

$$sl(T) = \text{ writhe of } xz\text{-projection}$$

Stabilization lowers sl by 2.

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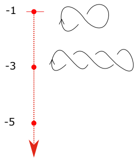
Example: Ray of transverse unknots



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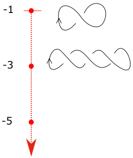

Unknot is **transversely simple**:
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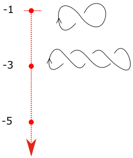

Unknot is **transversely simple**:
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$\exists$ non-transversely simple, e.g. $(2,3)$-cable of $(2,3)$-torus knot [Etnyre-Honda]

Unknotting in the Contact World

Legendrian Unknotting: ??

exists

never exists

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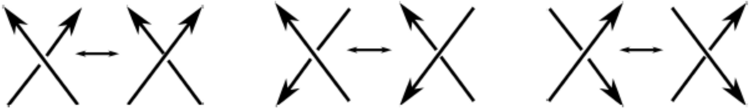
Transverse Unknotting: Use the following crossing changes



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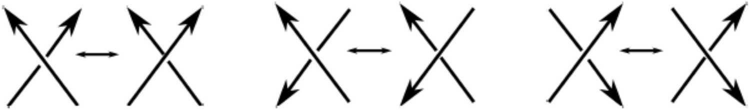

Transverse Crossing Changes

Rigid Crossing: $-$-down-down crossing

Transverse crossing changes gives us a path
in the space of immersed transverse curves.

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- $|sl(T') - sl(T)| = 2$.

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Q: Can every transverse knot be "unknotted" by allowable crossing changes?

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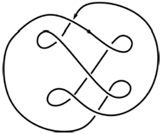

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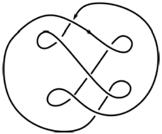

All rigid crossings!

After transverse isotopy, can we perform transverse crossing changes to convert to a transverse unknot?

Unknottable Transverse Representatives

A start to answering this:

Claim: Every smooth knot type has a transverse representative that can be converted to a transverse unknot by passing through self intersections.

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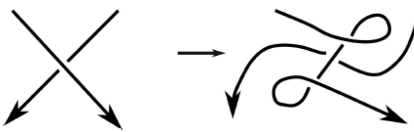
- Start with any transverse representative.
- Get another transverse representative after removing rigid crossings:



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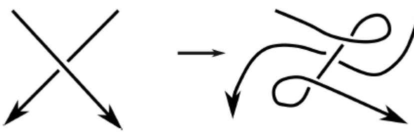

- Standard topological argument shows that crossing changes can be performed to get to an unknot.

Smooth Unknot vs. Transverse Unknots

Smooth World: there is a unique unknot.

Contact World: there are an infinite number of transverse unknots.

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Moving between transverse unknots

Claim: We can move between any two transverse unknots by transverse crossing changes.

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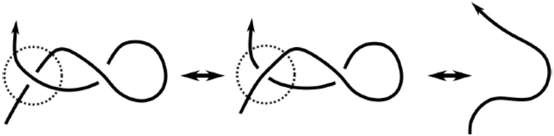
- All transverse unknots are related by stabilization.
- Stabilization/destabilization can be done by allowable crossing changes:



Moving between transverse representatives of fixed knot type

Theorem: If T, T' are transversal representatives of the same knot type $\mathcal{K}$, then can move between T and T' by transversal crossing changes in such a way that all transversal knots along the way are in $\mathcal{K}$.

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- After stabilization T and T' are transversely isotopic. [Fuchs-Tabachnikov]
- Stabilize via transverse crossing changes, transverse isotopy, destabilize via transverse crossing changes.

All Transverse Knots can be Unknotted

Corollary: For all knot types $\mathcal{K}$, any transverse representative T of $\mathcal{K}$ can be converted to a (and thus any) transverse unknot by transverse crossing changes.

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Corollary: For all knot types $\mathcal{K}$, any transverse representative T of $\mathcal{K}$ can be converted to a (and thus any) transverse unknot by transverse crossing changes.

- Earlier saw that there exists a representation T' of $\mathcal{K}$ with a projection that can be unknotted.
- By previous result, we can move from T to T' via transverse crossing changes.

Transverse Unknotting number

Definition: A transverse knot T has **transverse unknotting number** n if there exists a front diagram of T such that transversely changing n crossings in the diagram turns T into the transverse unknot U with $sl(U) = -1$, and there is no diagram of T such that fewer crossing changes would have produced the transverse unknot with self-linking number -1.

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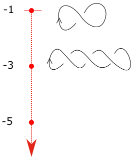

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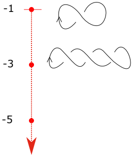

Notation: $U^{\pitchfork}_{-1}(T)$:

World of immersed transverse knots: the transverse unknotting number is the minimum number of times the knot must pass through itself in order to become the transverse unknot with self-linking number -1.

Lower Bound for Transversal Unknotting Number

Get upper bounds to $U^{\pitchfork}_{-1}(T)$ by constructions.

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Lower bound:

Lemma: Suppose T is a transverse knot in the smooth knot type $\mathcal{K}$. Then

$$\max\left\{u(\mathcal{K}), \left|\frac{sl(T)+1}{2}\right|\right\} \leq U^{\pitchfork}_{-1}(T),$$

where $u(\mathcal{K})$ is the smooth unknotting number.

Sample Calculation

Example: $\mathcal{K} = (2, -5)$-torus knot: transversely simple with $\overline{sl}(\mathcal{K}) = -7$.

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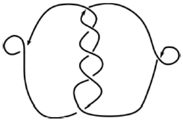
Max *sl* representative T: 

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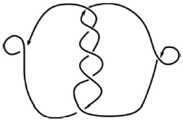

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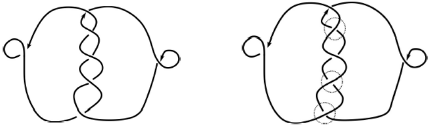

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For any transverse representative T' of $\mathcal{K}$,

$$u^{\pitchfork}_{-1}(T') = \left|\frac{sl(T')+1}{2}\right|.$$

Further Questions

Have established an unknotting number for transverse knots.

Q: What are some other questions related to unknotting in the transverse world?

Ancestor-Descendant Relationship

Knot Fertility and Lineage, Cantarella, Henrich, Magness, O'Keefe, Perez, Rawdon, and Zimmer, Journal of Knot Theory and Its Ramifications (2017).

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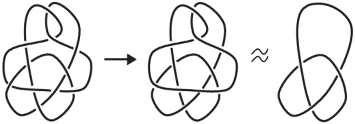

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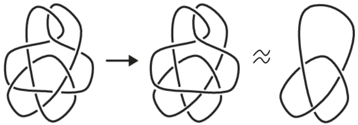
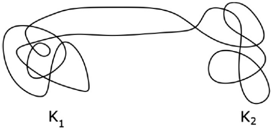


Examples:
11_a135 is an ancestor of the trefoil;

7_6 is fertile.

Properties of Ancestor-Descendant Relationship

- If don't require minimum crossings in diagram of ancestor, all knots are related by ancestor-descendant.



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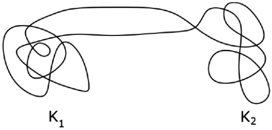

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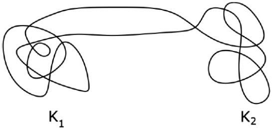

- Every knot is an ancestor of the unknot.

- The unknot is not an ancestor of any non-trivial knot.

Knot Lineage

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2. Hanaki [2020]: There are no alternating fertile knots with crossing number > 7.

3. Ito [2021]: A knot whose minimum crossing number c(K) is even and greater than 30 is not fertile.

New Questions

Q: What is the contact world analogue of this Ancestor-Descendant Relationship?

Transverse Family Trees

Defn: A sequence of transverse knots $(T_1, T_2, \ldots, T_n)$ is a **transverse family tree** if each T_{i+1} can be obtained from T_i by a single transverse crossing change.

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- **maximal** if each T_i has maximal self-linking number in its knot type.
- **increasing (decreasing)** if the self-linking numbers of T_i are strictly increasing (decreasing).

Properties of Transverse Family Trees

Given any smooth knot types K_1, K_2, there exists a transverse family tree $(T_1, \ldots, T_2)$ where T_1 is in the knot type of K_1, T_2 is in the knot type of K_2.

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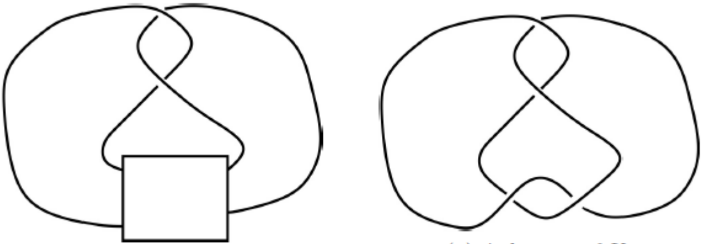
Q: What are examples of maximal transverse family trees?

Twist Family Trees



(A) A diagram of K_m. (B) A diagram of K_2

FIGURE 34. Twist knots.

Theorem 7.4 (Twist Knot Transverse Family Trees).

(1) *For any odd $m \geq 1$, there exists a maximal decreasing transverse family tree $(T_m, T_{m-2}, \ldots, T_1)$, where T_j is a transverse representative of the twist knot K_j.*

(2) *For any even $m \geq 2$, there exists a maximal decreasing transverse family tree $(T_m, T_{m-2}, \ldots, T_2)$, where T_j is a transverse representative of the twist knot K_j.*

(3) *There exists a transverse family tree $(T_m, U, T_{m-1}, U, \ldots, T_1)$, where T_j is a transverse representative of the twist knot K_j and U is a transverse unknot.*

Torus Knot Family Trees

Theorem 7.6 ((2, p)-Torus Knot Transverse Family Trees).

(1) *For all odd* $p \geq 3$*, there exists a maximal decreasing transverse family tree* $(T_{2,p}, T_{2,p-2}, \ldots, T_{2,3})$*, where* $T_{2,j}$ *is a transverse representative of the torus knot* $K_{2,j}$*.*

(2) *For all odd* $n \leq -3$*, there exists a maximal increasing transverse family tree* $(T_{2,n}, T_{2,n+2}, \ldots, T_{2,-3})$*, where* $T_{2,j}$ *is a transverse representative of torus knot* $K_{2,j}$*.*

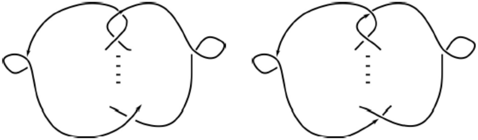


FIGURE 42. A transverse representative of a torus knot $K_{2,p}$ where p is odd, (a) when p is positive and (b) when p is negative. We will call each knot T_+ and T_- respectively.

Project Evaluation:

- Very accessible with knot theory background.

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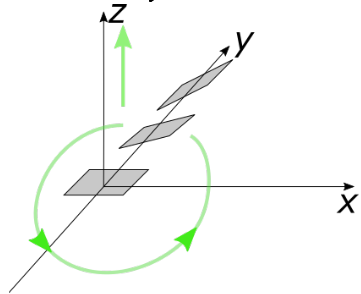
Final Comments:

Transverse Knots and Braids Connections

Transverse Knots from Braid Perspective

Standard Contact Structure: Symmetric Version



$$\xi_{sym} = \ker \alpha = \ker(dz + r^2 d\theta)$$

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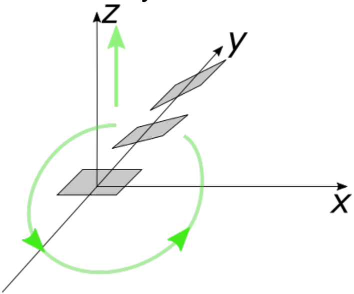

$$\xi_{sym} = \ker \alpha = \ker(dz + r^2 d\theta)$$

Any smooth closed braid B can be isotoped (through braids) to be a transverse knot.

Knots = Braids

Theorem: [Alexander, 1925] Any knot is isotopic to a closed braid.

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Theorem: [Markov, 1935] Two closed braid representatives X_-, X_+ of the same oriented knot type χ are related by a sequence of closed braid representatives of χ:

$$X_- = X_1 \to X_2 \to \cdots \to X_r = X_+$$

such that, up to braid isotopy, X_{i+1} is obtained from X_i by a single stabilization or destabilization.

Braids = Transverse Knots

Theorem: [Bennequin, 1983] Any transverse knot in $(\mathbb{R}^3, \xi_{sym})$ is transversely isotopic to a closed braid.

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If $T = \hat{\beta}$, can easily calculate sl from braid presentation:

$$sl(\hat{\beta}) = a(\beta) - n,$$

where n = braid index, $a(\beta)$ = algebraic crossing number of β.

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$$TX_- = TX_1 \to TX_2 \to \dots TX_r = TX_+$$

such that, up to braid isotopy, TX_{i+1} is obtained from TX_i by a single **positive** stabilization or destabilization.

Conclusion

There are lots of interesting topological problems that can be "contactified" to make interesting projects for undergraduate students.

Examples:

- Transversal Unknotting and lineage (today)
- Legendrian versions of multicrossing knots
- Constructions of "decomposable" Lagrangian fillings and cobordisms

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Thank you!