

Sage/Oscar Days for Combinatorial Algebraic Geometry

Lightning Talks Session 1

February 17, 2021

Terao's freeness conjecture for small arrangements in arbitrary characteristic

Lukas Kühne, Max Planck Institute for Mathematics in the Sciences, Leipzig

Motivated by singularity theory, Hiroakoi Terao introduced a module of logarithmic derivations associated to a hyperplane arrangement. This lightning talk is concerned with Terao's freeness conjecture which asserts that the freeness of this derivation module is determined by the underlying combinatorics of the arrangement. To investigate this conjecture, we have enumerated all matroids of rank 3 with up to 14 hyperplanes whose characteristic polynomial splits over the integers and saved it in a public database (matroid.mathematik.uni-siegen.de). Using the GAP package ZariskiFrames we have computed the moduli space and the free locus of the derivation module of each of these matroids as a quasi-affine set. As the main result, this yields a computational proof of Terao's freeness conjecture for rank 3 arrangements with up to 14 hyperplanes over arbitrary characteristic.

This talk is based on joint work with Mohamed Barakat, Reimer Behrends, Christopher Jefferson, and Martin Lerner."

Sage experimentation with stable Grothendieck polynomials

Jianping Pan, UC Davis

Grothendieck polynomials are representatives of Schubert varieties in the K-theory of the flag manifold. Taking stable limit, we obtain symmetric functions called the stable Grothendieck polynomials. Fomin and Kirillov gave a combinatorial construction of these functions, in terms of decreasing factorizations in the 0-Hecke monoid. Using Sage, we can quickly compute the Schur expansion of the stable Grothendieck polynomials. We also discovered a crystal structure on the decreasing factorizations of 0-Hecke monoid of those indexed by 321-avoiding permutations. Experimentation in Sage also helped us prove some properties of our star-insertion algorithm on decreasing factorizations of the 0-Hecke monoid. This includes an analogue of the Knuth equivalence, and connections to the Hecke insertion and the uncrowding algorithm on set-valued tableaux. We have implemented our crystal structure and the star insertion algorithm in Sage.

Deformation classes of bitangents to tropical quartic curves Marta Panizzut, TU Berlin

In this talk I introduce code in polymake to study deformation classes of bitangents to tropical quartic curves and their lift to real curves. The computations involve triangulations, secondary cones and hyperplane arrangements. This is ongoing work with Alheydis Geiger.

Variational GIT for Complete Intersections and a Hyperplane Section via Sage

Theodoros Papazachariou, University of Essex

Geometric invariant theory (GIT), pioneered by Mumford based on Hilbert's classical invariant theory, is an effective method to study the construction of quotients by group actions in algebraic geometry. Variational GIT includes the added complexity of a wall-chamber decomposition on the

moduli spaces, arising from a dependency on the linearization of the quotients. In this talk we demonstrate an algorithmic way to study quotients of pairs (S,H) by the $SL(n+1)$ action, where S is a complete intersection of k hypersurfaces of degree d and H is a hyperplane section, via Sage.

Matrix Schubert varieties and CM regularity

Colleen Robichaux, University of Illinois at Urbana-Champaign

We give an explicit formula for the degree of a vexillary Grothendieck polynomial. This generalizes a previous result of J. Rajchgot-Y. Ren-C. Robichaux-A. St. Dizier-A. Weigandt for degrees of symmetric Grothendieck polynomials. We apply our work to compute the Castelnuovo-Mumford regularity of certain matrix Schubert varieties. We also derive formulas for the regularity of Kazhdan-Lusztig varieties coming from open patches of Grassmannians.

Computing in the Derived Category of a Toric Variety

Mahrud Sayrafi, University of Minnesota, Twin Cities

This is a lightning talk about how exceptional collections and the associated quiver of sections makes the bounded derived category of coherent sheaves on P^n explicit. An application of this is an efficiently computable check for a vector bundle on P^n to split as a direct sum of line bundles.

A 'quantum equals classical' theorem for n-pointed Gromov-Witten invariants of degree one

Weihong Xu, Rutgers University

It is well known that there are 2 lines meeting 4 general lines in P^3 and that the computation can be done in the cohomology ring of $Gr(2,4)$. In this talk I invite you to think of this as a way to compute a 4-pointed (genus 0, degree 1) Gromov-Witten Invariant on P^3 . This is a first example of a general theorem I will state concerning flag varieties of Picard rank 1 defined by a long root. The 3-pointed case is known from the work of Perrin and others. This talk is based on joint work with Linda Chen, Angela Gibney, Lauren Heller, Elana Kalashnikov, and Hannah Larson