

• For my project, I think a lot about 4d hyperkähler ALE spaces:



• ALE stands for "asymptotic locally Euclidean".

• For each finite subgroup $\Gamma < \text{SU}(2)$, there is an ALE space X_Γ corresponding to Γ , and $X_\Gamma \cong \widetilde{\mathbb{C}^2/\Gamma}$.

• ALE spaces are first classified by Kronheimer in his thesis. He constructs them through a hyper-kähler reduction on a finite dimensional vector space.

• Relation with the McKay Correspondence:

For a finite subgroup $\Gamma < \text{SU}(2)$, the blowup diagram obtained by resolving the singularity of \mathbb{C}^2/Γ is the same as the McKay quiver of Γ , i.e., the irreducible representations of Γ correspond to the irreducible components of the exceptional set when resolving the singularity.

• What I did is to give a new gauge-theoretic construction of these spaces, realizing each of them as a moduli space of solutions to a system of equations, modulo a hyperkähler gauge group action.

• Future directions include extracting new insights on the McKay Correspondence via the new gauge-theoretic construction, generalizing this construction to get more hyperkähler spaces, etc.

Other things I'd like to learn more and think about include:

- Open book decompositions, Lefschetz fibrations, and contact structures
- Heegaard Floer homology, Knot Floer homology
- Instanton Floer homology
- Seiberg-Witten theory
- Hitchin's work on self-duality equations
- Trisections