Friday, February 5, 2021

AM Session

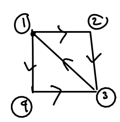
Speaker: Lauren Williams, Harvard University

Teaching Assistants: Jonathan Boretsky, Sunita Chepuri, Charles Wang

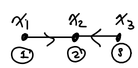
EXERCISES FOR THE ICERM CLUSTER ALGEBRA CLASS.

LECTURER: LAUREN WILLIAMS
TA'S: JONATHAN BORETSKY, SUNITA CHEPURI, CHARLES WANG

(1) Mutate the following quiver at vertex 1. Alternatively, mutate the quiver at vertex 2.



(2) Start with the following labelled seed and perform the following sequence of mutations: $\mu_1, \mu_3, \mu_2, \mu_1, \mu_3, \mu_2, \mu_1, \mu_3, \mu_2, \mu_1, \mu_3, \mu_2$. Compute the cluster variables you get at each step and make sure that they are Laurent polynomials in $\{x_1, x_2, x_3\}$ with positive coefficients.



(3) Verify that for any quiver Q and vertex k, $\mu_k^2(Q) = Q$.

(4) If T is a triangulation and T' is obtained by flipping at diagonal d, then $Q_T^{'} = \mu_d(Q_T)$. (Try verifying in some examples, then prove it.)

(5) Prove that for any $A \in Gr_{2,n}$ and for any $i < j < k < \ell$,
$$p_{ik}(A)p_{j\ell}(A) = p_{ij}(A)p_{k\ell}(A) + p_{i\ell}(A)p_{jk}(A).$$

(6) Draw the flip graph of the triangulations of a hexagon.

(7) (To do after the second lecture) Show that the *rectangles seed* gives a cluster structure on $\mathbb{C}[Gr_{k,n}]$. More specifically:
- Show that if one mutates at any mutable cluster variable, the new cluster variable is a *regular function* which is *coprime* to the old cluster variable (so that one can apply the Starfish Lemma).
- Show that one can obtain any Plücker coordinate from the rectangles seed by an appropriate sequence of mutations.

(8) (To do after the second lecture) Although the equation
$$P_{135}P_{246} - P_{134}P_{256} - P_{136}P_{245} - P_{123}P_{456} = 0$$
does not lie in the ideal generated by the exchange relations, show that we can multiply it by a monomial in the Plücker coordinates so that the result lies in the exchange ideal.