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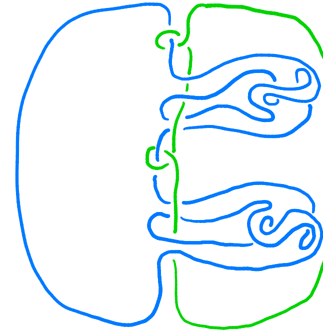
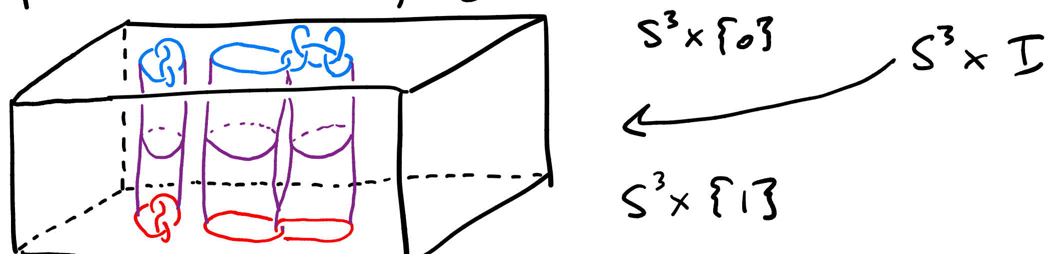


Since this is a longer talk, I can expand a bit:

Def. Two n-component links $L_1, L_2 \subset S^3$ are concordant if



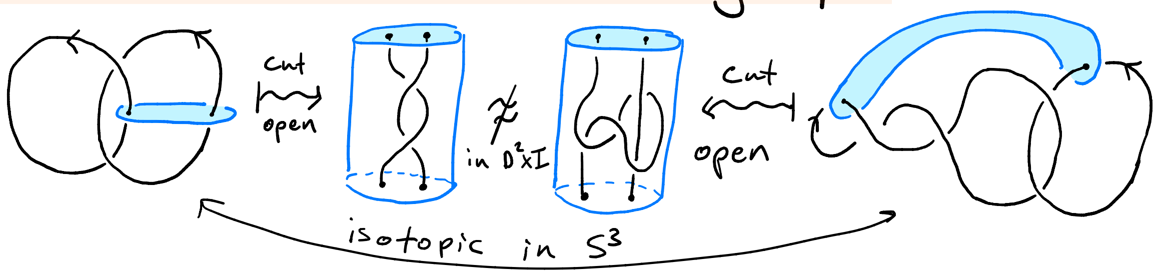
Note: homotopic $\underset{\not\Rightarrow}{\Leftarrow}$ link homotopic $\underset{\not\Rightarrow}{\Leftarrow}$ concordant $\underset{\not\Rightarrow}{\Leftarrow}$ isotopic

Why you should care: Every 3-manifold can be obtained by surgery on some link, and concordant links have homology cobordant complements (rel boundary)

Fact: $\{\text{Links} \subset S^3\} / \text{concordance}$ form a group, but not in an easy way

Aside: $\left(\{\text{Knots} \subset S^3\} / \text{concordance}, \#\right)$ is a group, and a mysterious one at that (despite being abelian)

One way to form a link concordance group:



Thm: (Le Dimet '88) These knotted and linked arcs in $D^2 \times I$ (called string links) form a group $\mathcal{C}_n$ up to (string link) concordance with stacking as the operation, and

pure braid group on n strands $\rightarrow$ $PB_n \leq \mathcal{C}_n$ $\leftarrow$ string link concordance group on n strands

Thm: (K. 20)
If $n>1$, $\mathcal{C}_n / \text{Normal closure}(PB_n)$ is non-abelian.

Not much is known beyond this!

Q:
- How "close" to being abelian is this? Is it solvable? Nilpotent?
- PB_n is torsion-free. What are the torsion elements of this quotient?
- Does $1 \longrightarrow Ncl(PB_n) \rightarrow \mathcal{C}_n \rightarrow \mathcal{C}_n / Ncl(PB_n) \rightarrow 1$ split for $n>2$?

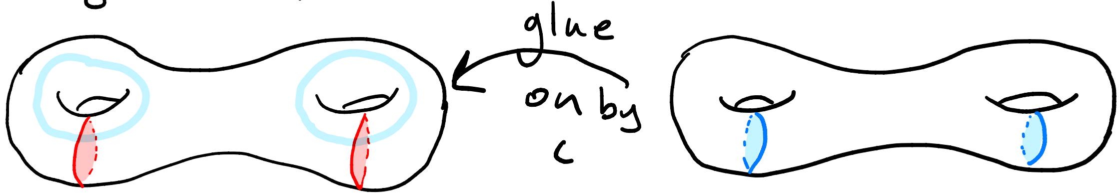
(DeCampos '05: splits for $n=2$)

Note: $\sigma = 1 \in \mathcal{C}_n$ if and only if $\hat{\sigma}$ is concordant to the unlink, but usually string link representatives are not unique.

Big Q:
- Is there a link concordance group G where each link has a unique representative?
- How about where the set map $\{\text{links} \subset S^3\} / \text{concordance} \longrightarrow G$ is a bijection?

Quickly on to something else I like:

Recall that concordant links lead to homology cobordant 3-manifolds in a few different ways, and that every 3-manifold can be split into two handlebodies glued together by a diffeomorphism (Called a Heegaard splitting)

$S^3 =$ 

Fact: (Morita '91) Every $\mathbb{Z}HS^3$ can be obtained by gluing by a diffeomorphism that acts trivially on H_1(gluing surface)

Thm: (WIP: Afton-K.-Lidman) Let Y be a $\mathbb{Z}HS^3$ with $Y = \mathcal{H}_g \cup_{\varphi \circ c} \mathcal{H}_g$ where $\varphi \in \mathcal{I}_{g,1} = \{\text{diffeos of } \Sigma_g \curvearrowright H_1(\Sigma_g) \text{ trivially}\} \leq \mathrm{Mod}_{g,1}$. Then

$$|d(Y)| \leq C \|\varphi\|$$

Useful homology cobordism invariant from Heegaard Floer theory

word length in $\mathcal{I}_{g,1}$

Thanks for listening and please come chat!

Since some of you are used to seeing my cats on zoom when I speak, here they are.