

Monday, February 1, 2021

PM Session

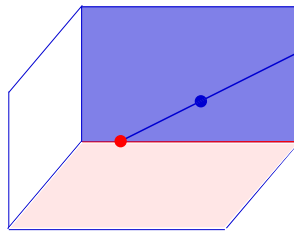
Speaker: Sara Billey, University of Washington

Teaching Assistants: Sean Griffin, Gleb Nenashev, Jianping Pan,
Colleen Robichaux, Anna Weigandt, Weihong Xu

Problem Set #1
due Monday, February 1, 2021

Recommended Problems: Give each of these problems a try. Help is available to those who ask.

1. What position is the purple flag in with respect to the red flag? What position is the red flag in with respect to the purple flag?



2. Recall that Bruhat order is defined as the partial order $v \leq w$ if and only if $X_v(R_\bullet) \subset X_w(R_\bullet)$. From the defining rank conditions on Schubert varieties, prove that Bruhat order can also be characterized as the transitive closure of

$$w < wt_{ij} \iff w(i) < w(j).$$

where t_{ij} is the transposition swapping i and j .

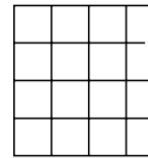
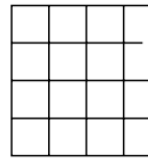
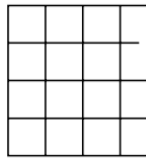
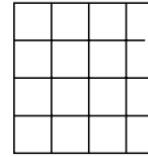
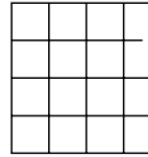
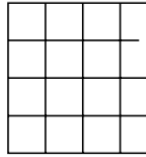
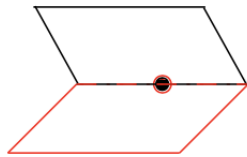
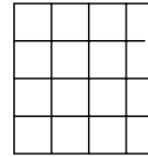
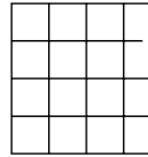
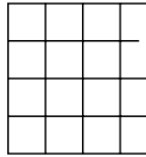
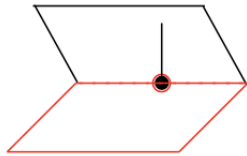
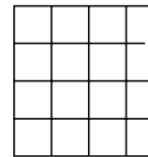
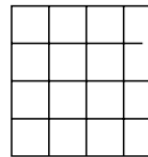
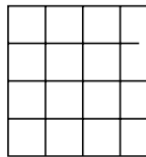
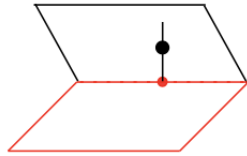
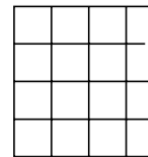
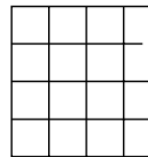
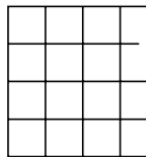
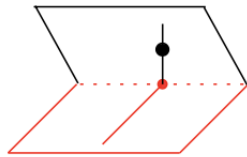
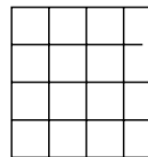
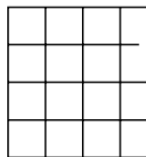
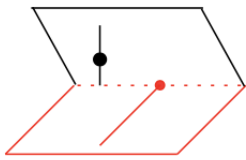
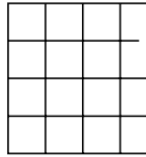
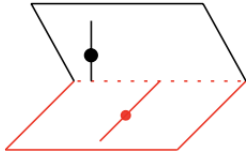
3. For each intersection below, determine its dimension and use shoebox pictures to draw/describe all flags in the intersections,

$$Y = X_{4213}(R_\bullet) \cap X_{3241}(B_\bullet) \cap X_{34123}(G_\bullet),$$

and

$$Z = X_{1342}(R_\bullet) \cap X_{2314}(B_\bullet) \cap X_{3124}(G_\bullet).$$

4. Draw a “purple” flag on the first picture below. Determine what position the three flags are in with respect to each other. As the black flag degenerates into the red flag, draw your purple flag at each step. Do you end up with multiple possibilities at any step?



Problem Set #2
due Monday, February 1, 2021

Recommended Problems: Give each of these problems a try. Help is available to those who ask.

1. Prove the Transition Equation from Monk's formula for Schubert polynomials.
2. Compute the Schubert polynomials \mathfrak{S}_{2134} and \mathfrak{S}_{3412} , multiply them, and expand the product into the basis of Schubert polynomials. Check your coefficients by drawing out the corresponding 0-dimensional intersections of Schubert varieties using the shoebox drawing from the first lecture.
3. Add the strings to these pipe dreams for $w = 7325614$ and compute its Schubert polynomial. Colored strings may help.

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++++++.  
++....  
+....  
++..  
++.  
..  
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```

```
++++++.  
++....  
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+...  
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++++++.  
++....  
+.+..  
+.+.  
+..  
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++++++.
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++++++.
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