Marithania Silvero - Universidad de Sevilla - marithania@us.es

scimeter.org/arxiv/clouds/

Khovanov homology

i

$Kh^{i,j}$()	0	1	2	3
9				$\mathbb{Z}$
7				$\mathbb{Z}/2$
5			$\mathbb{Z}$	
3	$\mathbb{Z}$			
1	$\mathbb{Z}$			

j

2

- What topological and geometric information can we get from $Kh(L)$?

Khovanov homology

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- Meaning of torsion. How to obtain *big* torsion groups.
- Properties of $Kh(L)$ when L belongs to a family.
- Given a Khovanov homology table T, $\exists\ L$ so that $Kh(L) \sim T$?

Geometrization of Khovanov homology

Given L, $\exists$ *space* X_L so that $H^*(X_L)$ or $H_*(X_L)$ coincides with $Kh(L)$?

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YES Lipshitz-Sarkar: Khovanov spectra (link invariant, stronger than $Kh(L)$.

$\longleftarrow i \longrightarrow$

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$Kh^{i,j}$(trefoil), i horizontal, j vertical:

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simplicial complex (combinatorial approach)

[González-Meneses, Manchón, S.]

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semi-simplicial set, equivalent 2-functors
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L semiadequate

$$X_L \sim_h \begin{cases} \bigvee_{p_1} S^{c-1} & \text{if } G_D \text{ is bipartite} \\ \bigvee_{p_1-1} S^{c-1} \vee \Sigma^{c-3}\mathbb{R}P^2 & \text{otherwise} \end{cases}$$

D diagram with no 

D not semiadequate $\Rightarrow$ $X_D \simeq$ wedge of spheres.

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Theorem [Jaeger, Vertigan, Welsh]:
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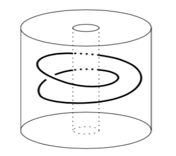
- Generalize to annular Khovanov homology / odd Khovanov homology.



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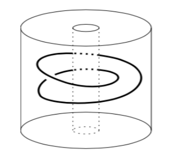

- Kauffman bracket skein modules:

M 3-manifold. Formal lineal comb. of links in M module some relations.

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  - different notions of positivity.
  - semiadequate links.
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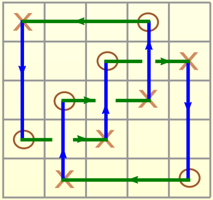
  Grid homology.



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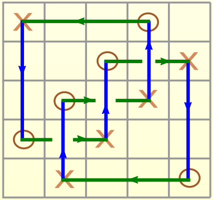

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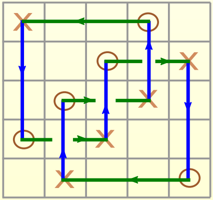

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