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Research Interest: Bi-orderability of Link Groups

A group is bi-orderable if there is a strict total order of its elements which is invariant under both left and right multiplication.

Examples: $\mathbb{Z}^n$, Free groups (Magnus)

Question: When is a link group bi-orderable?

Example: Hopf link has bi-orderable group

Link group is $\mathbb{Z}^2$ which is bi-orderable

Example: Trefoil does not have bi-orderable group

Knot group is $\langle a, b, t | tat^{-1} = b, tbt^{-1} = a^{-1}b \rangle$

If $1 < a$ then $a^{-1} < 1$ and $1 < t^3 a t^{-3}$

but $t^3 a t^{-3} = b^{-1} a^{-1} b$ so $1 < a^{-1}$.

Random Theorems

- (J) *If an oriented two-bridge link has an Alexander polynomial with (collectively) relatively prime coefficients and all real positive roots, then its link group is bi-orderable.*
- (Ito) *If a knot's Alexander polynomial has degree twice the knot's genus and no real positive roots, then the knot group is not bi-orderable.*
- (Clay-Rolfsen) *Knots with non-trivial L-space surgeries never have bi-orderable knot group.*

Given an n-strand braid β, the link formed the braid closure of β and its braid axis is the braided link, $\mathrm{br}(\beta)$, of β.

The braid β acts on a rank n free group F_n. The link group of $\mathrm{br}(\beta)$ is bi-orderable if and only if the action of β preserves a bi-ordering of F_n.

Question 1: Which links are braided links?

Question 2: When a link is a braided link, does the braid preserve a bi-ordering of the free group?