Locally dissipative solutions of the Euler equations

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Incompressible Euler and Navier-Stokes

$u : \mathbb{T}^3 \times [0, T] \to \mathbb{R}^3$
$p : \mathbb{T}^3 \times [0, T] \to \mathbb{R}$

$$\begin{cases} \partial_t u + \operatorname{div}(u \otimes u) + \nabla p = \varepsilon \Delta u \\ \operatorname{div} u = 0 \end{cases}$$

$\varepsilon > 0$ Navier-Stokes
$\varepsilon = 0$ Euler

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Energy conservation

For classical (i.e. regular enough) solutions

$$\partial_t \frac{|u|^2}{2} + \operatorname{div}\left(\left(\frac{|u|^2}{2} + p\right) u\right) = \varepsilon \left(\Delta \frac{|u|^2}{2} - |Du|^2\right) .$$

After integration in time

$$\frac{d}{dt} \frac{1}{2} \int |u|^2(x,t)\, dx = -\varepsilon \int |Du|^2(x,t)\, dx$$

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Energy for weak solutions

Question: Do weak solutions satisfy the energy identity? (Do the laws of conservation and momentum of continuum mechanics *for each fluid element* imply the law of conservation of energy?)

Scheffer 1993: No for Euler and even in 2 dimensions

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Admissible/dissipative solutions

The solutions constructed by Scheffer and Buckmaster-Vicol violate uniqueness and can have increasing energy.

A more reasonable notion of solution is a solution "à la Leray", i.e. a weak solution such that, in addition,

$$\frac{d}{dt}\frac{1}{2}\int |u|^2(x,t)\,dx \leq -\varepsilon \int |Du|^2(x,t)\,dx\,.$$

Energy space: $L^\infty_t L^2_x$ for Euler, $L^\infty_t L^2_x \cap L^2_t W^{1,2}_x$ for Navier-Stokes.

The Scheffer and Buckmaster-Vicol solutions do not belong to the energy space either.

Question: Does Leray's condition (or even just membership in the energy space) make solutions more reasonable?

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Completely open for Navier Stokes and for a good reason.

Since Leray solutions are regular on an open dense set of times, a negative answer is equivalent to the development of a singularity in finite time, starting from a smooth initial data (i.e. a negative answer to the Millennium Prize question).

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Admissible/dissipative solutions III

The negative result for Euler is “less surprising” because there is no compactness of (classical) solutions in the energy space.
D-Székelyhidi 2007 implements a construction technique known as “convex integration”.

A tenet in the literature: convex integration holds in absence of compactness. This tenet is incorrect

In a suitable intermediate regime between Euler and Navier-Stokes convex integration and compactness coexist.

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Consider the generalized Navier-Stokes equations with a “fractional dissipation”:

$$\begin{cases} \partial_t u + \operatorname{div}(u \otimes u) + \nabla p = -(-\Delta)^{\alpha} u \\ \operatorname{div} u = 0 \end{cases}$$

for $\alpha > 0$.

Leray solutions exist, are compact, and satisfy the weak-strong uniqueness for every $\alpha > 0$.

On the other hand

Theorem (Colombo-D-De Rosa 2017, De Rosa 2018)

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Scheffer 1977, Caffarelli-Kohn-Nirenberg 1982, suitable weak solutions are required to satisfy

$$\partial_t \frac{|u|^2}{2} + \mathrm{div}\left(\left(\frac{|u|^2}{2} + p\right) u\right) + \varepsilon |Du|^2 \leq \varepsilon \Delta \frac{|u|^2}{2}$$

distributionally.

Solutions of NS in the energy space satisfy $u \in L^{10/3}$ and $p \in L^{5/3}$ and thus $(|u|^2 + 2p)u$ is well defined. For Euler we impose in addition $u \in L^3$. The suitability condition for Euler is proposed by Duchon and Robert in 2000.

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Kolmogorov K41 theory

Should we expect energy conservation for solutions of Euler in general?

Kolmogorov's theory of turbulence. Typical solutions of Navier-Stokes are expected to dissipate energy at a rate independent of the viscosity

$$\frac{d}{dt}\int \frac{|u|^2}{2}(x,t)\,dx = \underbrace{-\varepsilon \int |Du|^2(x,t)\,dx}_{=O(1)}.$$

If this is correct the limit of most sequences of solutions with vanishing ε cannot be energy conservative of Euler.

Still open: produce an example of such a sequence.

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Onsager's conjecture

Conjecture (Onsager 1949)

For any $\alpha < \frac{1}{3}$ there are C^α weak solutions of Euler for which the *total* kinetic energy is not conserved.

Proved by Isett in 2016 after:

- D-Székelyhidi 2012 C^0 and $C^{1/10-}$
- Isett 2012 $C^{1/5}-$
- Buckmaster-D-Székelyhidi 2014 $L^1_t C_x^{1/3-}$
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One essential ingredient of all these proofs is some "convex integration scheme" which has its roots in the D-Székelyhidi C^0 paper.

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Vanishing viscosity

Question: Can a solution of Euler produced by “convex integration” be a vanishing viscosity limit?

Buckmaster-Vicol 2017: Yes if the solutions of NS are just weak.

Can it be a limit of Leray, or even suitable, weak solutions with vanishing viscosity?

Such a limit would need to be satisfy

$$\frac{d}{dt}\int \frac{|u|^2}{2}(x,t)\,dx \leq 0 \qquad (1)$$

$$\partial_t \frac{|u|^2}{2} + \mathrm{div}\left(\left(\frac{|u|^2}{2} + p\right) u\right) \leq 0\,. \qquad (2)$$

i.e. globally dissipative and locally dissipative

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Global and local dissipation

Theorem (Buckmaster-D-Székelyhidi-Vicol 2017)

For every $\alpha < \frac{1}{3}$ there are globally dissipative weak solutions of the Euler equations which belong to C^α and whose kinetic energy is not constant.

Theorem (Isett 2017)

For every $\alpha < \frac{1}{15}$ there are weak solutions of the Euler equations which belong to C^α, which are both locally and globally dissipative, and whose kinetic energy is not constant.

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A stronger form of Onsager's conjecture

Conjecture (Isett 2017)

There are $L^\infty_t C^{1/3}_x$ locally and globally dissipative solutions of the Euler equations whose kinetic energy is not constant.

Still far from the threshold $\frac{1}{3}$. Best exponent thus far:

Theorem (D-Kwon 2020)

For every $\alpha < \frac{1}{7}$ there are $L^\infty C^\alpha_x$ locally and globally dissipative solutions of Euler whose kinetic energy is not constant.

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[Giri-Kwon, in preparation] There are continuous entropy solutions of 3-dimensional compressible Euler with nontrivial dissipation of the corresponding entropy.

Compare to previous works, where "nonclassical solutions" have been constructed, but all of them are discontinuous ([D-Székelyhidi 2008], [Chiodaroli-D-Kreml 2013], [Chiodaroli-Kreml], [Klinberg-Markfelder], [Markfelder]).

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